Electron velocity distribution functions and Thomson scattering

by

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Biographical Sketch

The author graduated with great distinction from the Commonwealth Honors College at the University of Massachusetts Amherst, earning a Bachelor of Science in Physics and Mathematics. He began doctoral studies in the Department of Physics and Astronomy at the University of Rochester in 2015. In 2016 he was awarded the Frank J. Horton Fellowship supporting his research at the Laboratory for Laser Energetics with Professor Dustin Froula. He received a Master of Arts in Physics in 2017.

Presentations and Publications

The following publications were a result of work conducted during doctoral study:

First-Author Publications


Co-Author Publications


First Author Conference Presentations


• “Measuring electron distribution functions driven by inverse bremsstrahlung heating with collective Thomson scattering” Invited talk presented at Anomalous Absorption Conference 2019, Telluride CO
• “Novel techniques and uses of collective Thomson scattering” Invited talk presented at Laser Aided Plasma Diagnostics 2019, Whitefish MT

• “Measurements of arbitrary electron distribution functions using angularly resolved Thomson scattering” Contributed talk presented at 61st Annual Meeting of the American Physical Society Division of Plasma Physics 2019, Ft. Lauderdale FL


• “Concept for measuring electron distribution functions using collective Thomson scattering” poster presented at 59th Annual Meeting of the American Physical Society Division of Plasma Physics 2017, Milwaukee WI

• “Measuring Non-Maxwellian Distribution Functions Using Expanded Thomson Scattering” poster presented at HED Summer School 2017 San Diego CA
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Abstract

Statistical mechanics governs the fundamental properties of many body systems and the corresponding velocity distributions dictates most material properties. In plasmas, a description through statistical mechanics is challenged by the fact that the movement of one electron effects many others through their Coulomb interactions, leading to collective motion. Although most of the research in plasma physics assumes equilibrium electron distribution functions, or small departures from a Maxwell–Boltzmann (Maxwellian) distribution, this is not a valid assumption in many situations. Deviations from a Maxwellian distribution can have significant ramifications on the interpretation of diagnostic signatures, and more importantly in our ability to understand the basic nature of plasmas.

Optical collective Thomson scattering provides precise density and temperature measurements in numerous plasma-physics experiments. A statistically based, quantitative analysis of the errors in the measured electron density and temperature is presented when synthetic data calculated using a non-Maxwellian electron distribution function is fit assuming a Maxwellian electron distribution [A. L. Milder et al., Phys. Plasmas 26, 022711 (2019)]. In the specific case of super-Gaussian distributions, such analysis lead to errors of up to 50% in temperature and 30% in density. Including the proper family of non-Maxwellian electron distribution functions, as a fitting parameter, in Thomson-scattering analysis removes the model-dependent errors in the inferred parameters at minimal cost to the statistical uncertainty. This technique was used to
determine the picosecond evolution of non-Maxwellian electron distribution functions in a laser-produced plasma using ultrafast Thomson scattering [A. L. Milder et al., Phys. Rev. Lett. 124, 025001 (2020)]. During the laser heating, the distribution was measured to be approximately super-Gaussian due to inverse bremsstrahlung heating. After the heating laser turned off, collisional ionization caused further modification to the distribution function while increasing electron density and decreasing temperature. Electron distribution functions were determined using Vlasov-Fokker-Planck simulations including atomic kinetics.

A novel technique that encodes the electron motion to the frequency of scattered light while using collective scattering to improve the scattering efficiency at velocities where the number of electrons are limited was invented to measure non-Maxwellian electron distributions [A. L. Milder et al., in review Phys. Rev. Lett. (2021)]. This angularly resolved Thomson-scattering technique is a novel extension of Thomson scattering, enabling the measurement of the electron velocity distribution function over many orders of magnitude. Electron velocity distribution functions driven by inverse bremsstrahlung heating were measured to be super-Gaussian in the bulk \( (v/v_{th} < 3) \) and Maxwellian in the tail \( (v/v_{th} > 3) \) when the laser heating rate dominated over the electron-electron thermalization rate. Simulations with the particle code Quartz showed the shape of the tail was dictated by the uniformity of the laser heating. The reduction of electrons at slow velocities resulted in a \( \sim 40\% \) measured reduction in inverse bremsstrahlung absorption. A reduced model describing the distribution function is given and used to perform a Monte Carlo analysis of the uncertainty in the measurements [A. L. Milder et al., in review Phys. Plasmas (2021)]. The electron density and temperature were determined to a precision of 12\% and 21\%, respectively, on average while all other parameters defining the distribution function were generally determined to better than 20\%. It was found that these uncertainties were primarily
due to limited signal to noise and instrumental effects. Distribution function measurements with this level of precision were sufficient to distinguish between Maxwellian and non-Maxwellian distribution functions.
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Chapter 1

Introduction

Plasma is the most abundant state of matter in the universe being the main state of matter in stars and interstellar space. While examples of terrestrial plasmas are rare, lightning and aurora being two notable cases [1], they can still have an impact on our daily life through the solar wind [2–4] or industrial applications [5–7]. Furthermore there are numerous future technologies that rely on plasma, from extreme ultra-violet lasers to compact particle accelerators to fusion energy.

One of the main approaches to achieving laboratory fusion is laser driven inertial confinement fusion where lasers are use to directly or indirectly implode a capsule of fusion fuel. As the plasma ablates the solid density capsule, x-ray converter, or experimental hardware, a region of underdense (transparent to laser light) plasma is established. In this region, the laser can interact with the plasma or other laser beams, facilitated by the plasma [8, 9]. While necessary to deposit energy into the ablator, many of these laser plasma interactions degrade implosion performance. If harnessed laser plasma interactions can be used to alter implosion symmetry or tackle completely unrelated problems [10–12].

Whether the goal is improving fusion yield, making high-power lasers, or studying fundamental physics, an understanding of the plasma conditions, coupling, and evolu-
tion is paramount. Taking a step further back, even when lasers are not an important component, simply a way to create or diagnose a plasma, it is important to understand how the laser effects the plasma and the measurement. For most applications the interaction of lasers and plasmas is approximated by treating the plasma as a fluid. This has lead to numerous advances in the history of plasma physics. As our numerical tools, modelling capabilities, and measurement techniques improve, we are seeing more and more places where approximating a plasma as a fluid breaks down. Break downs often occur when a plasma is highly collisional [13] or where waves are non-linear [14]. The fluid approximation is built on the assumption that the plasma is locally in thermal equilibrium and therefor the distribution in velocities of electrons and ions can described using Maxwell-Boltzmann statistics [15]. In some cases, as we will show, this assumption cannot be made and leads to a misunderstanding of the plasma even where the fluid approximation is traditionally though to hold. In these cases non-Maxwellian electron and ion distribution functions must be considered.

The electron and ion distribution functions dictate the properties and evolution of a plasma. Many of these properties, including energy transport and laser coupling, are primarily determined by the electron velocity distribution function. The electron velocity distribution function is often assumed to be Maxwellian [16–19] or close to Maxwellian [20, 21], however the existence of non-Maxwellian electron distribution functions have significant ramifications on the behavior of a plasma, and the interpretation of diagnostic signatures. Consequences of non-Maxwellian electron distribution functions on laser absorption and laser-plasma instabilities were predicted as far back as the 1980s [22–26], but are often neglected in models and simulations due to a lack of experimental verification and the computation expense of including a kinetic solver. Uncertainties in the distribution function have implications across many areas of plasma
physics including magnetic and inertial confinement fusion, astrophysics, and space sciences.

In 1980, it was predicted that laser heating preferentially transfers energy to the slower electrons driving their velocity distribution to have a flat-top, or super-Gaussian shape [22]. It was shown that this reduction in slow electrons reduces the inverse bremsstrahlung heating rate and in subsequent years nearly all hydrodynamic models that include laser propagation have introduced a factor to adjust the laser absorption due to this effect [27, 28]. Challenges in measuring absorption and the electron distribution function [29–31] have made it difficult to verify these theories, although extensive computational work has been done over the last forty years [23, 24, 32–35].

These computational studies have explored the evolution of the distribution function resulting from inverse bremsstrahlung heating, including the consideration of the relatively small electron-ion collision rate of the fast electrons [36], thermal transport [24], and electron-electron collisions [32], which all tend to produce high-velocity electrons (tails) and a non-Maxwellian bulk of electrons. Despite the various considerations of these studies, and their physical interpretations, the modeling overwhelmingly supports the conclusion that inverse bremsstrahlung heating produces an electron distribution that is super-Gaussian in the bulk and Maxwellian in the tails.

In laser-produced plasmas, inverse bremsstrahlung heating is only one of the processes that can alter the electron distribution function. Thermal transport [37, 38], laser-plasma instabilities [39], and atomic kinetic processes [40] all provide competing mechanisms that shape the electron distribution function. A recent computational study has shown the impact of atomic kinetics on inverse bremsstrahlung heating and nonlocal thermal transport, through modifications of the electron distribution function [40]. In a separate study, non-Maxwellian electron distribution functions driven by thermal transport were shown to modify Landau damping of electron plasma waves
and enhance their corresponding instabilities [38]. Furthermore, most atomic physics models used to calculate x-ray emission for plasma characterization are built assuming a Maxwellian electron distribution and deviation from a Maxwellian modifies these calculations [23].

Although there have been numerous computational studies of kinetic effects in hydrodynamics [41], experiments have been challenged to isolate changes to the electron distribution function. In the 1990s, microwaves were used in low-temperature (∼1 eV), low-density (<10^{17} cm^{-3}) plasmas to investigate changes to the electron distribution function introduced by inverse bremsstrahlung heating [30]. Later in the decade, initial studies in laser plasmas suggested the existence of non-Maxwellian electron distribution functions using Thomson scattering [31]. More recently, Thomson-scattering experiments were able to show the effect of nonlocal thermal transport on electron distribution function [37].

Thomson scattering has been a workhorse diagnostic for temperature and density measurements in many areas of plasma physics [37, 42–50], but these studies have assumed Maxwellian distribution functions allowing the plasma conditions [16, 29, 31, 42, 45, 51–53] to be extracted from the spectrum scattered off electrons in the bulk (non-collective) or in the tails (collective) of the electron distribution functions [54]. In the non-collective regime, the power scattered at a particular frequency is proportional to the number of electrons with a velocity that Doppler shifts the frequency of the probe laser to the measured frequency. This provides a direct measurement of the electron distribution function, but in practice, the small scattering cross section of the electron and small number of electrons at high velocities limits this technique to measuring electrons in the bulk of the distribution function [29]. In the collective regime, the power scattered into the collective features is dominated by scattering from electrons propagating at velocities near the phase velocity of the electron plasma waves, which can
be significantly faster than the thermal velocity. In theory, a measurement of the complete scattering spectrum in either of these configurations could be used to determine the electron distribution function without an assumption on its shape, but in practice signal-to-noise, instrumental response, and dynamic range of instruments have limited measurements of the distribution function to the bulk [55–61] or a predetermined class of distribution functions [26, 30, 31, 37, 62–66].

While abandoning the assumption of Maxwell-Boltzmann statistics is neither universally appropriate or computationally tractable, understanding where the assumption breaks down and how it impacts the plasma and our understanding of the plasma could further propel the field, closing gaps between modeling and experiment and opening new opportunities to apply plasma to unique problems.

This Thesis will explore measurements of the electron distribution function in laser produced plasma with Thomson scattering. The sensitivity of electron temperature and density inferred from collective Thomson scattering to non-Maxwellian electron distribution functions, specifically super-Gaussian distribution functions, showed ignorance of the true distribution function leads to errors of up to 50% in temperature and 30% in density [62]. Ultrafast Thomson-scattering was used to make the first measurements of the interplay between inverse bremsstrahlung heating and ionization kinetics on the electron distribution function [67]. The preferential heating of the slow electrons by a laser, coupled with the redistribution of electron kinetic energy due to ionization, resulted in a non-Maxwellian electron distribution functions.

