1.B Improvements in the Coefficient for Inverse Bremsstrahlung Laser Absorption

Introduction

The classical coefficient for inverse bremsstrahlung (ib) laser absorption is proportional to a logarithmic factor, $\ln\Lambda_{ib}$, characteristic of Coulomb collisions. The argument Λ_{ib} generally is not calculated exactly but rather is estimated from physical considerations.[1] For low-density plasmas ($<10^{20}\text{cm}^{-3}$), $\ln\Lambda_{ib}$ is sufficiently large (>10) that the error in estimating Λ_{ib} should produce less than a $\sim 10\%$ variation in the logarithm, which is acceptable for most calculations. However, at the high plasma densities characteristic of short-wavelength laser irradiation (e.g., $\sim 9 \times 10^{21}$ cm^{-3} for 0.35-μm light), $\ln\Lambda_{ib}$ is <5, and uncertainties in $\ln\Lambda_{ib}$ can produce a 20% to 50% modification in the absorption coefficient. A more exact treatment of this term is presented here for the quantum-mechanical and classical limits of $\ln\Lambda_{ib}$, corresponding to low-Z and high-Z plasmas respectively.

For low-Z plasmas, a modified Born approximation is used to treat the electron-photon and electron-ion (e-i) interactions. The new features of the calculation are: (1) the time-dependent response of the plasma is modeled by including the plasma dielectric function as part of the Born-approximation treatment of the e-i interaction, and (2) an explicit treatment of ion-ion correlations replaces the usual assumption that the electrons scatter in an average electrostatic potential determined by the average positions of neighboring ions (e.g., a Debye-Hückel potential). This approach recovers the frequency dependence of $\ln\Lambda_{ib}$ obtained by Dawson and Oberman,[2] but it does not contain their indeterminate quantity k_{max}, which results from close e-i collisions. Close collisions are well described by the Born approximation in terms of the electron deBroglie wavelength.

The Born-approximation calculation is valid only for low-Z materials. For Z greater than ~ 10, any quantum mechanical treatment must include nonlinearities due to strong distortion of the electron wave function by the central ion, and in general a partial wave calculation is used. However, for the region of density and temperature of interest for laser absorption, the minimum-impact parameter for e-i collisions at high Z is no longer characterized by the deBroglie wavelength but by the classical-impact parameter for 90° scattering; thus, a quantum mechanical treatment is not required. The Coulomb logarithm could, in fact, be calculated using the classical nonlinear electron trajectory; that is the approach taken here for moderate-Z and high-Z plasmas. First, the logarithmic term for e-i scattering, $\ln\Lambda_{ei}$, is calculated for an average potential; then, a correction term (obtained from the Born approximation) is added to obtain the Coulomb logarithm for inverse bremsstrahlung, $\ln\Lambda_{ib}$. The Coulomb logarithm for e-i scattering ($\ln\Lambda_{ei}$) in a Debye-Hückel potential has been previously calculated over the quantum-mechanical and classical regions with the approximation $\Lambda_{ei} >> 1$, appropriate for moderate-Z ions ($Z < 25$). We extend those calculations to the high-Z region by including the effects of strong ion-ion correlations described by a nonlinear Debye-Hückel model, which merges smoothly with the previous large-Λ results. The electron trajectory in the resulting electrostatic potential is calculated numerically.

Physical Parameters and Definitions

Inverse bremsstrahlung is the process of light absorption induced by electron-ion collisions. The Coulomb logarithm generally is written in terms of the classical-impact parameters characterizing e-i scattering:

$$\ln\Lambda = \ln\,(b_{\max}/b_{\min}) + C \quad , \tag{1}$$

where $b_{\max}$ is the maximum impact parameter, $b_{\min}$ is the impact parameter for 90° scattering, and C is a number containing the remainder of the term, which is generally of the order of 1. For laser absorption, a correct calculation of $\ln\Lambda_{ei}$ should include (1) the response of plasma electrons to laser light in the presence of electron-ion scattering; (2) plasma shielding of interacting charged particles; (3) ion-ion correlations; and (4) nonlinear orbit dynamics or quantum-mechanical wave effects for close collisions. Various approximations have been used to determine the parameters in Eq. (1); no single approximation has determined all parameters self-consistently over the entire range of interest. (Of course, $\ln\Lambda$ would be well defined in a complete quantum mechanical calculation.)

The classical plasma calculation[2] for laser absorption has determined $b_{\max}$ in terms of the plasma Debye length λ_D and the laser frequency ω. Physically, these parameters play the following role. In a plasma, each ion is shielded by neighboring electrons and ions; for a low-Z to moderate-Z, high-temperature plasma the characteristic screening length is the Debye length

$$\lambda_D = [4\pi n_e e^2\,(1/T_e + Z/T_i)]^{-1/2} \quad , \tag{2}$$

where n_e is the electron density, T_e the electron temperature, T_i the ion temperature, and Z the ionic charge. Typically, T_e can be two to three times larger than T_i, as the e-i equilibration time can be much longer than the electron-heating time by inverse bremsstrahlung. The results below use $T_e = T_i$ for simplification, but the modification of the shielding distance for unequal temperatures is straightforward. Often, only the electron contribution to shielding is used [i.e., $Z = 0$ in Eq. (2)], which is based on the approximation of a uniform, ion background. But more realistic models, which include ion-ion correlations, show that the ion contribution to shielding can be dominant, as discussed below. For impact parameters much larger than the shielding distance, e-i scattering (and hence inverse bremsstrahlung) is negligible. Besides shielding, an additional factor enters into the determination of $b_{\max}$: the electron collision time should not be much longer than the period of the electromagnetic wave; otherwise, the interaction would be almost adiabatic and very little energy would be transferred to the electrons. The interaction time for an electron with an impact parameter b is roughly b/v_t, where $v_t = (T_e/m)^{1/2}$. Combining these two factors, the maximum impact parameter is approximated by

$$b_{\max} = \min\,(\lambda_{\mathrm{D}},\ v_t/\omega) \quad , \tag{3a}$$

which is characteristic of the detailed classical result.[2] Often only the high-frequency limit (low density) of the plasma calculation is quoted,[1] i.e., $b_{\max} = v_t/\omega$. This is not valid near the critical density where a majority of the laser light is absorbed. In this region, λ_{D} more closely characterizes the maximum impact parameter; it is approximately a factor $(Z+1)^{1/2}$ smaller than v_t/ω.

