Damping and Spatial Propagation of Oscillations in Weakly Collisional Plasma

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Abstract

Since Landau damping depends on the fine details of the distribution function near the phase velocity of a wave, the effect of including small collisions in otherwise collisionless plasma theory caused concern. This was presumably settled by Lenard and Bernstein[1] (LB) long ago. A similar concern arose about the phenomena of plasma echoes since this too depends on fine details of the electron distribution at velocities close to ω_a/k, where ω_a is a fixed antenna frequency and k is set by the antenna structure. Su and Oberman[2] (SO) considered this and concluded that there was a resulting decay of the echo signal as $\exp(-\beta x^3/v^5)$, where β is proportional to the electron collision rate. Recently, Ng *et al.*[3] (NG) questioned the validity of SO (and implicitly of LB) and suggested that SO improperly expanded about the Van Kampen continuum. All of this was motivated by an experiment by Skiff *et al.*[4] (Skiff) that directly measured the ion distribution function associated with ion sound waves downstream from an antenna. The measurements did not seem to agree with their LB-like analysis.

We have reexamined the NG–Skiff analysis and show that the SO result is correct. In doing so, we also obtain a new form of the LB dispersion relation, in terms of the incomplete gamma function. This is much simpler than the result in LB and allows a rapid calculation of the decay rates. We also show that the NG–Skiff analysis is inadequate for calculating the downstream decay from an antenna. This must be done either by a boundary-layer method like that of SO (but for the ion sound case) or by numerically inverting a new expression for the transformed distribution function, which we have derived in a compact and convenient form. To our knowledge, this has not yet been done. For this reason, it is premature to conclude that there is any discrepancy between experiment and the theory of oscillations in weakly collisional plasma.

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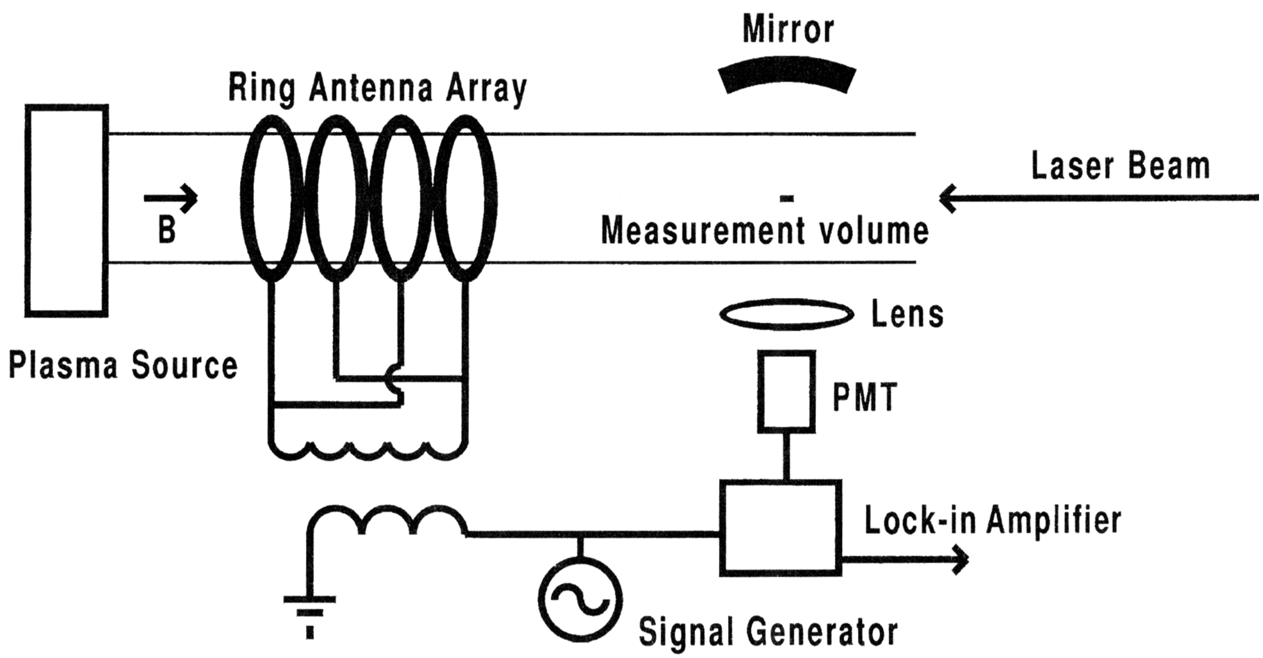
1. A. Lenard and Ira B. Bernstein, Phys. Rev. 112, 1456 (1958).
2. C. H. Su and C. Oberman, Phys. Rev. Lett. 20, 427 (1968).
3. C. S. Ng, A. Bhattacharjee, and F. Skiff, Phys. Rev. Lett. 83, 1974 (1999).
4. F. Skiff *et al.*, Phys. Rev. Lett. 81, 5820 (1998).

