Complex Ray Tracing and Cross-Beam Energy Transfer for Laser-Plasma Simulations

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Cross-beam energy transfer (CBET), which occurs when laser beams overlap in a plasma and drive ion-acoustic waves, may be responsible for a 50% decrease in hydrodynamic efficiency in OMEGA implosions.[1] A program was developed that models CBET for the simple case of two intersecting beams. The algorithms in this code followed a five-step process, which involved mapping ray trajectories to a grid, finding ray interactions, calculating gain coefficients for the interactions, and solving for final values through iteration.

A ray-tracing code was developed that propagates laser beams by representing them as bundles of rays and then evolving the rays in time according to the following set of differential equations:[2]

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = v_{\mathrm{g}}, \tag{1}$$

$$\frac{\mathrm{d}\mathbf{v}_{\mathrm{g}}}{\mathrm{d}t} = -\frac{c^2 \nabla n_{\mathrm{e}}}{2n_{\mathrm{c}}}, \tag{2}$$

where $\mathbf{x}$ is the position vector, t is time, and $\mathbf{v_g}$ is the group velocity vector, defined by

$$\mathbf{v_g} = \frac{\partial \omega}{\partial \mathbf{k}} = \frac{c^2 \mathbf{k}}{\omega}, \tag{3}$$

where ω is the angular frequency, $\mathbf{k}$ is the wave-number vector, and n_{c} is the critical density, defined by

$$n_{\mathrm{c}} = \frac{\omega^2 m_{\mathrm{e}} \varepsilon_0}{e_{\mathrm{c}}^2}. \tag{4}$$

These equations take into account the dispersion relation and the density profile of the background plasma. The energy deposited by the beams, as well as the beam intensities and electric fields, can be calculated and plotted onto a grid using the first-order linear interpolation method.

To map ray trajectories and find interactions, we store an array of each ray's coordinates along its trajectory. We also keep track of the gridlines it crosses and the cells through which it passes. Two rays are said to intersect if they are from different beams and they both pass through the same grid cell.

For each interaction, we calculate the gain coefficient from the following formula:[3]

$$\left(L_s^{ijkl}\right)^{-1} = \frac{e^2\left|E_{k0}\right|^2}{4m_\mathrm{e}c\omega_{ij}k_\mathrm{B}T_\mathrm{e}\left(1+3T_\mathrm{i}/ZT_\mathrm{e}\right)}\frac{n_\mathrm{e}}{n_\mathrm{c}}\frac{\omega_s}{\nu_\mathrm{ia}}P\left(\eta_{ijkl}\right), \tag{5}$$

$$P(\eta) = \frac{\left(\nu_\mathrm{ia}/\omega_s\right)^2\eta}{\left(\eta^2-1\right)^2+\left(\nu_\mathrm{ia}/\omega_s\right)^2\eta^2}, \tag{6}$$

$$\eta_{ijkl} = \frac{\omega_{kl}-\omega_{ij}-\left(\mathbf{k}_{kl}-\mathbf{k}_{ij}\right)\cdot\mathbf{u}}{\omega_s}, \tag{7}$$

where L_s^{ijkl} is the laser absorption length; e is the elementary charge; E_{k0} is the initial electric field of the pump ray; m_e is the electron mass; c is the speed of light; ω_{ij} and ω_{kl} are the frequencies of the seed beam and the pump beam, respectively; k_B is the Boltzmann constant; T_e is the electron temperature; T_i is the ion temperature; Z is the ionization state; n_e is the electron density; n_c is the critical density; ν_ia is the ion-acoustic wave energy damping rate; $\mathbf{k}_{ij}$ and $\mathbf{k}_{kl}$ are the seed and pump ray vectors, respectively; $\mathbf{u}$ is the plasma flow velocity; and ω_s is the acoustic frequency.

Once we determine the energy transfer for a single intersection, we must propagate the energy change to all downstream cells (see Fig. 1). After doing this for all possible ray intersections, we iterate the process, if necessary. This new program matches the results of Follett's CBET program,[3] which uses the same equations but different numerical algorithms; it also performs the calculations 10× faster.

