Optimization of Irradiation Configuration Using Spherical *t* Designs for Laser-Direct-Drive Inertial Confinement Fusion

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In laser-direct-drive inertial confinement fusion (ICF), a cryogenically cooled spherical shell of deuterium–tritium (DT) fuel covered with a plastic layer on the outside is irradiated by a number of laser beams. The laser irradiation ablates the outer plastic (ablator) layer, compressing the fuel to reach ignition conditions.[1–3] Because of the finite number of beams (e.g., 60 on the OMEGA Laser System[4]) the laser irradiation nonuniformity on the target surface leads to a nonuniform shell compression and reduction in the implosion performance.

The beam-overlap uniformity improves with increasing the number of beams N. Optimization of beam port locations to minimize drive asymmetries for a given number of beams is an important consideration in designing an ICF implosion facility. There have been two basic methodologies of obtaining beam configurations presented in literature. One is based on either augmenting or composing together regular polyhedrons.[4,5] Advantages of this method include a symmetric intensity distribution with minimized nonuniformity for a few select values of N. A disadvantage is the lack of a systematic extension to an arbitrary N. The other method is the charged-particle method, which uses a system of N particles constrained to a sphere that repel each other with a coulomb force (or another distance-dependent force). The beam configurations are chosen to correspond to particle configurations that minimize the potential energy.[6–8] The advantage of the method is its simplicity in obtaining beam configurations for arbitrary N. The disadvantage is a slow decay of nonuniformity with the number of beams as, e.g., $1\sqrt{N}$ in Ref. 8. Neither of the above methods offer a way of finding beam configurations that simultaneously eliminate spherical harmonic modes below a certain number, which has been recognized as an important strategy in designing the irradiation system.[5]

In this summary, we propose new beam configurations based on spherical t designs, which are studied in the area of mathematics known as spherical designs.[9] New configurations have the advantage of combining the following properties: all of the nonuniformity modes with $\ell \leq t$ are zero, nonuniformity amplitudes decrease strongly with the number of beams, and intensity distribution on spherical targets exhibit symmetric patterns. Computational methods developed in the field of spherical designs (see, e.g., Refs. 10 and 11) offer a systematic approach of obtaining such configurations for any number of beams[10] feasible for a direct-drive ICF facility.

The beam configuration of the OMEGA Laser System[4] is practically a 60-beam 9-design (a spherical t design with $t = 9$). It has all $\ell \leq 9$ exactly equal to zero, except for mode 6, which is close to zero. OMEGA is not a regular truncated icosahedron "soccer ball," which is only a 5-design, but rather a "stretched soccer ball" whose hexagonal faces have unequal sides A and B with the stretch factor $A/B = 1.2$ [see Fig. 1(a)]. The ratio $A/B = 1.21$ is quoted in Ref. 5 to eliminate mode $\ell = 6$ in the stretched soccer ball configuration. Values of the stretch factor in both OMEGA and Ref. 5 are within 0.5% from the exact value $A/B = 1.205285...$ that can be calculated following the method from Ref. 12, where it was recognized that the stretched soccer ball with the optimal stretch factor is a better approximation for a sphere than the regular soccer ball.

Remarkably, a 10-design exists for a set of 60 points,[13] which we will call $T60$. The $T60$ configuration eliminates ℓ modes up to and including mode 10 and is shown in Fig. 1(b) along with the $M60$ configuration [Fig. 1(c)] obtained in Ref. 6 using the charged-particle method. Although $T60$ has [3,3]+ symmetry group and is the union of five snub tetrahedrons,[13] there are no

