Impact of Boundary Sharpness on Temporal Reflection in Dispersive Media

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In this summary, we investigate the impact of the finite rise time of a temporal boundary inside a dispersive medium used for temporal reflection and refraction of optical pulses. We develop a matrix approach in the frequency domain for analyzing such temporal boundaries and use it to show that the frequency range over which reflection can occur is reduced as the rise time increases. We also show that total internal reflection can occur even for shallow boundaries. This feature suggests that temporal waveguides can be realized through cross-phase modulation, even when pump pulses have relatively long rise and fall times. In past studies, the moving temporal boundary was assumed to be infinitely sharp such that the refractive index changes instantaneously at the boundary location. In practice, any temporal boundary will have a finite rise time. One expects the results obtained for a sharp boundary to remain valid as long as the rise time is much shorter than other time scales of interest (such as the width of the pulse being reflected at the boundary). It is not known, however, how the results obtained for a sharp boundary need to be modified when the rise time of the temporal boundary is non-negligible.

We assume that a temporal boundary, moving at the speed V_B, has been created inside the dispersive medium using a suitable technique (e.g., cross-phase modulation with a pump pulse) so that the refractive index of the medium differs by a small amount Δn on the two sides of the boundary. In most previous studies on temporal reflection, $s(t)$ is taken to be a step function of the form $h(t{-}T_B)$, assuming an infinitely sharp boundary located at $t = T_B$. In this work, we consider temporal boundaries with a finite rise time T_r; in particular, $s(t)$ was considered to be a super-Gaussian of order $m, e^{-[(t-T_B)/T_0]2m}$, where $T_r \sim T_0/m$.

For a sharp boundary with $T_r = 0$, it is known that a pulse splits into two parts after it arrives at the boundary, which can be identified as the reflected and transmitted parts.[1] Their spectra are shifted from the spectrum of the incident pulse in such a way that the reflected part never crosses the boundary. We extend this approach to temporal boundaries of arbitrary shapes by making a reasonable approximation. We divide the boundary region into N segments, each of finite duration such that $s(t)$ can be treated as a constant inside the segment. In other words, we replace the actual shape of the boundary with a staircase. This can be done for a boundary of any shape if we make N large enough that $s(t)$ does not vary much inside each segment. Consider one spectral component of the pulse before the first segment with the frequency $\omega = \omega_0 + \delta_0$. It propagates as a plane wave $A_0 e^{i(Kz-\delta_0 t)}$. As this plane wave traverses the boundary region, its frequency changes from one segment to the next, but K remains the same because of momentum conservation at any temporal boundary.[1] At the same time, a reflected wave is produced with a shifted frequency. As a result, two plane waves exist in the nth segment, where $s(t) = s_n$ is a constant.

Our approach is similar to that used for calculating the reflectivity of a stack of multiple dielectric layers.[2] We use the transfer and propagation matrices to cross all segments, starting from the far end of the last segment. The resulting matrix of the entire temporal boundary is the product of $2N + 1$ matrices. In terms of the four elements of this matrix, the incident, reflected, and transmitted waves are related as

$$\begin{pmatrix} A_{\text{in}} \\ A_{\text{R}} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_{\text{T}} \\ \text{O} \end{pmatrix}.$$

The reflectivity and transmissivity of the temporal boundary are given by $R = |M_{21}/M_{11}|^2$ and $T = |1/M_{11}|^2$, where the matrix elements can be computed using the group velocity, the group velocity dispersion, and the frequency shift across the boundary. One interesting feature of an infinitely sharp temporal boundary is that the analog of total internal reflection (TIR) can occur, i.e., it is possible to have the situation where the faster-propagating probe pulse never overtakes the slower-moving pump because the frequency of the probe pulse changes. We can normalize the parameter $(\omega_0/c)\ \Delta n$ to the minimum value for TIR to occur. Physically, this normalized parameter B represents the value of the index change relative to the value required for TIR to occur.

As seen in Fig. 1, TIR occurs for Δf values below 0.3 THz. The reason it ceases to occur for larger values of Δf is related to a larger speed mismatch between the wave relative to the moving boundary. As Δf increases beyond 0.3 THz, change in the propagation constant continues to increase, which decreases the reflectivity further. The rate of decrease depends on the boundary's rise time, and it becomes more rapid as T_r increases (or m decreases). For a Gaussian-shaped boundary with $m = 1$, the reflectivity becomes nearly a step function of Δf. In practical terms, for such boundaries, a narrowband signal is either totally reflected or fully transmitted, depending on its frequency. We considered two cases with $B = 0.95$ and $B = 1.05$, where $B = 1$ is required for TIR to occur. In the first case ($B = 0.95$), 96% of the pulse energy crosses the boundary. For $B = 1.05$, all the energy is reflected because the index change across the boundary is larger by 10% and exceeds the TIR threshold of $B = 1$. These results show that a relatively small change in the refractive index can produce large changes in the transmitted energy of a probe pulse when pump pulses are used to create a moving temporal boundary using the Kerr nonlinearity of an optical fiber.

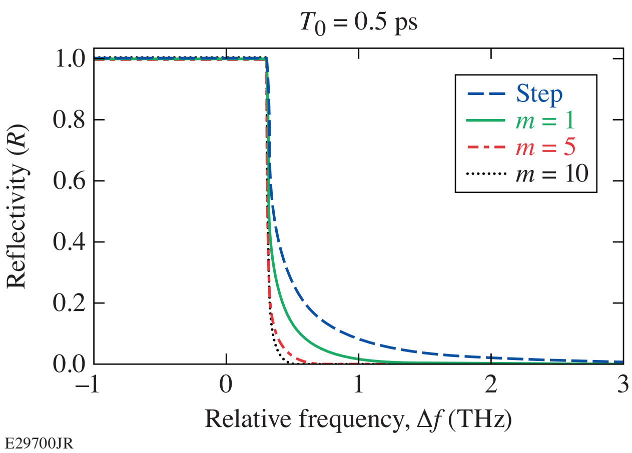

Figure 1
Dependence of the reflectivity spectrum on the rise time of a temporal boundary ($T_0 = 0.5$ ps) is shown using three values of m. The dashed curve shows, for comparison, the case of a step-function boundary.

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