A technique and diagnostic were invented that uses the angular dependence of Thomson scattering to simultaneously access the non-collective and collective nature of plasmas. This angularly resolved Thomson scattering allowed the first measurements of complete electron distributions without any assumptions on their shape or the underlying physics that produced them [68]. This first-principles measurement showed that
during significant heating by the laser beams, the distributions had a super-Gaussian shape in the bulk \((v < 3v_{th})\) with a Maxwellian tail \((v > 3v_{th})\). The super-Gaussian bulk is associated directly with inverse bremsstrahlung heating while particle simulations show the isotropy of the heating plays a role in accurately predicting the high-velocity tail \([69]\). A super-Gaussian + Maxwellian model was developed to accurately represent the measured distributions. Markov-chain Monte Carlo analysis was performed using this reduced model to identify a confidence region on the measured electron distribution functions. A reduction in laser absorption was measured when the electron distributions were determined to be super-Gaussian, due to the depleted number or low-velocity electrons.
Chapter 2

Theory

This chapter gives the background plasma physics required for understanding the work of this thesis. The goal of the chapter is to establish the foundational importance of the electron distribution function in the behavior and evolution of plasmas. In Section 2.1 the distribution function is defined and shown to govern plasma evolution through the Boltzmann equation. Section 2.2 shows that Maxwellian and non-Maxwellian electron distribution functions are solutions to the Boltzmann equation. In Section 2.3 the electrostatic waves in a plasma are derived in the fluid and kinetic pictures and shown to change with distribution function.

2.1 Kinetic description of a plasma

At the most fundamental level a plasma is a collection of charged particles. It can therefore be described by

\[ F_q(r, v, t) = \sum_{j=1}^{N_q} \delta(r - r_j(t))\delta(v - v_j(t)). \]  

(2.1)
Each particle $j$ of the species $q$ has a discrete velocity $v$ and position $r$ at the time $t$, the plasma is described as the combination of these individual particles. The evolution of this collection of particles is determined by taking the time derivative of this equation yielding the Klimontovich equation[70],

$$\frac{\partial F_q}{\partial t} + v \cdot \frac{\partial F_q}{\partial r} + a \cdot \frac{\partial F_q}{\partial v} = 0. \tag{2.2}$$

The acceleration $a$ includes all forces internal and external. While perfectly exact, as it includes all forces and all particles discretely, these equations are generally intractable, a solution would require the behaviour of all particles to be calculated. To obtain a useful equation, two simplification must be made. First to the acceleration. As the plasma is made of charged particles it is assumed that they are mainly electrons and atomic ions. It is possible to consider the ions of more complicated molecules, but these will disassociate at a sufficient level of ionization yielding atomic ions. Therefore, the dominant forces acting on the particles will be electromagnetic,

$$a = \frac{q}{m} \left( E + \frac{v}{c} \times B \right) \tag{2.3}$$

where $E$ and $B$ are the total electric and magnetic fields.

The second simplification is to $F_q$. The complicated feedback of the fields on the particles, and back on the fields is only tractable numerically. To obtain some closed form solutions the particle description ($F_q$) is averaged over an ensemble of replica systems,

$$f_q(r,v,t) = \langle F_q(r,v,t) \rangle. \tag{2.4}$$

This ensemble averaged function is known as the distribution function. It can be interpreted as the probability distribution function of finding a particle of species $q$ with
position $\mathbf{r}$ and velocity $\mathbf{v}$ at the time $t$. This probabilistic approach has the benefit of providing a continuous, differentiable function. With the additional step of dividing this distribution function into an average (or slowly varying) and fluctuating (quickly varying) part,

$$f_q = f_{0q} + f_{1q},$$

Eq. 2.2 can be written,

$$\frac{\partial f_{0q}}{\partial t} + \mathbf{v} \cdot \nabla f_{0q} + \frac{q_q}{m_q} \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \nabla_v f_{0q} = -\frac{q_q}{m_q} \left\langle \left( \mathbf{E}_m^m + \frac{\mathbf{v}}{c} \times \mathbf{B}_m^m \right) \cdot \nabla_v f_{1q} \right\rangle. \tag{2.6}$$

The fields have been split into quickly varying microscopic fields ($\mathbf{E}_m^m, \mathbf{B}_m^m$) and average ($\mathbf{E}, \mathbf{B}$) fields. The left side of Equation 2.6 generally describes collective effects, while the right is associated with collisional effects. This equation is known as the Boltzmann equation and is often further simplified by assuming the collisions are negligible. This collisionless assumption gives an equation known as the Maxwell-Boltzmann or the Vlasov equation,

$$\frac{\partial f_{0q}}{\partial t} + \mathbf{v} \cdot \nabla f_{0q} + \frac{q_q}{m_q} \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \nabla_v f_{0q} = 0. \tag{2.7}$$

## 2.2 Sources of Maxwellian and non-Maxwellian plasmas

With the distribution function established (Eq. 2.4) and the equation governing its evolution (Eq. 2.6), the natural question arises, what is the form of $f_{q0}$. A common
assumption is the equilibrium Maxwellian distribution,

\[ f_q = \sqrt{\frac{m_q}{2\pi T}} \exp\left\{ -\frac{m_q v^2}{2T} \right\}, \quad (2.8) \]

as this is the equation for an ideal gas in thermal equilibrium, a very similar system. This distribution can be derived directly from the micro-canonical ensemble in statistical mechanics[71], but it is informative to derive this distribution function from the Boltzmann equation. There are multiple approaches to this solution, the one presented follows the Boltzmann H-theorem[15] but there are also solutions relying on Lenard-Balescu equation, Fokker-Planck equation, and BBGKY hierarchy.

### 2.2.1 Derivation of Maxwellian distribution function

The simplest form of the collision operator, the right side of Eq. 2.6, is described by considering various forms of collisions. The lowest order term is interactions of a single particle with it’s own microscopic field, but particles do not interact with such a field. The next term is the electromagnetic interaction of 2 particles, further terms involve interactions of escalating numbers of particles. Arguing that the interactions involving the least particles are the most probable, the higher order terms are dropped and only the 2 particle interactions considered.

Let a particle \( j \) with a location \( \mathbf{r} \) and velocity \( \mathbf{v}_j \) interact with particles \( i \) within a cylindrical shell of radius \( b \) and thickness \( db \) during the time \( dt \). The probable number of \( i \) particles is,

\[ 2\pi f_i(\mathbf{r}, \mathbf{v}_i, t)g_{ij}b \, db \, dt \quad (2.9) \]

where \( g_{ij} = |\mathbf{v}_i - \mathbf{v}_j| \) is the differential speed of the particles. The integral over the impact parameter \( b \) and velocity \( \mathbf{v}_i \), gives the total number of interacting particles in the
time $dt$. Considering the probability of the particle $j$ being at this location, the number of particles that will be knocked out of their free streaming motion is,

$$2\pi \int \int f_j f_i g_{ij} b \, db \, d\mathbf{v}_i,$$  \hspace{1cm} (2.10)

dthis assumes there is no correlation between the positions and velocities of all particles.

Considering the opposite problem, the number of particles that will be knocked into this free streaming motion is,

$$2\pi \int \int f'_j f'_i g_{ij} b \, db \, d\mathbf{v}_i,$$  \hspace{1cm} (2.11)

where the prime denotes the precollision quantities that will become the unprimed quantities after collision. Liouville’s theorem was used to equate the volume element for primed and unprimed quantities. The Boltzmann equation becomes,

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j + \frac{q_a}{m_q} \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \nabla_v f_j = 2\pi \sum_i \int \int [f'_j f'_i - f_i f_j] g_{ij} b \, db \, d\mathbf{v}_i.$$  \hspace{1cm} (2.12)

To find an equilibrium solution of the Boltzmann equation for a single component plasma, the Boltzmann H-function is defined,

$$H(t) = \int \int f(r, \mathbf{v}, t) \ln f(r, \mathbf{v}, t) \, d\mathbf{r} \, d\mathbf{v}.$$  \hspace{1cm} (2.13)

Taking the time derivative,

$$\frac{dH}{dt} = \int \int \frac{\partial f}{\partial t} \ln f \, d\mathbf{r} d\mathbf{v} + \int \int \frac{\partial f}{\partial t} \, d\mathbf{r} d\mathbf{v},$$  \hspace{1cm} (2.14)
and if the number of particles is conserved then the second term vanishes,

\[ \frac{dH}{dt} = \iint \frac{\partial f}{\partial t} \ln f \, dr \, dv. \quad (2.15) \]

Multiplying the Equation 2.12 by \( \ln f \) and integrating over \( r \) and \( v \),

\[
\iint \frac{\partial f_j}{\partial t} \ln f_j \, dv_j + \iint \ln f_j v_j \cdot \nabla f_j \, dv_j + \iint \ln f_j \frac{q_j}{m_q} \left( E + \frac{v}{c} \times B \right) \cdot \nabla v \, f_0 q \, dv_j
\]

\[ = 2\pi \iint \ln f_j [f'_j f'_i - f_i f_j] g_{ij} b \, db \, dv_i \, dv_j. \quad (2.16) \]

Assuming the plasma is uniform in space, at least locally, \( v \cdot \nabla f \to 0 \). In order for the number of particles to be finite \( f(v) \to 0 \) as \( v \to -\infty, \infty \). Therefore, the second and third terms vanish,

\[
\frac{dH}{dt} = \iint \frac{\partial f_j}{\partial t} \ln f_j \, dv_j = 2\pi \iint \ln f_j [f'_j f'_i - f_i f_j] g_{ij} b \, db \, dv_i \, dv_j. \quad (2.17) \]

The two particle collisions must be symmetric with respect to time, i.e. for a set of initial and final velocities the initial velocities are recovered if the collision is reversed,

\[
\iint \ln f_j [f'_j f'_i - f_i f_j] g_{ij} b \, db \, dv_i \, dv_j = \iint \ln f_j [f'_j f'_i - f_i f_j] g'_{ij} b' \, db' \, dv_i \, dv_j \quad (2.18) \]

The definitions of differential speed \( g_{ij} \), impact parameter \( b \), and volume element are also time symmetric

\[ g_{ij} = g'_{ij} \quad b = b' \quad dv_i dv_j = dv'_i dv'_j. \quad (2.19) \]
Since these forms are equivalent, they also equal half their sum

\[
\frac{dH}{dt} = \frac{1}{2} \iiint (\ln f_j - \ln f'_j)[f'_j f'_i - f_i f_j] g_{ij} b \, db \, d\mathbf{v}_i d\mathbf{v}_j \tag{2.20}
\]

The same arguments apply to the collision being particle symmetric, it does not matter which particle is called \( j \).

\[
\frac{dH}{dt} = \frac{1}{4} \iiint (\ln f_j + \ln f_i - \ln f'_j - \ln f'_i)[f'_j f'_i - f_i f_j] g_{ij} b \, db \, d\mathbf{v}_i d\mathbf{v}_j \tag{2.21}
\]

Rearranging the logarithm,

\[
\frac{dH}{dt} = \frac{2\pi}{4} \iiint (\ln f_j f_i - \ln f'_j f'_i) [f'_j f'_i - f_i f_j] g_{ij} db \, d\mathbf{v}_i d\mathbf{v}_j. \tag{2.22}
\]

The integrand is analogous to \(-(x - y) \ln x/y\), which is \( \leq 0 \). So,

\[
\frac{dH}{dt} \leq 0 \tag{2.23}
\]

The goal is to obtain a steady state solution, implying for \( t \to \infty \) \( dH/dt = 0 \), or \( \ln f'_j + \ln f'_i = \ln f_j + \ln f_i \). Therefore, \( \ln f \) must be a summational invariant of the symmetric two particle collision. For elastic collisions, these invariants are mass \( (m) \), momentum \( (mv) \), and kinetic energy \( (mv^2/2) \).

\[
\ln f = \alpha m + \beta \cdot (mv) - \gamma m^2 \frac{v^2}{2} = \alpha m + \frac{m \beta \cdot \beta}{2 \gamma} - \frac{m \gamma (v - \beta)}{\gamma} \tag{2.24}
\]

and

\[
f(r, v) = c \exp \left\{ \left[ -\frac{m \gamma}{2} (v - \frac{\beta}{\gamma})^2 \right] \right\} \tag{2.25}
\]
The constants $c, \beta, \gamma$ can be identified from the moments of the distribution.

\[
 n = \int f dv \tag{2.26}
\]
\[
 \bar{v} = \frac{1}{n} \int vf dv \tag{2.27}
\]
\[
 \frac{3}{2} T = \frac{1}{n} \int v^2 f dv \tag{2.28}
\]

These moments are independent of the shape of the distribution and are physical properties that can be identified from the definition of the distribution function Eq. 2.4.

