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# Alexander N. Tait

Allendale Columbia School

Rochester, New York

Advisor: Wade Bittle

# **Laboratory for Laser Energetics**

University of Rochester

Rochester, New York

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# Abstract

An electro-magnetic pulse (EMP) is produced when a high intensity laser strikes an inertial confinement fusion target and free electrons are displaced.<sup>1</sup> Electric and magnetic fields produced can disrupt or damage nearby electronic diagnostics and controls.<sup>2</sup> Electric (D dot) and magnetic (B dot) field probes are used to measure the EMP. This project developed a MATLAB<sup>™</sup> routine and a graphical user interface to analyze data from the field probes. The routine converts time domain, raw-voltage data into the frequency domain and then to absolute magnetic and electric field strength. Plots of field strength versus frequency are valuable in assessing the EMP mitigation effects of shielding and filtering since their effectiveness is frequency dependent. The project also entailed calculating important EMP metrics as well as verifying theoretical predictions of EMP behavior. Predictions were verified relating to laser beam tubes connected to the target chamber acting as cylindrical wave guides and the electromagnetic resonance of the spherical OMEGA target chamber. Several experiments were conducted to measure the propagation cut-off frequency of the beam tubes, to test the responsivity of the field probes, and to verify calculations within the MATLAB routine.

## 1. Introduction

The Laboratory for Laser Energetics (LLE) is a facility devoted to the study of the interaction of intense radiation with matter utilizing high powered laser systems. Currently, the primary experimental system at LLE is the OMEGA laser. OMEGA consists of 60 beams distributed spherically around a target chamber capable of delivering 30 kilo-Joules to a fusion target. The four-beam OMEGA EP (enhanced performance) laser system, which will come online in 2008, will support ultra short, ultra high intensity laser experiments for a variety of

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millimeter-scale targets. The Multi TeraWatt Laser (MTW) is a moderately-sized laser system employing a single beam. The MTW laser is used primarily to study laser pulses similar to those that will be generated by the OMEGA EP.

When high-powered lasers strike the experimental targets on these laser systems, electrons are displaced. These electrons generate an electro-magnetic pulse (EMP), a powerful burst of broadband radio waves. It is vital to study and understand EMP because it can induce currents within conductors and electronic circuits. Induced current can interfere with, disrupt, or damage electronic diagnostics and controls. Interference causes noise which can reduce the accuracy and reliability of electric signals both for control and measurement purposes. Disruptions cause temporary functionality failures, which may render the affected equipment useless for at least the duration of the pulse. Damage can occur when the EMP power coupled into an electronic component exceeds its maximum rating. It is possible to protect equipment against EMP by electro-magnetic shielding and filtering. Completely enclosing equipment in metallic structures provides shielding against all electro-magnetic waves; however, EMP can still couple to external wires and be conducted into this housing. One way to attenuate EMP-induced electrical signals while leaving desired signals unaffected is by use of frequency selective electronic filters. Electronic filters present different impedances for different frequencies. If the spectral attributes of desired and interference signals are known, filters can effectively isolate and pass the desired signal. The purpose of this project was to develop a tool to analyze spectral aspects of the EMP event.

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## 2. EMP Data Collection and Reduction

EMP is measured by electric or magnetic field antenna's called field probes. These field probes, which can be mounted wherever a field measurement is desired, produce a voltage output directly proportional to the change in the field strength.<sup>3</sup> This voltage is applied to an oscilloscope which digitizes the signal and stores the data. The raw data from the oscilloscope is of voltage versus time. Two file formats are utilized to store the raw data. The comma separated variable (CSV) format is a spreadsheet format. It contains numeric data as ASCII text with rows and columns delimited by commas. This format is the standard used by the oscilloscope. To archive data in a more memory efficient form, the hierarchal data format (HDF) file format is used. Unlike CSV files, HDF files contain metadata with important field probe and shot information.

