Characterization of Cryogenic Deuterium-Tritium Target Motion

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Abstract

Knowledge of target motion characteristics is important for maximizing the efficiency of cryogenic target implosions in internal confinement fusion. Target motion data from two high-speed video cameras situated inside the target chamber of the OMEGA laser system was analyzed using MATLAB. It was shown that targets follow elliptical paths through space. A time-domain parametric elliptical model of low frequency target motion was created that can predict the location of targets at the time of a shot. Using this model, vibration characteristics including resonance frequencies, amplitudes, and phase angles were extracted. Damping was estimated using the exponential decay method with the amplitudes of these ellipses. It was found that cryogenic targets typically damp within four seconds, meaning that it is possible to optimize timing of the retraction of the thermal shroud such that target motion at the time of a laser shot is minimized.

1 Introduction

The Laboratory for Laser Energetics of the University of Rochester operates the OMEGA laser system and is one of several large laboratories experimenting with internal confinement fusion using this facility. On OMEGA, a cryogenic spherical capsule containing the deuterium and tritium isotopes of hydrogen is compressed and heated by sixty ultraviolet laser beams to ignite the following reaction and produce energy [1]:

$$ {}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + 17.59\ MeV. \qquad (1) $$

This capsule is a plastic shell 5 to 20 μm thick, with a layer of cryogenic deuterium and tritium under the surface approximately 100 μm thick [2].

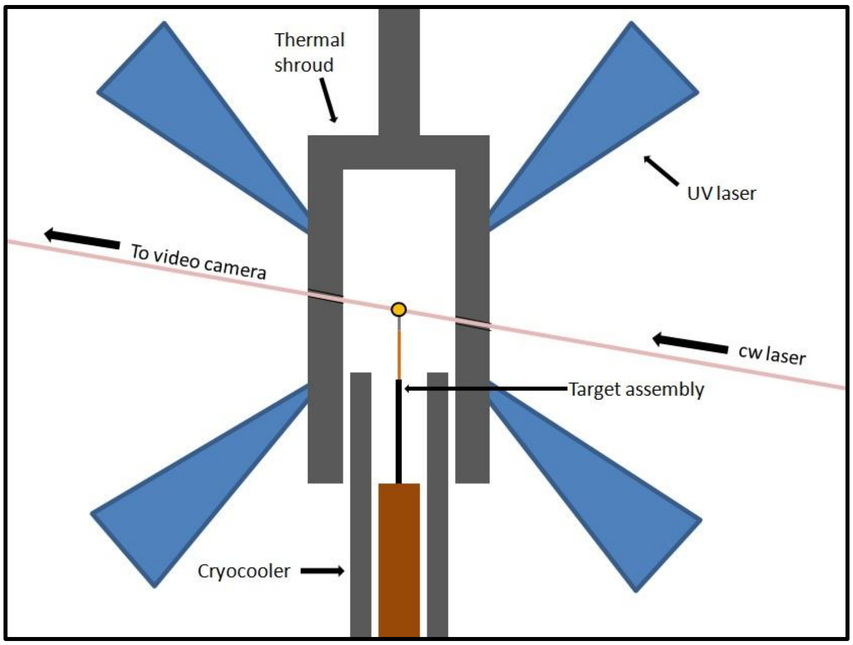

Fig. 1 The geometry of the OMEGA target chamber
For simplicity, four of the sixty ultraviolet lasers which irradiate a cryogenic target when fired on OMEGA are shown. Feedback from a cw laser which illuminates the target provides high-speed video cameras information about target position.

In order to compress the deuterium and tritium evenly and maximize the fusion energy produced, uniform distribution of the laser radiation on the surface of the capsule is necessary. This requires that the capsule be as close to target chamber center as possible, preferably less than or equal to 10 μm away. However, there are numerous sources of vibrational excitation present in the target chamber, as depicted in Fig. 1. Cryocoolers, necessary to keep the capsule below 20 K, create a steady state vibration observed to be around 4 Hz. The thermal shroud which insulates the target capsule from the surrounding environment transiently excites the target on all axes when it is retracted prior to the laser shot [2]. Since off-center targets will not be shocked and compressed evenly by a laser pulse, knowledge of target vibration characteristics is important for providing information on target location and motion behavior at the time of a laser shot. Target damping, the rate of decay of vibration amplitude, is particularly important since targets with high damping will be closer to the target chamber center at shot time.

In the OMEGA target chamber, two high speed video cameras monitor the position of targets constantly during the time period before a laser shot. The most important data is visible in the 50-70 milliseconds after the retraction of the thermal shroud but before a shutter blocks each camera to prevent damage from laser shot debris, since the target is damping from vibrational excitation during this period. This data is returned as graphs of target location vs. time on each axis as well as radial distance. Using MATLAB, programs were written to model the path of target motion and extract vibration characteristics for various types of targets.