Solving for these constants we obtain the distribution function,

\[
 f(r, v) = \frac{1}{n} \left( \frac{m}{2\pi T} \right)^{3/2} \exp\left\{ -\frac{mv^2}{2T} \right\}. \tag{2.29}
\]

this is the 3-D version of the Maxwellian distribution (Eq. 2.8) with the density included.

From this derivation the solution of the Boltzmann equation has been identified as a Maxwellian. It is also evident that the source of this distribution function is binary single species collisions. For a plasma of electrons, this is electron-electron collisions, and for a plasma of ions, this is ion-ion collisions. In a plasma with ions and electrons the argument is made that these collisions happen on such a short time-scale that both the ions and electrons are able to maintain Maxwellian distributions with different temperatures.

The Maxwellian distribution function only depends on velocity, while the general definition of a distribution function depends on time, space, and velocity. This is a result solving for a distribution that is stationary in the asymptotic time limit, and locally uniform in space. For processes that occur on a time-scale longer than electron-electron collisions, it is useful to make these assumptions as the electron-electron collisions will
establish a new equilibrium. However, in a case where there is a spatial, or temporal variation of significance these assumptions break down the distribution function must be rederived.

This derivation required 8 assumptions, approximations, and simplifications.

(1) Forces are electromagnetic

(2) Particles can be represented by an ensemble average

(3) The distribution function can be separated into a slow and fast component

(4) The collision are limited to two particles

(5) There is only one type of particle

(6) Collisions are perfectly elastic

(7) The distribution function is uniform in space

(8) There are no external fields

It is possible to solve this problem relaxing some of these assumptions, but it is not surprising that these do not always hold true. Most of these assumptions can be summarized as saying there are no sources or sinks of energy. One of the simplest ways to violate these assumptions is if there are collisions other than binary single species collisions. This reintroduces additional terms on the right side of the Boltzmann equation and depending on their form allows new and unique solutions to this equation.

2.2.2 Inverse bremsstrahlung heating and Langdon effect

For the purposes of this thesis, non-Maxwellian distributions arising from inverse bremsstrahlung heating will be examined, but some other potential sources will
be described in the next section. The distribution function resulting from inverse bremsstrahlung heating is referred to as the Langdon distribution or the DLM (Dum-Langdon-Matte) distribution.

Inverse bremsstrahlung is the main heating mechanism in under-dense laser produced plasmas and an important mechanism across a wide range of plasma physics. Inverse bremsstrahlung occurs when an electron oscillating in the electric field of the laser collides with an ion, it retains some of the momentum gained from the laser field, thereby transferring energy from the field to the electron.

This argument was first proposed in a 1980 paper by Langdon[22], and the following derivation follows the argument in the paper. With the presence of a laser field the Maxwellian electron distribution oscillates relative to the ions and if the relative velocity is \( V \) then the electrons will experience a drag force \( \propto V^{-2} \), this will have a greater effect on electrons with small \( V/v_{th} \), where \( v_{th} = \sqrt{T_e/m_e} \) is the electron thermal velocity. Electrons with faster velocities will feel a smaller drag and while some gain energy other loose energy. The resulting velocity dependence of the electrons can be found by solving the Boltzmann equation (Eq. 2.6) including electron-ion collisions in an oscillating electric field.

Assuming the electron density and electric field are uniform in space the Boltzmann equation is,

\[
\frac{\partial f}{\partial t} + \frac{e}{m_e} E \cdot \nabla_v f = A \nabla_v \cdot \left[ \frac{v^2 I - vv}{v^3} \cdot \nabla_v f \right] + C_{ee}(f). 
\]  

(2.30)

The particle charge has been identified as the electron charge \( (q = e) \), the mass is the electron mass \( (m_e) \) and the spatial gradient was dropped under the uniform assumption. The collision term has been broken in two, with the second term being the electron-electron collision operator as derived in the previous section, and the first term is a form of the Fokker-Plank operator known as the Lorentz collision operator[72] describing the
electron-ion collisions. In this operator, \( A = 2\pi n_e Z e^4 / m_e^2 \ln \Lambda \), \( Z \) is the ionization state, and \( n_e \) is the electron density.

Expanding the distribution function in Legendre function gives

\[
\begin{align*}
\frac{\partial f_0}{\partial t} - eE \frac{1}{m_e} \frac{\partial}{\partial v} (v^2 f_1) &= C_0, \\
\frac{\partial f_1}{\partial t} - eE \left( \frac{\partial f_0}{\partial v} + \frac{2}{5v^3} \frac{\partial}{\partial v} (v^3 f_2) \right) &= -\frac{2A}{v^3} f_1 + C_1.
\end{align*}
\]

(2.31)

Here, the expansion was truncated at \( f_2 \) implying that \( v_{osc}^2 / v_{th}^2 \ll 1 \). Considering that the time dependence of \( f_1 \) is mainly high-frequency, \( C_0 \) will be evaluated for \( f_0 \) giving,

\[
\frac{\partial f_0}{\partial t} = \frac{2A}{v^3} \frac{1}{3} \frac{1}{v^2} \frac{\partial}{\partial v} \left( \frac{g \frac{\partial f_0}{\partial v}}{v} \right) + C_0(f_0)
\]

(2.32)

with

\[
g(v) = \frac{1}{[1 + \omega^{-2} \tau_{ei}^{-2}(v)](1 + v_{th}^6 / v^6)}
\]

(2.33)

where \( 1 / \tau_{ei}(v) = 2A / v^3 \) is the electron scattering rate and \( v_{th} \) is defined such that \( \omega \tau_{ei}(v_{th}) = 1 \). The factor \( g(v) \) accounts for the differing response of fast and slow electrons. If the collisions are treated as a perturbation of the entire distribution, then \( g(v) = 1 \).

This can be solved either by Laplace transform [73] or with the self similarity \( \xi = 3v^5 / Av_{th}^2 \) [74] to find the distribution function,

\[
f_m(v) = C_m \exp \left[ - \left( \frac{v}{a_m v_{th}} \right)^m \right],
\]

(2.34)

where

\[
C_m = \frac{1}{4\pi \Gamma(3/m)(a_m v_{th})^3}
\]

(2.35)
and

\[ a_m = \frac{\Gamma(3/m)}{\Gamma(5/m)} \]  

(2.36)

The result is a super-Gaussian shape, with super-Gaussian order \( m \). Langdon[22] showed that a maximum order of 5 is obtained if inverse bremsstrahlung collisions are completely dominant. The normalization constant also allows this distribution function to maintain the physical meaning of the first three moments, preserving the definitions of density, flow, and temperature. This shape is consistent with the physical interpretation of the slow electrons being heated to velocities near the thermal velocity, as this results in the flattening and widening of the peak of the distribution.

Fokker-Planck simulations, performed by Matte et al., showed that the degree to which the distribution is flattened depends on the relative strength of inverse bremsstrahlung heating and thermalization due to electron-electron collisions, which was parameterized as \( \alpha = Zv_{osc}^2/v_{th}^2[23] \). \( v_{osc} = eE_0/m_e\omega \) is the velocity of electrons oscillating in the laser field. This enabled the development of a heuristic scaling law relating the Langdon parameter (\( \alpha \)) to the super-gaussian order,

\[ m(\alpha) = 2 + \frac{3}{1 + 1.66/\alpha^{0.724}}. \]  

(2.37)

Other Fokker-Planck simulations showed an increased range of validity for this model. While this derivation assumed \( v_{osc}^2/v_{th}^2 \ll 1 \), the result is valid even when \( v_{osc}^2/v_{th}^2 \approx 1[74] \). Another important finding was that inverse bremsstrahlung heating only induces a small anisotropy. The anisotropic component of the pressure tensor is only \( \frac{1}{45} v_{osc}^2/v_{th}^2 \) [75].

While only inverse bremsstrahlung heating will be discussed here, this distribution
function can also arise as a result of ion turbulence, as shown by C. T. Dum[34]. Hence the alternate name Dum-Langdon-Matte distribution.

While there is consensus on the shape and underlying physics of a super-Gaussian distribution function, questions remain about the high-energy electrons. Some of the physics considered has been the relatively small electron-ion collision rate of the fast electrons [36], thermal transport[24], parametric instabilities[23], and electron-electron collisions[32]. All of these proposed mechanisms result in a Maxwellian tail, but often with a different temperature than the bulk distribution. The various formalisms also differ in where the distribution transitions from super-Gaussian to Maxwellian and the shape of the transition region.

One model of particular relevance to this work reexamined the effects of the small anisotropy in inverse bremsstrahlung[32]. This anisotropy exists because the polarization of the laser gives a direction to the initial oscillations of the electrons, resulting in a modified collision operator. This leads to an altered tail in the isotropic distribution function,

\[
f(x) = A \exp \left[ \frac{-x}{x_0(n(x))} \right]^{n(x)}
\]

\[
x_0 = \sqrt{\frac{3 \Gamma(3/n)}{\Gamma(5/n)}} \quad n = 2 + \frac{m-2}{1 + (x/x*)^9}
\]

\[
x^* = Z^{1/2(m-1)} \left[ \frac{3 \Gamma(3/n)}{\Gamma(5/n)} \right]^{m/4(m-1)}
\]

(2.38)

In this form of the distribution function, \( m \) is still the conventional super-Gaussian order, as for small \( x \) the distribution will have a super-Gaussian shape with the order \( m. n \) handles the transition from super-Gaussian to Gaussian. The transition point \( x^* \) is dependent on the ionization state, since that impacts the relative strength of electron-ion collisions and electron-electron collisions.
2.2.3 Other non-Maxwellian distribution functions

There are many other non-Maxwellian distribution functions known to exist in plasmas. Most of them are unstable and will decay back to a Maxwellian but there is potential for some to be sustained as long as there is a significant source or sink of energy.

Possibly the most commonly used and observed type of non-Maxwellian distributions are the bi-Maxwellian distributions. These can take the form of a Maxwellian with different temperatures in two spatial dimensions, a common result in the presence of a magnetic field[76, 77]. Or there can be two distinct population present with different apparent temperatures due to heating non-uniformity[78]. Similarly there many other distributions including bump-on-tail (from a beam in a plasma)[79, 80] and two stream[81, 82] that can be described as a sum of Maxwellians. While these are sometimes thought of as Maxwellian or modified Maxwellians, they can alter the damping and resonance of plasma waves.

Other more exotic distributions, such delta-function-like distribution resulting from field ionization[63, 83] or the complicated spectra of non-linear waves[84, 85], tend to be treated more kinetically. This ensures the consequences of the distribution function are treated self-consistently but is computationally expensive and is often limited to problems with a short time-scale.