The choice of λ_{D} as the shielding length is only valid when it is much larger than the average-ion radius R_o, defined as $(4\pi n_i/3)^{-1/3}$. For high-Z plasmas, λ_{D} can become smaller than R_o, and strong ion-ion correlations must be considered for evaluating the plasma shielding. In this case, often the larger of R_o and λ_D is used.[3,4] This condition will be denoted here by an asterisk, i.e.,

$$b^*_{\max} = \min\,[\max(\lambda_{\mathrm{D}},R_o),\ v_t/\omega] \quad . \tag{3b}$$

The minimum impact parameter $b_{\min}$ in Eq. (1) is left indeterminate in the classical-plasma calculation.[2] It is often approximated by the impact parameter b_{90} for 90° scattering of an electron in a Coulomb potential,

$$b_{90} = Ze^2/mv^2 \quad , \tag{4}$$

where v is the electron velocity. If b_{90} is smaller than approximately the deBroglie wavelength, then quantum-mechanical effects must be considered. Typically, the quantum-mechanical "minimum-impact parameter" is defined as[5]

$$\lambda_q = \hbar/2mv \quad . \tag{5}$$

The parameter $b_{\min}$ becomes

$$b_{\min} = \max\,(b_{90},\ \lambda_q) \quad , \tag{6}$$

and is evaluated here at the effective velocity given by the energy relation

$$\frac{1}{2}\,mv^2 = \frac{3}{2}\,T \quad . \tag{7}$$

The region where $b_{\min} = \lambda_q$ will be denoted here as quantum mechanical, and the remaining region, will be called classical.

It is convenient to define a standard Coulomb logarithm, $\ln\Lambda_s$ to compare with the new results discussed below. We use

$$\ln\Lambda_s = \ln(b_{\max}/b_{\min}) \quad , \tag{8}$$

with Eqs. (3) and (6) defining the impact parameters, and with C from Eq. (1) set equal to zero. We denote the classical and quantum-mechanical limits of Λ_s as Λ_c and Λ_q respectively. The classical limit of Eq.(8) is

$$\ln\Lambda_c = \ln(12\,\pi n_e\,\lambda^3_{\mathrm{D}}) \quad , \tag{9a}$$

which has been evaluated near the critical density n_c with $\lambda_D < v_t/\omega$ and with the approximation $Z \approx Z + 1$. For high density, Λ_c is modified by Eq. (3b), in which the average-ion radius is used as the shielding distance

$$\Lambda_c^* = \Lambda_c \cdot \max(1, R_o / \lambda_D) \quad . \tag{9b}$$

In the quantum mechanical limit, Eq. (8) becomes

$$\ln\Lambda_q = \ln(\sqrt{12\, mT} / \hbar k_D) \quad , \tag{10}$$

where $k_D = 1/\lambda_D$.

Boundaries characterizing the different regions are sketched in Fig. 30.6, for Z versus T, at a plasma electron density of 9×10^{21} cm^{-3}. Temperatures around 1 keV are typical of laser-irradiated plasmas. The boundary between the quantum-mechanical and classical regions is determined by the condition $b_{90} = \lambda_q$. Except for the lowest-Z materials, $\ln\Lambda$ is in the classical region and can be determined by classical-orbit dynamics. Even for CH ($Z \sim 3$) at $T < 1$ keV, $\ln\Lambda$ is nearly classical. For moderate-Z and high-Z materials, Fig. 30.6 shows that approximations based on a linearized Debye-Hückel model may not be adequate, as $\lambda_D < R_o$. At high Z, approximations based on $\Lambda >> 1$ are questionable.