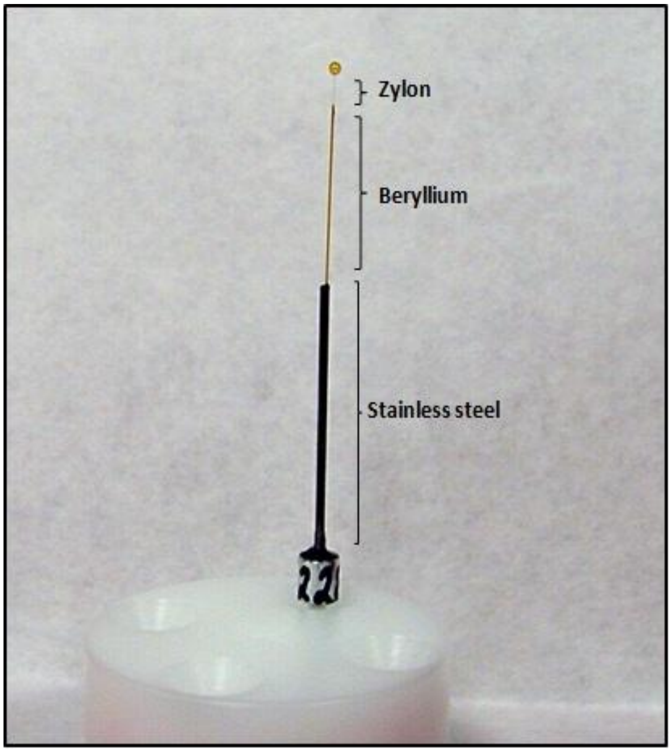

Fig. 2 Anatomy of a Type 1 mass equivalent fusion target
The highly flexible synthetic polymer Zylon permits excessive target vibration, reducing laser shot efficiency.

Fig. 2 displays a Type 1 mass equivalent fusion target assembly, one of several types used on OMEGA for laser fusion experiments. The base of the stalk is composed of stainless steel and beryllium, which are important for their strength and relative immunity to radiation. The tip of the stalk is made of Zylon, a highly flexible synthetic polymer. Though this flexibility guards against the danger of having the capsule snap off from the stalk, it allows a greater range of vibrational motion for the target. Type 1 targets have a fairly low natural frequency in comparison with some other target types, which have different metals and polymers in their stalks and are less mobile but more fragile.

For the purposes of analyzing target motion, some targets were tested without a laser shot, yielding data on target motion during free vibration in the OMEGA target chamber. The mass of the deuterium-tritium fuel significantly affects target motion since it changes the natural frequency of targets. Therefore, targets with a mass equivalent of plastic at 20 K were used for laboratory testing in order to exactly simulate the conditions of a cryogenic target.

There are several methods for measuring the vibration characteristics of targets in the laboratory. One, called the half-power bandwidth method, uses a shaker test to convert the response acceleration to an applied force on an empty or mass equivalent target into a frequency-domain function, called a frequency response function. This method allows calculation of resonance frequency, response amplitude, and a damping estimate. Another method, called the logarithmic decrement or exponential decay method, excites a target at its known resonance frequency and uses the resultant oscillating damping curve to calculate damping (ζ) via the formula:

$$\zeta = \frac{\delta^2}{\sqrt{4\pi^2+\delta^2}} \tag{2}$$

where δ, known as the logarithmic decrement, is described by:

$$\delta = \frac{1}{n}\ln\frac{x_0}{x_n} \tag{3}$$

where x_0 is the maximum amplitude and x_n is the amplitude of the nth peak from the maximum amplitude of the oscillating damping curve [3].

Previously, the vibration characteristics of cryogenic fusion targets had never been characterized. As a result of this work, a time-domain parametric model of target motion was created by processing high-speed video camera data with MATLAB, allowing target location at the time of a laser shot to be calculated. Target damping during the time shortly before a laser shot was estimated for the first time using the exponential decay method, and it was shown that targets may damp to steady state motion within one to three seconds after thermal shroud retraction. A new routine for thermal shroud retraction allowing for minimum target motion at the time of a laser shot has been implemented as a result of this characterization. Another potential solution to excessive target motion concerning alternative target choices has also been considered based on models of the target motion path.

2 Research Methods

The target-stalk system is a viscoelastically damped multiple degree of freedom oscillator. However, motion at the capsule is more easily and still accurately modeled as a single degree of freedom oscillating system on each axis [4]. This means that the target can be treated as any other oscillating system with a restoring force proportional to displacement and a damping force proportional to velocity. Such systems are typically modeled by the homogeneous second-order linear differential equation on each axis:

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + {\omega_0}^2 x = 0 \qquad (4)$$

where $x(t)$ is the position function on the respective axis of the capsule, γ is the damping constant for that axis, and ω_0 is the resonance frequency on that axis [2]. The solution to this equation gives both a real and complex term, which describes the amplitude, frequency, and phase angle of target oscillation.

By analyzing camera data of target motion, these parameters which characterize target vibration can be extracted. Position data from the two high-speed video cameras situated inside the OMEGA target chamber is sampled at 2000 Hz. The data is based in “OMEGA-space”, a complicated linear basis for 3-space which is appropriate for the geometry of the cameras and target chamber, since the camera views are not perfectly orthogonal. Therefore, the data from these cameras is passed through a change-of-basis algorithm to convert it into a measurement of the coordinates X, Y, and Z relative to the origin, which is the center of the target chamber. The radial distance from the origin of the target is also measured.

