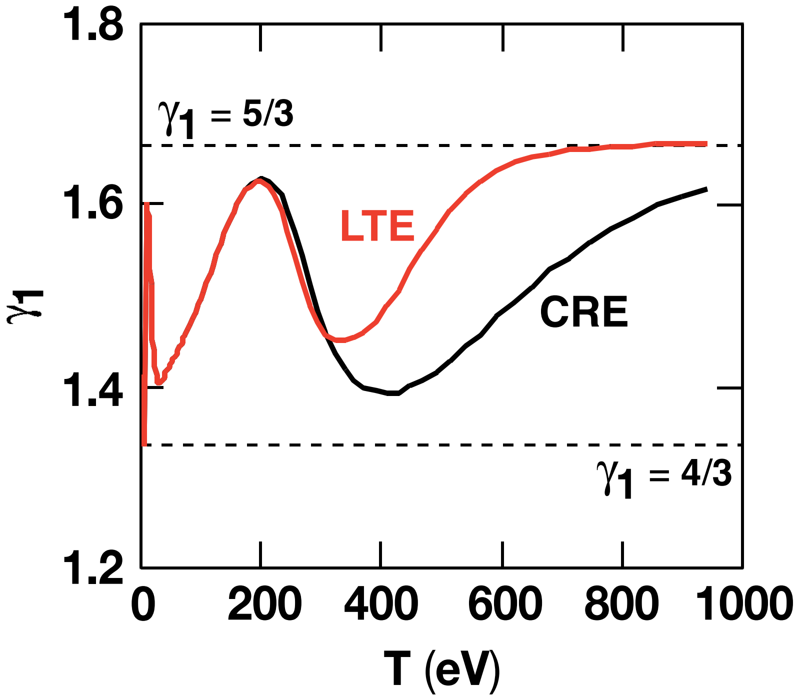
Non-LTE Speed of Sound, Irreversibility, and Thermodynamic Consistency

Aluminum at one-tenth solid density



R. Epstein and W. Fong
University of Rochester
Laboratory for Laser Energetics

34th Anomalous
Absorption Conference
Gleneden Beach, OR
2–7 May 2004

Summary

Non-LTE modification of the speed of sound in a plasma follows the modification of the ionization

- The speed of sound in radiating plasma is obtained from a self-consistent collisional-radiative (CR) model based on the non-equilibrium thermodynamics of irreversible radiation emission.
- The speed of sound and the adiabatic indices are modified through CR effects on the equation of state.
- Time-dependent ionization kinetics causes dispersion and damping of the sound waves.
- The model is formulated in terms of a single ionization transition, but it accepts equivalent ionization parameters from larger models.

Outline

- Thermodynamics, nonequilibrium and irreversibility
- Modification of adiabatic compressibility
- Results: adiabatic index and the speed of sound
- Time-dependent kinetics, dispersion, and damping

Non-LTE EOS effects appear as modified adiabatic indices and modified sound speed in many applications

- R. M. More *et al.** have considered the response of multilevel atomic systems to be small departures from radiative LTE.
- Shock compression is sensitive to $\delta\gamma$, e.g., maximum $\rho_2/\rho_1 = (\gamma + 1)/(\gamma - 1)$.
- Astrophysical self-gravitating objects (or layers) collapse when ionization drives γ below 4/3.
- Sensitive ionization transitions occur below the "CRE density" (e.g., 1/10 solid for Al, near solid for Ti) in expanding plasmas in lab astrophysics, z-pinches, x-ray lasers, interpulse ablation coronas, etc.
- Irreversible relaxation is due to finite relaxation times (even in LTE) damp and disperse sound waves and other transient, particularly near $\omega\tau_s \sim 1$.

---

*R. M. More, T. Kato, S. B. Libby and G. Faussurier, JQSRT 71, 505–518 (2001).