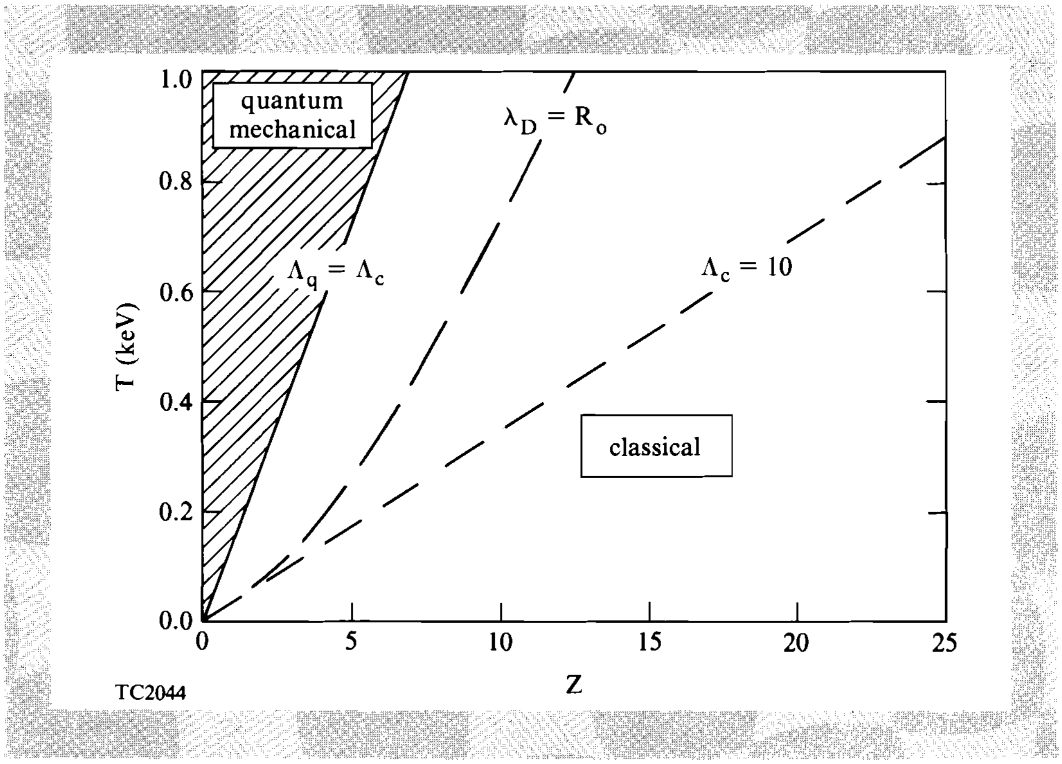

Fig. 30.6
Different regions and parameters characterizing $\ln\Lambda \sim \ln(b_{max}/b_{min})$. In the shaded region, the deBroglie wavelength, Eq. (5) determines b_{min}; in the remainder b_{90}, Eq. (4), is the appropriate minimum impact parameter. For Z greater than ~ 25, the Debye length λ_D, Eq. (2) becomes comparable to the average-ion radius R_o and strong ion correlations become important in determining b_{max}; approximations based on $\Lambda >> 1$ may become inaccurate.

Although the quantum-mechanical region (low Z) is relatively small, it is of considerable importance because low-Z ablators are required for direct-drive laser fusion. In this region, an accurate expression for $\ln\Lambda$ can be obtained relatively simply by using the Born approximation. This approximation is applicable[6] when the kinetic energy of the interacting electrons is much larger than the potential energy at approximately a deBroglie wavelength from the ion, i.e., $1/2\, mv^2 > Ze^2/(\hbar/mv)$, or, using Eqs. (5) and (7):

$$T > 35\, Z^2 \text{ eV} \quad , \tag{11}$$

corresponding to a low-Z, high-temperature plasma. (This condition is equivalent to $b_{90} > \lambda_q$.) As discussed later in **Results**, the Born approximation determines all parameters in the interaction: the classical result for $b_{\max}$ is recovered in the Born approximation when the Coulomb potential is modified by the plasma dielectric function; $b_{\min}$ is obtained in terms of the deBroglie wavelength; and $C \sim -1$. Results similar to these were obtained by Cauble and Rozmus[7], who used a modified Coulomb potential that phenomenologically accounted for quantum-wave effects in close electron-ion collisions.

The Born-approximation model is applicable to laser absorption for CH ($Z \sim 3$), but is invalid for SiO_2 ($Z = 10$) and for higher-Z materials at keV temperatures characteristic of laser-plasma interactions. We extend the calculation of $\ln\Lambda$ into the higher-Z region by relating the inverse-bremsstrahlung Coulomb logarithm $\ln\Lambda_{ib}$ to the logarithm for electron-ion scattering, $\ln\Lambda_{ei}$, in a shielded electrostatic potential. The Born approximation shows this relation to be

$$\ln\Lambda_{ib}(\text{Born}) = \ln\Lambda_{ei}\,(\text{Born}) + \frac{1}{2} + \frac{1}{Z}\, O\,(1/2) \tag{12}$$

near the critical density. The term 1/2 is the result of averaging $\ln\Lambda_{ib}$ over all ion positions, compared to simply using an average electrostatic potential (Debye-Hückel) in the calculation of $\ln\Lambda_{ei}$, as discussed by Hubbard and Lampe.[8] Equation (12) is extrapolated into the high-Z region, beyond the validity of the Born approximation, according to

$$\ln\Lambda_{ib} = \ln\Lambda_{ei} + [\ln\Lambda_{ib}(\text{Born}) - \ln\Lambda_{ei}\,(\text{Born})] \quad , \tag{13}$$

which is similar to Eq. (7) in Ref. 9. This extrapolation is probably the largest source of uncertainty for high Z. The term 1/2 makes a 25% contribution to $\ln\Lambda_{ib}$ for $Z = 50$. Using Eq. (13), $\ln\Lambda_{ib}$ can be determined by calculating e-i scattering in a spherically symmetric potential. An expression for $\ln\Lambda_{ei}$ that spans the quantum-mechanical and classical limits has been obtained by Williams and DeWitt,[9] for moderate Z. However, their results depend on the approximate solution by Liboff[10] for electrons scattering in a linearized Debye-shielded potential with $\Lambda \gg 1$, and is not appropriate for high-Z plasmas.

For high-Z materials with Λ less than ~ 10, the potential around an ion can no longer be described by the linearized Debye-Hückel model, and stronger ion-ion correlations must be considered. Such correlations were examined by Cauble and Rozmus,[7] but with a model that produces only the quantum-mechanical minimum-impact parameter. This is valid only at low Z, where the strong ion correlations occur at very low temperatures; these conditions are not characteristic of the laser absorption region. In this article, strong ion-correlation effects are examined at the higher temperatures achieved in coronal high-Z plasmas. A nonlinear Debye-Hückel (NLDH) model[11,12] is used to prevent the close approach of neighboring ions, which is the main effect of strong correlations. The Coulomb logarithm is evaluated by using the classical electron trajectory in the NLDH self-consistent electrostatic potential. This model is convenient for considering electrons and ions at different temperatures, and it merges smoothly with the low-Z (large-Λ) results of Liboff for a linearized Debye-shielded potential.

The starting point for the calculation is Boltzmann's equation for the change in the electron distribution function f due to inverse bremsstrahlung:

$$\frac{\partial f}{\partial t} - \frac{e}{m} E \cdot \nabla_v f = \int d^3\Delta v$$

$$\times \left[W(v-\Delta v \Rightarrow v)\, f(v-\Delta v) - W(v \Rightarrow v+\Delta v)\, f(v)\right] \quad . \qquad (14)$$

The electric field E and the two-body interaction W can take on different meanings according to the particular model of the laser-plasma interaction. Three models are considered:

(a) The first model, by Dawson and Oberman[2], treated all e-i scattering as a self-consistent electrostatic potential, which was included in E together with the laser electric field. Close two-body interactions were considered negligible, and the term W was set equal to zero. This approach is able to calculate the collective plasma effects but not the close e-i encounters, which are reflected by an indeterminate quantity $k_{\max}$ in the effective Coulomb logarithm.