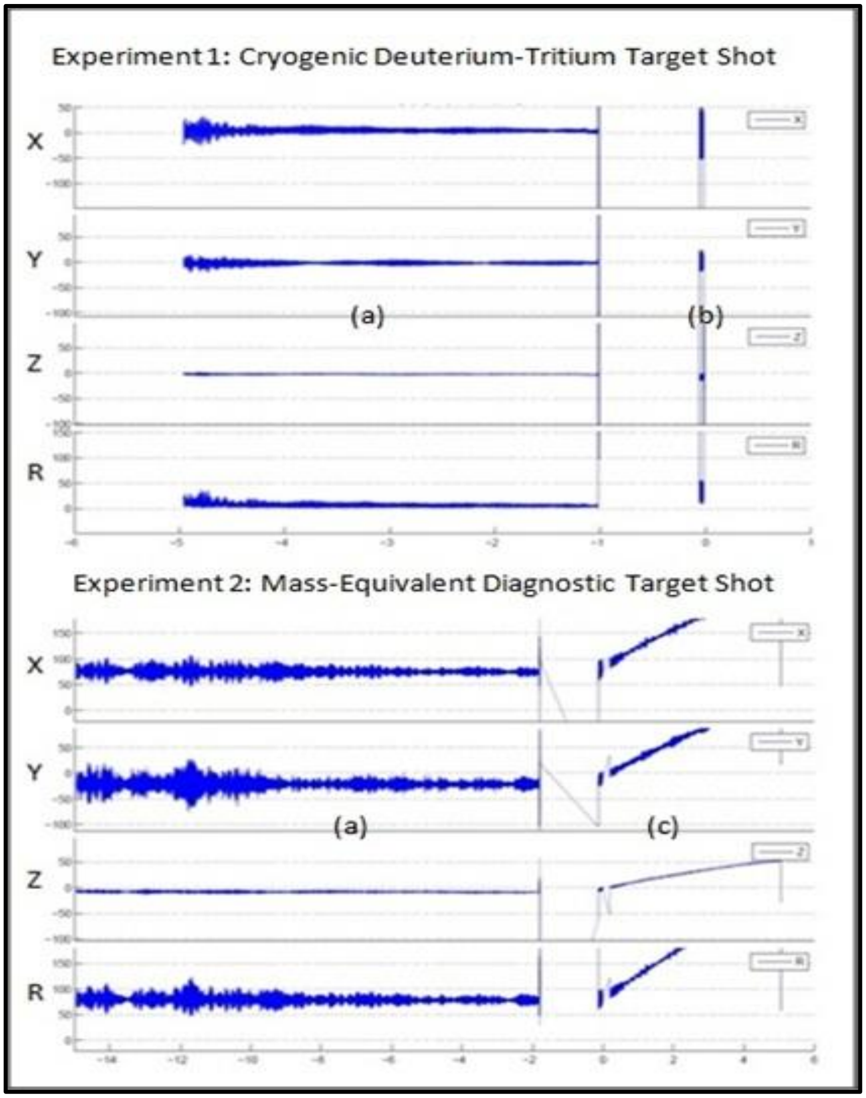

Fig. 3 Transient data taken by two high-speed video cameras situated inside the OMEGA laser system
Data from two different shots are displayed for all three standard axes as well as radius. In region (a), the target is experiencing steady state vibration caused by background machinery. In region (b), the thermal shroud has been lifted and the target is exposed to the video cameras. In region (c), the target has not been hit by a laser shot but instead its vibrations have been allowed to damp out.

Different target data is available depending on the general experiment that was performed at the time of the laser shot. Fig. 3 shows two different data printouts from different experiments. The data show oscillation on all axes due to the various excitation sources previously discussed. When a laser shot is desired, the only data available is the steady state vibration of the target as well as target motion in the 50-70 ms window after the thermal shroud has been retracted but before the shot. In Experiment 1, the thermal shroud is retracted between Fig. 3 regions (a) and (b), resulting in the significantly increased amplitude of oscillation in region (b) which has not been able to damp down to the steady state amplitude of region (a). A shutter blocks the cameras at the time of the shot, that is, at the end of Fig. 3 region (b). The brevity of this time window illustrates the necessity to characterize target damping. Occasionally, as in Experiment 2, diagnostic tests are being performed which do not require a laser shot. Data from these tests are useful because they feature broad enough time windows to provide information about target damping on all axes.

Interestingly, as seen in Fig. 3 region (c), the measured target position drifts away from the origin, which in the case of this specific target, CRYO-ME-2Q11-01-42, is likely the result of thermal expansion. This presents a problem because targets should obey an oscillating damping curve according to their equation of motion, permitting use of the exponential decay method to estimate damping, but thermal expansion distorts this curve. Additionally, the measured position of the target is displaced from the origin from the beginning of the time record. A program was written in MATLAB to break down data into 50 ms segments, and the means of these data segments were subtracted out to standardize and normalize the data. The top three graphs of Fig. 4 show the raw data from the two high-speed video cameras which has been adjusted in the bottom three graphs. The periodic nature of these curves may not be immediately apparent due to the high volume of data in each graph. Fig. 5 displays a close-up view of target motion on one axis to confirm that targets are indeed oscillating about the origin. Since the data is periodic on all axes, this indicates that target motion is characterized as revolution around the origin.