The thermodynamics of an ideal plasma is modified to include entropy production due to irreversible radiation

Thermodynamics: $dE = dQ - pdV, \quad TdS = dE + pdV - AdN^+$

Ideal gas: $pV = NkT, \quad E = \frac{3}{2}NkT + \int_0^{N^+} \chi(n)\,dn$

Model I: 2-species ionization $\quad e^- + I^Z \rightarrow 2e^- + I^{Z+1} : \quad K_f = n_e C_f$

$$\frac{dN^+}{dt} = K_f N^0 - K_r N^+ = 0 \qquad \left.\begin{array}{l} 2e^- + I^{Z+1} \rightarrow e^- + I^Z \\ e^- + I^{Z+1} \rightarrow hv + I^Z \end{array}\right\} : \quad K_r = n_e{}^2 C_r + n_e R_r$$

$$\delta S = k\,\delta\,\ln\left[\left(\frac{K_f}{K_r}\right)^{N^+}\right], \qquad A = kT\,\ln\left(1 + \frac{R_r}{n_e C_r}\right)$$

Model II: phenomenological $\quad A = BkT$

The adiabatic compression index and the ionization law are constrained by thermodynamic consistency

1st law: $$\gamma_1 = \frac{5/2 + \eta_T\left(5/2 + \chi/kT\right) + \eta_V\left(\chi/kT\right)}{3/2 + \eta_T\left(3/2 + \chi/kT\right)}; \quad p \propto \rho^{\gamma_1}$$

$$\eta_T = \frac{T}{N^+}\left(\frac{\partial N^+}{\partial T}\right)_V, \qquad \eta_V = \frac{V}{N^+}\left(\frac{\partial N^+}{\partial V}\right)_T {}^{*}$$

Entropy as function of state: $$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$$ constrains η_T and η_V

Model I: 2-species ionization $\eta_T/\eta_V = \left(3/2 + \chi/kT\right)\left(1 + f\right) - f/2$, $f = R_r/\left(n_e C_r\right)$

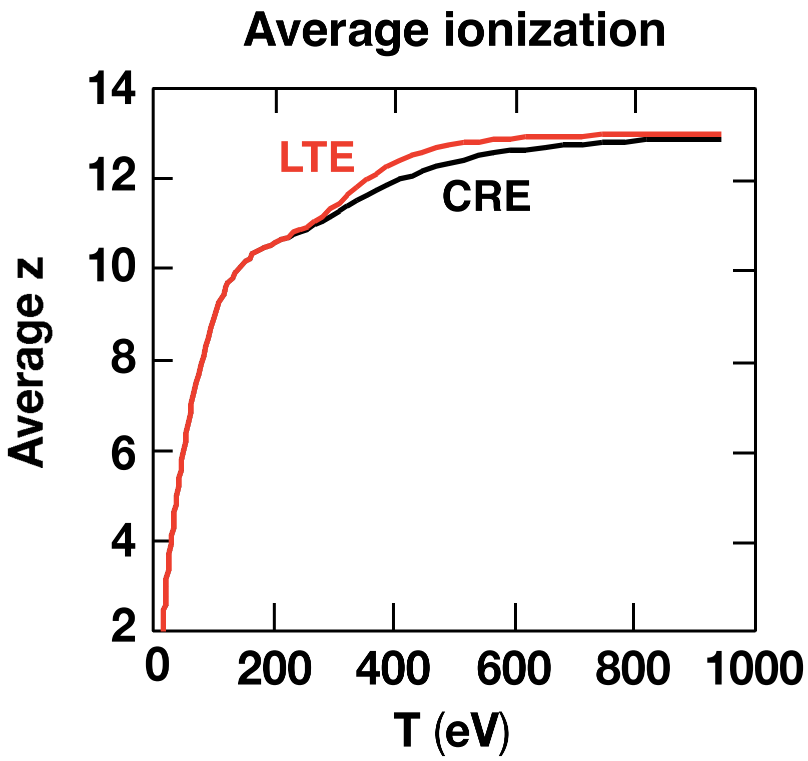
Model II: phenomenological $\eta_T/\eta_V = 3/2 + \chi/kT$

