Including Emissivity in the Analysis of Implosion Radiographs

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Abstract

In order to achieve ignition conditions in inertial confinement fusion (ICF) experiments, it is necessary for the imploding target to reach very high densities. To measure the properties of the target while it implodes, X-ray radiography is employed. The target is backlit, and the resulting radiograph is recorded using a pinhole camera. The optical thickness profile of the target is then inferred from the intensity profile recorded in the radiograph. In the simple analysis for obtaining the radial opacity profile of the target from the optical thickness profile in the radiograph, the target is treated as an absorbing spherical shell, with no emissivity. However, it may be necessary in some cases to be able to account for a significant amount of self-emission from the target. By including both an emitting center and an emitting outer shell in a simple model of an imploded target, we show that visible self-emission at the outer edge of a radiograph indicates a corresponding emission contribution to the apparent absorption seen in the inner part of the radiograph and that the apparent and actual absorption contribution can differ significantly. The relationship between this difference and the visible self-emission may provide the means for correcting for self-emission in obtaining radial density distributions from radiograph analyses.

Introduction

The Laboratory for Laser Energetics at the University of Rochester is pioneering nuclear fusion by conducting laser driven, direct-drive inertial confinement fusion experiments. In order to achieve ignition, or a state in which fusion reactions are self-sustaining, it is necessary for the plastic coated deuterium-tritium (DT) targets to reach densities of several hundred grams per cubic centimeter and temperatures of approximately 100 million degrees Celsius[1]. Given that high temperatures and densities must be reached to achieve fusion conditions, it is desirable to know the density of the DT target during the implosion.

Using Radiography to Determine Density

In free-free absorption of x-rays, the opacity, κ_{ff} of an object is directly related to its density ρ by the equation

$$\kappa_{ff} = \frac{k\rho^2}{(k_B T)^{1/2}(h\nu)^3}\,, \tag{1}$$

where k is a constant, T is the temperature, k_B is Boltzmann's constant and hν is the photon energy. The temperature may be modeled with a simulator, and the photon energy is a known property of the backlighter, but the value of opacity must be measured to be able to solve for the density.

If the opacity is constant, it is related to the optical thickness, τ, of the object along a backlighter ray path by the equation

$$\tau = \kappa L \tag{2}$$

where L is the length of propagation of the backlighter ray. By employing radiography and observing the attenuation of the backlighter intensity, the optical thickness is calculated using

$$\tau = \ln\left(\frac{I}{I_0}\right), \tag{3}$$

where I is the final intensity observed in the radiograph and I_0 is the initial backlighter intensity, in arbitrary units.

Because of self-emission from the high temperature target, the final intensity may include not only the backlighter radiation, but a contribution from the target as well. Thus, the apparent optical thickness, τ_{app}, and the actual optical thickness, τ_{act}, are unequal and must be related to each other so that τ_{act} can be inferred from measurements of τ_{app}.

Radiative Transfer and Optical Thickness

The solution to the equation of radiative transfer for constant emissivity and opacity, given by

$$I = \frac{\varepsilon}{4\pi\kappa} + \left(I_0 - \frac{\varepsilon}{4\pi\kappa} \right) e^{-\tau}, \tag{4}$$

is used to model the change in intensity as a beam travels through an absorbing and emitting object, where I_0 is initial intensity, I is final intensity, ε is the emissivity, κ is the opacity, and τ is the optical thickness.[2] When an object either emits or absorbs, simplified forms may be used. For an object of thickness L with emissivity only,

$$I = I_0 + \frac{\varepsilon L}{4\pi}, \tag{5}$$

and, with opacity only,

$$I = I_0 e^{-\kappa L} \tag{6}$$

The target we modeled consisted of three distinct shells, which either emitted or absorbed exclusively, as shown in Fig. 1. We calculated the corrections for obtaining the actual optical thickness profile due to the opacity alone from the apparent optical thickness one would infer from the radiograph by assuming that the net effect of opacity and emissivity was entirely due to opacity. To accomplish this, four distinct possibilities for the path of the backlighter emissions must be studied, as is shown in Fig. 2.