A data series in the time domain can be converted to the frequency domain. This process, called the Fourier transform, can represent time domain data as a sum of sinusoids of scaled amplitude and phase with different frequencies. Plotting the amplitude of each sinusoid against its frequency reveals which frequencies are present and their magnitudes within the time sampled data. Every signal has a unique Fourier transform, and every Fourier transform has a corresponding time domain signal. Equation 1 presents the Fourier transform pair of a continuous signal.<sup>5</sup>

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$
(1a)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$
 (1b)

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where  $F(\omega)$  is the Fourier transform of f(t) and  $\omega$  is defined as the radian frequency,  $2\pi f$ . Since a sampled data waveform provides a finite number of discrete data points, the discrete Fourier transform pair (Equation 2) is used.<sup>6</sup>

$$X_{n} = \sum_{k=0}^{N-1} x_{k} e^{-i\omega_{n}t_{k}} \qquad n = 0, 1, 2, \dots, N-1$$
(2a)

$$x_{k} = \frac{1}{N} \sum_{n=0}^{N-1} X_{n} e^{i\omega_{n}t_{k}} \qquad k = 0, 1, 2, \dots, N-1$$
(2b)

$$\omega_n t_k = \left(\frac{2\pi}{N}\right) nk \tag{2c}$$

Here, N is the number of sample points,  $x_k$  is the time domain function, k is a discrete time counting variable,  $X_n$  is the frequency domain function, and n is a discrete frequency counting variable. The discrete Fourier transform is similar to a continuous Fourier transform, but it has some additional interesting properties, which will be discussed in section 3.3.

Using voltage versus frequency data, absolute electric or magnetic (depending on the type of field probe) field strength versus frequency can be calculated. This data is stored and saved for later assessment of EMP mitigation techniques.

### 3. The MATLAB EMP Analysis Routine

MATLAB<sup>™</sup> is a high-level mathematical programming language, provided by The MathWorks, Inc., that uses matrices as fundamental objects. It offers sufficient user and system compatibility to facilitate the development of a self-contained program with a graphical user

interface (GUI). This project developed an EMP Measurement Analysis Routine using the MATLAB language.

#### **3.1 Importing Field Probe Measurement Files**

The first function of the EMP Analysis Routine is to read and import probe files stored by an oscilloscope. Voltage versus time data is stored in one of two formats.

The CSV format is stored directly by the oscilloscope. Different oscilloscope models create variations in the header information placement within the file. Some contain header information in the same two columns as the data values. In this case, the file has a total of two columns, and data starts in the first row without alphabetic characters. Other CSV files contain header information next to the data values. In this form, the file has five columns, and data starts at row 1, column 4. The final CSV storage convention encountered does not include header information. The MATLAB routine constructs a map of empty cells and cells with alphabetic characters to indicate where actual numeric values are located in each particular CSV.

The HDF format is specifically designed for scientific data set storage. Raw scope data sets to be archived are converted by a separate software task to an HDF file for efficient archival memory utilization. MATLAB has two built-in functions for importing HDFs. The function "hdfread(filename)" returns a vector of integers that are proportional to the y-values measured by an oscilloscope. Another function "hdfinfo(filename)" returns a structure that contains the scale factor for y-values, the time interval between samples, and experiment type. At this point, voltage versus time data has been imported into the MATLAB routine from either supported format. It is necessary to associate data with information about the particular experiment and probe so the analysis routine can produce correctly scaled and well-labeled results.

The probe structure, shown expanded in Figure 1, is defined and used throughout the EMP Analysis Routine to keep track of all of the aspects of each probe data set. Information fields are found using different methods, depending on the imported file format. HDF files specify most of the necessary information in their own information structure; however, CSV files



Figure 1. An expansion of the probe structure convention

have no useful metadata. Probe structure information from CSV files relies on the diligence of the experimentalist in placing that information in the filename. For example, if the string 'dB' is located in the filename, the program infers that the signal attenuation is equal to the number preceding 'dB.' Using the GUI, the user can manually enter or change a probe's information fields. Once data is imported and the probe structure identified, the program can analyze the raw voltage waveform.



