Anomalous Stimulated Raman Scattering and Electron Acoustic Waves in Laser-Produced Plasmas: A Linear Model

Stimulated Raman scattering (SRS), an instability in which light is scattered by and amplifies electron plasma waves, has long been a concern in laser-fusion research because of its potential to reduce absorption and implosion efficiency. This concern is enhanced by the unsatisfactory state of the theory, particularly in accounting for the prevalence of anomalous SRS—scattering from plasma waves that according to conventional theory should be very heavily Landau damped.[1–5] Furthermore, scattering from waves satisfying the electron-acoustic (EA) dispersion relation (see below), which according to conventional theory should also be very heavily damped, has recently been identified.[2,4] These observations imply that Landau damping in laser-produced plasmas is often lower by an order of magnitude than would be predicted by the conventional linear theory of waves in a Maxwellian plasma.[5]

Since nonlinear Berstein–Green–Kruskal (BGK)[6] waves are undamped and under some conditions satisfy the EA dispersion relation,[7] they have recently been invoked to account for these phenomena.[2,4,8] However, the observed EA wave scattering (as well as much of the anomalous SRS) is at low levels, so that the fundamental question is how the associated plasma waves are amplified from small amplitudes (noise) despite prohibitive linear damping, rather than how they behave at large amplitudes. While nonlinear analysis has a place in modeling large-amplitude processes such as saturation, it seems appropriate to seek a linear explanation of the small-amplitude aspects of anomalous SRS and EA waves.

We begin by observing that the undamped nature of BGK waves stems not from their nonlinearity but rather from the fact that they represent a local flattening of the distribution function at the wave phase velocity (since the trapped distribution must be an even function of velocity in the wave frame). Our model of anomalous SRS posits a local flattening of the distribution function resulting from Landau damping of thermal noise driven by SRS. We shall see that such a locally flattened distribution function (LFDF) supports linear waves with reduced damping and enhanced SRS growth. Flattening resulting from Landau damping is a well-known process and represents a nonlinear modification of the original Maxwellian but does not imply the formation of BGK waves. In fact the evolution of the flattening is usually analyzed using so-called quasi-linear (that is, nonlinear only in the sense that it involves products of linear wave quantities) theory,[9] as a result of the damping of a continuum of linear waves. For our purposes the significant properties of the flattening (its location, width, etc.) can be ascertained from linear theory, and a detailed nonlinear analysis of the dynamics of the flattening process is unnecessary. We first demonstrate that a LFDF arbitrarily close to a Maxwellian supports linear undamped plasma waves. We take a model LFDF of the form

$$f(u) = f_0(u) + f_1(u) + f_2(u), \tag{1a}$$

where

$$f_0(u) = \frac{1}{\sqrt{\pi}} e^{-u^2}, \tag{1b}$$

$$f_1(u) = -f_0'(u_0)(u-u_0)e^{\frac{(u-u_0)^2}{(\Delta u)^2}}, \tag{1c}$$

and

$$f_2(u) = \frac{1}{3}\left[\beta - f_0''(u_0)\right]\left[(u-u_0)^2 - \frac{1}{2}(\Delta u)^2\right]e^{\frac{(u-u_0)^2}{(\Delta u)^2}}. \tag{1d}$$

Velocities are normalized to $\sqrt{2}v_T$, where v_T is the electron thermal velocity. The model thus comprises a Maxwellian that has been "flattened" near $u_0 = v_0/\left(\sqrt{2}v_T\right)$ over the range Δu, so that $f'(u_0) = 0$ and $f''(u_0) = \beta$, an arbitrary parameter. Note that $\int_{-\infty}^{\infty} f(u)\,du \equiv 1$ and that $f(u)$ is an analytic function of u that becomes arbitrarily close in L_p norm to the Maxwellian $f_0(u)$ as Δu is made small.