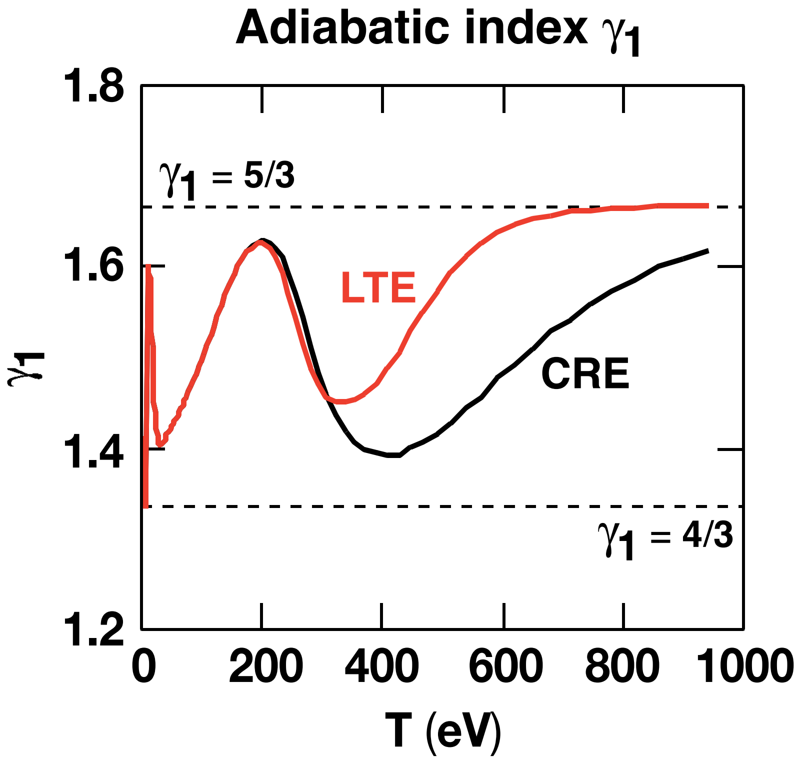
Speed of sound: $$c_S{}^2 = \gamma_1 \frac{p}{\rho}$$

* Cf. J. P. Cox and R. T. Giuli (1968)

Non-LTE modification of the plasma compressibility follows the modified ionization temperature dependence