As diagnostic techniques become better, non-Maxwellian effects are becoming more apparent and a better candidate for explaining the complex interaction occurring in plasma experiments. Consequently, there has been an increased interest in fast ions (magnetically confined plasmas), non-Maxwellian distribution in the solar wind, and hot ions escaping the fuel in inertial confinement fusion.
2.3 Waves in a plasma

The most common approach to describing plasmas and their behavior is through the various waves that can propagate through them. By solving wave equations it is possible to understand how energy will be absorbed, reflected, carried or transformed. The waves in a plasma can be found using the moments of the Vlasov equation (Eq. 2.7) or by directly interrogating the Vlasov equation.

2.3.1 Waves in the fluid description

The fluid description of a plasma uses a set of equations that are analogous to the equations for incompressible fluids. Integrating the Vlasov equation with respect to velocity gives the continuity equation, which describes the conservation of particles,

\[
\frac{\partial n_q}{\partial t} + \nabla (n_q v_q) = 0.
\] (2.39)

Taking the first moment of the Vlasov equation with respect to velocity yields the force equation, which describes the momentum conservation in the plasma,

\[
\frac{\partial}{\partial t}(n_q v_q) + \nabla (n_q v v) = \frac{q}{m} \left( E + \frac{v_q}{c} \times B \right) n_q.
\] (2.40)

As is seen with the first two equations, each moment equation depends on the next, known as the closure problem. One way to close this system, is by truncating the series with,

\[
\nabla (n_q v v) = \frac{1}{m_q} \nabla P_q + n_q v_q v_q,
\] (2.41)

reducing the tensor \( v v \) to pressure and the average velocity \( v_q \). The pressure can be
described by one of the common thermodynamic relations. Such as the adiabatic approximation where, $\nabla P_q = 3T_q \nabla n_q$.

These equations can be solved for plane wave solutions where only the electrons move by considering small perturbations of the density, velocity, and electric field,

\[
\begin{align*}
    n &= n_0 + n_1 e^{-i\omega t + ikx}, \\
    v &= v_1 e^{-i\omega t + ikx}, \\
    E &= E_1 e^{-i\omega t + ikx}.
\end{align*}
\]

Substituting into the moment equations and Gauss’ law, $\nabla E = 4\pi \rho$ gives,

\[
\begin{align*}
    n_1(-i\omega) + n_0 v_1(ik) &= 0, \\
    m_e n_0 v_1(-i\omega) &= -3T_e n_1(ik) - e_n E_1, \quad (2.43) \\
    ik E &= -4\pi e n_1.
\end{align*}
\]

solving for $\omega$, we find the classic Bohm-Gross dispersion relation for electron-plasma or Langmuir waves,

\[
\omega^2 = \omega^2_{pe} + 3k^2 v^2_{th},
\]

where $\omega^2_{pe} = \frac{4\pi e^2 n_e}{m_e}$ is the electron plasma frequency.

The same procedure can be followed considering the motion of the electrons and ions. The continuity and momentum equations will become,

\[
\begin{align*}
    n_{i1}(i\omega) &= n_{i0} v_{i1}(ik) \\
    n_{e1}(i\omega) &= n_{e0} v_{e1}(ik) \\
    m_i n_{i0} v_{i1}(-i\omega) &= -\gamma_i T_i n_{i1}(ik) - e Z n_{i0} E_1 = -\gamma_e T_e n_{e1}(ik) - e n_{e0} E_1.
\end{align*}
\]

Here, $m_e$ has been taken to zero since it is much smaller than $m_i$. The densities have
also been related as $n_e = Zn_i$, where $Z$ is the ionization state. $\gamma_{e,i}$ are the electron and ion degrees of freedom, also known as the adiabatic index or polytrope index. The momentum equations can be rearranged to give

$$\frac{\omega}{k}\frac{n_{i0}}{n_{i1}}v_{i1} = \frac{\omega^2}{k^2} = \frac{\gamma_i T_i + \gamma_e ZT_e}{m_i}$$  \hspace{1cm} (2.47)$$

where the density ratio is found from the continuity equation. This is the dispersion relation for ion-acoustic waves. These are sound-like waves that are propagated by electrostatic interactions instead of the collisional interaction of sound waves. This electrostatic interaction means waves exist even for negligible ion temperature.

2.3.2 Waves in the kinetic description

In the fluid description, the Fourier treatment of waves was applied to the momentum equations. It is possible to directly apply this to treatment to the Vlasov equation and Poisson’s equation. The Fourier transformed equations are,

$$-\omega \tilde{f} + ikv_z \tilde{f} + \frac{qs}{m_s} \Phi \frac{\partial f_0}{\partial v_z} = 0$$ \hspace{1cm} (2.48)$$

and

$$k^2 \Phi = -4\pi q_s \int_{-\infty}^{\infty} \tilde{f} d^3v$$ \hspace{1cm} (2.49)$$

where the tilde denotes the transformed variable. The first equation can be solved for $\tilde{f}$ and substituted into the second equation to yield the general dispersion relation,

$$D_s(k, \omega) = 1 - \frac{\omega_{ps}^2}{k^2} \int_{-\infty}^{\infty} \frac{dF_s0/\partial v_z}{(v_z - \omega/k)}dv_z.$$  \hspace{1cm} (2.50)$$
Here, $F_{0\theta}$ is the distribution projected onto the $\hat{z}$ direction. Since there is freedom of aligning the Cartesian grid, it has been defined so $\hat{z} = \hat{k}$.

Unlike in the fluid case, the kinetic dispersion relation has explicit dependence on the distribution function. If the distribution function is taken to be Maxwellian and the damping is weak, then the kinetic dispersion relation reduces to the fluid dispersion relation,

$$D_s(k, \omega) = 1 - \frac{\omega_{ps}^2}{\omega^2 - \gamma_s C_s^2 k^2}.$$  \hspace{1cm} (2.51)

This illustrates the implicit assumption in the fluid description that the distribution function is Maxwellian. This assumption arises in the closure of the moment equations. Since the moment equations were closed with a thermodynamic description of the pressure thermal equilibrium, and therefore a given relation between temperature and velocity, is imposed.

### 2.3.3 Effect of non-Maxwellian distribution functions

From the kinetic dispersion relation (Eq. 2.50), it can be seen that significant changes to the distribution function will modify the dispersion relation for the waves. This means both a modified resonance and modified damping. It is possible to write new dispersion relations for certain waves and certain distribution function. As an example, Afeyan et al.\cite{25} found the dispersion relation for the ion-acoustic waves where the electrons have a super-Gaussian distribution function,

$$\frac{\omega^2}{k^2} = \frac{3\Gamma^2(3/m)}{\Gamma(1/m)\Gamma(5/m)} \frac{\gamma_i T_i + \gamma_e ZT_e}{m_i}$$  \hspace{1cm} (2.52)

Generally, this is a problem that must be treated numerically.

This information about the waves is contained in the dielectric function or suscep-
tibility,

\[ \varepsilon = 1 + \chi_e + \chi_i \]

\[ \chi_{e,i}(\mathbf{k}, \omega) = \int_{-\infty}^{\infty} d\mathbf{v} \frac{4\pi e^2 n_{e,i}}{m_{e,i} k^2} \frac{\mathbf{k} \cdot \frac{\partial f_{e,i}}{\partial \mathbf{v}}}{\omega - \mathbf{k} \cdot \mathbf{v} - i\gamma'} \] (2.53)

which has the same dependence on the distribution function.
Chapter 3

Thomson Scattering

This chapter gives an overview of Thomson scattering and its application to measuring plasma conditions and the electron distribution function. The goal of this chapter is to give a physical and mathematical description of Thomson scattering and introduce the newly developed angularly resolved Thomson-scattering technique. In Section 3.1 the fundamentals of Thomson scattering are discussed along with discussion of the non-collective Thomson-scattering regime. Section 3.2 describes the case where the electron motion is correlated leading to collective Thomson scattering. Section 3.3 shows how non-Maxwellian distribution functions effect the spectra and provides a detailed discussion of its effect on inferred plasma conditions. Section 3.4 describes the novel angularly resolved Thomson scattering technique and its advantages.

3.1 Non-collective Thomson scattering

Thomson scattering is the elastic scattering of photons off free electrons. In other words it is the low energy version of Compton scattering, where the electron recoil is ignored since the photon energy is much less then the rest mass of the electron ($2\hbar\omega_0 \ll m_e c^2$). This scattering process results in a Doppler shifted photon and an electron whose mo-
momentum is unchanged by the interaction. For a plasma, where there is an ensemble of electrons, the scattered photons contain information about the number and velocities of all the electrons.

Photons are supplied by a probing laser beam. The power scattered to a location $R$, within the solid angle $d\Omega$, by a plasma can be written,

$$P_s(R, \omega_s) d\omega_s d\Omega = \frac{P_0 r_0^2}{A 2\pi} \left| \hat{s} \times (\hat{s} \times \hat{E}_0) \right|^2 \left(1 + 2 \frac{\omega}{\omega_0}\right) N S(k, \omega). \quad (3.1)$$

Here the first term describes the energy density of the probe beam with the probe power ($P_0$) and the cross sectional area of the beam ($A$). The classical electron radius ($r_0 = 2.8179 \times 10^{-13}$ cm) comes from the Poynting vector. The second term is the scattering efficiency, based on the observation direction ($\hat{s}$) and probe polarization direction ($\hat{E}_0$), since Thomson scattering is a dipole scattering process the emission is maximal when observing perpendicular to the polarization. The third term is the first order relativistic correction, which accounts for the “relativistic headlight” and $v \times B$ effects for electrons up to $\sim 5$ keV [86]. The final term is the number of electrons in the scattering volume $N$ and the spectral density function, which describes the electron density fluctuation spectrum.

$$S(k, \omega) = \lim_{\gamma \to 0, V \to \infty} \frac{2\gamma}{V} \left\langle \left| \frac{n_e(k, \omega - i\gamma)}{n_e0} \right|^2 \right\rangle. \quad (3.2)$$

This description of spectral density function shows the relation to density perturbations, but a more useful form can be found. The spectral density function can be solved in two regimes the non-collective and collective. In the non-collective regime ($1/k\lambda_D < 1$), the scale length is smaller than the electron Debye length ($\lambda_D$), the clas-
sical shielding length. At this scale length the plasma cannot support waves, so the fluctuation spectrum can be simply related to the distribution function.

\[ S(k, \omega) = \frac{2\pi}{k} f_e \left( \frac{\omega}{k} \right) \]  

Figure 3.1: Non-collective Thomson-scattering spectra from plasmas with electron density $10^{13}$ cm$^{-3}$ and electron temperatures 200 eV (red) and 400 eV (blue).

Figure 3.1 shows the Thomson-scattered spectrum for two different temperatures. The width of the spectrum can be related to electron temperature through the distribution function. This allows the temperature to be measured in experimental data by matching the width of the scattered spectrum. The density can also be measured from the amplitude of the spectrum. This requires the diagnostic to be absolutely calibrated.

Non-collective Thomson scattering was first used experimentally in 1963 [87] and remains an important tool for measuring temperatures especially in the magnetic fusion community [52, 66]. The relatively long integration times (can be microseconds long) available in magnetically confined experiments allow the low Thomson-scattering efficiency to be overcome. This techniques is also well suited for the relatively low density
and temperatures ($n_e \sim 10^{13}$ cm$^{-3}$, $T_e \sim 100$ eV), which give larger electron Debye lengths.

Another important application of non-collective Thomson scattering is x-ray Thomson scattering. Here, the extremely short wavelengths of the x-ray probe keeps experiments in the non-collective regime. This technique can be used experiments where the plasma is opaque to optical light, allowing applications in warm dense matter and other high-energy density sciences [56, 57].