The corresponding dielectric function for waves of frequency ω and wave number k is given by

$$\varepsilon(k,\omega) = 1 - \frac{1}{2(k\lambda_D)^2} \int_{-\infty}^{\infty} \frac{f'(u)}{u - \frac{\omega}{\sqrt{2}kv_T}} du. \qquad (2)$$

Substituting (1a)–(1d) into (2) gives

$$\begin{aligned} \varepsilon(k,\omega) &= 1 + \frac{1}{(k\lambda_D)^2}\left[1 + \Omega Z(\Omega)\right] \\ &+ \frac{u_0 e^{-u_0^2}}{(k\lambda_D)^2}\left[2y + \left(2y^2 - 1\right)Z(y)\right] \\ &+ \frac{\Delta u}{(k\lambda_D)^2}\left[\frac{\sqrt{\pi}}{2}\beta + \left(1 - 2u_0^2\right)e^{-u_0^2}\right] \\ &\times \left[\frac{2}{3}\left(y^2 - 1\right) + \left(\frac{2}{3}y^3 - y\right)Z(y)\right], \end{aligned} \qquad (3)$$

where Z denotes the plasma dispersion function, $\Omega \equiv \omega / \left(\sqrt{2}kv_T\right)$, and $y \equiv (\Omega - u_0)/\Delta u$. The first two terms on the right of (3) give $\varepsilon_0(k,\omega)$ the dielectric function for a Maxwellian.

For each value of $k\lambda_D$ there are an infinite number of roots of (3). For the Maxwellian case, the least-damped root is usually referred to as “the” electron plasma wave; the other roots are very heavily damped and have little physical significance.[10] The flattening in the LFDF case introduces a new set of roots corresponding to waves with phase velocities near u_0. The least damped of these has much smaller damping than the plasma wave for large $k\lambda_D$. It is of interest to determine the conditions under which the damping of this root vanishes. In that case Ω is real, and

$$Z(\Omega) = i\sqrt{\pi}e^{-\Omega^2} - 2\ \mathrm{Daws}(\Omega),$$

where the Dawson function is defined as

$$\mathrm{Daws}(x) \equiv e^{-x^2}\int_0^x e^{t^2} dt = \frac{1}{2}\sqrt{\pi}e^{-x^2}\,\mathrm{erfi}(x).$$

Then for $\Omega = u_0$ we see from (3) that $\mathrm{Im}(\varepsilon) = 0$, and the condition for a root as $\Delta u \to 0$ is

$$1 + \left[1 - 2\Omega\,\mathrm{Daws}(\Omega)\right] / (k\lambda_D)^2 = 0,$$

which is just the dispersion relation for electron-acoustic waves in a Maxwellian plasma. Electron-acoustic modes were proposed in 1962 by Stix,[11] who speculated that a particular relative drift velocity of electrons and ions might cancel $\mathrm{Im}\left[\varepsilon(\omega,k)\right]$ and allow such modes to exist. As this seemed rather contrived, EA modes attracted little interest and were dropped from the second edition of Stix’s book.[12] The experimental observation of scattering from EA waves[4] therefore came as somewhat of a surprise, and at first they were not recognized as such.[13] We will return to EA waves after investigating anomalous SRS in a LFDF.

We combine in the usual way[14] the electromagnetic equations for the pump and scattered light waves with the dielectric function for the plasma waves (3) to obtain the dispersion relation for SRS backscatter:

$$\begin{aligned} D(k,\omega) &\equiv \varepsilon(k,\omega)\left[(\omega - \omega_0)^2 - (k - k_0)^2 c^2 - \omega_p^2\right] \\ &- \left[1 - \varepsilon(k,\omega)\right]\frac{k^2 v_{\mathrm{osc}}^2}{4} = 0. \end{aligned} \qquad (4)$$

Here k_0 and ω_0 are the wave number and frequency of the pump (laser) wave, ω_p is the electron-plasma frequency (a homogeneous plasma and fixed ions are assumed), and $v_{\mathrm{osc}} \equiv eE_0/m\omega_0$ is the electron quiver velocity in the laser field E_0.

We begin by studying a Maxwellian (unflattened) plasma, substituting $\varepsilon_0\ (k,\omega)$ in (4). Temporal growth rates of the SRS instability are found by solving (4) for ω as a function of k. The growth rate is a maximum at the resonance where both factors of the first term in (5) are small. Figure 92.29 shows normalized growth rates $\gamma / \left(\sqrt{2}\,kv_T\right)$ for a laser intensity of 10^{15} W/cm^2, temperature of 4 keV, and various values of the normalized electron-plasma density $n/n_c = \omega_p^2/\omega_0^2$, where n_c is the critical density. In the absence of damping, the growth rate would be $\gamma_0 / \left(\sqrt{2} k v_T\right) \sim 10^{-2}$, so it is evident that Landau damping has substantially decreased the growth rate. Note that the resonant value of $k\lambda_D$ increases and the resonance becomes broader as the density decreases.