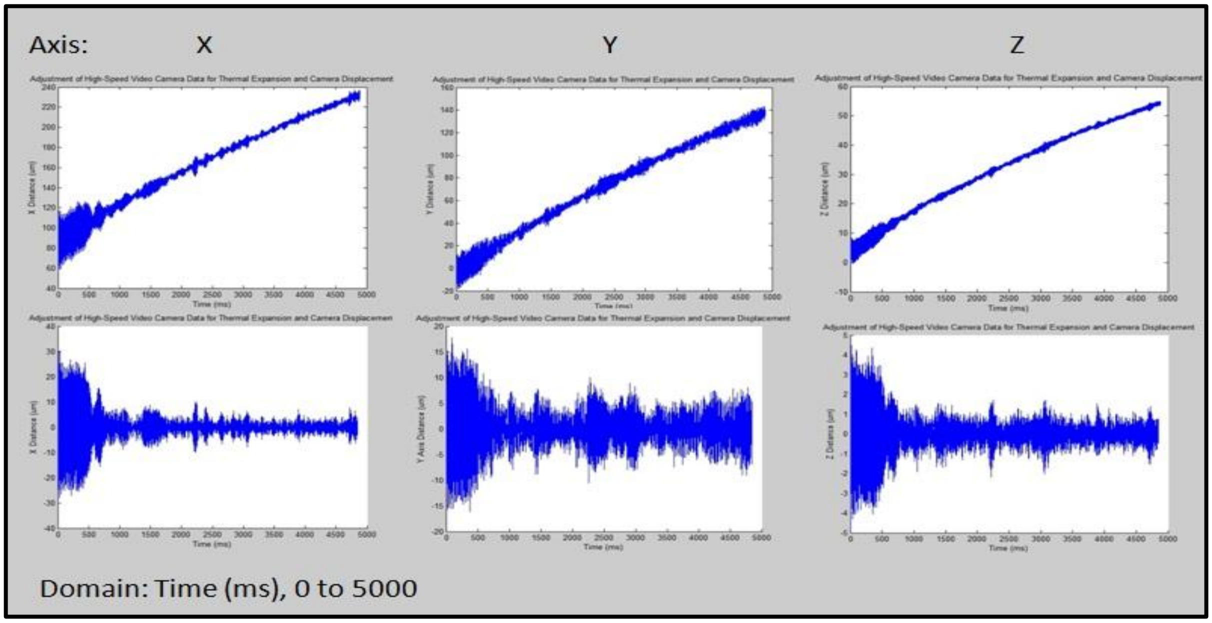

Fig. 4 Adjustment of high-speed video camera data to account for thermal expansion and camera displacement
By applying a MATLAB program, position data was adjusted from the raw data on the top three graphs to form the oscillating damping curves shown in the bottom three graphs. Position is measured in microns from the origin and is typically under 100 microns.

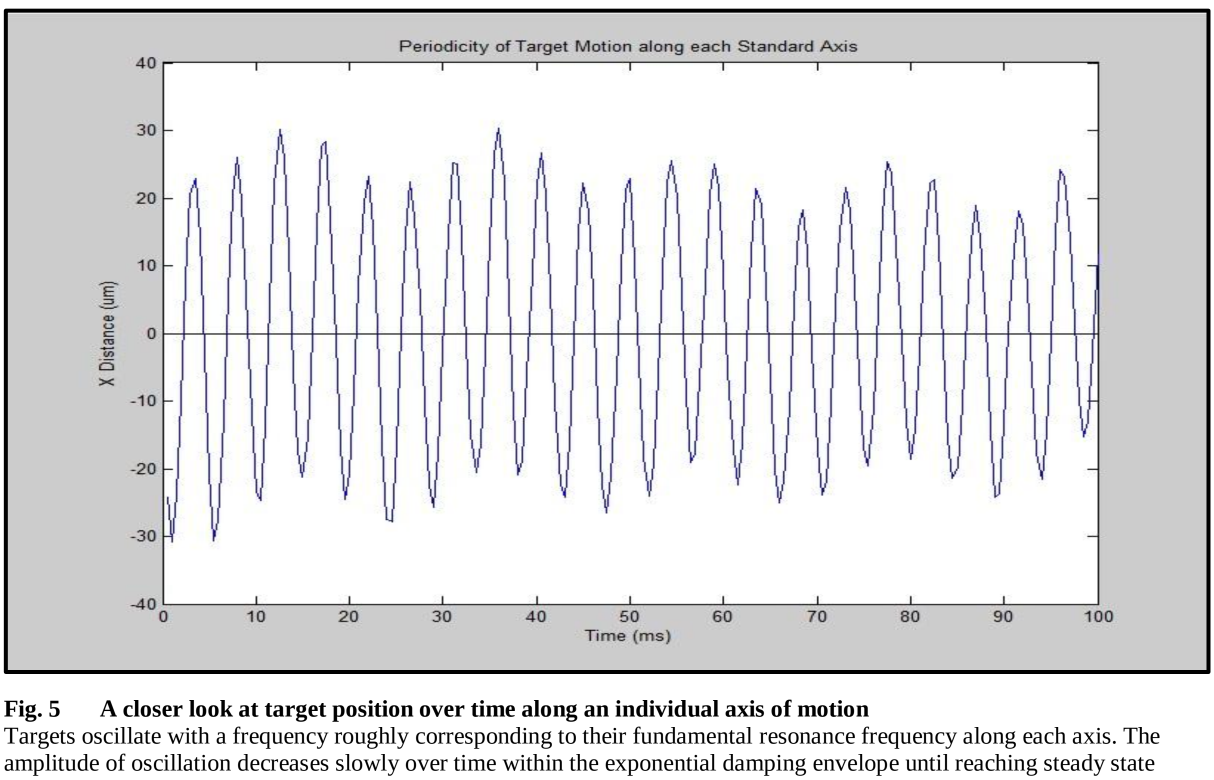

Fig. 5 A closer look at target position over time along an individual axis of motion
Targets oscillate with a frequency roughly corresponding to their fundamental resonance frequency along each axis. The amplitude of oscillation decreases slowly over time within the exponential damping envelope until reaching steady state vibration as shown in the bottom graphs of Fig. 4

3 Results and Analysis

3.1 3-dimensional transient cryogenic target motion modeling

As stated, position data from the two high-speed video cameras situated inside the OMEGA target chamber were imported into MATLAB for further analysis. Fast Fourier Transforms, a method of extracting frequency components from time-domain data, were performed to identify fundamental frequencies as shown in Fig. 6. Table 1 presents the results of this analysis for some targets which were used in laser shots and diagnostic tests.