### 3.2 Collective Thomson scattering

In the collective regime, the scattering scale length is larger than the electron Debye length ($1/k\lambda_D > 1$) and correlated motion of electrons must be considered. Correlated electron motion, or thermal fluctuations, are heavily damped at the smaller scale length. The scattering off correlated electrons interferes constructively resulting in peaks in the spectrum that correspond to thermal fluctuations in the plasma. This transition occurs around $1 > ZT_e/T_i$ for ion-acoustic waves. Here, the spectral density function can be written

\[
S(k, \omega) = \frac{2\pi}{k} \left| 1 - \chi_e \frac{\omega^2}{k^2} \right|^2 f_{e0}(\omega/k) + \frac{2\pi Z}{k} \left| \chi_e \chi_i \frac{\omega^2}{k^2} \right|^2 f_{i0}(\omega/k) \tag{3.4}
\]

where $f_{e0}$ and $f_{i0}$ are the one-dimensional (1-D) electron and ion distribution functions, $Z$ is the ionization state, and $\chi_{e,i}$ are the electron and ion susceptibilities. This collisionless form factor assumes there is no magnetic field, the measurement is non-perturbative, and ignores interactions of three or more bodies. The real and imaginary
parts of the ion and electron susceptibility are given by[88],

\[\chi_{Re} = -\frac{Z}{k^2\lambda_{Di,e}^2} \mathcal{R} \int_{-\infty}^{\infty} \frac{\partial f_{i,e}/\partial x'}{x' - x} dx',\]

\[\chi_{Im} = -\frac{Z\pi}{k^2\lambda_{Di,e}^2} \frac{\partial f_{i,e}}{\partial x'} \bigg|_x.\]

For the electron susceptibility \(Z = 1\). The pole in the real component of the susceptibility gives rise to the peaks mentioned earlier, when the imaginary part is sufficiently small.

![Figure 3.2: Collective Thomson-scattering spectra from plasmas with (blue) \(n_e = 2 \times 10^{19} \text{ cm}^{-3}\) and \(T_e = 300 \text{ eV}\) and (red) \(n_e = 4 \times 10^{19} \text{ cm}^{-3}\) and \(T_e = 400 \text{ eV}\). Ion-acoustic wave features have been cut off (actual amplitude \(\sim 150\)) in order to visualize the electron-plasma wave features.](E29561J1)

Figure 3.2 shows the Thomson-scattered spectrum for two different temperatures and densities. Two sets of peaks can be seen in the spectrum. The outer peaks are associated with electron-plasma waves, while the inner peaks are associated with ion-acoustic waves. For low frequency (\(\omega\)) the second term in Eq. 3.4 dominates and the resonance follows the dispersion relation of ion-acoustic waves. The peak separation for these inner peaks can be related to \(\sqrt{\frac{ZT_e}{m_i}}\) while their width is dependent on
With some knowledge about the ionization state and the temperature equilibration, ion-acoustic Thomson scattering is a useful tool for temperature measurements in many laboratory experiments. This feature can be orders of magnitude brighter than the electron-plasma wave feature or non-collective scattering, making ion-acoustic Thomson scattering the easiest version to measure.

At high frequency ($\omega$) the first term in Eq. 3.4 dominates and the resonance follows the dispersion relation for electron-plasma waves. The peak separation of these outer electron-plasma wave features is primarily based on density. This can be seen with the Bohm-Gross dispersion relation (Eq. 2.44) where the lowest order term in temperature is simply $\omega_{pe}$, which is proportional to electron density. The width of these peaks reflects the damping of the wave, and arises from the imaginary part of the electron susceptibility, which depends on the slope of the distribution function. The temperature dictates the width of the distribution and therefore influences the slope at a particular $k\lambda_D$. This width can be physically interpreted as Landau damping of the thermal fluctuation.

Early work using the electron-plasma wave feature mainly focused on the peak separation as a way to measure electron density [19]. More recently experiments measuring the entire collective spectrum have matched the spectral shape in order to derive electron temperature and density [86]. When possible the electron and ion features have been measured simultaneously allowing measurements to be made of electron density, electron temperature, ion temperature, and ionization with a single technique. This generally requires separate diagnostics to measure the electron and ion features due to their signal strengths and the required spectral resolution.
3.3 Effects of non-Maxwellian distribution functions

Revisiting the spectral density function (Eq. 3.4), the dependence of the spectrum on plasma conditions can be seen to originate from the dependence on the electron and ion distribution functions. The standard interpretations of peak separations and widths ascribe physical intuition to parts of the spectrum, but rely on the standard dispersion relations. As seen in Sec. 2.3 these dispersion relations inherently assume the distribution function is Maxwellian.

Thomson scattering is sensitive to more than just the bulk properties, it can measure the distributions themselves. Here, the focus will be on the electron distribution function and how it can be measured by focusing on the high-frequency spectrum.

3.3.1 Changes to the spectral shape

![Figure 3.3](image.png)

Figure 3.3: Theoretical spectra for (a) non-collective and (b) collective Thomson scattering from a Maxwellian plasma (blue) and a non-Maxwellian plasma with super-Gaussian order 5 (red). The circles represent synthetic data generated from the theoretical curves. These spectra are for plasmas with (a) $n_e = 1 \times 10^{13}$ cm$^{-3}$ and $T_e = 400$ eV and (b) $n_e = 4 \times 10^{19}$ cm$^{-3}$ and $T_e = 400$ eV.

Figure 3.3 shows how the non-collective and collective spectra change in response to the distribution function. These specific changes are a consequence of the type of
Figure 3.4: Distribution function (a) and its derivative (b) with respect to $x$. A Maxwellian is shown in blue, while a super-Gaussian of order 5 is shown in red. A super-Gaussian of order 5 calculated when including Maxwellian tails (Eq. 2.2.2) is shown in orange. This is likely going to become part of the the previous figure as part c/d.

The collective peaks show three modifications going from a Maxwellian to super-Gaussian electron distribution function: (1) Thinner peaks, (2) Steeper falling edge to the outside of the peak, (3) enhanced scattering in the central region between the peaks.
The super-Gaussian electron distribution function has a steeper slope above $2v_{th}$, which leads to less Landau damping and results in the thinner peaks and a steeper falling edge. The increased central region is due to the increased number of electrons in the region of $1 - 2v_{th}$.

While both regimes are sensitive to the electron distribution function, experimental limitations hamper their practicality for measuring the distribution function. The small electron scattering cross section restricts non-collective Thomson scattering to measuring the bulk of the distribution function where the scattering exceeds thermal background. Laser heating, filamentation and other laser-plasma instabilities, caused by an intense probe beam, typically constrain measurements to a couple orders of magnitude. Collective Thomson scattering has an improved signal-to-noise but is less sensitive the the distribution function away from the peaks of the spectrum. These regions away from the peaks also tend to be noisy as the signal is low. These restriction have limited previous distribution function measurements to the bulk of the distribution function [55].

### 3.3.2 Changes to the inferred conditions

Previous Thomson-scattering experiments [37, 42–50] were limited in their ability to measure the complete electron-plasma wave feature with high signal to noise. As a result the features of a non-Maxwellian distribution function may have gone unnoticed or ignored. As will be shown in this section, the assumption that the distribution function is Maxwellian can lead to significant errors in the inferred plasma conditions. These errors can be mitigated by including the true distribution function in the accessible fit space.

To determine the error in inferred plasma conditions synthetic Thomson scattering
spectra were fit with the collisionless spectral density function (Eq. 3.4). The synthetic spectra (Fig. 3.3), were produced by down-sampling a calculated spectrum to match the spectral resolution (2 nm/resolution unit) of a typical Thomson-scattering diagnostic [89] and scaled so the final signal-to-noise ratio would be 10 at the peak of the signal. Poisson noise was added to each resolution unit by drawing a random number from a Poisson distribution whose mean was the original number of counts in that bin. Adding noise in this way approximates a realistic diagnostic signal; any further sources of noise or reduction in the signal-to-noise ratio would increase the uncertainty in the analysis, but have no significant effect on the errors found here. Synthetic data for a variety of plasma conditions and super-Gaussian distribution functions (Eq. 2.34) was generated in this manner.

As mentioned in Section 3.2, the standard approach to determining plasma conditions from Thomson-scattering data is to fit the spectrum with the collisionless spectral density function (Eq. 3.4). The distribution function was assumed to be Maxwellian and a 2-D least-squares statistical analysis was performed across the simulated spectrum by varying the electron density ($n_e$) and temperature ($T_e$). The sum of the squared deviation from the data was used as a fit metric ($\Delta^2$), thereby treating each point equally. This fit metric was minimized using an interior-point algorithm [90].

To determine the uncertainty in the fit parameters, each synthetic data set was fit multiple times and each fit was performed with a new realization of the Poisson noise and a new random initial guess. The distribution of the fit parameters, from the set of fits, was used to determine the uncertainty in the fit parameters (Fig 3.5 inset). The average value of the parameter and one standard deviation are reported as the inferred parameter and its uncertainty.
Errors in super-Gaussian plasmas

Figure 3.5: Percent error in (a,c) temperature and (b,d) density as a function of the normalized phase velocity ($\alpha$) when the fit model assumes a Maxwellian electron distribution function and the true electron distribution function is (a,b) super-Gaussian or (c,d) super-Gaussian with a Maxwellian tail (Eq. 2.2.2). The absolute difference between the inferred and actual parameter dividend by the actual parameter (percent error) is calculated for a range of phase velocities. The values for 4 different super-Gaussian orders are plotted in different colors with error bars that represent the standard deviation of 100 fits.

Figure 3.5 (a,b) shows that the inferred temperature and density can differ from the actual values by 50% and 30%, respectively. As expected, there is no error when fitting Maxwellian data with Maxwellian theory, but this provides a benchmark for the analy-
sis and quantifies the general uncertainties in the fitting process. For non-Maxwellian plasmas \((m > 2)\), the percent error in temperature increases as a function of the super-Gaussian order and normalized phase velocity \((\alpha = v_\phi / v_{th})\). The statistical uncertainty in the inferred parameters [error bars in Fig. 3.5 (a,b)] remains small. This reflects the strength of the fits and shows that the error is due to changes in the distribution functions not variability in fitting. The same trend of increasing error as a function of super-Gaussian order is calculated for density. However, the error decreases or stays relatively constant as a function of the scattering parameter.

The errors in the electron temperatures result from differences in the Maxwellian and non-Maxwellian electron distribution functions (Fig. 3.4) at velocities around the phase velocities of the electron-plasma waves. Over the range of phase velocities studied \((\alpha \simeq 2 \rightarrow 4)\), the super-Gaussian distributions have fewer electrons and a smaller slope than the Maxwellian electron distribution functions. This consequently reduces the Landau damping of the probed fluctuations, resulting in narrower peaks in the synthetic Thomson-scattering spectra. To compensate for the narrower peaks, the spectra calculated with the Maxwellian electron distribution functions (fits) have a reduced temperature (i.e. smaller Landau damping) than was used to generate the synthetic spectrum. The reduced electron temperature requires an increased electron density to maintain the resonant (peak) location of the electron-plasma wave features. These trade-offs are illustrated in the Bohm-Gross dispersion relation, \(\omega^2 = \omega_{pe}^2 + 3k^2v_{th}^2\).

As the phase velocities move farther into the tail of the electron distribution function, Landau damping decreases faster in the non-Maxwellian than in the Maxwellian calculations. This results in increased error as a function of phase velocity and super-Gaussian order, eventually leveling off as the damping rate in all cases approaches zero. The downward trend in density error with increasing phase velocity is also due to this apparent temperature. As the electron temperature decreases, a smaller change in elec-
tron density is required to maintain the resonant (peak) location of the electron-plasma wave features.