(b) A second approach, based on the Born approximation, places both the laser electric field and e-i collisions into the term W, in terms of an inverse-bremsstrahlung transition rate, and E is set equal to zero. Close e-i collisions are now treated accurately (within the range of validity of the Born approximation), and the collective plasma effects of (a) are recovered by using the plasma dielectric function to modify the vacuum Coulomb potential around an ion. There are no indeterminate parameters in this model,[13] but its validity is limited to very low ionic charge.

(c) The third model assumes that electron oscillation in the laser electric field does not modify e-i scattering and can be separated from it: the laser electric field is included in E, and e-i scattering (in an electrostatic potential) is in the term W. This model is used for high-Z plasmas.

Results

1. Low Z

Two features characterize the Coulomb logarithm for laser absorption in low-Z materials: (1) the plasma shielding distance λ_D is sufficiently large that the ω/v_t contribution to b_{max} can introduce an ω dependence to Λ [Eq. (3a)]; and (2) the impact parameter for 90° electron scattering is sufficiently small that quantum-wave effects can contribute to b_{min} [Eq. (6)]. The first effect has been calculated by Dawson and Oberman.[2] Both effects are simultaneously addressed by the Born approximation result. (A schematic of the processes considered in the Born approximation is shown in Fig. 30.7.) Using the Born approximation, with a Coulomb potential modified by the plasma dielectric function, we obtain

$$\begin{aligned}\ln \Lambda_{ib}\,(\text{Born}) = {} & \ln\Lambda_q + \ln[(Z+1)^{1/2}/\bar{\omega}] - \gamma + \frac{1}{2}\ln(4/3) \\ & + \frac{1}{2Z}\, e^{\bar{\omega}^2/2}\, E_1(\bar{\omega}^2/2) \\ & - \frac{Z+1}{2Z}\, e^{\bar{\omega}^2/2(Z+1)}\, E_1[\bar{\omega}^2/2(Z+1)] \quad , \end{aligned} \tag{15}$$

where $\ln\Lambda_q$ is the standard quantum-mechanical Coulomb logarithm defined in Eq. (9), γ is Euler's constant (γ = 0.577), E_1 is the exponential integral, and $\bar{\omega} = \omega/\omega_p$ (where ω_p is the local plasma frequency; or, in terms of the critical density, $\bar{\omega}^2 = n_c/n$). To compare the Born-approximation result with Dawson and Oberman,[2,14] we examine the two limits: (1) absorption near the critical density [$\bar{\omega} \sim 1$ in Eq. (15)], and (2) absorption at very low density ($\bar{\omega} >> 1$). It is the latter limit that is most often quoted,[1] but it is the former that is most relevant to laser-fusion experiments.

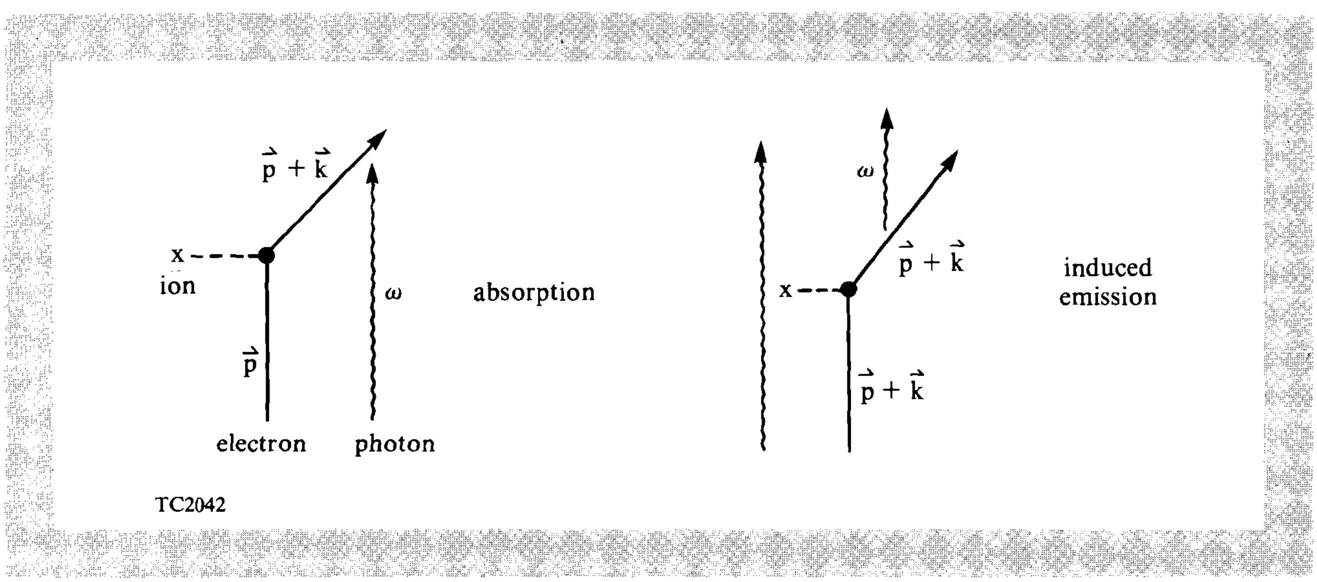

Fig.30.7
Schematic of the processes contributing to inverse bremsstrahlung in the Born approximation. Momentum transfer by the photon is neglected.