Table 1 Analysis of frequency components of cryogenic fusion targets

Target Name	CRYO-2094-1830			CRYO-ME-2Q11-01-42		
Axis	X	Y	Z	X	Y	Z
Fundamental Resonance Frequency	156.3 Hz	148.4 Hz	437.5 Hz	215.2 Hz	215.6 Hz	215.7 Hz

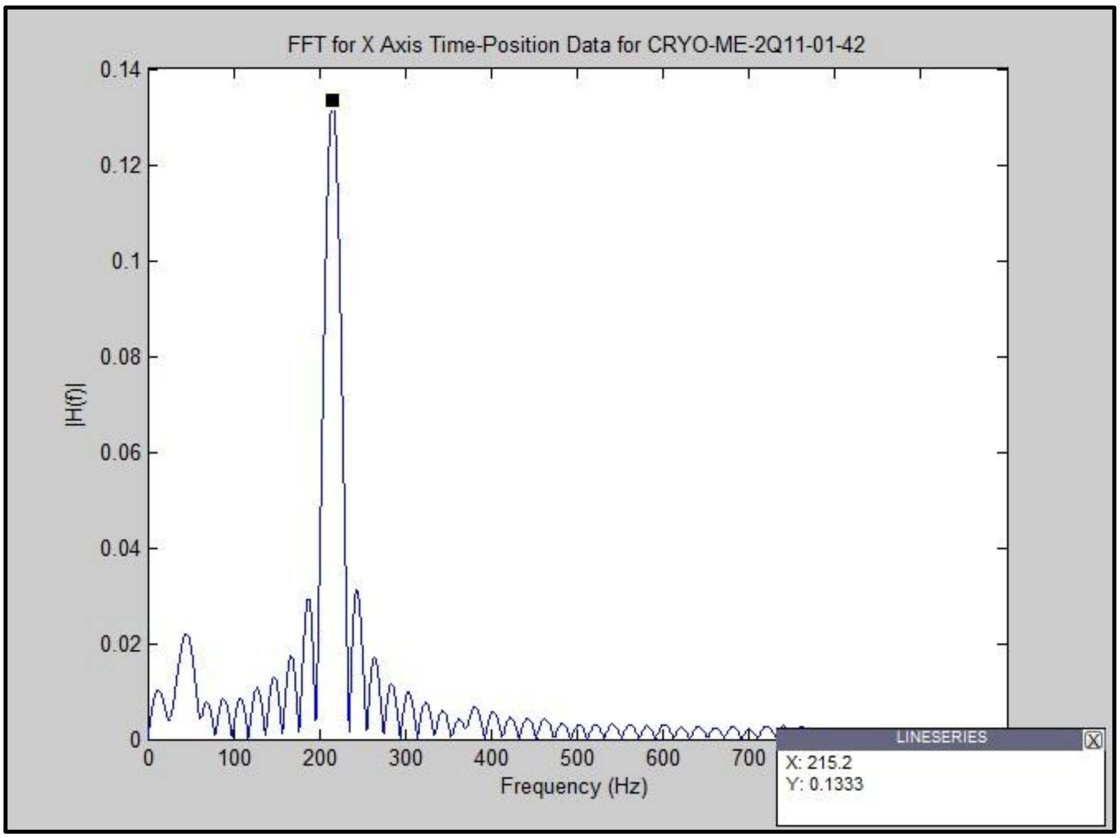

Fig. 6 Fast Fourier Transform of X axis position of CRYO-ME-2Q11-01-42
Position data is processed through the FFT algorithm to yield frequency components of target motion and thus fundamental resonance frequencies. The fundamental frequency of this target, CRYO-ME-2Q11-01-42, is seen to be approximately 215.2 Hz on the X axis.

The two targets in Table 1 are Type 1 and 1d targets, respectively. These targets have relatively low fundamental frequencies on their X and Y axes. Because the Z axis is oriented longitudinally along the stalk of the target, it is sometimes characterized by much higher frequencies, as in the case of CRYO-2094-1830. Because the video camera sampling frequency is 2000 Hz, the highest frequency level possible to measure, called the Nyquist frequency, is 1000 Hz. Some other types of targets such as Type 4 and Type 6 targets are made of different materials and have fundamental frequencies around 1000 Hz, so it is difficult to extract vibration characteristics for these as well.

Target oscillation on each axis is usually at approximately the same frequency, as shown in Table 1, but has different amplitudes and different phase angles. The target's path of motion as a result of these characteristics is that of an arbitrarily oriented ellipse in 3-dimensional space. Given that the standard ellipse is described by the parametric equations $x = a\cos t$, $y = b\sin t$, $z = 0$, one can use three rotation matrices to find the coordinates of any ellipse in 3-space:

If $a > b$:

$$\hat{x}''' = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} a\cos t \\ b\sin t \\ 0 \end{bmatrix} \quad (5)$$

If $a < b$:

$$\hat{x}''' = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} a\cos t \\ b\sin t \\ 0 \end{bmatrix} \quad (6)$$

where ϕ, θ, and ψ are the angles of rotation in space about the Z, Y, and X axes respectively, *a* and *b* are either the semimajor or semiminor axes, and *t* is time [5]. There are two possible equations as shown because the order in which the rotation matrices should be applied varies depending on the major axis of the ellipse.