These errors of 50% in temperature and 30% in density are for extreme changes to the electron distribution function, but even for small changes in the shape of the distribution function, the errors in temperature and density are larger than the statistical uncertainty of $\sim 5\%$ that is typically reported [54, 91] and can be a limiting factor in determining plasma conditions.

**Errors in super-Gaussian with Maxwellian tail plasmas**

As discussed in Section 2.2.2, both Fourkal et al. [32] and Brunner et al. [24] have proposed that the distribution function is super-Gaussian only at small velocities and trends toward Maxwellian at large velocities. To understand the impact of the tails on inferred plasma conditions Fig. 3.5 (c,d) repeats the analysis shown in Fig. 3.5 (a,b), but includes the modified non-Maxwellian electron distribution functions. The errors are reduced in this case, but when the phase velocities are small, errors of up to 30% in temperature and 13% in density are found. The trend in error with super-Gaussian order is preserved, but the trend with phase velocity is reversed. Density errors still show the trend of increased error with super-Gaussian order, but the error rapidly decreases with increasing phase velocity until $\alpha \simeq 2$, where the error effectively drops to zero.

The trends in error are again due to deviations of the true distribution functions from a Maxwellian electron distribution function. For the calculations presented in Fig. 3.5 (c,d), the effective ionization state ($Z$) was taken to be 1. This results in a small range for $x^* \simeq 1.57 \rightarrow 1.60$ when $m = 3 \rightarrow 5$. The reduction in errors calculated when including Maxwellian tails (Eq. 2.2.2) is a result of a convergence between the Maxwellian and non-Maxwellian electron distribution functions around the phase velocity of the electron-plasma wave. In the case where the distribution function has
Maxwellian tails, decreasing super-Gaussian order or increasing phase velocities results in less local deviation from a Maxwellian electron distribution function, at the phase velocity of the probed fluctuations. Therefore, the resonant frequency and width of the electron-plasma wave features are closer to the expected quantities, reducing the inferred errors. This trend sheds light on what might happen for arbitrary electron distribution functions: the greater the deviation from Maxwellian, especially in slope, the greater the possibility for error in inferred plasma conditions.

Figure 3.5 illustrates a limited range of phase velocities. For smaller phase velocities, the distribution function has such a strong effect on the scattered spectra that it is not possible to reproduce key features of the Thomson-scattered spectra with a Maxwellian model. For larger phase velocities, the chosen spectral resolution (2 nm/resolution element) is on the order of the width of the peak.

The two cases presented, Fig. 3.5 (a,b) and (c,d), represent the limiting cases for the effects of adding Maxwellian tails to the super-Gaussian electron distribution functions. As the ionization state is increased, the errors will transition from those presented in Fig. 3.5 (c,d) to those presented in Fig. 3.5 (a,b). However, it is worth noting that this is the result only for this formulation of super-Gaussian electron distribution functions with Maxwellian tails, and previous work [30, 35] has shown good agreement with a purely super-Gaussian distribution function.

**Utilization of the complete high-frequency spectrum**

It is common to infer plasma parameters from a single electron-plasma feature [54]. In laser-produced plasmas there can be significant light scattered by other means at frequencies near one of the electron-plasma wave resonances [92], reducing the signal-to-noise ratio of the feature. However, in conditions where both electron-plasma wave features can be measured, the Thomson-scattered spectrum provides more information
Figure 3.6: Region where fits to the Stokes and anti-Stokes peaks yield different plasma conditions as a function of the super-Gaussian order. Error bars represent the uncertainty due to simulation resolution. Within the shaded region, different plasma conditions (namely electron temperature) are determined by fitting to synthetic Stokes and anti-Stokes shifted spectra, with a model that assumes a Maxwellian electron distribution function.

about the shape of the distribution function. Figure 3.6 shows the conditions at which fits, that assume a Maxwellian electron distribution function, to the Stokes and anti-Stokes electron-plasma wave features yield different plasma conditions (red shaded region). The Stokes and anti-Stokes shifted electron-plasma wave features were fit independently, and determined to yield the same plasma conditions when the temperature was in agreement to one standard deviation of 100 fits. Here, temperature was used because inferred density was found to be nearly the same between the two fits. Below the points, the two fits yield different temperatures. This indicates missing physics from the fit model; it is therefore possible to identify the distribution function as non-Maxwellian, but without further analysis it is not possible to identify the shape of the
distribution function. Above the points, both fits yield the same plasma conditions but with the errors discussed in the previous sections.

The behavior in Fig. 3.6 is due to the competition between super-Gaussian order and the impact of the spectral resolution on the inferred temperature. For large super-Gaussian order the two peaks give the same temperature once the damping becomes so small that the width of the peaks are not resolved by the instrument. This leads to a decrease in the boundary, $\alpha$, with super-Gaussian order, as increasing the super-Gaussian order decreases the damping on the wave (Fig. 3.4). When the super-Gaussian order is small, it becomes impossible to see the difference in the damping between the two peaks because the change in the damping, along with the resulting inferred temperature, falls below the precision of the measurement due to the imposed limited spectral resolution. Because this curve is highly dependent on spectral resolution, only the overall trend is meaningful.

Figure 3.7 shows that a global minimum in the fit metric can be found by changing the super-Gaussian order in the analysis. There is a well-defined minimum in the fit metric when data generated with $m = 4$ is fit with $m = 2$ [Fig. 3.7 (a, e)]. However, the fit metric gets smaller as the super-Gaussian order goes to the correct value (third column). This improvement is due to the shape of the spectrum not just the width and peak location, which can be seen in the top images [Fig. 3.7 (a, b, c, d)]. As the super-Gaussian order goes to the correct solution ($m = 4$), the slope on the small wavelength side of the peak steepens to match the data and the flatter region on the longer wavelength side rises to match the data. Additionally, the images on the bottom show the location of the minimum shifting toward the true values.

Figure 3.7 provides insight into the error bars in Fig 3.5. Multiple local minima can be seen for each super-Gaussian order in Fig. 3.7. While none of the local minima in Fig. 3.7 are low enough for the gradient decent minimizer to stop, this is not the case at
Figure 3.7: Images of the fit-metric as a function of temperature and density at $m = 2, 3, 4,$ and 5. The value of the fit metric is shown on a logarithmic scale with the smallest value representing the “best” fit. Each image of the fit-metric space is paired with a lineout of the data and the fit that corresponds to the minimum in the fit space. The data was generated with electron density $10^{20}$ electrons/cm$^3$, temperature of 1.2 keV, and $m = 4.0$ ($\alpha = 2.33$).

other scattering parameters. The multiple minima increase the uncertainty of the fit and correspondingly increase the error-bars. This effect occurs when the spectral feature becomes smaller than 3 resolution elements.

Allowing the distribution function, via super-Gaussian order, to change as a parameter in the fitting routine can eliminate model-dependent errors with minimally increased statistical uncertainty. Generally, when the super-Gaussian order of the distribution function is allowed to change, errors fall below 10%. There is a corresponding increase in the uncertainties due to the added fit parameter, and uncertainties go from a few percent to $\sim 10\%$. This is also the case for the distribution functions with Maxwellian tails, and it does not matter if one or both electron-plasma wave features are used.

Figure 3.8 shows that including the true distribution function in the available fit
Figure 3.8: Synthetic scattered spectrum for electron density $10^{20}$ electrons/cm$^3$, temperature of 2 keV, and $m = 5.0$. A fit to the data using a non-Maxwellian model (red) and a fit using the standard Maxwellian model (yellow) are shown.

Space allows for elements of the synthetic spectrum to be reproduced that were not possible using Maxwellian analysis. The Maxwellian analysis is not capable of achieving the correct peak width or slope around $v/v_{th} = 2.5$. Maxwellian analysis under-predicts the signal in the range from $0 < v/v_{th} < 1.5$. When super-Gaussian order is added as a fit parameter, it becomes possible to match these spectral features for smaller phase velocities where they become more prominent. This is the case in Fig. 3.8 where $\alpha = 1.8$ putting it to the left of the data in Fig. 3.5.

Assuming Maxwellian electron distribution functions in Thomson-scattering analysis was shown to produce significant errors in the inferred plasma temperature and density. This error was shown to vary with with the distribution function around the phase velocity of the probed fluctuation, as seen with the variation based on distribution function and normalized phase velocity. Including the correct distribution function in accessible fit space, as done here with super-Gaussian order, can eliminate model-dependent errors with minimally increased statistical uncertainty.
Because the errors vary with the local distribution function it is expected that other changes to the distribution function (non-Maxwellians other than super-Gaussian) will also produce errors in the inferred parameters. Additionally in this study it was possible to match the true distribution function because it was known a priori, but in an experiment it is not as simple to distinguish between different distribution functions.

### 3.4 Angularly resolved Thomson scattering

As was seen in the previous sections, it is possible to determine the distribution function from the collective or non-collective Thomson scattering spectrum. However, there are multiple constraints that make a more robust technique advantageous. This is achieved by angularly resolving the Thomson-scattering spectrum.

Angularly resolved Thomson scattering can measure the electron distribution function over many orders of magnitude by simultaneously collecting light scattered from fluctuations with a large range of wavevectors. The probed wavevector is related to the scattering angle by

\[ k^2 = k_0^2 + k_s^2 - 2k_0 k_s \cos \theta, \]

where the scattering angle (\( \theta \)) is angle between the probe beam (\( k_0 = \frac{2\pi}{\lambda_0} \)) and the scattering directions (\( k_s = \frac{2\pi}{\lambda_s} \)). \( \lambda_0 \) and \( \lambda_s \) are the probe laser wavelength and scattered wavelength in the plasma, respectively. Therefore, the Thomson-scattering spectrum dispersed in scattering angle measures a range of \( k \)-vectors or a range of \( 1/k\lambda_D \). Each of the angles probed yields a spectrum with a different range of frequencies with optimal signal-to-noise, which correspond to different locations on the distribution function. These redundant spectra with varying optimal probing velocities enable a detailed reconstruction of the distribution function.

The probed frequencies or velocities can span a large range on the distribution function and can be targeted to elucidate specific physical processes. By measuring the distribution function over the velocity range \( 1 - 5v_{th} \) it is possible to measure the bulk and
tail of the distribution function. This removes the need to assume a shape for the bulk or tail as was required in the previous sections analysis.
Chapter 4

Diagnostics

This chapter discusses the laser systems, targets, and diagnostics used for measuring Thomson scattering. The general structure of a Thomson-scattering experiment is introduced with descriptions of various Thomson-scattering diagnostics. Section 4.1 gives a brief overview of the OMEGA and MTW laser systems where this work was conducted. The gas jet target is discussed in Section 4.2. In Section 4.3 various forms of Thomson scattering are discussed with escalating complexity. Details are given for imaging, time-resolved, and the new angularly resolved Thomson-scattering systems.

4.1 Laser systems

Experiments were performed at the Laboratory for Laser Energetics on two laser systems, the Multi-Terawatt (MTW) laser [93] and the OMEGA Laser System[94].

4.1.1 OMEGA

The OMEGA Laser System has been operational since 1995 and consists of 60 beams, split into three legs of 20 (Fig. 4.1). Two fiber lasers are used to generate the laser pulses, allowing the pulses to be temporally shaped. The beams go through a process
Figure 4.1: Schematic of the OMEGA laser system. A single oscillator is split into 3 legs, amplified, each leg is split into 5 and amplified again as the beam propagate from the center of the image to the right. At the end of the room each of the beams is split into 4 giving the final 60 beams, which propagate back down the exterior of the laser bay and are amplified 3 more times. The vacuum spacial filters (grey tubes) are supported by a structural scaffold (blue). The pass through the shield wall, and are frequency triples (color change from red to teal). Each of the beams is directed into the target chamber on the left (grey sphere surrounded by blues scaffold, a section has been cut away to show the interior).
of being amplified and split, first into 3, then 5, then 4 to achieve the 60 beams. A different fiber source can be used for one leg allowing up to 20 beams to have a different pulse shape. The beams are amplified by a series of neodymium doped glass rod and disc amplifiers pumped by flash lamps. The beams are capable of achieving a max energy of 500 J in 1 ns, but the power and delay of individual beams can be adjusted independently.

