A Generalized Forward Fit for Neutron Detectors with Energy-Dependent Response Functions

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To date, most analyses of neutron time-of-flight (nTOF) data from inertial confinement fusion experiments have focused on the relatively small range of energies corresponding to the primary neutrons from D–D and D–T fusion. These analyses have therefore employed instrument response functions (IRF's) corresponding to monoenergetic 2.45- or 14.03-MeV neutrons. For analysis of time-of-flight signals corresponding to broader ranges of neutron energies, accurate treatment of the data requires the use of an energy-dependent IRF. This work describes interpolation of the IRF for neutrons of arbitrary energy, construction of an energy-dependent IRF, and application of this IRF in a forward fit via matrix multiplication.

The measured nTOF signal includes effects from the detector's IRF such that the relationship between the neutron energy spectrum and the measured signal is not immediately obvious. One method of interpreting an nTOF signal is the forward-fit technique,[1] which involves the convolution of a model neutron energy spectrum with the detector IRF. In the absence of bright, pulsed monoenergetic neutron sources, the total neutron IRF cannot be directly measured. The total neutron IRF is instead constructed by convolving a measured x-ray response with an energy-dependent neutron interaction response. The measured x-ray response accounts for the detector's response to a short pulse of incident energy, while the energy-dependent neutron interaction response accounts for neutron transport through the detector and can be calculated using a particle transport code such as MCNP.[2]

The oscilloscope-recorded nTOF signal $m(t)$ can be understood as the superposition of the detector's response to a neutron of incident energy E, weighted by the number of neutrons incident on the detector at that energy. The variable t represents the time scale recorded on the oscilloscope, while t' represents the neutron's time of arrival at the detector. If the number of neutrons detected per unit energy is given by $\mathrm{d}N/\mathrm{d}E[E(t')]$, the detector's temporal response for a given incident neutron energy is given by $R[E(t'), t-t']$, the Jacobian describing the conversion from neutron energy to nTOF is $\mathrm{d}E/\mathrm{d}t'$, and the detector's calibration constant is C, then the measured signal can be written as a Fredholm integral of the first kind given by

$$m(t) = C\int_0^{t'} \frac{\mathrm{d}N}{\mathrm{d}E}\left[E(t')\right]\frac{\mathrm{d}E}{\mathrm{d}t'}\left[E(t')\right]R\left[E(t'), t-t'\right]\mathrm{d}t'. \tag{1}$$

The equation for the forward fit with an energy-dependent IRF can also be written as a sum such that Eq. (1) becomes

$$m(t_k) = \sum_{i=0}^{k} P\left[E(t_i')\right]R\left[E(t_i'), t_k - t_i'\right].$$

This sum can be represented by the matrix multiplication $\vec{m} = \mathbf{T}\vec{P}$, where $\vec{P}$ is the vector of prediction of length N_p, $\vec{m}$ is the vector of measured values of length $N_m = N_p + N_r - 1$, and $\mathbf{T}$ is a Toeplitz matrix of the response vector[3] with shape $N_m \times N_p$ given by

$$T = \begin{bmatrix} r_{0,0} & 0 & \cdots & 0 & 0 \\ r_{0,1} & r_{1,0} & \cdots & \cdots & \cdots \\ r_{0,2} & r_{1,1} & \cdots & 0 & 0 \\ \cdots & r_{1,2} & \cdots & r_{N_{p-1},0} & 0 \\ r_{0,N_r-1} & \cdots & \cdots & r_{N_{p-1},1} & r_{N_p,0} \\ r_{0,N_r} & r_{1,N_r-1} & \cdots & \cdots & r_{N_p,1} \\ 0 & r_{1,N_r} & \cdots & r_{N_{p-1},N_{r-2}} & \cdots \\ 0 & 0 & \cdots & r_{N_{p-1},N_{r-1}} & r_{N_p,N_{r-2}} \\ \cdots & \cdots & \cdots & r_{N_{p-1},N_r} & r_{N_p,N_{r-1}} \\ 0 & 0 & 0 & \cdots & r_{N_p,N_r} \end{bmatrix}. \tag{2}$$

Note that the first index of each matrix element corresponds to energy $E\{t'_i\}$, while the second index corresponds to time $t_k - t'_i$ (i.e., the index of a specific entry within the array of $R[E(t'_i), t_k - t'_i]$. Each column represents a monoenergetic IRF. It is clear that several monoenergetic IRF's must be generated in order to construct the Toeplitz matrix since each column of the matrix represents a response function of a different energy. This is best accomplished by generating a representative set of neutron interaction responses, convolving them with the measured x-ray IRF, and interpolating the total IRF.

Uncertainty in the IRF is introduced mainly by the uncertainty in the measured x-ray IRF. Minimization must be carried out in order to include both Poisson-distributed uncertainties from the number of neutrons detected as well as Gaussian-distributed uncertainties from digitization noise. Details of the error propagation that is necessary to construct the correct χ^2 fit metric are included in the full length paper.[4]

As an example of the application of this method, an analysis of synthetic data relevant to T–T fusion experiments at the Omega Laser Facility is discussed. This example is used to illustrate the differences between a forward fit that uses an energy-dependent IRF and a forward fit that uses a monoenergetic IRF. Use of the energy-dependent IRF results in an accurate inference of the fit parameters of interest. The inferred masses of the ^{5}He ground state and first excited state are minimally affected since the mass is related to the mean neutron energy. Use of the monoenergetic 2.45-MeV (DD) IRF affects the inferred width and magnitude of the ground state significantly (>20% change). Use of the monoenergetic 14.03-MeV (DT) IRF affects the inferred magnitude of the ground state slightly, but barely affects the inferred width of this state. The inferred ^{5}He mass distributions are shown in Fig. 1.