Figure 2. A sample plot of a raw voltage versus time waveform

### 3.2 Analysis of the Time Domain

Although the primary purpose of the routine is to analyze the frequency spectrum of EMP, other important measurements, such as the peak voltages, attack time, pulse duration, and energy, are made in the time domain. For these metrics to be determined accurately, it is useful to know the peak voltages and the pulse start and end time. The peak voltages are found to be the maximum and minimum values of the data. EMP pulses from the OMEGA laser experiments generally take the form of a fast rising and exponentially decaying complex combination of sinusoids as illustrated in Figure 2. To find the start of the pulse or the head, a sample of pre-shot noise is taken. The head is defined as the point where the voltage exceeds twice the noise level. The attack time or the rise time is useful in assessing how abruptly EMP reaches its maximum voltage  $V_{max}$ . To find the end of the pulse or tail, it is assumed that voltage decays exponentially with time. The program makes a moving average envelope of the waveform and fits an exponentially decaying function to it. The tail is defined to be where the envelope decays to less than 1% of the maximum voltage or approximately  $V_{max} * e^{-5}$ . Once the head and tail are known,

the pulse duration is defined as the difference between these times. The EMP energy that the probe intercepts is one of the most useful tools for assessing the magnitude of EMP effects. Energy is calculated with the equation:

$$W = \int \frac{V(t)^2}{R} dt \tag{3}$$

where W is energy and R is 50 ohms ( $\Omega$ ), the standard termination resistance presented to the probe by the data digitizing oscilloscope.

#### **3.3 Transformation into Field Versus Frequency Data**

The principal function of the EMP analysis routine is to convert field probes' measured voltage versus time data into a spectrum of electric or magnetic field strength versus frequency. Since the data consists of a sampled data set of a finite number of points, the discrete Fourier transform is used. When computing this transform for large data sets, the fast Fourier transform algorithm is used to save some computational repetition. MATLAB contains a built-in function "fft" (Fast Fourier Transform) which does most of the rigorous mathematics for the EMP routine.

The properties of discrete Fourier transforms have several characteristics. Because new information is neither gained nor lost in the transform, discrete Fourier transformed time domain data results in the same number of data points as the raw data. However, since the time domain data is real but the frequency domain data is complex, N discrete sample points yield N/2 discrete frequencies, each with a unique real and imaginary component. This means that the frequency domain result of a discrete Fourier transform is between 0 Hertz (Hz) and one-half of the inverse of the sample interval at which the raw data was collected, or half of the original Fourier transform data set. The routine therefore only keeps the first half of the transform result.

The inverse of the sampling interval divided the number of data points yields the frequency interval between Fourier data points. When setting oscilloscope sampling properties, the interrelationship between the maximum measurable frequency, the resolution between points, and the size of the resulting data file must be kept in mind.

Once data is in the form of voltage versus frequency, the conversion to electric or magnetic field strength versus frequency is a linear algebraic operation. Determining which field to find depends on the type of probe (D-dot or B-dot) that made the initial measurements. Every field probe has an effective area that dictates the sensitivity of the probe to the field. The effective area of a probe, which is specified by the manufacturer, is important for determining the absolute field strength. Equations 4 and 5 present the linear relations used to calculate electric and magnetic field strengths versus frequency from voltage output as a function of frequency. Equation 4 is the relation between the D-dot probe response and the electric field strength.<sup>4</sup>

$$E(f) = \frac{V(f)}{2\pi \cdot f \cdot A_{eq} \cdot R \cdot \epsilon_0}$$
(4)

where E is the electric field strength, f is frequency,  $A_{eq}$  is the effective area, R is the resistance of the probe loading (100  $\Omega$ ), and  $\epsilon_0$  is the permittivity of free space. Equation 5 is the equivalent relation for the B-dot probe and the magnetic field strength.<sup>4</sup>

$$H(f) = \frac{V(f)}{2\pi \cdot f \cdot A_{eq} \cdot \mu_0}$$
(5)

where H is the magnetic field strength and  $\mu_0$  is the permeability of free space. The inverse frequency term in each equation results in the field strength appearing to approach infinity as f

approaches zero. The 1/f issue at low frequencies presents certain problems for automated field strength versus frequency analysis. This issue is discussed further in section 3.4.