In the high-frequency limit ($\bar{\omega} >> 1$) appropriate for low-density absorption, both exponential integrals in Eq. (15) approach zero, leaving

$$\ln \Lambda_{ib} \approx \ln(4\ T/\hbar\omega) - \gamma \quad . \tag{16}$$

(The same result has been obtained for bremsstrahlung emission[15] in the Born approximation with $\hbar\omega << T$, for a pure Coulomb potential.) To compare with the Dawson-Oberman result $\ln\Lambda_{DO}$ the indeterminate quantity k_{max} ($= 1/b_{min}$) in Ref. 2 is replaced by the suggested quantum mechanical expression, $k_{max} = (mT)^{1/2}\ /\ \hbar$. The difference from Eq. (16) is found to be $\ln\Lambda_{ib} - \ln\Lambda_{DO} \approx 0.75$, representing a 15% correction for conditions attained in short-wavelength laser irradiation of CH, characterized by $\ln\Lambda \approx 5$.

For the region around the critical density ($\bar{\omega} \sim 1$), Eq. (15) reduces to

$$\ln \Lambda_{ib} \approx \ln\Lambda_q + \frac{1}{2}\ln(2/3) - \frac{1}{2}\gamma - \frac{1}{2Z}\ln(Z+1) \ , \tag{17}$$

where we have also assumed $Z^{1/2}\ /\ \bar{\omega} >> 1$. This should be compared with the $\omega << \omega_p$ case of Dawson in Ref. 14, $\ln\Lambda_D$, evaluated at $\omega = \omega_p$. (When the effects of ion shielding are included, the results for large and small ω are no longer equal at $\omega = \omega_p$ as they were in Ref. 1, which used only electron shielding. The $\omega << \omega_p$ result is the one that best approximates the correct solution for $\omega = \omega_p$.) Again, the suggested replacement $k_{max} = (mT)^{1/2}\ /\ \hbar$ was used, with the same result for the difference in solutions: $\ln\Lambda_{ib} - \ln\Lambda_D \approx 0.75$. In the region that dominates laser absorption, the ω dependence of $\ln\Lambda_{ib}$ is found to be negligible.

Equation (17) is similar to the result obtained by Cauble and Rozmus[7], who did not use the Born approximation but rather a modified Coulomb potential that approximates quantum effects at small distances. Their resulting Coulomb logarithm differs from the one here by only ~ 0.1 for $Z = 3$. Cauble and Rozmus note that there is a substantial difference between their results (with linear, Debye-Hückel ion correlations) and the Dawson-Oberman results from Ref. 2, which indeed did not include the ion contribution to shielding. However, if comparison had been made with Ref. 14 instead, where Dawson has removed the assumption of a random ion distribution and imposed Debye-Hückel correlations, then very little difference would have been found (using the $\omega << \omega_p$ result for the region around the critical density). The remaining difference could be removed by modifying the choice of k_{max}, which does not depend on ion correlations.

The relationship between $\ln\Lambda_{ib}$ and the classical high- and low-frequency limits is shown in Fig. 30.8, over the density range from 0.1 n_c to n_c, for a 1-keV plasma with $n_c = 9\times10^{21}$ cm^{-3}. Although $\ln\Lambda_{DO}(\omega << \omega_p)$ was derived for $n > n_c$, it is evaluated here at the subcritical density indicated. Over this density range, where laser absorption predominantly occurs, $\ln\Lambda_{ib}$ is well approximated by the

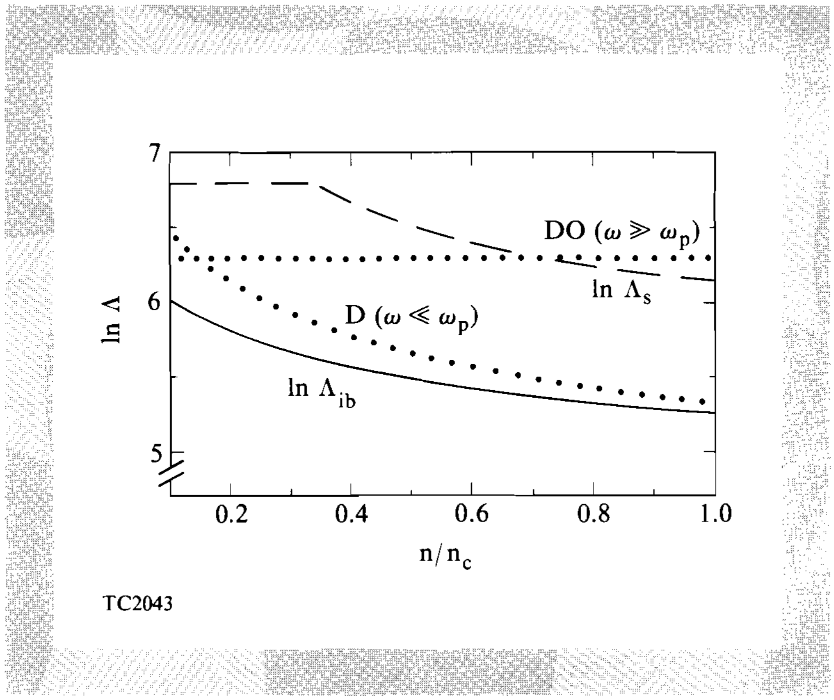

Fig. 30.8
The density dependence of $\ln\Lambda_{ib}$ (Born) for $Z = 3$, $T = 1$ keV in terms of the critical density n_c (9×10^{21} cm^{-3}). Compared are Eq. (8) for $\ln\Lambda_s$, the Dawson and Oberman (DO) result[2] for $\omega >> \omega_p$, and Dawson's (D) result[15] derived for $\omega << \omega_p$ but evaluated at the $\omega > \omega_p$ density indicated. Over the region shown, $\ln\Lambda_{ib}$ is best approximated by the $\omega << \omega_p$ result. Only at densities below $\sim 0.1\ n_c$ does $\ln\Lambda_{ib}$ reach the $\omega >> \omega_p$ result.

$\omega << \omega_p$ result. Only at densities well below 0.1 n_c does $\ln\Lambda_{ib}$ approach the high-frequency limit. Use of the high-frequency limit around n_c results in a $\sim$20% error for low Z.