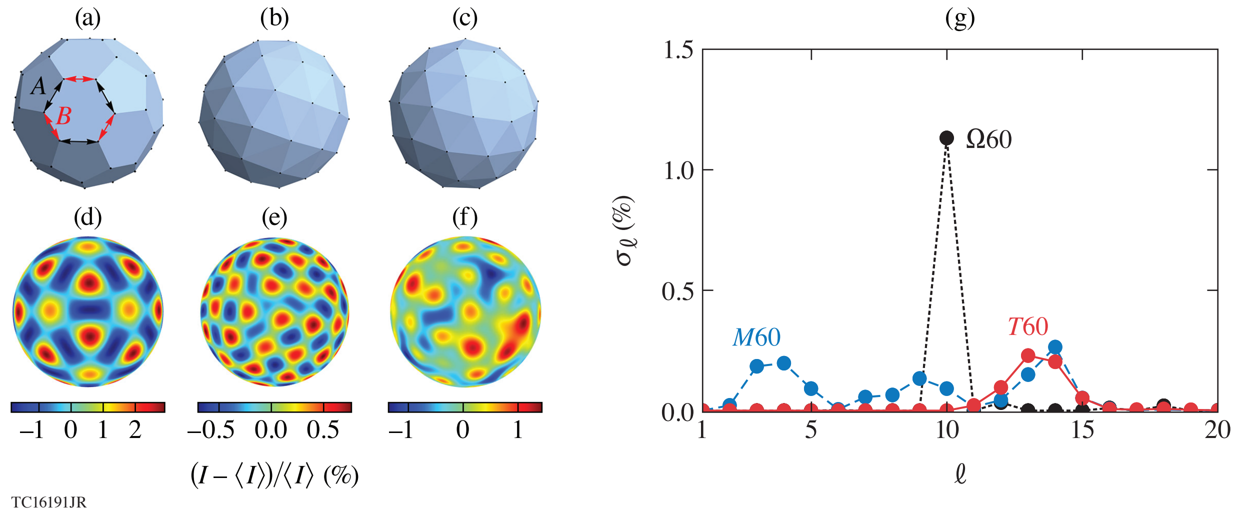

Figure 1
Beam configurations of (a) OMEGA, (b) $T60$, and (c) $M60$. Absorbed intensity distributions for (d) OMEGA, (e) $T60$, and (f) $M60$ configurations. (g) Plot of rms nonuniformity σ_ℓ as a function of mode number ℓ for OMEGA (black curve), $T60$, (red curve), and $M60$ (blue curve) configurations.

visually discernible symmetries in $T60$ point locations (much like in $M60$) in comparison to the aesthetically pleasing OMEGA laser. Figures 1(d)–1(f) show the intensity distributions produced by 60 laser beams from the OMEGA, $T60$, and $M60$ configurations, respectively. As an example, for Fig. 1, the beams were chosen to have the following profiles,

$$I_a(\theta) = I_0 \exp\left[-(\sin\theta/0.635)^{4.19}\right] \cdot \max(\cos\theta, 0), \tag{1}$$

which correspond to profiles produced by SG5-650 phase plates[14] on 450-μm-diam targets on the OMEGA Laser System. Similar to OMEGA, $T60$ produces a symmetric illumination pattern [compare Figs. 1(d) and 1(e)], while $M60$ produces less-regular illumination pattern [see Fig. 1(f)]. Out of the three configurations, the 10-design configuration produces the lowest intensity variation amplitude. Figure 1(g) shows the ℓ-mode spectra, σ_ℓ versus ℓ, corresponding to configurations in Figs. 1(a)–1(c). One can see dominant mode 10 and zero modes $\ell < 10$ for OMEGA. $M60$ has significant amplitude in a broad range of modes from $\ell = 2$ to $\ell = 14$. $T60$ has modes $\ell = (11$ to $15)$ with amplitudes that are similar to that of $M60$ but, as expected, has zero modes $\ell < 11$.

Future direct-drive ICF laser systems will look to improve uniformity, which means increasing t in t-design configurations using the least number of beams N. Finding t designs with the smallest N for a given t happens to be one of the main problems in spherical design theory. It was conjectured that the spherical t designs exist for any t and require at least number of points $N \approx (1/2)t^2$ (see Ref. 13), where t designs are listed for every $N \le 100$. The authors' website in Ref. 13 also contains point coordinates for t designs when $N = 12m$, $m = 1,2,3...$ with the number of points up to $N = 240$ and $t = 21$. Below, we will use these designs as an example. It is worth noting that numerical methods have been developed that make it possible to calculate t designs with $t \le 1000$ and corresponding $N \le 10^6$ (see, e.g., Ref. 10).

Figures 2(a)–2(d) show the intensity distribution for four $N = 12m$ spherical t designs with $t = 11$, 13, 16, and 21 and $N = 72$, 96, 144, and 240, which we call $T72$, $T96$, $T144$, and $T240$, respectively. Similar to $T60$, the $N = 12m$ configurations with a higher number of beams have intrinsic symmetry groups and show symmetric intensity distribution patterns. Note a dramatic reduction of the nonuniformity amplitude with t. Figure 2(e) shows ℓ-mode spectra σ_ℓ for the five $N = 12m$ t designs, four designs from Figs. 2(a)–2(d), and $T60$ from Fig. 1(b). One can see zero amplitude of modes $\ell \le t$ and a sharp decrease of the σ_ℓ for large ℓ for all t designs.