### **3.4 Frequency-Dependent Analysis**

Voltage and field versus frequency data is mainly interpreted visually. The EMP analysis routine can, however, automatically locate multiple spectral peaks. This peak picker function encounters two main difficulties. The first is where to differentiate between 1/f noise and real field peaks. The second is how to define the difference between one and two discrete peaks. A self-iterative function picks peaks based on the specified number of peaks and buffer interval. The buffer interval defines the minimum allowable interval between adjacent peaks, so the user has a level of control over what constitutes a distinct peak. Currently, the only solution to filtering off 1/f noise is through manual truncation of spectral data below a selected frequency.

At this stage in the study of laser generated EMP, it is unclear as to which aspects of the frequency domain are the most important. The EMP analysis routine provides a basic foundation for the application of further analytical methods.

### 3.5 User Oriented Interface and Output

Apart from efficient spectral analysis of EMP data, the analysis routine gives the user control over how data is analyzed and how results are saved. A front end GUI gives the user this control, and serves to bundle the entire routine into an intuitive, user-friendly program. Data can be opened and imported in the GUI one probe at a time. The GUI itself (Figure 11) is divided into two areas. The top half displays probe structure information associated with the current probe, such as name, model, and effective area, and allows this information to be modified. The

bottom half displays a plot of the raw digitized voltage, the Fourier transform, or the field magnitude with certain important points highlighted as specified by the user. The panel below the plot controls which plot is shown, and the panel to the right of the plot controls which parts of the analysis are displayed on the plot. For example, the option to highlight the peaks of a raw voltage versus time waveform is available when the "Raw Data" option is selected. When a spectral plot is displayed (as in Figure 11), the user can set parameters for a peak picker method(Section 3.4) and immediately see the result of changing each parameter. The GUI not only simplifies the operation of the routine, but it also adds to the routine's overall capability.

By pressing a button in the lower right corner of the GUI, the analysis can be previewed, saved, or directly printed in the form of a standard, two-page output report (Figure 12). The first page of a report is headed by all the information associated with the given probe. It also shows the calculated scalar metrics of the time domain (Section 3.2) and the coordinates of the top "n" peaks of both the Fourier and field data. The parameters needed for picking peaks, which are specified by the user of the GUI, are used in determining these coordinates. The second page of a typical report exhibits three plots: raw, Fourier, and field plots. Analysis reports are initialized as MATLAB figures. If "preview" is selected, the figure representing the analysis report is displayed on the screen. If "print" is selected, that figure is exported to a postscript file and sent to the selected printer. If "save" is selected, the figure is exported to a postscript file and converted, using a system distiller, to a portable document file (PDF).

A final option for saving the results of an analysis is configured to save probe information and scalar metrics in a spreadsheet format along with point-by-point field data versus frequency. This format will facilitate the future study of EMP events.

## 3.6 Preliminary Work with Multiple Data Set Analysis

The analysis routine was primarily developed to work with individual probe data sets. Because of this limitation the original EMP analysis program can not automatically be used to efficiently examine data from many shots in order to spot trends or gather statistical information. The same tools used for analyzing individual probe files can be intuitively expanded to deal with multiple probe files. Since MATLAB relies on the use of matrices, a data structure that holds an arbitrary number of probes is identical to one that holds a single probe. Thus, the functions "import\_multiple" and "spreadsheet\_of\_all" were developed to import a vector of probe structures and to make a MATLAB workspace spreadsheet respectively. The spreadsheet function, "spreadsheet\_of\_all" also takes the names of which fields are of interest. For example, if the user only wants to know the shot number and EMP energy, the resultant spreadsheet will consist of a shot number column and the corresponding EMP energy column. Although these two functions are unrefined on their own, they can form the building blocks of future multiple field probe data analysis development.



### 4. Field Probe Verification Experiments

Two experiments were performed during the course of this project to verify the responsivity of the field probes and the accuracy of the EMP analysis routine. The premise of the

experiments was to place the field probes in a field of known amplitude and frequency and to compare the resultant measurements with a calculated, theoretical result. The experimental setup is shown in Figure 3.