With this in mind, a brute force algorithm was created to solve for the three spatial angles and semimajor/minor axes of the least-squares ellipse. However, such an algorithm was computationally intensive, slow to run, and of questionable precision. Therefore, a genetic algorithm called *TransformerEvolution* was created to solve for the same five ellipse parameters by evolving toward the minimum σ, where:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\sqrt{(x_i - x_2)^2 + (y_i - y_2)^2 + (z_i - z_2)^2}}{N}} \qquad (7)$$

where *N* is the number of position data points, (x_i, y_i, z_i) are the coordinates of a data point, and (x_2, y_2, z_2) are the coordinates of the closest point on an ellipse along the geodesic, that is, the shortest possible path, to the elliptical model. Essentially, this solves for the true least-squares ellipse by varying the parameters of potential elliptical models until the average distance from all data points on the model is minimized. This algorithm, the results of which are displayed in Fig. 7, both precisely and efficiently produces one of the ellipse equations *(5)* or *(6)* as a model of cryogenic deuterium-tritium target motion. These two similar equations are necessary because alignment along the correct axis is a required starting condition for this genetic algorithm to run

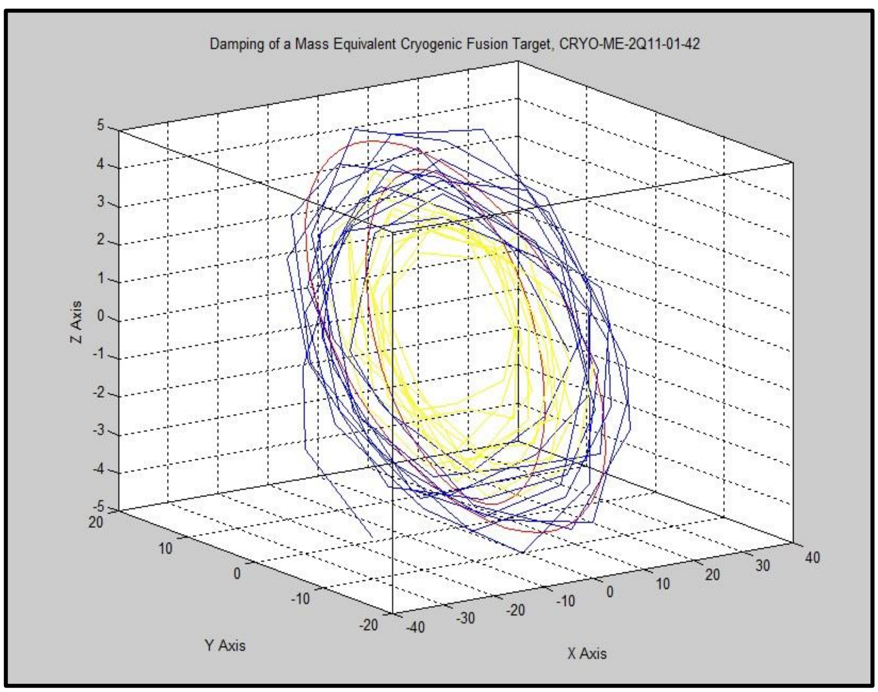

Fig. 7 3-dimensional visualization of the damping of a cryogenic target motion path
This path of motion of this target, CRYO-ME-2Q11-01-42, diminishes in size while rotating through space. Two elliptical models calculated at different points in time, depicted here in red, are accurate representations of target motion path, overlaid on the data shown in blue and yellow. Note that the magnitude of the scale on each axis varies.

accurately. This model is extremely useful because it is in the time domain, so one can extrapolate an accurate prediction of target location in the time regions where the high-speed video cameras in OMEGA are unable to record data simply by increasing or decreasing the value of t as given by the ellipse equations. As t increases, calculating new best-fit elliptical models shows that the values of a and b decrease while the values of ϕ,ψ, and θ also change. This describes the path's rotation on all coordinate axes as the size of the target motion path decreases, due to the variation of damping along each axis of the target.

When a mass equivalent target is used in an experiment without a laser shot, there is enough data to calculate a reliable damping estimate with the exponential decay method. The average radii of elliptical models for the target path were treated as peaks on a damping curve, as displayed in Fig. 8. The average radii of the elliptical models were used in place of the initial data for target radius because the models discretize the data and also eliminate the mathematical difficulties involved with oscillating damping curves. This model shows that it takes between one and three seconds for a typical target to damp to steady state