A shield wall (Fig. 4.1) separates the target area and laser bay. Prior to passing through the shield wall the beams are frequency tripled from the lasing wavelength of the nd:glass amplifiers, 1053 nm, to 351 nm. Each beam is directed into the target chamber by a set of two mirrors. Prior to entering the target chamber the beams are smoothed by polarization rotation (DPRs) and distributed phase plates (DPPs) and focused by a lens to the center of the target chamber (TCC). The phase variations added by the DPP break the beam into hundreds of small speckles, removing any large scale spatial non-uniformity from the amplifiers. The DPRs double the number of speckles by adding a small angle between the two polarization components. These combine with the spectral smoothing (SSD) supplied prior to amplification to smooth the beam profiles. SSD adds a time dependent wavelength shift to the beams causing the speckles to move from chromatic aberration as the wavelength shifts.

The beams enter the chamber in a hexagonal-pentagonal pattern similar to a soccer ball. Beams are placed symmetrically with each beam having another directly opposed. To prevent the beams from counter propagating up opposing beam lines and damaging optics, the energy per beam is limited to 200 J for under-dense plasma experiments.

Beam 25 can be redirected through one of the pentagon centered ports, P9 and frequency doubled or quadrupled, instead of the standard tripling. This beam was used as the Thomson-scattering probe.

Diagnostics on OMEGA are permanently attached to a port or inserted with one of
ten inch manipulators (TIMs). TIMs allow diagnostics to be easily inserted into the chamber and swapped from one experimental campaign to the next.

### 4.1.2 Multi-Terrawatt Laser

![Diagram of Multi-Terrawatt Laser](image)

Figure 4.2: (a) Schematic of the Multi-Terrawatt (MTW) laser. An OPCPA front-end is amplified with Nd:glass amplifiers before being recompressed. (b) Picture of the underdense plasma chamber (UDP) where experiments are conducted. Shown at atmospheric pressure with multiple access doors open. Ports on the sides of the chamber allow diagnostics to be places externally.

The Multi-Terrawatt (MTW) laser employs a hybrid chirped pulse optical-parametric amplification (OPCPA) system [Fig. 4.2 (a)]. An initial oscillator is temporally stretched using a set of grating and amplified by combining the beam with a pump beam in a non-linear crystal. This OPCPA system can operate at 5 Hz with millijoules of energy. A Nd:glass amplifier rod and disc pumped by flash lamps are used to increase the energy to 20 J. A grating compressor removes the chirp added by the stretcher compressing the pulse as short as 1 ps.

A dedicated experimental chamber (UDP) is maintained for underdense plasma experiments [Fig. 4.2 (b)]. This rectangular vacuum vessel, roughly 2 m × 1 m × 0.5
m, houses the transport and target, while diagnostics are built on tables surrounding the chamber. Diagnostics and configuration of the chamber are rebuilt based off the needs of each experimental campaign.

Figure 4.3: Optical model of the UDP chamber from the modeling software FRED, as built during this work. The vacuum chamber (blue) is surrounded by 5 optical tables (gray). The path of the MTW beam (red) and IR alignment laser (pink) are traced through the chamber along with any back-scattered IR light (dark red). The path of the $2\omega$ probe (green) and green alignment laser (yellow) are shown.

Figure 4.3 shows an optical model of the chamber and diagnostics as built for this work. The 1.053 µm, $P = 0.08$ TW, 60 ps square pulse MTW laser (red) enters the chamber through a port in the bottom left. An CW IR alignment laser (pink) was matched to MTW’s entry vector. The first mirror turns the IR beam to the right while letting a 15% leak through. The main beam was propagated around the chamber and
focused to the interaction volume slightly to the left of the center of the chamber. The main beam was focused ∼1.4 mm before the interaction volume using an \( f/9 \) geometry. The beam was relayed out to the table on the top left where the focal spot was reimaged. This focal spot diagnostic was used to measure the diameter of the approximately Gaussian focal spot, 200 µm \((r = 100 \text{ µm})\) [Full width at half maximum (FWHM)], which provided a spatially \( A = \pi r^2 \) averaged intensity of \( I = P/A = 2.5 \times 10^{14} \text{ W/cm}^2 \) at the interaction volume in vacuum. A pellicle along the final focusing vector reflected a small amount of light to the pointing and centering diagnostics, table on the right, aiding in alignment. The pellicle also reflected any light backscattered from the interaction volume (dark red) up to an energy meter and IR spectrometer.

The initial 15% leak of the main beam was reflected by a second leaky mirror. The leak through this second leaky mirror was used to diagnose the pointing and centering of MTW’s path into the chamber. The reflected light was frequency doubled with a KDP crystal to produce a \( 2\omega \) probe with the same 60 ps square pulse as the MTW beam and a linear chirp \((0.05 \text{ nm/ps})\). The \( 2\omega \) probe was split in two; one part being used for interferometry (light green) and the other for Thomson scattering (dark green). The portion split for the Thomson scattering probe was transported with a k-mirror periscope; a series of three mirrors that rotated the polarization [95]. Both green beams utilized trombone delay stages, a set of mirror mounted on a movable platform to increase or decrease the path length, this allowed all beams to be precisely timed relative to each other. During the experiment the probe was delayed 40 ps from the heater beam, resulting in a 20 ps temporal overlap of the beams. A pick-off of the Thomson-scattering beam was taken to measure the converted energy. The Thomson-scattering probe beam was focused at \( f/20 \) to the interaction volume at \( 43^\circ \) to the main beam. The probe’s focal spot was imaged onto a camera, on the bottom right table, to ensure the probe was properly aligned. A green alignment laser (not shown) could be injected onto the
same path as the Thomson-scattering probe, while a second green alignment laser (yellow) could be injected from the bottom right onto the path of the Thomson-scattering collection.

The Thomson-scattered light (teal), from a 60 µm × 60 µm × 40 µm volume, was collected by an \( f/3 \) optic at 80° relative to the probe beam (resulting in a 100° scattering angle). The light passed through a 532 nm dielectric notch filter with a spectral width of 28 nm to reject the portion of the spectrum associated with ion acoustic waves, or other scattering of the probe beam. The collimated light was imaged to the 75 µm input aperture of an ultrafast streaked spectrometer system [96] by an \( f/4 \) achromat.

### 4.2 Gas jet

The target for experiments on both the OMEGA and MTW laser systems was the gas plume of a supersonic gas jet [97]. A gas jet provides a uniform density of neutral gas over a few millimeters with a sharp drop-off at the edge. This is useful for basic plasma and laser-plasma experiments where a uniform density is desirable, and superior to other techniques, such as gas bags and tubes which have a solid density film that must be ablated first. This gas-jet system also utilizes a electromagnetically controlled fast opening valve, suited for many gases, in order to supply gas at high pressure without discharging large volumes of gas.

An analytic model of the gas jet’s density profile was derived based off a compressible ideal gas in a supersonic converging/diverging geometry[97]. Figure 4.4 shows this geometry. Gas released from the reservoir accelerates toward the throat of the nozzle, where the flow velocity equals the speed of sound at the throat. After leaving the throat the gas undergoes isentropic adiabatic expansion leading to the supersonic velocities. Exiting the nozzle the gas is traveling supersonically with a Mach number (\( M \)) related to
Figure 4.4: Sketch of a converging/diverging nozzle. In the converging section the flow is subsonic, in the diverging section it is supersonic. Image reproduced from Hansen et al. [97].

The gas travels supersonically out of the nozzle, but only expands at the speed of sound ($c_s$) transverse to the flow. This maintains a cone of gas with steep edges. The uniform density within the cone at a distance $L$ from the nozzle is given by,

$$\rho(L) \approx \rho_0 \frac{1}{\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{1}{\gamma - 1}}} \left(\frac{D_{exit}}{D_{exit} + 2\frac{L}{M}}\right)^2.$$  \hspace{1cm} (4.2)

For this work, nozzles with an exit diameter of 1 mm were used on MTW with the interaction volume 1 mm above the nozzle. On OMEGA, a 2mm exit diameter was used with the interaction volume (TCC) 2 mm above the nozzle. These nozzles had a Mach number of 3 for monoatomic gases (Ar and Kr) and 2.6 for diatomic gases (H$_2$ and N$_2$).
4.3 Thomson-scattering experimental setups

A typical Thomson-scattering system consists of three components: probe beam, scattered light relay, and spectrally resolved detector. Figure 4.5 shows a general Thomson-scattering setup with these components. The probe provides a source of photons to be scattered, the relay collects scattered light and images it to a spectrometer, which disperses the light in wavelength space, so the spectrum discussed in Chapter 3 can be observed and recorded.

The power scattered (Eq. 3.1) provides an insight into one of the main difficulties in Thomson scattering. Plugging in some characteristic numbers for a laser plasma ($L = 100 \mu m$, $\int d\Omega = 10^{-2}$ sr, $n_e = 10^{18-20}$ cm$^{-3}$) the power ratio $P_s/P_0$ is roughly $10^{-12}$ –
With the input power being limited to $\sim 10^{11}$ W, to prevent filamentation, a sensitive diagnostic is required. Additionally, it is advantageous to remove all other sources of light that might degrade the signal-to-noise ratio (SNR).

A probe beam with a wavelength unique to other beams in the experiment is often used to isolate the Thomson scattering spectrum. While it is possible for the heating beam to act as the probe beam, often referred to as self-Thomson-scattering, in multi-beam experiments the scattering from overlapping beams can make data difficult to interpret. A common choice is to use a low harmonic of the heater beams. A separate probe beams provides more control over the timing, positioning, and power of the probe. Independent control of the probe beam’s power can be used to prevent filamentation. Increasing the probe power to get a stronger Thomson-scattering signal is not always an option as high power can lead to pondermotive filamentation, which can break up or divert the beam.

An aperture stop is a useful addition to the relay system. While not strictly required, it limits the collection volume, allowing the collection volume to be matched to the probe’s size. This prevents collecting background light from a larger volume than Thomson-scattering light. The aperture stop also allows the measured volume to be smaller than the whole plasma. Gradients in plasma conditions as a function of time or space can blur the Thomson-scattering features and a small scattering volume helps limit these issues. The relay system as a whole must be achromatic over the wavelength range of interest. This often requires special achromatic lenses or reflective transport for collective electron plasma wave measurements, which can span hundreds of nanometers [89].

Some other considerations in the design and analysis of Thomson scattering, are the instrumental response and finite aperture effects [98]. Instrumental response is the combination of geometric and diffractive performance of the optical system. For a
spectrometer system, the spectral response can be measured by collecting light from a line emitter such as a mercury lamp. This provides known spectral lines with negligible spectral width, therefore the measured spectral width will reflect the instrumental response. For systems that collect with small f-numbers, or have probes focused with small f-numbers, there is a significant range of angles that are collected. The scattering angle, the angle between the probes direction and the scattering direction, can vary as a function of location on the collection optic. This means the measured spectrum is integrated over some range of scattering angles. Finite aperture effects are important to consider if large collection optic are used to collect more light.

The spectrometer was coupled to a camera to record data. Since the camera is capable of measuring along two independent axes, there is an opportunity to gain more information about the plasma by utilizing this second dimension, the first being frequency. The following sections will examine various versions where the second axis is space, time, or scattering angle.