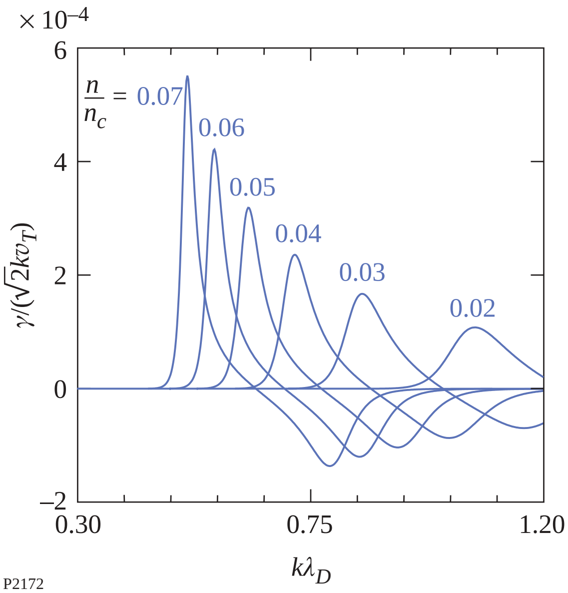

Figure 92.29
The SRS growth rates in a Maxwellian plasma with an incident laser intensity of 10^{15} W/cm^2 and an electron temperature of 4 keV.

Since monotonic distribution functions support only damped waves, we may expect the localized distortion of the distribution function resulting from damping of the SRS-driven plasma waves to continue until there is a point of inflection at the wave phase velocity. Such a situation can be represented by a LFDF with $\beta = 0$. To model the flattening effect of SRS we choose the flattening velocity $u_0 = \mathrm{Re}\left(\Omega_{\max}\right)$, where $\Omega_{\max}$ is the root corresponding to the peak growth rates in Fig. 92.29. The effects of interest are insensitive to Δu as long as it is small, so we take $\Delta u = 0.1$. Results for a density of $n/n_c = 0.05$ are shown in Fig. 92.30(a). The upper line represents the electromagnetic mode, effectively undamped in hot, low-density plasma, and the lower curve represents the least-damped plasma mode. In the absence of the pump these two modes would propagate independently, but the coupling due to the pump produces the SRS instability near the resonance at $k\lambda_D \cong 0.63$. Owing to the reduced Landau damping of the plasma mode resulting from the flattening, the peak SRS growth rate is larger than in Fig. 92.29 for the same plasma conditions by a factor of ~3. Note that the flattening has also resulted in an undamped electron-acoustic mode at $k\lambda_D \cong 0.5$. This mode does not satisfy the wave-vector-matching conditions for Raman scattering, however, so it is not driven.

While substantially reduced by the flattening, the Landau damping of the SRS plasma mode in Fig. 92.30(a) is still significant. This damping will continue the net transfer of particles from velocities below u_0 to velocities above u_0 until the wave is undamped. We may thus expect the flattening to overshoot, resulting in a small secondary maximum of the distribution function near u_0. This situation can be modeled by a nonzero value of β; the value of β giving an undamped plasma wave at the SRS matching wave vector is

$$\beta = \frac{3}{\sqrt{\pi}}\left[\frac{1+\left(k\lambda_D\right)^2 - 2u_0 \,\mathrm{Daws}\left(u_0\right)}{\Delta u} + \frac{2}{3}\left(2u_0^2 - 1\right)e^{-u_0^2}\right].$$

For the parameters of Fig. 92.30, $\beta \cong 2.54$, and the resulting plot is seen in Fig. 92.30(b). The reduction in damping of the plasma mode has increased the peak SRS growth rate as seen at $k\lambda_D \cong 0.63$ in Fig. 92.30(b) by a factor of ~3 over the SRS growth rate in Fig. 92.30(a). Note however that, owing to the secondary maximum of the distribution function, the EA mode has now become unstable, with an even larger growth rate than the SRS. This suggests that the EA wave may quickly grow to large amplitudes and disrupt the SRS process. Such a quench-