2. High Z

For high Z, where Λ is less than $\sim$10, we use Eq. (13) to calculate $\ln\Lambda_{ib}$ from e-i scattering in an average, self-consistent, electrostatic potential. In this region, corresponding roughly to $Z > 25$, Fig. 30.6 shows that it would be questionable to use λ_D as the shielding distance, or to use approximations dependent on $\Lambda >> 1$. Here we evaluate $\ln\Lambda_{ei}$ by numerically calculating the electron trajectory in the nonlinear Debye-Hückel potential to determine the relationship between the impact parameter b and the scattering angle θ. The relation between the scattering cross section σ and the Coulomb logarithm for e-i scattering is given by

$$\ln \Lambda_{ei} = \frac{1}{8} \int dv\ v\, f_o(v) \int_{-1}^{1} \bar{\sigma}(\theta,v)\ (1-\cos\theta)\ d\cos\theta \ / \int f_o\ v\ dv \ . \quad (18)$$

The dimensionless quantity $\bar{\sigma}$ is defined as

$$\bar{\sigma} = \sigma(\theta,v) \ / \ \sigma_c(180°,v) \ , \quad (19)$$

in terms of $\sigma_c = (Ze^2/2\ mv^2)^2$, the cross section for 180° scattering in a pure Coulomb potential. The first velocity moment of the distribution function is required for inverse bremsstrahlung. (Other e-i processes such as electron diffusion would require higher-order moments.) Using $\sigma\ d\cos\theta = b\ db$, the double integral in Eq. (18) is calculated numerically, and finally, $\ln\Lambda_{ib}$ is evaluated from $\ln\Lambda_{ei}$ using Eq. (13).

The electrostatic potential $V(r)$ is calculated from Poisson's equation, without linearization:

$$\nabla^2 V = -4 \pi e (Z n_i - n_e) \quad ; \tag{20}$$

$$n_e = Z \langle n_i \rangle \exp(eV/T_e) \quad ; \tag{21}$$

$$n_i = \langle n_i \rangle \exp(-eZV/T_i) \quad ; \tag{22}$$

with the boundary conditions $V(\infty) = 0$ and $V(r \Rightarrow 0) = Ze/r$. The Fermi-Dirac form of the electron distribution function was also used because n_e can become large (and degenerate) near the nucleus; however, the degeneracy effect on $\ln\Lambda$ was found to be insignificant for the conditions considered here. If Eqs. (20) through (22) are linearized in terms of V, the usual Debye-Hückel shielding length is obtained. However, linearization is not valid within the average ion-sphere radius R_o, where V becomes large. Near the central ion, Eq. (22) forces the ion density of neighboring ions to rapidly approach zero, and only electrons remain for shielding.

The classical trajectory for an electron scattering in an arbitrary potential $V(r)$ is given by[16]

$$\theta = \pi + 2 \int_o^{u_o} \left[1 - V\left(\frac{1}{u}\right)/E - (bu)^2 \right]^{-\frac{1}{2}} b \, du \quad , \tag{23}$$

where θ is the scattering angle, β is the impact parameter, and u is the inverse radius between the electron and the ion. The upper limit to the integral is given by the zero of the square-root factor and corresponds to the distance of closest approach.

This NLDH model for the ions is valid for values of the ion-ion coupling parameter Γ ($= Z^2 e^2 / R_o T$) less than ~ 1. For $Z = 50$, $T = 0.5$ keV and $n_e = 9 \times 10^{21}$, we have $\Gamma = 6$, which suggests that NLDH may be only marginally applicable. To test the sensitivity of $\ln\Lambda_{\rm ib}$ to the model, an alternate potential was tried: the potential was determined assuming a uniform electron density, which does not permit neighboring ions inside R_o. (The NLDH model does permit a small amount of neighboring ions to penetrate R_o.) For $Z \sim 50$, there was less than about a 2% difference between the models for the calculation of $\ln\Lambda_{\rm ei}$, and both gave values about a factor of 2 higher than the linearized Debye-Hückel model result. (For low Z, the NLDH model reproduces the linearized results of Liboff to within a few percent, while the uniform electron model is $\sim 50\%$ lower and would not be applicable in this region.) This suggests the applicability of using the NLDH model to calculate $\ln\Lambda_{\rm ei}$ over the entire classical region, at the conditions considered here.

To test the numerical procedure, comparison was made with the free-free Gaunt factor calculated by Lamoureux *et al.*[17] They performed a quantum-mechanical partial-wave calculation of bremsstrahlung emission, produced by 1-keV electrons in a Ce ($Z = 55$) plasma at an ion density of 8.6×10^{21} cm^{-3}. The Gaunt factor G is related to the Coulomb logarithm by $G = (\pi^{1/2}/3) \ln\Lambda$.

Lamoureux *et al.* observe that G is relatively insensitive to the shapes of potentials with roughly the same range, as above. Their effective $\ln\Lambda$ in the soft-photon limit is 1.2. The classical model used here is in close agreement, predicting 1.3 for the NLDH potential.

The NLDH results for $\ln\Lambda_{ib}$, as a function of Z, are presented in Fig. 30.9 for T = 1 keV and in Fig. 30.10 for T = 0.5 keV (both are at the critical density 9 × 10^{21} cm^{-3} for 0.35-μm light). Also shown in the figures are (1) $\ln\Lambda_{ei}$, from Liboff's calculations, corresponding to the moments in Eq. (18); (2) $\ln\Lambda_s$, defined in Eq. (9a), which uses