Furthermore, a new ray-tracing method was investigated, namely complex ray tracing, which represents a laser beam with only five rays: a chief ray (a.k.a., base ray), two waist rays, and two divergence rays. The electric field or intensity at any point can be calculated by finding the distance between the point and three of the rays along a line perpendicular to the chief ray.[4]

We tried two different approaches to implement this technique: In the first approach (cell by cell), we traced all five rays at the beginning and then went cell by cell to calculate the intensity for each cell. While this method was very accurate, it was very slow when the number of cells became large. The second method (update outward along the chief ray) started by tracing the waist and divergence rays; then, while the chief ray was traced, we updated the intensities outward from the chief ray. This method worked quickly and accurately: it ran 4× faster than the standard ray-tracing method and produced a smoother plot.

With this program, we reproduced a basic version of Young's double-slit experiment, showing that complex ray tracing can model additional effects such as diffraction and interference. In the future, this work will be implemented into the 3-D hybrid fluid-kinetic code *CHIMERA*.

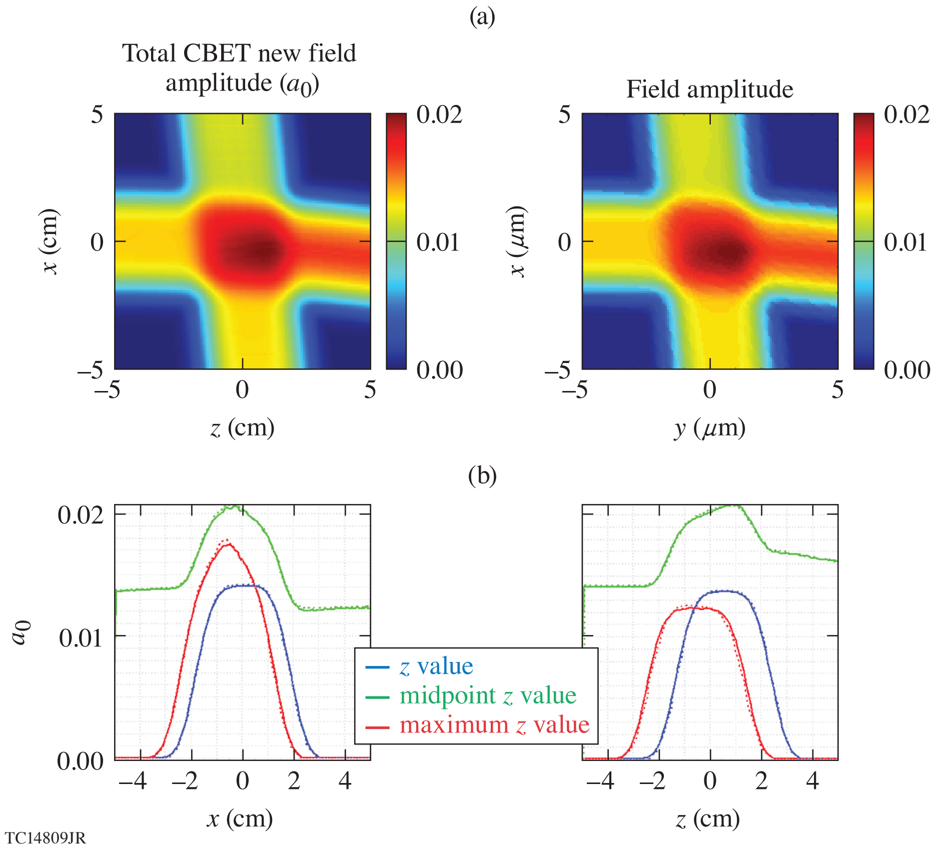

Figure 1
Comparison of Follett's CBET code[3] with the developed code. (a) Our results are on the left and Follett's results are on the right. Energy from the upward-traveling beam has been transferred to the rightward-traveling beam. (b) Our results are shown as solid lines and Follett's results are shown as dotted lines. The blue, green, and red lines show the electric-field profiles taken at the minimum z, midpoint z, and maximum z values, respectively.

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3. R. K. Follett *et al*., Phys. Rev. E **98**, 043202 (2018).

4. H. Yu *et al*., Appl. Opt. **55**, 6530 (2016).