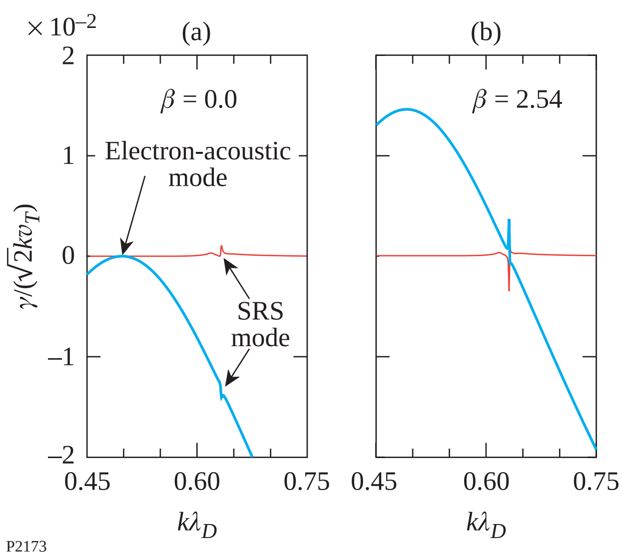

Figure 92.30
(a) The imaginary parts of the two least-damped roots of the SRS dispersion relation for a flattened distribution with $u_0 = 1.71$, corresponding to the peak of the SRS growth curve in Fig. 92.29 for $n/n_c = 0.05$. (b) The same distribution function, but with β increased to give zero damping of the plasma mode at the SRS resonance.

ing mechanism may be related to the intermittent interruptions in the SRS reflectivity seen in simulations [Fig. 1(a) of Ref. 15] and may help to explain why anomalous SRS reflectivities saturate at much lower levels than standard (small $k\lambda_D$) SRS reflectivities, though both have similar initial growth rates (see, e.g., Fig. 4 of Ref. 5). It is also noteworthy that the SRS growth rate peak in the $\beta = 0$ case is much narrower than in the original Maxwellian and consists of a sharp peak with a smaller subsidiary peak at slightly smaller $k\lambda_D$; this strongly resembles the form of the plasma-wave spectrum seen in SRS simulations [Fig. 1(b) of Ref. 15].

This analysis of anomalous SRS has been based on a locally flattened Maxwellian, an idealization capturing the essential aspects of a distribution function modified by Landau damping while remaining mathematically tractable. However, the Maxwellian itself represents an idealization; in a real plasma, thermal and other fluctuations are continually causing local, temporary distortions in the form of the distribution function. In particular, we may expect that such local distortions will on occasion result in a reduction or elimination of the slope of the distribution function. As noted above, such a distortion may represent an arbitrarily small deviation from the Maxwellian and still allow propagation of undamped EA waves. As the slope of the distribution function relaxes back to its equilibrium value, such a wave would ordinarily become heavily Landau damped and disappear. However, if it satisfies the frequency and wave-number-matching conditions for SRS, it will be driven just as the plasma wave in the case of anomalous SRS, resulting in amplification of the EA wave, maintenance of the local flattening by Landau damping, and the production of scattered light.

Such scattering has recently been seen at low levels in experiments,[4] though it was attributed to a nonlinear EA-like BGK mode. In fact, Refs. 4 and 8 suggest that such waves are the source of anomalous SRS and, on that basis, propose limiting SRS reflectivities in NIF targets by modifying plasma parameters so that $k\lambda_D > 0.53$ for the SRS-driven BGK wave (such waves are limited to $k\lambda_D < 0.53$). However, experiments in which SRS originates from low densities indicate that this nonlinear interpretation is implausible. To see this it is useful to combine the EA dispersion relation with the SRS resonance conditions to obtain a relation between the scattered light wavelength and the density at which scattering occurs. This relation is shown in Fig. 92.31 for the plasma parameters of the experiments in Ref. 3 (laser wavelength 351 nm, electron temperature $T_e \cong$ keV). Note that EA-wave scattering does not occur at densities below 0.08 critical; this is related to the $k\lambda_D < 0.53$ restriction (at lower densities the Debye length becomes too large to satisfy this condition for the SRS k). At higher densities there are two possible scattered wavelengths; the upper solution corresponds to plasma-wave scattering and the lower to EA-wave scattering.

Two types of experiments are described in Ref. 3. In the first, an interaction beam is focused on a relatively homogeneous plasma formed by an exploding foil; the density in the interaction region is (from hydro simulations) about 0.2 critical. From Fig. 92.31 it is seen that plasma waves at this density produce scattered light at about 650 nm, and that is what is seen in the experiment. No EA-wave scattering (at about 450 nm) is detected. In the second type of experiment, however, the interaction beam is focused on a plasma derived from a solid target, so that a full range of densities down to vacuum is represented in the interaction region. The SRS spectrum in these experiments extends from 420 to 540 nm, indicating that low densities are necessary to produce scattering at these wavelengths. The absence of SRS at higher densities is likely due to the steeper density gradients there that hydro simulations show in the solid-target experiments; such gradients would equally rule out EA waves at these densities as the source of the scattering. The scattering is, however, consistent with anomalous SRS from plasma waves in densities ≲0.04 critical. At such densities and temperatures (~4 keV) EA

