The Role of Baroclinicity in the Kinetic Energy Budget

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The role of baroclinicity, which arises from the misalignment of pressure and density gradients, is well known in the vorticity equation, yet its role in the kinetic energy budget has never been obvious. We have shown that baroclinicity appears naturally in the kinetic energy budget after carrying out the appropriate scale decomposition. Strain generation by pressure and density gradients, both barotropic and baroclinic, also results from our analysis. These two processes underlie the recently identified mechanism of "baropycnal work,"[1–3] which can transfer energy across scales in variable density flows. We also provide numerical evidence from high-resolution (i.e., 1024 × 1024 × 1024) direct numerical simulations (DNS's) of compressible turbulence (Fig. 1).

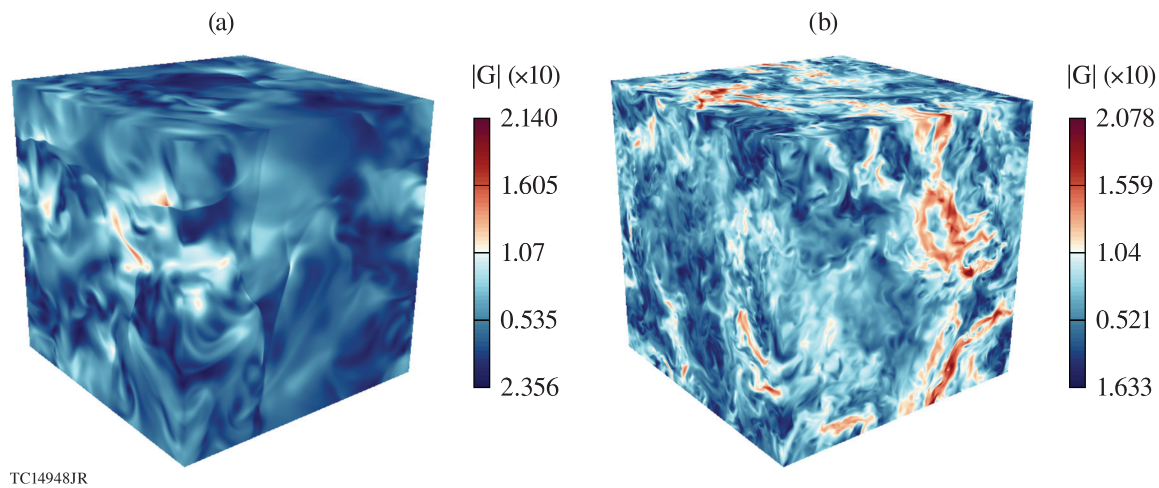

Figure 1
DNS of turbulent flows at varying levels of compressiblity using our DiNuSUR code.

To analyze the dynamics of different scales in a compressible flow, we use the coarse-graining approach. It has proven to be a natural and versatile framework for understanding and modeling scale interactions (e.g., Refs. 1–6) and is closely related to well-established physics techniques, including macroscopic electromagnetism,[7,8] where coarse graining of microscopic charge and current densities coupled with gradient expansions yield the macroscopic polarization **P** and magnetization **M** as well as higher-order multipole contributions. It is also closely related to the renormalization group (RG), especially "real-space RG,"[9,10] where a coarse-grained field is like a "block spin" and coarse-grained equations are analogous to "effective Hamiltonian/action" for the block spins with running coupling constants that depend on the scale parameter ℓ. Our approach is also intimately related to large eddy simulation (LES) in turbulence modeling.[4,11] Equations governing the dynamics of different scales can be derived relatively easily, allowing for a direct analysis of processes at those scales both analytically and using data from simulations or experiments.

For any field $a(x)$, a coarse-grained or (low-pass) filtered field, which contains modes at scales $>\ell$, is defined in n dimensions as

$$\bar{\mathbf{a}}_\ell(x) = \int d^n \mathbf{r}\, G_\ell(\mathbf{r})\, \mathbf{a}(\mathbf{x}+\mathbf{r}), \tag{1}$$

where $G(r)$ is a normalized convolution kernel and $G_\ell(r) = \ell^{-n} G(r/\ell)$ is a dilated version of the kernel having its main support over a region of diameter ℓ. The scale decomposition above is essentially a partitioning of scales in the system into large ($>\ell$), captured by $\bar{a}_\ell$, and small ($<\ell$), captured by the residual $\bar{a}'_\ell = a - \bar{a}_\ell$.