Schematic of the experimental apparatus

Figure 1

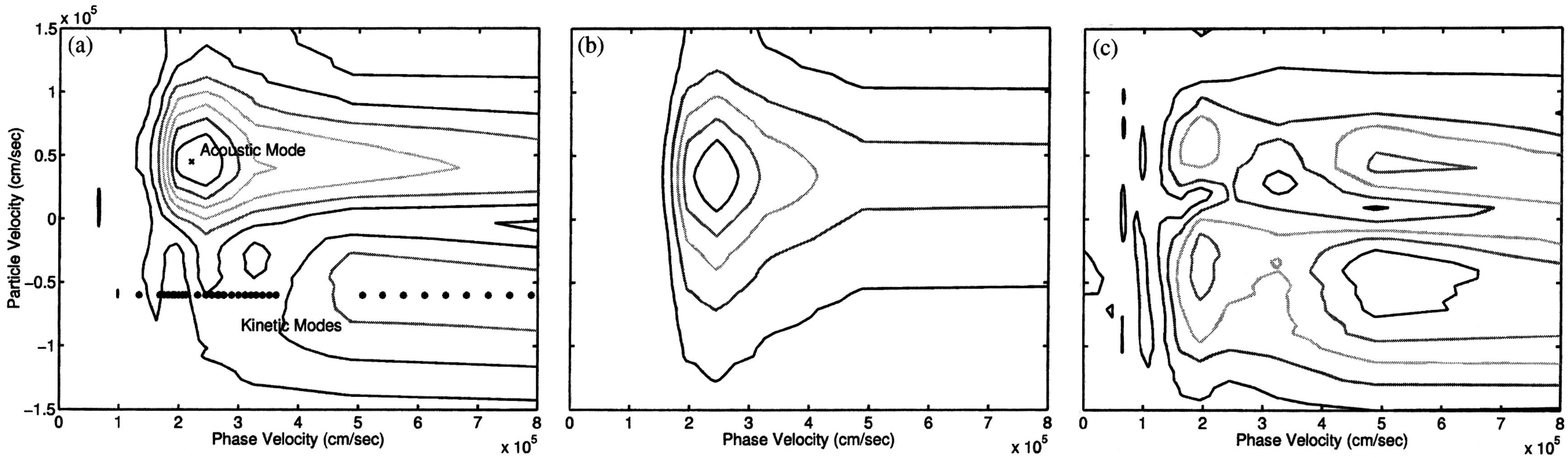


F. Skiff *et al.*, "Linear Kinetic Modes in Weakly Collisional Plasma," Phys. Rev. Lett. 81, 5820 (1998)

Amplitudes of the ion response function (from SVD analysis)