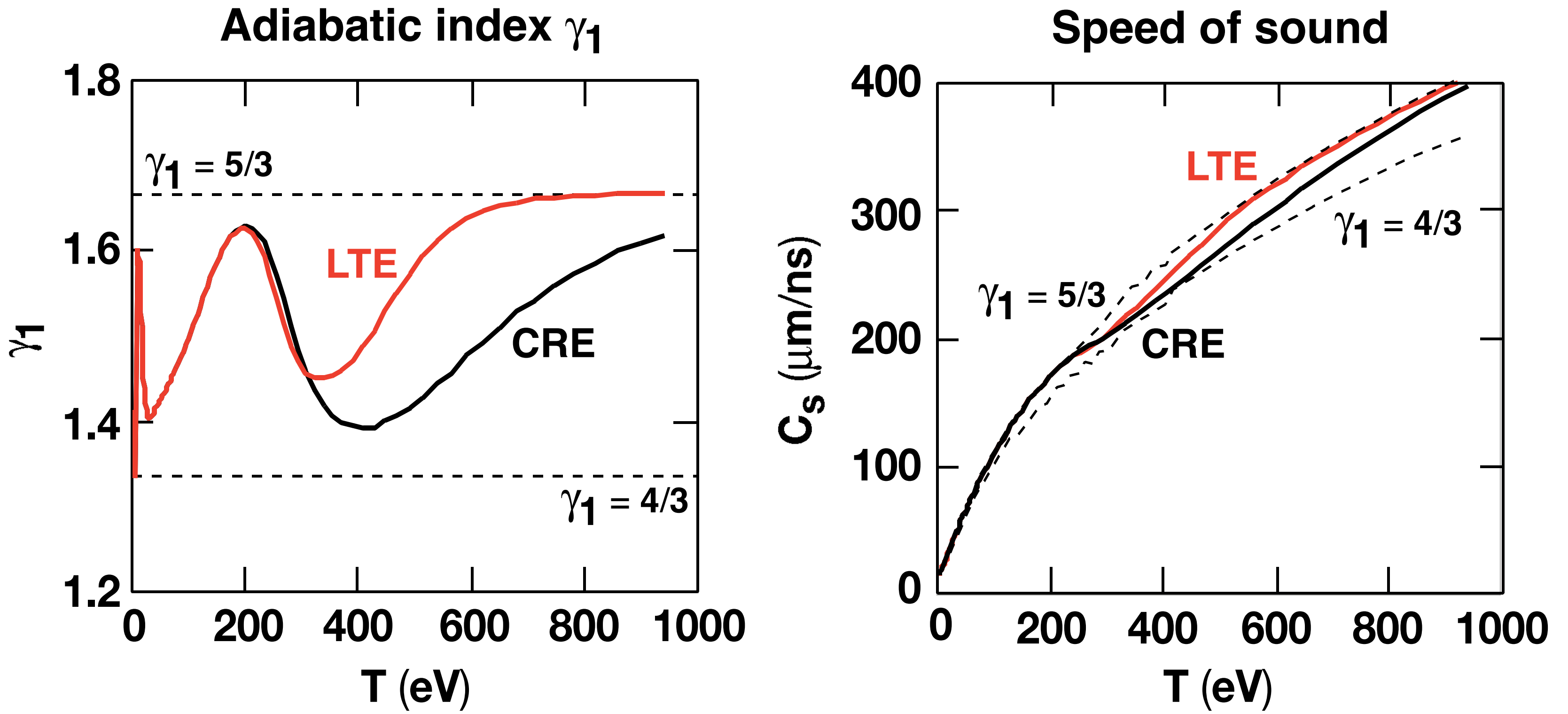
Aluminum at one-tenth solid density





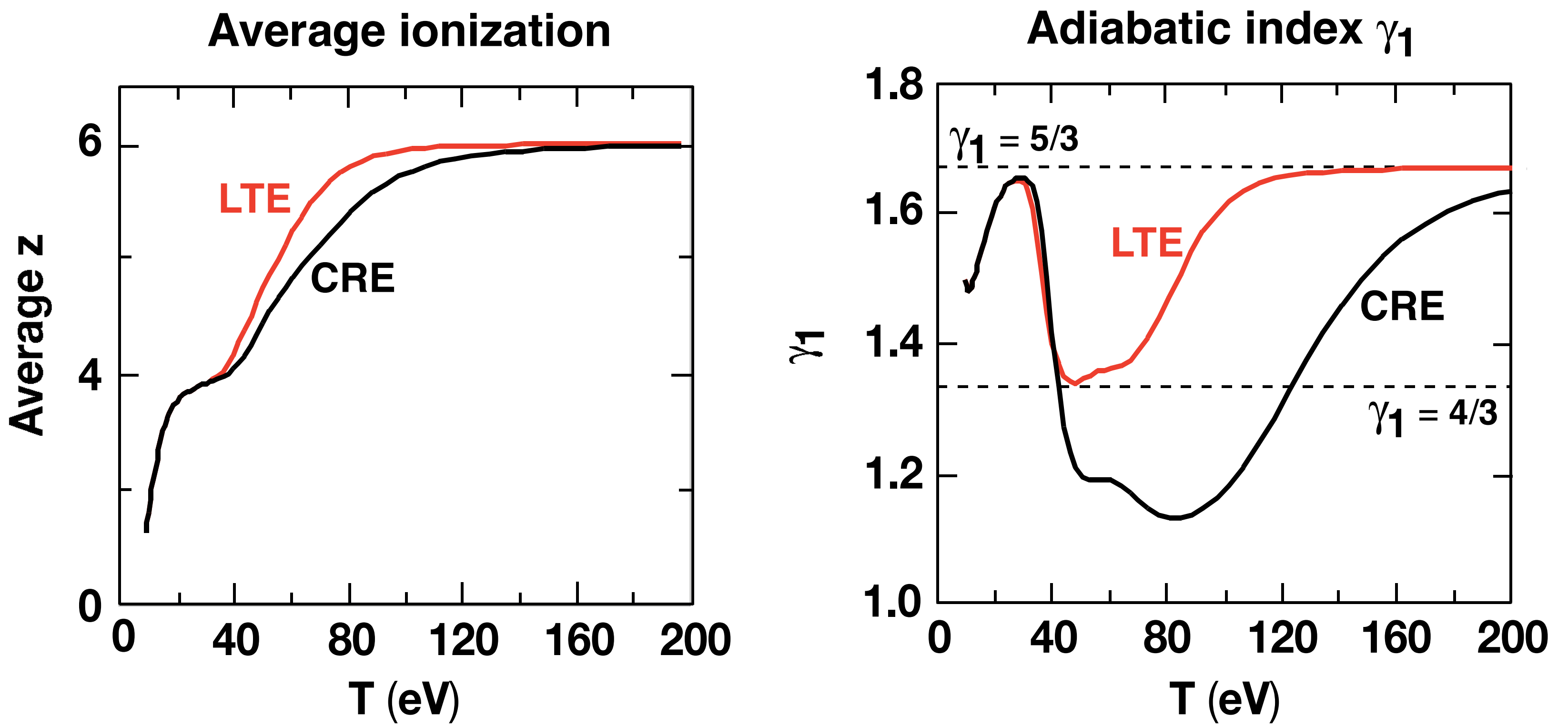
Non-LTE modification of the plasma speed of sound reflects the corresponding modification of the compressibility