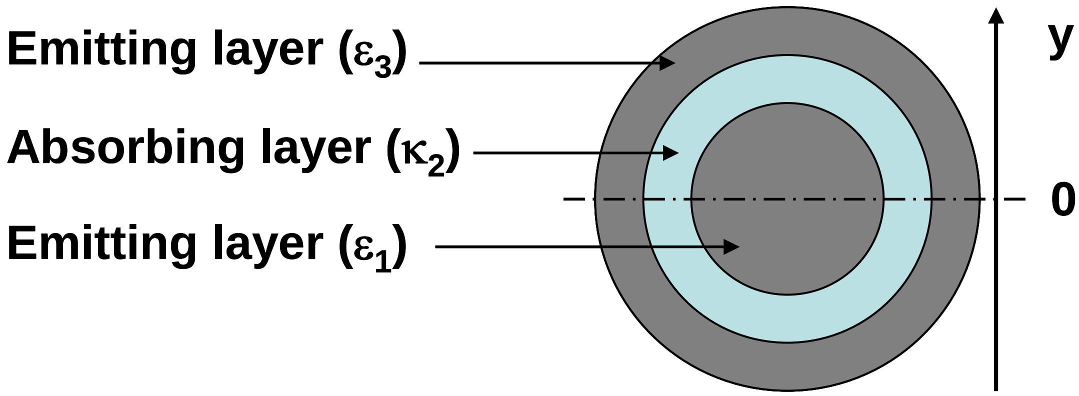

Fig. 1 The target is modeled with an emitting layer surrounding an absorbing layer, all surrounding an emitting core.

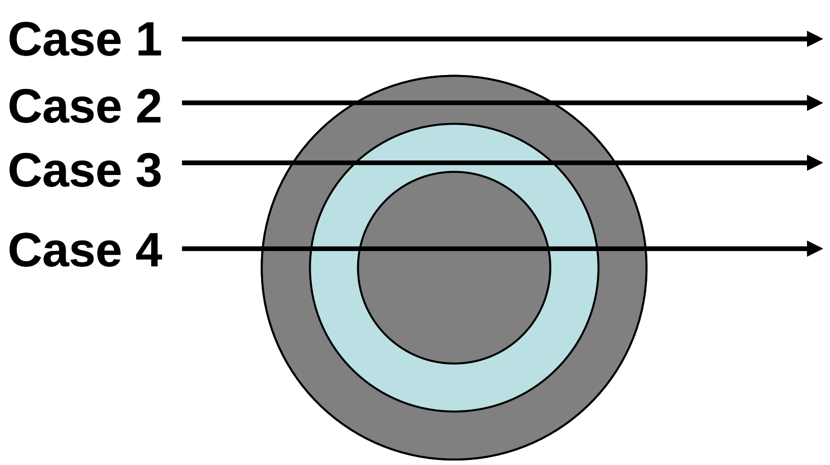

Fig. 2 Four possibilities for the path of backlighter emissions

In each case, the final intensity is modeled based on the emissivity and opacity of each layer of the target, the succession of path segments along the propagation path as it passes through each layer of the target, and the initial intensity of the backlighter. The four distinct cases are Case 1,

$$I = I_0 , \tag{7}$$

Case2,

$$I = I_0 + \frac{\varepsilon_3 L_3(y)}{4\pi} \;, \tag{8}$$

Case 3,

$$I = \left(I_0 + \frac{\varepsilon_3 L_3(y)}{4\pi} \right) e^{-\kappa_2 L_2(y)} + \frac{\varepsilon_3 L_3(y)}{4\pi} , \tag{9}$$

and Case 4,

$$I = \left[\left(I_0 + \frac{\varepsilon_3 L_3(y)}{4\pi} \right) e^{-\kappa_2 L_2(y)} + \frac{\varepsilon_1 L_1(y)}{4\pi} \right] e^{-\kappa_2 L_2(y)} + \frac{\varepsilon_3 L_3(y)}{4\pi} . \tag{10}$$

By dividing both sides of the equations by the initial intensity, the equations become expressions for $e^{-\tau_{app}}$. For Case 1, we have

$$e^{-\tau_{app}} = 1, \tag{11}$$

for Case2,

$$e^{-\tau_{app}} = 1 + \frac{\varepsilon_3 L_3(y)}{4\pi I_0} , \tag{12}$$

and for Case 3,

$$e^{-\tau_{app}} = \left(1 + \frac{\varepsilon_3 L_3(y)}{4\pi I_0} \right) e^{-\tau_{act}} + \frac{\varepsilon_3 L_3(y)}{4\pi I_0} , \tag{13}$$

where

$$\tau_{act} = \kappa_2 L_2(y) . \tag{14}$$

For Case 4, the ray propagates through the absorbing layer twice, so, for this case alone,

$$\tau_{act} = 2\kappa_2 L_2(y) , \tag{15}$$

and

$$e^{-\tau_{app}} = \left[\left(1+\frac{\varepsilon_3 L_3(y)}{4\pi I_0}\right)e^{-\tau_{act}/2} + \frac{\varepsilon_1 L_1(y)}{4\pi I_0}\right]e^{-\tau_{act}/2} + \frac{\varepsilon_3 L_3(y)}{4\pi I_0}. \tag{16}$$