Figure 2

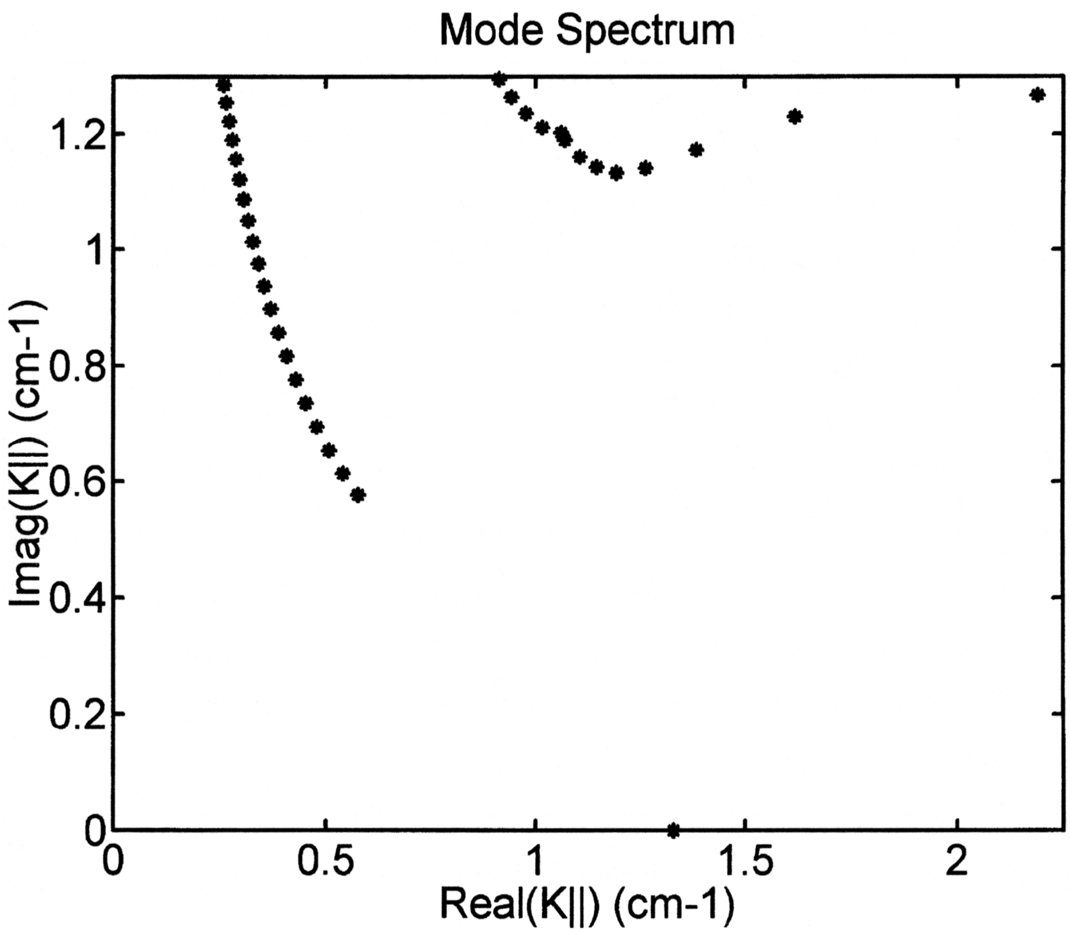
Linear ion response amplitude contours



F. Skiff *et al.*, "Linear Kinetic Modes in Weakly Collisional Plasma," Phys. Rev. Lett. 81, 5820 (1998)

Complex-k_z eigenvalues of the ion L-B equation for real ω

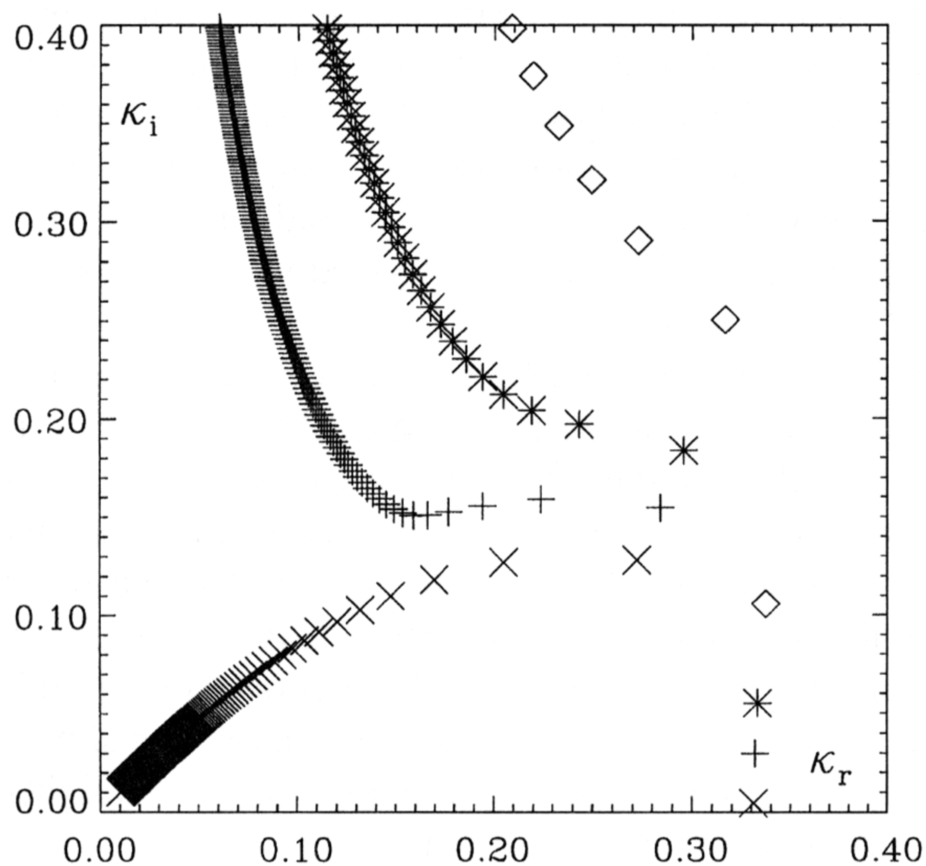
Figure 3



F. Skiff *et al.*, "Linear Kinetic Modes in Weakly Collisional Plasma," Phys. Rev. Lett. 81, 5820 (1998)

Complex-κ eigenvalues for the electron L-B equation for real ω

Figure 4



C. S. Ng *et al.*, "Kinetic Eigenmodes and Discrete Spectrum of Plasma Oscillations in a Weakly Collisional Plasma," Phys. Rev. Lett. 83, 1974 (1999).