Aluminum at one-tenth solid density



Non-LTE modification of the plasma compressibility follows the modified ionization temperature dependence

Carbon at one-tenth critical density ($z \cong 5$)

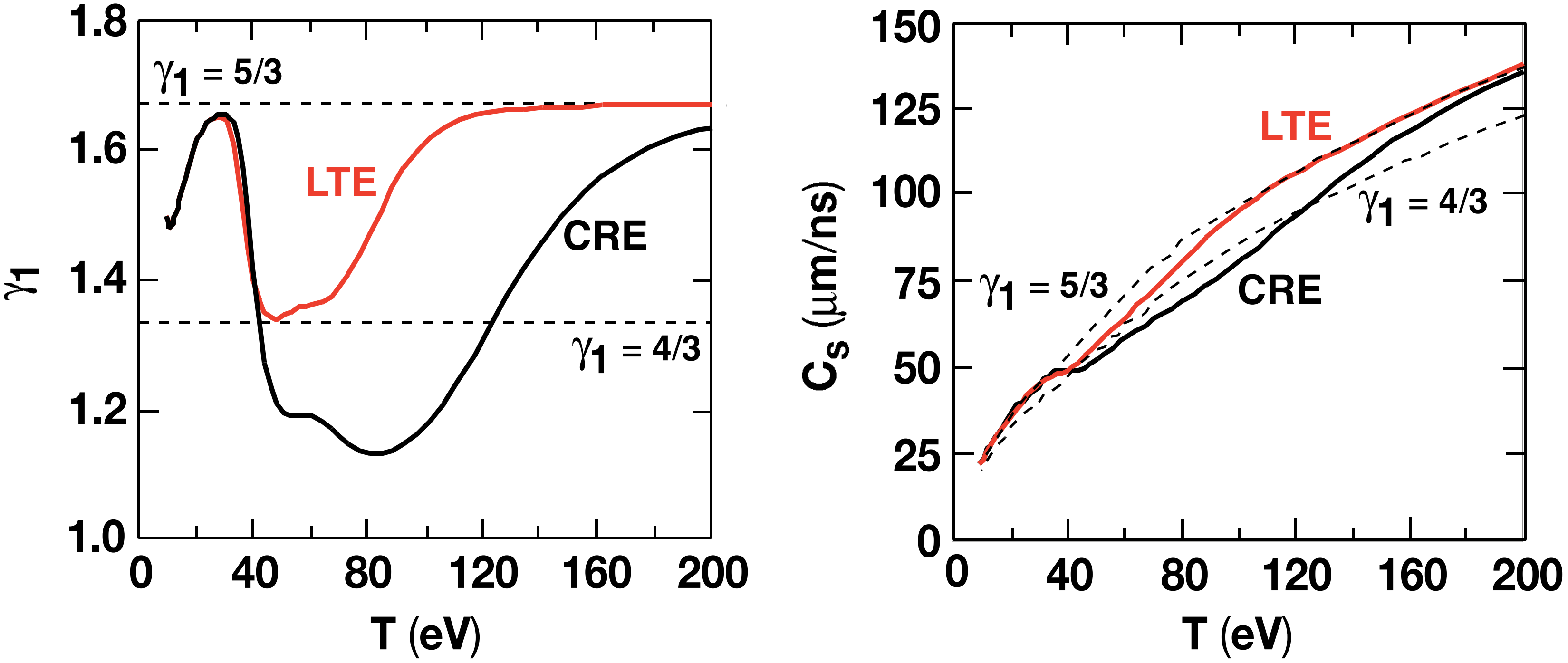


Average ionization

Adiabatic index γ_1

Non-LTE modification of the plasma speed of sound reflects the corresponding modification of the compressibility

Carbon at one-tenth critical density (z = 5)


Adiabatic index γ_1

Speed of sound

The finite plasma equilibration time introduces sound wave dispersion and damping

$$\frac{dN^+}{dt} = n_e C_f N^0 - \left(n_e{}^2 C_r + \overset{\approx 0}{\cancel{n_e R_r}}\right) N^+$$

Apply method of R. Haase (1969) in LTE

Adiabatic relaxation time:

$$\tau_S{}^{-1} = \left[\frac{\partial}{\partial N^+}\left(\frac{dN^+}{dt}\right)\right]_{S,V} = n_e C_f\left[\frac{1+4z}{2z} + \frac{(3/2+\chi/kT)^2}{3(1+z)}\right]$$

$$n_e C_f = 1.75\ \text{ps}^{-1}\left(\frac{kT}{500\ \text{eV}}\right)^{-3/2}\left(\frac{n_e}{n_c}\right)\frac{\exp(-\chi/kT)}{(\chi/kT)}$$

Sound wave: $N^+ - N_0^+ \propto \exp[i(\omega t - kx)]$

$$c_S{}^2 = \frac{c_A{}^2 + i\omega\tau_S c_N{}^2}{1+i\omega\tau_S} \begin{cases} \omega\tau_S \ll 1,\ c_S = c_A = \sqrt{\dfrac{\gamma_1 p}{\rho}} & \dfrac{c_S k}{\omega} \approx 1 - \dfrac{i\omega\tau_S}{2}\left(\dfrac{5}{3\gamma_1} - 1\right)^* \\ \omega\tau_S \gg 1,\ c_S = c_N = \sqrt{\dfrac{5p}{3\rho}} & \dfrac{c_S k}{\omega} \approx 1 - \dfrac{i}{2\omega\tau_S}\left(1 - \dfrac{3\gamma_1}{5}\right) \end{cases}$$

* Cf. Zel'dovich and Raizer II (1967); A. H. Nelson and M. G. Haines (1969)

Summary/Conclusions

Non-LTE modification of the speed of sound in a plasma follows the modification of the ionization

- The speed of sound in radiating plasma is obtained from a self-consistent collisional-radiative (CR) model based on the non-equilibrium thermodynamics of irreversible radiation emission.
- The speed of sound and the adiabatic indices are modified through CR effects on the equation of state.
- Time-dependent ionization kinetics causes dispersion and damping of the sound waves.
- The model is formulated in terms of a single ionization transition, but it accepts equivalent ionization parameters from larger models.