The actual optical thickness is then solved for algebraically in Cases 3 and 4, as those are the only cases impacted by absorption. For Case 3, we have

$$e^{-\tau_{act}} = \frac{e^{-\tau_{app}} - \dfrac{\varepsilon_3 L_3(y)}{4\pi I_0}}{1+\dfrac{\varepsilon_3 L_3(y)}{4\pi I_0}}, \tag{17}$$

and for Case 4,

$$e^{-\tau_{act}/2} = \frac{-\dfrac{\varepsilon_1 L_1(y)}{8\pi I_0} + \sqrt{\left(e^{-\tau_{app}} - \dfrac{\varepsilon_3 L_3(y)}{4\pi I_0}\right)\left(1+\dfrac{\varepsilon_3 L_3(y)}{4\pi I_0}\right) + \left(\dfrac{\varepsilon_1 L_1(y)}{4\pi I_0}\right)^2 / 4}}{1+\dfrac{\varepsilon_3 L_3(y)}{4\pi I_0}}. \tag{18}$$

Equation (18) gives the correct result in the limit of zero emissivity,

$$\lim_{\varepsilon_1,\varepsilon_3 \to 0} e^{-\tau_{act}} = e^{-\tau_{app}}. \tag{19}$$

The chosen problem was to consider the effect of emissivity ε_3 on the Case 3 result. We assume that we are given a radiograph and that we wish to infer the opacity κ_2 from the Case 3 portion of the radiograph. If an emissivity ε_3 is present, then it will be visible in the Case 2 portion of the radiograph, according to Eq. (12). If ε_3 can be inferred using Eq. (12), then ε_3 can be used in Eq. (17) for Case 3 to obtain τ_{act} from τ_{app}. The opacity κ_2 is then obtained from τ_{act} using Eq. (14). Although it is clear in principle that a procedure such as this will succeed, we have not yet worked out the details of how this would work in practice. However, we have modeled the effect of ε_3 on τ_{act} using Eqs. (11) through (18). To demonstrate by example the effects of emissivity on a radiograph, pairs of simulations were run, one in which there was no self emission and one in which there was significant self emission along the backlighter path in Layer 3. One such result is shown in (Fig. 3).