Skiff has a clever way to measure the ion velocity distribution associated with ion-sound waves in a weakly collisional plasma

- Experimental setup
  - See adjacent Fig. 1 (ion-sound waves are generated).
  - Detect f_i by phase-locked, laser-influenced fluorescence.
  - $n_e \cong 10^9$ cm^{-3}, T_e = 2 eV, T_i = 0.07 eV, ω_a = 10 MHz, B = 1 kG
  - Obtain f_i (v_z, z, ω_a) at 6150 points in z, v_z space.
  - Spatial Fourier analysis yields f_i (v_z, ω_a/k_z, ω_a).
  - There seems to be a single $k_\perp$ in all cases.
  - Contour amplitudes of $|f_i|$ are shown in Fig. 2(a).

Singular-value decomposition yields a dominant ion-sound mode and residual "kinetic" modes

- **SVD analysis yields $f_i = \Sigma A_n g_n(z) h_n(v_z)$.**
- **The largest-amplitude mode is at the expected ion-sound phase velocity, and involves the bulk of the ion distribution [see Fig. 2(b)].**
- **Other SVD modes appear above the noise level. Their amplitudes vary as ω_a is changed, but their phase velocity is unchanged [see Fig. 2(c)].**

Comparison was then made with a theory

- They use a model ion–ion collision operator (á la Lenard–Bernstein) and assume the electrons respond adiabatically:

$$\frac{\partial f}{\partial t} + \vec{v} \bullet \nabla f + \frac{\vec{V}}{c} \times \Omega_i \vec{i}_z \bullet \nabla_v f - c_s^2 \nabla \int f\,dv \bullet \nabla_v f^0 = \beta \nabla_v \bullet \left(\vec{v}\, f + v_t^2 \nabla_v f \right)$$

- $\dfrac{\beta}{\omega_a} \cong 0.05 \qquad \dfrac{\omega_a}{\Omega_i} \cong 1.3 \qquad \dfrac{k_\perp v_\perp}{\Omega_i} \cong 0.05$

- They solve this linear equation for the eigenmodes and eigenvalues. For real ω they find complex values of k_z. (see Fig. 3; note $k_\perp$ fixed)

- One obtains a virtually undamped “acoustic” mode and many damped “kinetic” modes.

Skiff noted at least three discrepancies with theory

- The observed spatial decay of the kinetic modes was much less than that predicted by the imaginary part of the k_z's.
- The velocity distribution of the kinetic modes only qualitatively resembled that of the calculated kinetic modes.
- The calculated spatial damping rate of the kinetic modes varied as $\beta^{1/2}$, while the classic Su–Oberman theory of echo damping has a decay rate that varies as β.
- Skiff concluded that the Su–Oberman theory must be inadequate since the ion-sound analysis is analogous to the electron plasma wave analysis.

Based on Skiff's experiment, Ng *et al*. revisited the case of electron waves in a weakly collisional plasma

- They concluded that Su and Oberman <u>are</u> wrong and attributed that to expansion about the unperturbed Van Kampen continuum.
- They start again with Lenard–Bernstein model collision operator and consider eigenmodes of the following equation:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} - \frac{e}{m}\frac{\partial f_0}{\partial v}E = \beta\frac{\partial}{\partial v}\left(vf + v_t^2\frac{\partial f}{\partial v}\right)$$

$$\frac{\partial E}{\partial z} = -4\pi e\int f\,dv$$

- They solve for the eigenvalues and eigenmodes by Fourier transforms in x and t, and by an expansion in Hermite polynomials in v. The result, for real ω, is shown in Fig. 4. A most interesting difference exists between the more strongly damped roots in the collisional case and those in the collisionless case (Landau). There is very little difference, however, in the least-damped roots.

First, we consider the electron plasma wave case (Ng versus Su–Oberman)

- The dispersion relation solved by Ng is identical to that of Lenard–Bernstein (this is easy to show). L–B found that the least-damped mode decays only slightly faster than the Landau value. They did not calculate other modes, and indeed their DR is not in a convenient form.
- We have obtained a new expression for this DR by using a Fourier transform in velocity. This is an extremely convenient form (expressed in terms of the incomplete gamma function), and one obtains all the roots shown in Fig. 4 via a PC and Mathematica.
- Unfortunately, the roots of this DR are mostly irrelevant in determining the spatial variation downstream of an antenna (the Su–Oberman echo problem).