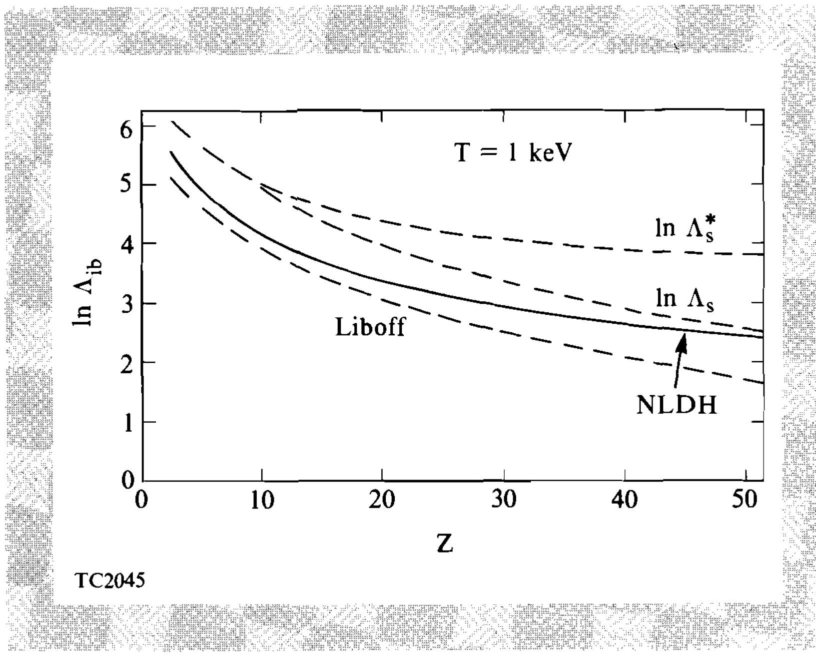

Fig. 30.9
$\ln\Lambda_{ib}$, using nonlinear Debye-Hückel (NLDH) ion correlations, compared with the approximate $\ln\Lambda_{ei}$ of Liboff,[10] $\ln\Lambda_s$ of Eq. (9a), which uses λ_D as the shielding length, and with $\ln\Lambda_s$*, which uses the maximum of the ion-sphere radius R_o and λ_D as the shielding length. At higher Z (or lower temperature), both $\ln\Lambda_s$ and Liboff's result would become negative. The results are for n_e = 9 × 10^{21} cm^{-3} and T_e = 1 keV. Equation (24) is a good approximation to the NLDH results.

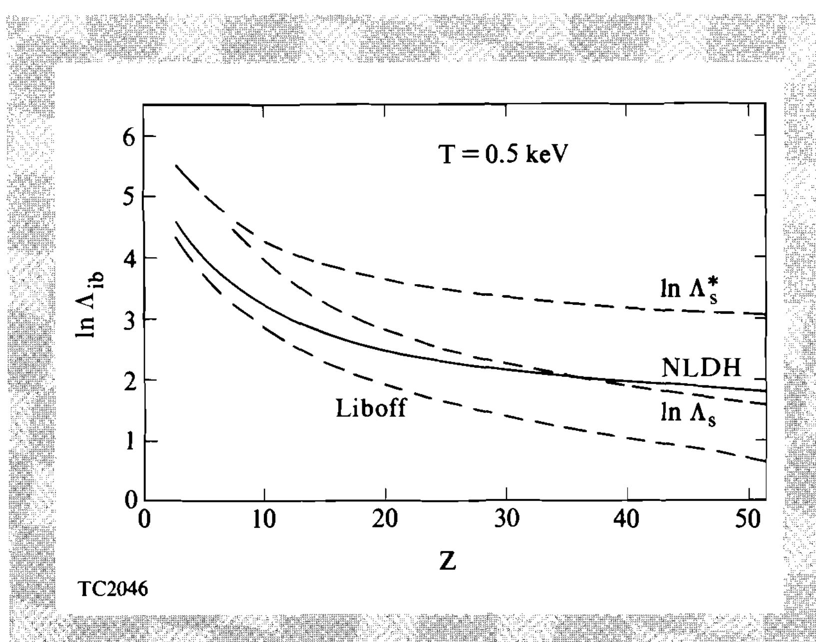

Fig. 30.10
Same as Fig. 30.9, but with T_e = 0.5 keV.

the Debye length λ_D as the shielding distance; and (3) $\ln\Lambda_s{}^*$, which uses the average-ion radius as the shielding distance whenever it is larger than λ_D [Eq. (9b)]. We make the following observations: The deviation between $\ln\Lambda_s$ and $\ln\Lambda_s{}^*$ becomes apparent for Z greater than ~ 10, corresponding to the region of $R_o > \lambda_D$ in Fig. 30.6. Both $\ln\Lambda$ (Liboff) and $\ln\Lambda_s$ have the wrong functional form in this region, which would become negative at higher Z or lower T. Nevertheless, over the region shown, $\ln\Lambda_s$ is able to approximate the NLDH results to within $\sim 10\%$.

The often-quoted high-frequency limit of Dawson and Oberman is not shown in the figures. Effectively, it uses only electron shielding for $b_{\max}$ and is related to $\ln\Lambda_s$ by

$$\ln\Lambda(\omega >> \omega_p) \approx \ln\Lambda_s + 1/2\ln_e(1+Z)-1.$$

For $Z = 50$, it would be in error by about 50% compared to $\ln\Lambda$(NLDH).

The NLDH solution decreases very slowly with Z, and does not fall much below 2. One result of the NLDH calculation is to support the use of $\max(R_o, \lambda_D)$ as the effective shielding distance. The $\ln\Lambda_s{}^*$ curve, which has this constraint, very closely follows the functional form of $\ln\Lambda$(NLDH) into the high-Z, low-temperature region. An approximation to $\ln\Lambda$(NLDH), to within a few percent, is

$$\ln \Lambda_{ib}\,(\text{NLDH}) = \ln \Lambda_s^* - 1.25 \tag{24}$$

for the conditions of temperature and density considered here, including the quantum-mechanical region. The nonlogarithmic term 1.25 contains all the details of the calculation. It corresponds to a $\sim 50\%$ effect at high Z.

For high Z, $\ln\Lambda_{ib}$ is obtained by adding ~ 0.5 to $\ln\Lambda_{ei}$ [Eqs. (12) and (13)]. This attempt to reduce the effect of different ion configurations to an average electrostatic potential represents a 25% variation for $Z \sim 50$. It is probably the greatest source of uncertainty in the calculation, and further investigation is needed.