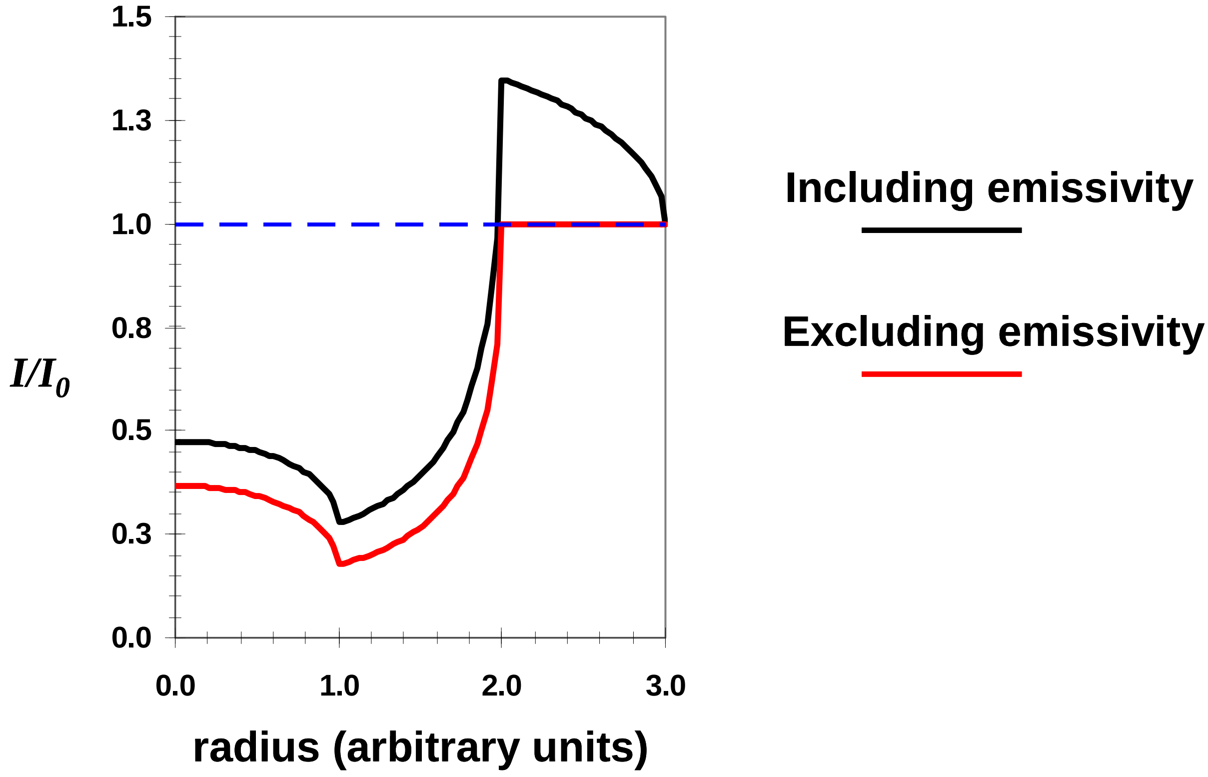

Fig. 3 Radial optical thickness profiles with and without including a strong, thick emitting outer layer.

Figure 3 shows the radial transmission profile of a radiograph of an absorbing shell with an inner radius of 1 and a thickness of 1, in arbitrary units. The red curve shows the radiograph that would be obtained with a shell with a central optical thickness of 1. The black curve is the radiograph of the same absorbing shell with a very thick outer emitting layer with the same thickness as the absorbing layer. The emissivity is an arbitrary value chosen so that its maximum intensity on the radiograph, at a radius in the image plane corresponding to the inner radius of the emitting layer, is nearly equal in magnitude to the intensity lost at the center of the radiograph due to absorption. The choice of a central optical thickness of 1 is a good intermediate value for radiography, since it is a robust signal representing a loss of most of the backlighter intensity, and, at the same time, the minimum intensity recorded in the radiograph is far enough above zero transmission so that the emission signal will not easily overwhelm the absorption signal. For the radiograph to be interpreted as an absorption shadow, the intensity should, as much as possible, be the result of the absorption of the backlighter intensity, without a large modification due to self-emission. If the transmission of the absorbing layer is very small, then small contributions due to self emission can make a relatively large contribution to the intensity. In this example, the apparent increase in the transmission due to the emitting layer is about 23%, which is quite modest, considering the strength and size of the emitting layer. Examples such as this can serve as benchmarks for accepting or rejecting radiographs to be analyzed in terms of absorption alone. Figure 4 shows two examples where the reduced size and strength of the emitting layer results in smaller corrections.