The antenna problem requires the solution of a driven inhomogeneous kinetic equation

- The relevant equations are

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} - \beta\frac{\partial}{\partial v}\left(v f + v_t^2\frac{\partial f}{\partial v}\right) = \frac{e}{m}\frac{\partial f_0}{\partial v}E$$

$$\frac{\partial E}{\partial z} = \frac{\partial E_a}{\partial z} - 4\pi e\int f\,dv,$$

where

$$E_a = |E_a|\exp(i\omega_a t)\delta(z)$$

- After Fourier transforms, and keeping only the driven terms, this is

$$(\omega_a - kv)f + i\beta\frac{\partial}{\partial v}\left(v f + v_t^2\frac{\partial f}{\partial v}\right) = i\frac{e}{m}\frac{\partial f_0}{\partial v}E$$

$$ikE = |E_a| - 4\pi e\int f\,dv$$

- It is now necessary to solve the first equation for f.

The solution of this equation requires boundary-layer theory

- It is straightforward to obtain f in the form

$$f = \frac{e}{m} E\, g(k, v, \omega_a),$$

where g is highly peaked at $v = \omega_a/k$

- Upon substitution in Poisson's equation, one solves explicitly for E. The end result for f itself is

$$f = \frac{e}{m} E_a\, g(k, v, \omega_a) \Big/ \left[1 + \frac{\omega_p^2}{k^2} \int g(k, v, \omega_a)\, dv\right].$$

- The spatial variation of f then follows from the Fourier inverse

$$f(z, v, \omega_a) = E_a \int_{-\infty}^{+\infty} \frac{\frac{e}{m} g(k, v, \omega_a) e^{ikz}}{D(k, \omega_a)}\, dk,$$

where D represents the denominator of f above.

The dominant contribution comes from the numerator, and this is the Su–Oberman result

$$f(z, v, \omega_a) \propto \exp\left(-\beta v_t^2 \left|z^3\right| / \left|v^5\right|\right)$$

- The small contributions of the denominator are weakly related to the zeros of D in the complex k-plane and represent decaying roots of the L–B relation. These are explicitly ignored by Su–Oberman, as dictated by echo experiment conditions.

- Su and Oberman never use the Van Kampen modes. They solve a driven problem. Their result is correct.

The calculation necessary for comparison with Skiff's experiment has not yet been carried out

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- There is a long history of calculation of the decay rate of ion-sound waves in weakly collisional plasma (much of it incorrect!).
- For $T_e >> T_i$, one must include both ion-ion collisions and electron-ion collisions, since it is electron Landau damping that dominates in the collisionless case.
- The ion kinetic equation is proportional to E and then the Poisson equation brings in f_e. Hence, one must necessarily solve the electron kinetic equation as well. This equation is in the form requiring a boundary-layer theory solution, as in the electron plasma wave case. (Improper solution of this equation was the origin of the earlier errors in the literature.)

The spatial dependence of f_i will take the following form

$$f_i(z, v, \omega_a) = \int_{-\infty}^{+\infty} \frac{g_i(k, v, \omega_a)}{D(k, \omega_a)} e^{ikz} dk,$$

where

$$D(k, \omega_a) = 1 - \frac{\omega_{pe}^2}{k^2} \int g_e(k, v, \omega_a) dv - \frac{\omega_{pi}^2}{k^2} \int g_i(k, v, \omega_a) dv.$$

- We do not expect the contribution of the numerator (akin to Su–Oberman) to be important. Now it is the large terms in the denominator that will dominate. This has not yet been carried out. It may be easier to do it numerically by initially taking Fourier transforms in velocity as well.
- At this point, one cannot conclude that there is any significant discrepancy between theory and the Skiff experimental results.

Summary and conclusions

- Skiff has measured $f_i(z, v_z)$ associated with an ion-sound wave driven by an antenna in a weakly collisional plasma.
- Comparison with an “incomplete” theoretical model seemed to indicate major differences and led to questions about earlier plasma-wave echo damping work by Su and Oberman.
- We show that Su and Oberman’s results remain correct.
- We do obtain a new and simplified form of the dispersion relation for electron plasma oscillations in weakly collisional plasma.
- We describe the theoretical calculation that must be done for comparison with Skiff’s measurements.
- At the moment, there is insufficient evidence to indicate a difference between theory and experiment.