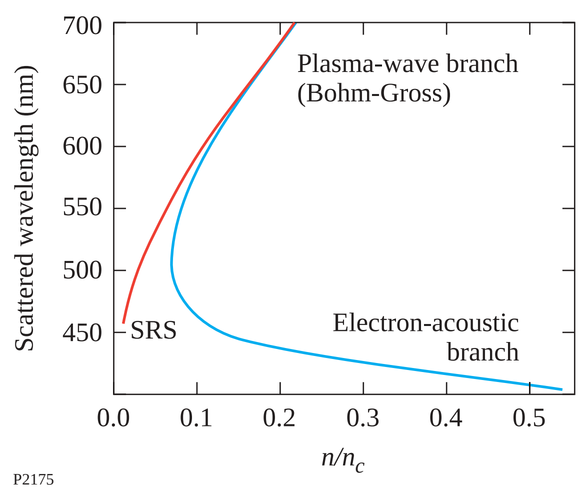

Figure 92.31
The scattered wavelength as a function of density for incident laser wavelength 351 nm for EA scattering and SRS. At long scattered wavelengths both waves approach the Bohm-Gross dispersion relation but diverge at shorter wavelengths (larger $k\lambda_D$).

scattering does not exist, and reduced damping of the SRS plasma wave ($k\lambda_D \sim 1$) is required to account for the dominance of these wavelengths in the observed spectrum. Local flattening of the distribution function as described above therefore seems the most satisfactory explanation for these results.

In summary, the model presented here shows that linear waves in a locally flattened distribution function can account for both anomalous SRS and EA-wave observations. Several predictions of the linear model differ qualitatively from those of the nonlinear approach. Among these: the linear model accommodates coexistence of SRS and EA waves, as seen in Refs. 4 and 13, and the occurrence of SRS at very low densities, as in Ref. 3. The linear model also suggests that proposals[2,4,8] to eliminate SRS in the NIF by altering plasma parameters so that $k\lambda_D > 0.53$ may not be successful.

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REFERENCES

1. A. Simon, W. Seka, L. M. Goldman, and R. W. Short, Phys. Fluids **29**, 1704 (1986) and references therein.
2. D. S. Montgomery *et al.*, Phys. Plasmas **9**, 2311 (2002).
3. S. P. Regan, D. K. Bradley, A. V. Chirokikh, R. S. Craxton, D. D. Meyerhofer, W. Seka, R. W. Short, A. Simon, R. P. J. Town, B. Yaakobi, J. J. Caroll III, and R. P. Drake, Phys. Plasmas **6**, 2072 (1999).
4. D. S. Montgomery *et al.*, Phys. Rev. Lett. **87**, 155001 (2001).
5. J. C. Fernandez *et al.*, Phys. Plasmas **7**, 3743 (2000).
6. I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).
7. J. P. Holloway and J. J. Dorning, Phys. Lett. A **138**, 279 (1989); J. P. Holloway and J. J. Dorning, Phys. Rev. A **44**, 3856 (1991).
8. H. A. Rose and D. A. Russell, Phys. Plasmas **8**, 4784 (2001).
9. E. M. Lifshitz and L. P. Pitaevskifi, *Physical Kinetics*, 1st ed., Course of Theoretical Physics, Vol. 10 (Pergamon Press, Oxford, 1981), pp. 199–205.
10. These correspond to the alternative definition of electron-acoustic modes; see Ref. 11.
11. T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962), p. 218. The term “electron-acoustic wave” is also often used to refer to the infinite set of solutions to the Landau dispersion relation other than the least-damped one [e.g., D. C. Montgomery, *Theory of the Unmagnetized Plasma* (Gordon and Breach, New York, 1971), p. 69]. Since in the past all these modes were usually mentioned only to be dismissed, many authors confuse or conflate the two definitions; our usage conforms to that of Stix.
12. T. H. Stix, *Waves in Plasmas* (American Institute of Physics, New York, 1992).
13. J. A. Cobble *et al.*, Phys. Plasmas **7**, 323 (2000).
14. J. F. Drake *et al.*, Phys. Fluids **17**, 778 (1974).
15. H. X. Vu, D. F. DuBois, and B. Bezzerides, Phys. Plasmas **9**, 1745 (2002).