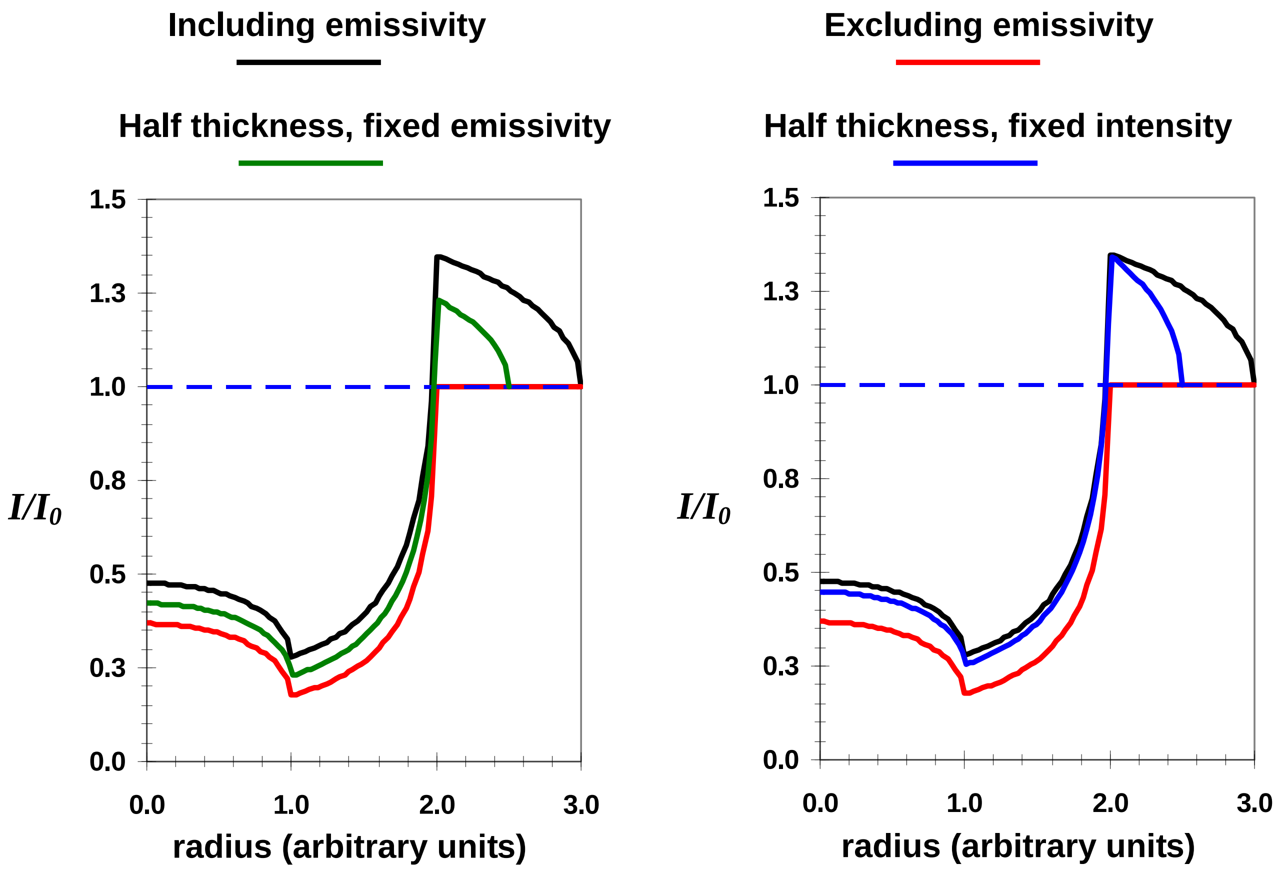

Fig. 4 Radial optical thickness profiles showing the change in the apparent optical thickness due to the effects of an emitting layer. The green curve, when compared with the black curve, show a reduced effect when the emitting layer is reduced to half the thickness of the emitting layer producing the black curve. The red and black curves are identical to those in Fig. 3. The blue curve shows that reducing the thickness of the emitting layer reduces the effect on the apparent transmission, even if the apparent intensity of the emission is kept constant.

A thinner emitting layer can result in a radiograph with a smaller self-emission modification, one that may be ignored, possibly, depending on the desired degree of precision. In Fig. 4, the green curve shows that the emission effect in the radiograph is reduced nearly in proportion to the reduction in the thickness of the emitting layer, keeping the emissivity at the same value as used for the result shown in Fig. 3, represented here again by the black curve. In this case, the volume of the emitting layer has been reduced to only slightly larger than the volume of the absorbing layer, which is still larger than what would seem reasonable in a well-designed radiography experiment. Nevertheless, the resulting 10% emissivity effect in the radiograph can be considered modest, perhaps even negligible by some standards of precision. The blue curve is a similar example of a reduced self-emission effect with a thinner emitting layer, but the emissivity in this example has been increased to keep the intensity of the emission ring in the radiograph constant. These results illustrate how the apparent thickness and intensity of the Case 2 portion of the radiograph together indicate the effect of self-emission on the apparent Case 3 transmission.

Conclusions and Future Work

Based on the examples shown above, it is clear that a correction may be needed to infer the opacity profile of an object from its radiograph when self-emission is seen in the radiograph, although there may be cases where the correction can be neglected, to an acceptable degree of approximation. Experiments involving radiography should be designed to minimize the effects of self-emission, and examples such as those shown here can be a useful guide.

If this work were to be continued, one could consider the application of the Abel transformation to infer opacity profiles from radiographs.[3] Considering the equation of radiative transfer as a means to move from the opacity distribution of the target itself to the shadow pattern it creates, the Abel transform can be thought of as a means to infer the opacity profile of a target based on the shadow created by it. Just as a term is included in the solution to the equation of transfer, Eq. (4), to account for self-emission, it would be useful if a simple correction to the Abel transform could be found to account for self-emissions from the target.

References

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