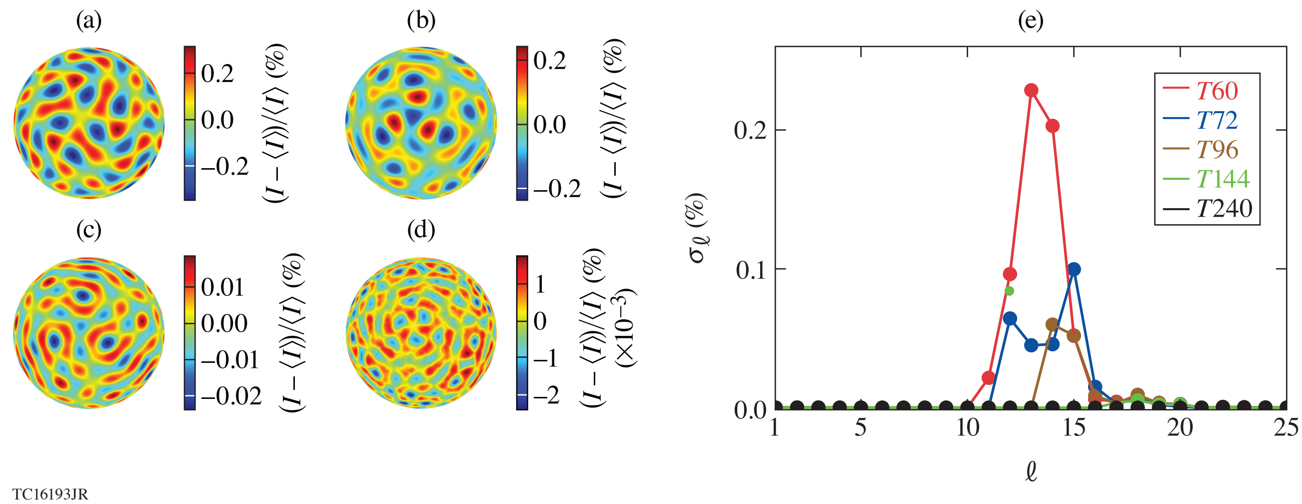

Figure 2
Intensity distributions for (a) $T72$, (b) $T96$, (c) $T144$, and (d) $T240$ configurations. (e) Plot of rms nonuniformity σ_ℓ as a function of mode number ℓ for $T60$ (red), $T72$ (blue), $T96$ (brown), $T144$ (green), and $T240$ (black) configurations.

In this summary, we have proposed a new approach to systematically obtain beam configurations for laser-direct-drive ICF systems that are based on t designs from the area of mathematics known as spherical designs. The t-design beam configurations offer the following advantages: (a) they eliminate all nonuniformity modes $\ell \le t$, where t increases with the number of beams as $\sim\sqrt{2N}$; (b) the rms nonuniformity drops rapidly with the number of beams as a high power of N or close to exponentially with N; and (c) t designs with intrinsic symmetries show symmetric intensity-distribution patterns (although, it may have only aesthetic benefits). We envision that future laser-direct-drive ICF facilities will use t-design beam configurations with a number of beams that will be determined by the nonuniformity requirements. As a final note, spherical t designs can be used, more generally, in applications where a uniformity of an action on a sphere applied at discrete points is required, e.g., a uniformity of pressure applied with a system of identical actuators.

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1. J. Nuckolls *et al.*, Nature **239**, 139 (1972).
2. J. D. Lindl, Phys. Plasmas **2**, 3933 (1995).
3. R. S. Craxton *et al.*, Phys. Plasmas **22**, 110501 (2015).
4. *LLE Review Quarterly Report* **39**, 113 (1989).
5. M. Murakami, Appl. Phys. Lett. **66**, 1587 (1995).
6. M. Murakami *et al.*, Phys. Plasmas **17**, 082702 (2010).
7. M. Murakami and D. Nishi, Matter Radiat. Extremes **2**, 55 (2017).
8. W. Trickey *et al.*, Front. Phys. **9**, 784258 (2021).
9. P. Delsarte, J. M. Goethals, and J. J. Seidel, Geom. Dedicata **6**, 363 (1977).
10. M. Gräf and D. Potts, Numer. Math. **119**, 699 (2011).
11. R. S. Womersley, in *Contemporary Computational Mathematics–A Celebration of the 80th Birthday of Ian Sloan*, edited by J. Dick, F. Y. Kuo, and H. Woźniakowski (Springer, Cham, 2018), pp. 1243–1285.
12. J. M. Goethals and J. J. Seidel, Nieuw Archief voor Wiskunde **XXIX**, 50 (1981).
13. R. H. Hardin and N. J. A. Sloane, Discrete Comput. Geom. **15**, 429 (1996).
14. W. Theobald *et al.*, Phys. Plasmas **29**, 012705 (2022).