A known field was created using various fixed frequency, half-wavelength vertical dipole antennas.<sup>7</sup> The particular antennas used were 1.88, 1.00, 0.6, and 0.3 meters long corresponding to frequencies of 80, 150, 250, and 500 MHz, respectively. At their half wavelength frequency, the dipoles generate the electric and magnetic fields along their horizontal axis given theoretically by Equations 6 and 7.<sup>7</sup>

$$E = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}} \cdot I_0}{2\pi r} \tag{6}$$

$$H_{\phi} = \frac{I_0}{2\pi r} \tag{7}$$

where r is the distance between the dipole antenna and probe and  $I_0$  is the amplitude of the current applied to the dipole.



Figure 4. The circuit diagram for a matching network. The input signal comes from the left and the dipole signal exits to the right. For the matching requirements of the 80 MHz dipole the series component is an inductor and the shunt component is a capacitor.

The current source for each experiment was provided by a voltage generator of 50  $\Omega$  source impedance. However, the impedance of the dipole antennas differs from 50  $\Omega$  at the half wavelength frequency. This impedance mismatch causes the signal current to be reduced slightly

due to signal reflection. It was calculated that this attenuation effect was negligible (less than one decibel or 10%) for all of the antennas except the 80 MHz dipole. Therefore, a matching network circuit (Figure 4) was built to interface between the source and 80 MHz dipole. The matching network is designed by proper analytic selection of components to reactively match the dipole impedance to 50  $\Omega$  at the half wavelength frequency.

The first experiment was to measure the coupling between dipole and probe. This was performed with a two-port network analyzer, which compares a signal sent out to the dipole to a received signal from the field probe. Coupling is measured in decibels or dB, which is a scaled logarithmic ratio of the two signals. It can be calculated theoretically at a certain separation distance by determination of the field strength at the field probe location produced by the dipole driven by the network analyzer source using Equations 6 and 7. The probe response to this field is calculated using Equation 4 or 5, depending on the probe type.





Upon comparing the calculated and measured couplings in Figure 5, it is apparent by the close correlation of corresponding data points that the experimental setup was free of significant sources of error.

A second experiment was performed with the same dipole and field probe configuration as the first, except that an oscilloscope was connected to the probe output and an EMP pulse simulator was connected to the dipole input. The EMP pulse simulator uses a resistor-inductorcapacitor (RLC) resonator circuit to create an exponentially decaying 80 MHz sinusoidal signal when triggered, so it was only used with the 80 MHz dipole antenna. Initially, the pulse simulator was connected directly to the oscilloscope, and a dipole source drive waveform was recorded and saved. This waveform was converted into the frequency domain via the EMP analysis routine and then converted into electric (Equation 7) or magnetic (Equation 8) field strength versus frequency at the field probe location utilizing the dipole field equations.



Figure 6. Peaks of theoretical and measured magnetic field magnitude versus frequency. The measured field multiplied by two is

The EMP source was reconnected to the dipole and a field probe was positioned to measure actual electric or magnetic fields a specific distance from the dipole. The MATLAB routine was utilized to convert the scope data to field plots. The peaks of the theoretical magnetic field and measured magnetic fields are plotted in Figure 6. The frequency shift of 1 MHz is most likely produced by a slight shift in the RLC resonator characteristics when connected to the dipole antenna. Also, the measured field magnitude was roughly a factor of two less than the expected field magnitude. Similar results were obtained with the electric fields. This factor of two discrepancy has not been isolated yet. More work must be done to fully understand the source of this disparity.



Figure 7. A plot of the number of EMP related OMEGA system problems versus the external field probe energy for shots 47713-47723.

### 5. Verified EMP Event Effects

The EMP analysis routine has already been used to verify several hypotheses about EMP event effects for gathered field probe data sets. Based on work done previously at the LLE, it is suspected that the numbers of EMP generated control and diagnostic electronics system problems are related to EMP energy. This project was one of the first to analyze the measured EMP energy on the OMEGA laser. Figure 7 confirms the hypothesis that system failures are

related to EMP energy. As the measured EMP energy increases, the number of failures also increases.