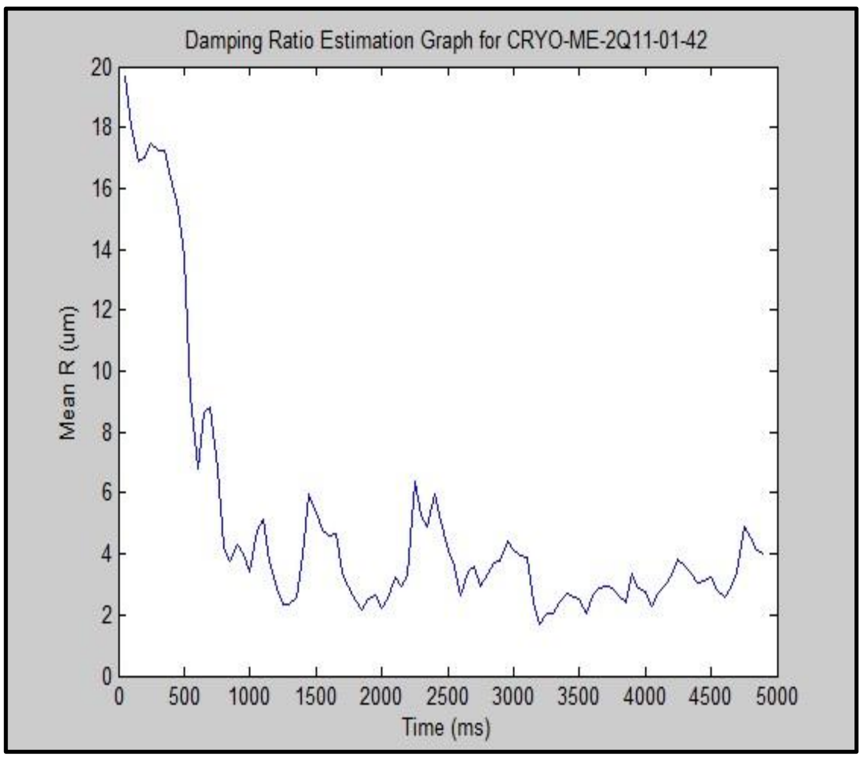

Fig. 8 Radial damping of cryogenic fusion target CRYO-ME-2Q11-01-42
This target damps to steady state motion within one to three seconds after excitation by the thermal shroud. Using the logarithmic decrement method with two points on this damping curve, a damping estimate of 0.38% was produced, which also implies a time for 1/e decay.

vibrational motion (which has a mean radius of approximately 3.3187 μm for CRYO-ME-2Q11-01-42). This is critically important in its potential for minimizing target vibration at the time of a laser shot. Designing a new routine for retraction of the thermal shroud which takes place over several seconds instead of approximately one second before the laser shot should, based on this work, be a successful way to address excessive target motion.

This data also lends further insight to the damping characteristics of targets under cryogenic conditions. It was found that the damping estimate actually decreases under the complex effects of the decreased temperature. This was demonstrated with the Type 1d target CRYO-ME-2Q11-01-42 with fundamental resonance frequency of approximately 207.5 Hz, where the damping estimate decreased from 0.90% of critical damping (half-power bandwidth method from a shaker test at room temperature) to 0.38% of critical damping (exponential decay method with target at 19 K). The half-power bandwidth method and exponential decay method have previously been found to give approximately equal damping estimates at room temperature [2].

3.2 Optimizing target design as a solution to excessive target motion

There is an alternative possibility for addressing excessive target motion, which considers targets that exhibit decreased amplitude of oscillation under excitation. Low frequency targets

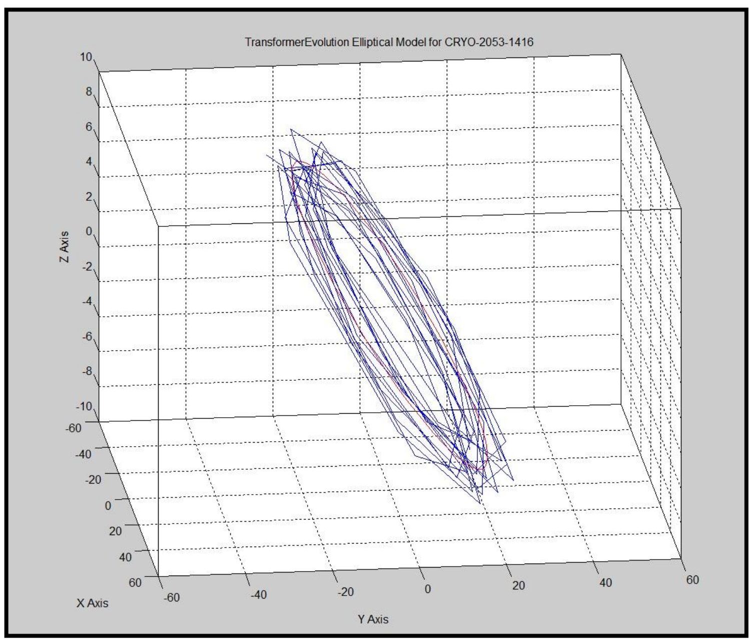

Fig. 9 Parametric elliptical model of the motion path of a low fundamental frequency target, CRYO 2053-1416
The red ellipse is the target motion path model created by the evolutionary algorithm *TransformerEvolution*, overlaid on the data plot (in blue). Low frequency targets such as this often exhibit a high radius of motion, around or greater than 30 μm.

with fundamental resonance frequencies around or below 200 Hz tend to feature regular elliptical paths which can be easily modeled as described above and shown in Fig. 9. However, both initial camera data and elliptical models clearly show a high average radius of motion for these targets which is detrimental to the efficiency of a laser shot.

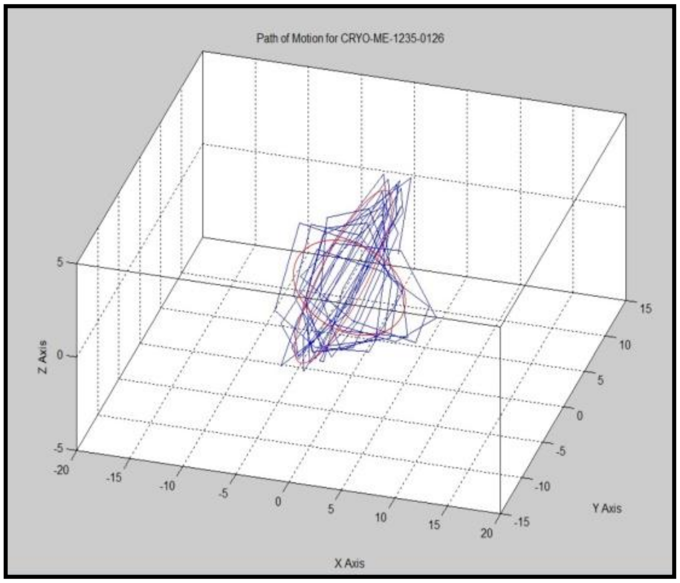

Fig. 10 Degeneration of distinct elliptical motion path around a threshold frequency of 300 Hz
Two separate elliptical motion paths are visible in this data as the phase angles of the motion components change about halfway into the time record. Average radius also significantly decreases.