The budget for the large-scale kinetic energy can be easily derived[3] from the compressible momentum equation:

$$\partial_t\, \bar{\rho}_\ell \frac{\left|\tilde{\mathbf{u}}_\ell\right|^2}{2} + \nabla \cdot \mathbf{J}_\ell = -\Pi_\ell - \Lambda_\ell + \bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell - D_\ell + \epsilon_\ell^{\text{inj}}, \tag{2}$$

where $J_\ell(x)$ is space transport of large-scale kinetic energy, $\bar{P}_\ell \nabla \cdot \bar{u}_\ell$ is large-scale pressure dilatation, $D_\ell(x)$ is viscous dissipation acting on scales $>\ell$, and $\epsilon_\ell^{\text{inj}}(x)$ is the energy injected due to external stirring. The $\Pi_\ell(x)$ and $\Lambda_\ell(x)$ terms account for the transfer of energy across scale ℓ.

Using the property of scale locality,[1] we have derived a model of Λ that shows how it transfers energy by two processes: barotropic and baroclinic generation of strain S from gradients of pressure and density ρ:

$$(\text{const})\,\ell^2 \rho^{-1} \left\{\left[\nabla_\rho \left(\nabla P\right)^T\right] : \mathbf{S}\right\} \tag{3}$$

and baroclinic generation of vorticity ω:

$$(\text{const})\,\ell^2 \rho^{-1} \left(\nabla_\rho \times \nabla_P\right) \cdot \omega. \tag{4}$$

While the role of pressure and density gradients in generating vorticity is well recognized, their role in strain generation has been less emphasized in the literature.

To our knowledge, this is the first direct demonstration of how baroclinicity enters the kinetic energy budget, which arises naturally from our scale decomposition and the identification of Λ as a scale-transfer mechanism (Fig. 2). Baroclinicity is often

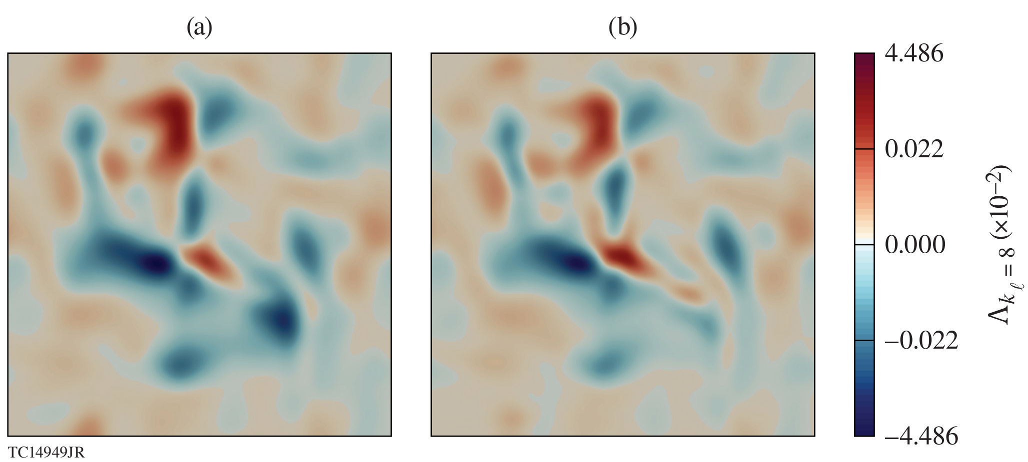

Figure 2
Visualization of a slice from our 3-D flow of the true Λ and its model that we derived, showing excellent pointwise agreement.

analyzed within the vorticity budget but its role in the energetics has never been obvious. The need for a scale decomposition in order for Λ and, as a result, baroclinic energy transfer to appear in the kinetic energy budget is similar to the scale transfer term Π, which appears in the budget only after decomposing scales due to energy conservation. In the same vein, the appearance of baroclinicity in the vorticity equation can be interpreted as being a consequence of an effective scale decomposition performed by the curl operator $\nabla\times$, which is a high-pass filter.

This material is based upon work supported by the Department of Energy National Nuclear Security Administration under Award Number DE-NA0003856, the University of Rochester, and the New York State Energy Research and Development Authority. This research was funded by DOE FES grant number DE-SC0014318. H. Aluie was also supported by NASA grant 80NSSC18K0772 and DOE grant DE-SC0019329. Computing time was provided by the National Energy Research Scientific Computing Center (NERSC) under Contract No. DE-AC02-05CH11231.

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