Another expectation is that the OMEGA target chamber acts as a resonator that 'rings' at a specific frequency related to the chamber geometry when excited by an EMP impulse. The chamber is a hollow metal structure and will therefore tend to support a specific wavelength of electro-magnetic waves. Of the waves bouncing back and forth in the chamber, one frequency, called the resonant frequency, and its harmonics will constructively interfere and sustain until being diminished by losses. The OMEGA target chamber is approximately a spherical cavity with a radius of 1.54 meters. The equation for the lowest order resonant frequency of a sphere is:

$$f_R = \frac{1}{2.29 \, a \sqrt{\mu_0 \epsilon_0}} \tag{9}$$

where  $f_R$  is the resonant frequency in Hz, a is the radius of the structure,  $\mu_0$  is the permeability of free space, and  $\epsilon_0$  is the permittivity of free space. Consequently, the OMEGA target chamber

has a resonant frequency of about 85 MHz. Figure 8 illustrates the close proximity between this theoretical resonant frequency (indicated by the vertical line) and the largest peak of a typical intra-chamber field plot. A slight difference in frequency can be accounted for by the fact that the OMEGA chamber is not a perfect sphere. A great number of metallic diagnostic and control structures reside within the chamber, each one producing some modifying effect to the overall resonant frequency.

The EMP analysis routine has also confirmed the prediction that metal cylindrical beam tube structures intended to guide laser beams into the target chamber act as electro-magnetic waveguide structures for EMP energy. Laser beam tubes, which are connected to the OMEGA, MTW, and OMEGA EP target chambers, have circular cross sections and, therefore, should act as waveguides with a defined low frequency cut-off for any electro-magnetic wave propagation through them. That means that an EMP event that is coupled through these beam tubes should exhibit a frequency below which the magnitude of the field will drop off. This cutoff frequency is related to the specific beam tube cross sectional dimensions by Equation 10 which provides the cut-off frequency in Hz for the lowest order mode of propagation.

$$f_c = \frac{1.841}{2\pi a \sqrt{\mu_0 \epsilon_0}} \tag{10}$$

where  $f_c$  is the high-pass cutoff frequency in Hz and a is the radius of the cylindrical waveguide structure. Utilizing equation 10, the OMEGA focus lens assemblies of radius 0.1715 meters have a cutoff frequency of 513 MHz, the OMEGA EP beam tubes of radius 0.298 meters have a cutoff frequency of 294 MHz, and the MTW beam tube has a cutoff frequency of 1.38 GHz. When these particular frequencies are highlighted on field versus frequency plots for the MTW (Figure 9) and OMEGA (Figure 10), a correlation is noted. Above the particular cutoff frequency, field



strength increases abruptly. Figure 9b is a plot of the top ten peaks for multiple MTW shots. Figure 10 shows a similar high-pass characteristic for OMEGA external field probes. The fact that most spectral peaks for the external OMEGA probes exist above the focus lens assembly tube cut-off indicates that these 60 ports may provide a major source for the internal to external chamber EMP coupling.

Figure 10. A plot of the top ten peaks frequencies over OMEGA shots 48408 - 48414 obtained automatically by the MATLAB analysis routine. The theoretical cutoff frequency of the OMEGA focus lens structures (~ 513 MHz) is shown as a vertical green line.



# 6. Conclusion

The EMP measurement analysis routine will continue to be a powerful tool for studying EMP events caused by high-intensity lasers. It greatly streamlines and simplifies analysis in the time and frequency domains by employing the technical efficiency of the MATLAB programming language. The analysis routine has already been used to verify several EMP event effects. Future utilization of this routine in the study of EMP events at the Laboratory for Laser Energetics will lead to a better understanding of EMP and how to mitigate electronic disruptions associated with it.

Figure 11. A snapshot of the Analysis GUI



Figure 12. An example output report PDF. The first page contains numerical results, and the second page displays graphical results.



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