This conclusion concerning the 14.03-MeV IRF is, however, only applicable for this specific detector and this specific combination of nuclear states. It is not possible to know whether any monoenergetic IRF would be an acceptable approximation for the energy-dependent IRF unless the widths and mean energies of the relevant nuclear states are already relatively well known. The use of a monoenergetic IRF to approximate the energy-dependent IRF is therefore not recommended for use with experimental data that spans a wide range of energies, especially if the analysis in question attempts to infer parameters of nuclear states that are currently poorly understood or parameters related to complicated combinations of nuclear states. The use of an energy-dependent IRF is most essential for the analysis of nuclear states with smaller widths, especially if (1) there are several states of relatively narrow widths spread across a wide range of energies, (2) there are several nuclear states located at low energies, and/or (3) there are several relatively sharp features in the nTOF data. Neutron spectra related to backscattered neutrons produced in cryogenic DT experiments as well as neutron spectra produced from inelastic reactions between DT neutrons and ^{7}Li are two such applications for which analyses of recent OMEGA data are underway.

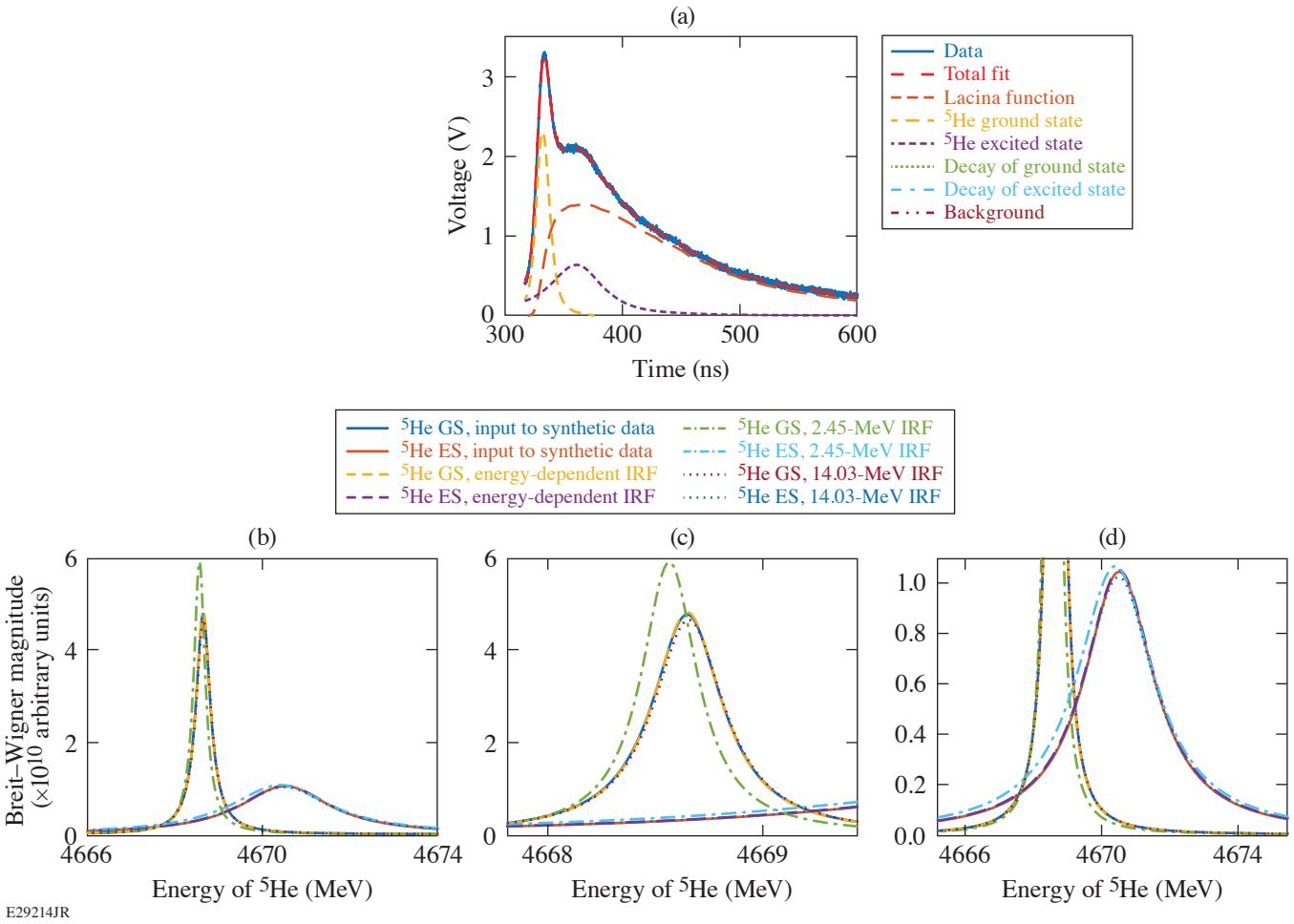

Figure 1
(a) The synthetic TT data and an example of the components of the forward fit are shown along with the inferred ^{5}He mass distributions. (b) The total mass distribution is shown along with (c) two close views of the inferred ground state and (d) the inferred first excited state. The forward fit with the energy-dependent IRF infers the mean energies, width, and magnitudes of the input mass distributions to within a few percent. There is little change to the inferred first excited state regardless of the choice of IRF because it is very wide (2.5 MeV). There is >20% change to the inferred ground state width when the 2.45-MeV monoenergetic IRF is used, but little change when the 14.03-MeV IRF is used. This result is an artifact of this combination of nuclear states and this specific detector setup, as there is only a 300-ps difference between the 8.5-MeV IRF (i.e., the approximate neutron energy where the ^{5}He ground state is located) and the 14.03-MeV IRF for this detector.

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