Summary

The "Coulomb logarithm" for laser absorption has been calculated for conditions achieved in short-wavelength laser irradiation: $n_e \sim 10^{22}$ cm^{-3} and $T \sim 1$ keV. At these conditions $\ln\Lambda_{ib}$ is < 5, and uncertainties in previously used models can produce variations in this term of 20% to 50%.

For low-Z materials, $\ln\Lambda_{ib}$ was calculated quantum mechanically using a modified Born approximation. Collective plasma effects were included by multiplying the e-i interaction term by the plasma dielectric function. Unlike the classical calculation,[2] the "minimum impact parameter" was well determined and, of course, related to the deBroglie wavelength. The effective "maximum impact parameter" was the same as the classical result. The ω dependence of $\ln\Lambda_{ib}$ was found to be negligible near the critical density n_c (Fig. 30.8), where absorption predominantly occurs. Use of the often-quoted high-frequency limit of $\ln\Lambda_{ib}$ in this region can lead to a ~20% error. Near n_c, $\ln\Lambda_{ib}$ is found to be closely related to $\ln\Lambda_{ei}$, the Coulomb logarithm for electrons scattering in a shielded electrostatic potential around an ion; Eq. (13) was used to extrapolate that relationship beyond the range of validity of the Born approximation into the high-Z region.

For Z greater than ~10, the minimum impact parameter is no longer quantum mechanical (Fig. 30.6) and is determined by the distance of closest approach for the classical electron trajectory around an ion. From the trajectory, an effective e-i Coulomb logarithm $\ln\Lambda_{ei}$ was calculated using Eq. (18), and Eq. (13) was then used to determine $\ln\Lambda_{ib}$. To bridge the classical and quantum-mechanical regions, the results of Williams and DeWitt[9] were used. For moderate-Z plasmas, where the approximation $\Lambda_{ei} >> 1$ is applicable, a previously calculated expression[10] was used for $\ln\Lambda_{ei}$.

However, at high Z, the calculation of $\ln\Lambda_{ei}$ does not permit approximations based on $\Lambda >> 1$ or the use of λ_D as the shielding length (Fig. 30.6). We have extended the calculation into the high-Z region by using the NLDH model. The dominant high-Z effect is that neighboring ions are strongly repelled at distances smaller than the average-ion radius R_o. The NLDH model was found to reproduce results for $\ln\Lambda_{ei}$ at high Z (as calculated from the uniform electron model), and to also merge smoothly to moderate-Z results (calculated by Liboff[10]). Use of Eq. (13) to relate the average of $\ln\Lambda_{ib}$ over all ion configurations to the result obtained from an average, spherical, electrostatic potential is probably the greatest source of uncertainty in the calculation, and the resulting error requires further investigation.

Results for $\ln\Lambda_{ib}$ are shown in Fig. 30.9 at $n_e = 9 \times 10^{22}$ cm^{-3} for $T = 1$ keV, and in Fig. 30.10 for $T = 0.5$ keV. A good fit to the numerical results for all Z is given by Eq. (22). The calculation supports the use of replacing λ_D by R_o as the effective shielding distance, Eq. (3b), whenever $\lambda_D < R_o$. The nonlogarithmic term 1.25 in Eq. (24) represents a 50% correction at high Z.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Energy Office of Inertial Fusion under agreement No. DE-FC08-85DP40200, and by the Laser Fusion Feasibility Project at the Laboratory for Laser Energetics, which has the following sponsors: Empire State Electric Energy Research Corporation, General Electric Company, New York State Energy Research and Development Authority, Ontario Hydro, and the University of Rochester. Such support does not imply endorsement of the content by any of the above parties.

REFERENCES

1. Examples of $\ln\Lambda$ are contained in *NRL Plasma Formulary*, edited by David L. Book (Naval Research Laboratory, Washington, DC, 1983); T. W. Johnston and J. M. Dawson, *Phys. Fluids* **16**, 722 (1973).
2. J. Dawson and C. Oberman, *Phys. Fluids* **5**, 517 (1962).

3. R. S. Cohen, L. Spitzer, and P. Routly, *Phys. Rev.* **80**, 230 (1950).

4. J. Albritton, Laser Program Annual Report 84, UCRL-50021-84 (Lawrence Livermore National Laboratory, Livermore, CA, 1984), pp. 3-50.

5. S. Ichimaru, *Basic Principles of Plasma Physics* (W.A. Benjamin, MA, 1973).

6. D. Bohm, *Quantum Theory* (Prentice Hall, NJ, 1951), pp. 551–555.

7. R. Cauble and W. Rozmus, *Phys. Fluids* **28**, 3387 (1985).

8. W. B. Hubbard and M. Lampe, *Astrophys. J. Suppl. Ser.* **18**, 297 (1969).

9. R. Williams and H. DeWitt, *Phys. Fluids* **12**, 2326 (1969).

10. R. Liboff, *Phys. Fluids.* **2**, 40 (1959).

11. R. Cowan and J. Kirkwood, *J. Chem. Phys.* **29**, 264 (1958).

12. S. Skupsky, *Phys. Rev. A* **21**, 1316 (1980).

13. (a) H. W. Wyld and D. Pines, *Phys. Rev.* **127**, 1851 (1962); (b) The formalism of Ref. 13(a) was used in: S. Skupsky, *Phys. Rev. A* **16**, 727 (1977).

14. J. Dawson, in *Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Wiley, NY, 1969), Vol. I, p. 1.

15. As quoted in: I. Shkarofsky, T. Johnston, and M. Bachynski, *The Particle Kinetics of Plasmas* (Addison-Wesley, MA, 1966), p. 233.

16. H. Goldstein, *Classical Mechanics* (Addison-Wesley, MA, 1965), Chap. 3.

17. M. Lamoureux, I. J. Feng, R. H. Pratt, and H. K. Tseng, *J. Quant. Spectrosc. Radiat. Transfer* **27**, 227 (1982).