When a target's fundamental resonance frequency is increased, the characteristics of a target's path of motion change drastically. Fig. 10 illustrates the path of motion of a Type 1c target, CRYO-ME-1235-0126, with fundamental resonance frequency of approximately 302.5 Hz. When the fundamental frequency of a target approaches 300 Hz, there is a distinct breakdown in the regular elliptical motion path into two ellipses. Since some targets change path very suddenly and may even increase their radius of motion, it is concluded that this effect is due to the higher fundamental frequencies of these targets. Higher frequency targets, above 400 Hz, do not exhibit a visible elliptical path at all. As shown in Fig. 11, a high

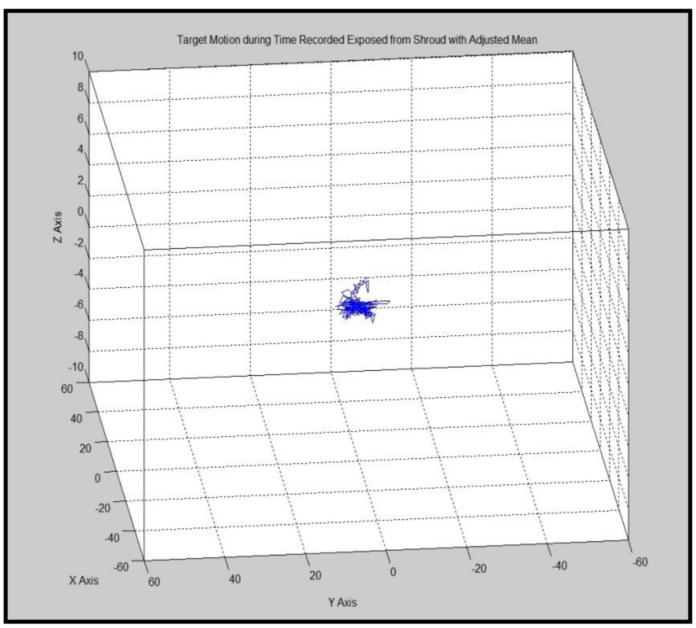

Fig. 11 Plot of motion path of a high fundamental frequency target
The target motion path exhibits a drastically decreased average radius of motion. Though the motion path appears noisy, tracing the curve shows the presence of periodic components.

fundamental frequency target's path of motion is extremely small as well as noisy. However, a program written in MATLAB to trace and highlight data points revealed significant periodicity to the target paths. The degeneration of the elliptical motion path around 300 Hz implies that these paths are unlikely to be elliptical. However, the path size approaches the sensitivity of the high speed camera data, limiting analysis. Nevertheless, this presents a second solution to excessive target motion. Using targets with high fundamental frequencies can minimize a target's motion simply because these targets do not exhibit high amplitudes of oscillation under excitation by the sources of vibration in the OMEGA target chamber. The average radius for these targets was calculated as an arithmetic mean of data, since 50-75 ms is not enough time for any significant damping to occur. A comparison of average radii for Type 1 (low fundamental frequency) and Type 4 and 6 (high fundamental frequency) targets measured is given below in Table 2 as an exact quantification of the potential of high frequency targets.

Table 2. Comparison of average radii for high and low fundamental frequency targets

	Average Radius	σ (Standard Deviation)
Type 1	33.30 μm	8.97
Type 4/6	3.98 μm	0.70

However, high fundamental frequency targets tend to be more fragile and are often lost during the process of filling the target with deuterium-tritium fuel and transporting it to the OMEGA target chamber for a laser shot.

4 Conclusions

Data from the OMEGA target chamber were used to analyze cryogenic target paths of motion and vibration characteristics. A time-domain parametric elliptical model was created based on this data to further characterize target behavior under excitation. Using this model, the damping ratio of cryogenic fusion targets at shot time was measured for the first time. It was shown that a cryogenic mass equivalent target can damp under free vibration in one to three seconds. This model also has the potential to provide information about target position at the time of a laser shot. A new routine has recently been implemented on OMEGA which retracts the thermal shroud approximately four seconds before a laser shot, which has been numerically justified by this work. It is expected that the additional time after retraction will permit targets to damp fully and increase shot efficiency.

This data is also important in terms of helping to optimize target selection. Though targets with low fundamental resonance frequencies tend to follow a regular elliptical path, they have a greatly increased average radius from the origin. Targets with high fundamental resonance frequencies have decreased average radii, which suggests that these targets are superior in internal confinement fusion experiments because they permit greater uniformity of laser irradiation. The fragility of these targets makes them less desirable for practical laboratory use, however. Future work will build upon these models to design a target which is strong, stable, and less prone to vibrational motion. For future designs, the exponential decay method can be used in conjunction with the elliptical models described in this paper to calculate the necessary delay between thermal shroud retraction and time of the laser shot, with the ultimate goal of improving the efficiency and thus usable energy yield of internal confinement fusion.

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