### 4.3.1 Imaging Thomson scattering on OMEGA

An image of the plasma is presented to the aperture stop and the input of the spectrometer, if the image is maintained in the non-dispersed direction with an imaging spectrometer, then the resulting data measures the Thomson scattering as a function of wavelength and one spacial dimension. The spatially-resolved Thomson-scattering diagnostic on OMEGA [95] simultaneously resolved both the high-frequency (electron-plasma wave) and low-frequency (ion-acoustic wave) features. This system has two sections, a TIM based collection system, and a fixed optical table with the spectrometers and cameras. The all reflective collection system gathers an $f/10$ cone of scattered light using an off-axis section of a Schwarzschild objective, and relays the light from
the target chamber to the diagnostic table. The light is split between two spectrometers. A high-spectral-resolution 1.0-m-focal-length Czerny-Turner spectrometer was used to resolve the ion-acoustic wave spectrum, while a 0.3-m-focal-length Czerny-Turner spectrometer was used to resolve the electron-plasma wave spectrum. Both legs utilized Princeton Instruments PiMax-3 time-gated CCDs with a minimum gate time of 3 ns. With the grating used for this work, the electron and ion spectrometers had a spectral resolution of 3.3 nm and 0.066 nm FWHM respectively. Both had a spatial resolution of 50 µm.

Figure 4.6(a) shows the spatially-resolved ion-acoustic wave spectrum along the path of the probe beam. Spatially-resolved Thomson scattering provided a measurement of the plasma conditions from a series of 50 µm × 200 µm × 50 µm volumes of plasma along the propagation of the probe beam with a constant scattering angle of 60°.

The plasma conditions along the path of the probe beam [Fig. 4.6(b)] were determined using the standard collisionless dynamic form factor (Eq. 3.4) [42], with a super-Gaussian electron distribution function (Eq. 2.34). The collisionless form factor
was simultaneously matched to the ion-acoustic wave and electron-plasma wave spectra to obtain the plasma conditions. To first order, the electron temperature and density were determined from the electron plasma wave features, while the ionization state was determined from the ion-acoustic wave feature.

### 4.3.2 Time resolved Thomson scattering on OMEGA

The OMEGA Thomson scattering system can be coupled to a Rochester optical streak system (ROSS) to measure the Thomson-scattered light as a function of time [89]. A streak camera takes a line image presented to the entrance slit, and images it to a photocathode, converting the light to electrons. The electrons are accelerated down the streak tube and a time varying voltage is applied to a set of deflection plates, causing electrons arriving at different times to land in different positions on a phosphor screen. The light emitted by the phosphor screen is recorded by a CCD.

![Figure 4.7](image-url): (a) Temporally resolved electron-plasma wave spectrum and (b) ion-acoustic wave spectrum without background subtraction.

The temporally resolved Thomson-scattering diagnostic on OMEGA simultaneously resolved both the high-frequency [electron-plasma wave, Fig. 4.7 (a)] and low-frequency [ion-acoustic wave, Fig. 4.7 (b)] features from a 100 µm × 50 µm × 50 µm...
volume of plasma at TCC. The temporally resolved system operates at the same scattering angle as the spatially resolved system ($\theta = 60^\circ$) [89].

4.3.3 Ultrafast time resolved Thomson scattering on MTW

Ultrafast streak cameras are capable of picosecond temporal resolutions, however spectrometers limit this resolution. Light dispersed by a grating is reflected or transmitted at an angle with respect to the input beam, resulting in a different optical path length for light from the top and bottom of the grating. This pulse front tilt (PFT) can degrade the temporal resolution of a streaked spectrometer depending on the required spectral resolution. In order to measure Thomson scattering with picosecond resolution, the pulse front tilt must be optimized with the required spectral resolution.

Figure 4.8: (a) A schematic of the experimental setup on MTW is shown with the heater (red) and probe (green) beams incident on the gas jet. The heater beam was imaged on a focal-spot diagnostic. Thomson-scattered light was collected by an $f/3$ optic at $80^\circ$ relative to the probe beam (resulting in a $100^\circ$ scattering angle). (b) Thomson-scattering spectrum measured from a plasma heated by an intensity of $2.5 \times 10^{14}$ W/cm$^2$. The heater beam begins at $t=0$ ps and the probe beam at $t=40$ ps.

A $f/3$ pulse-front-tilt compensated spectrometer was coupled to an ultrafast optical streak camera (ROSS P820) [Fig. 4.8 (a)]. The spectrometer used an echelon to trade unrealized resolving power for improved temporal resolution [96]. The echelon was
created by stacking a series thin mirror segments with an offset between their faces. This staircase-like reflective optic separated the beam into pieces with varying delay, counteracting the pulse-front tilt induced by the grating. Using an echelon, degraded the spectral resolution to match the resolution the streak camera, as set by the electron optics. The temporal response function was measured with a fully compressed pulse from the Multi-Terawatt laser system to be 2.2 ps FWHM [17]. The spectral instrument response (1.27 nm FWHM) and the spectral dispersion were measured using the emission lines of a mercury lamp. The spectral instrument response was dominated by the diameter of the entrance aperture of the spectrometer.

Figure 4.8 (b) shows a Thomson-scattering spectrum measured from an argon plasma. The angle between the Thomson-scattering probe beam and the collection optics (80°) was chosen to maximize the ability to determine the shape of the electron distribution function. The electron plasma wave spectral features were clearly resolved along with the light scattered between the features, which results from the relatively large Landau damping (\(k\lambda_D \sim 0.6\)). The spectra throughout have been summed over two resolution units in wavelength and time.

### 4.3.4 Angularly resolved Thomson scattering on OMEGA

A diagnostic was invented to measure the angularly resolved Thomson-scattering spectrum discussed in Section 3.4. Figure 4.9 shows the angularly resolved Thomson-scattering instrument, which collected light from a small collection volume and ultimately imaged it with wavelength dispersed along one axis and scattering angle along the other. The large primary mirror was located \(\sim 100\) mm from the plasma and collected light over 108° in scattering angle, the angle between the probe vector and collection vector. In the tangential plane, referred to as the spectral direction, light was
Figure 4.9: (a) Experimental setup. The Thomson-scattering probe laser (green) and the view from the temporally or spatially resolved Thomson-scattering instrument (black) are shown. (b) The wave vectors probed by the angularly resolved instrument. (c) A wire-frame model of the angularly resolved Thomson-scattering instrument with ray fans, originating in the collection volume, showing the path of sagittal (green) and tangential (blue) rays.

integrated over 11.5°. In this direction the angular information was not preserved and therefore did not contribute to the diagnostics 1° angular resolution. Two wires were placed over the primary mirror to aid in calibration of the angular axis, resulting in small dips in the spectra near 40° and 110° (not shown in Fig. 4.9).

The primary rectangular section of a spherical mirror that was approximately 200 mm × 50 mm (primary mirror) reflected the collected light back onto an aspheric secondary mirror in an off-axis Schwarzschild configuration, collimating the collected light. The aspheric secondary mirror corrected spherical aberration due to the extreme working angle of the secondary mirror. The light was imaged through a field stop at
10× magnification, which in combination with the probe beam localized the collected light to a 50 $\mu$m $\times$ 50 $\mu$m $\times$ 100 $\mu$m volume in the plasma. The first two dimensions were set by the 500-$\mu$m diameter field stop, while the final dimension was set by the focal-spot diameter of the probe beam. A conjugate correcting lens was placed next to the field stop to align the focal plane of rays with the same scattering angle coming from different locations in the plasma, to the focal plane of rays in the spectral direction. A dielectric notch filter blocked a set of wavelengths (522 nm-542 nm) to remove the Thomson-scattering signal from ion-acoustic waves, which were 10-100× brighter.

An image rotating periscope aligned the angular and spectral axes with the axes of the CCD camera. The periscope allowed the collection optics to be aligned properly relative to the probe beam while aligning the spectrometer to the CCD.

A cylindrical lens focused the rays in the spectral direction presenting a line image to the spectrometer input that was dispersed in angle along one direction and focused in the other. The angular dispersion is inherent to the near field presented to the cylindrical lens. In collimated space the location of a ray is dependent on the angle it leaves the source. By focusing in one dimension with a cylindrical lens, this angular information was kept in one dimension and collapsed to a near diffraction limited spot in the other dimension. The spectrometer section collimated the light with an achromatic doublet lens, dispersed it in wavelength with a 400 grooves/mm transmission grating, and a second achromatic doublet focused the light onto a PI-Max4 gated CCD, which was capable of a minimum integration time of 500 ps.

Figure 4.10 shows angularly resolved Thomson-scattering data from six different experiments. The collective Thomson-scattering peaks from electron plasma waves can be seen as a function of angle. The amplitude of the features decrease, and the features become broader as the scattering angle increases. For each data shot, a second shot was taken to record a background spectrum by maintaining the same plasma conditions and
Figure 4.10: Angularly resolved Thomson-scattering data from six shots. The data was taken in (a,b,c,f) krypton, (d) hydrogen, and (e) argon. Plasma conditions for each shot are given in Table 4.1.

turning off the probe beam. The background spectrum has been subtracted from the results presented in Fig. 4.10.

The gas type, heater beam intensity, measured density, and electron temperature for each shot are shown in Table 4.1. The density was fairly consistent across the shots, showing a shot-to-shot rms variation of 13%. Shot 95831 was the only shot with the delayed probe configuration (probing after the heating lasers were turned off), providing more time for the plasma to expand and cool. The variation in electron temperatures was due to differences in laser intensities and heating efficiencies.
Table 4.1: Gas, heating laser intensity, and plasma conditions for six shots. The uncertainty represents the standard deviations from Monte Carlo analysis on these measurements (see Section 5.2.1).

<table>
<thead>
<tr>
<th>Shot Number</th>
<th>Gas</th>
<th>$I_{UV}^{\text{total}}$ ($10^{15}$ W/cm²)</th>
<th>Temperature (keV)</th>
<th>Electron Density ($10^{20}$ cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95826</td>
<td>Krypton</td>
<td>2.8</td>
<td>1.34 ± 0.23</td>
<td>0.466 ± 0.058</td>
</tr>
<tr>
<td>95830</td>
<td>Krypton</td>
<td>1.2</td>
<td>0.99 ± 0.24</td>
<td>0.408 ± 0.061</td>
</tr>
<tr>
<td>95831</td>
<td>Krypton</td>
<td>0.84</td>
<td>0.50 ± 0.11</td>
<td>0.349 ± 0.045</td>
</tr>
<tr>
<td>94475</td>
<td>Argon</td>
<td>2.8</td>
<td>1.04 ± 0.13</td>
<td>0.418 ± 0.043</td>
</tr>
<tr>
<td>94479</td>
<td>Hydrogen</td>
<td>2.6</td>
<td>0.499 ± 0.078</td>
<td>0.471 ± 0.043</td>
</tr>
<tr>
<td>94481</td>
<td>Krypton</td>
<td>2.8</td>
<td>1.16 ± 0.14</td>
<td>0.428 ± 0.042</td>
</tr>
</tbody>
</table>
Chapter 5

Results

This Chapter shows the results of experiments measuring the electron distribution function and its effect on laser absorption. Electron velocity distribution functions have been measured over many orders of magnitude for the first time. Over the course of this work, experimental and analysis techniques progressed from the ability to infer the distribution function in single-angle Thomson scattering, to matching Thomson scattering with simulated distribution functions, to direct measurements of distribution functions using angularly resolved Thomson scattering. Each improvement provided increasing detail about the distribution function and the processes occurring within the plasma. Section 5.1 discusses the single-angle Thomson scattering results, both the ability to infer super-Gaussian order (Sec. 5.1.1) and measurements of ionization kinetics by matching data with simulations (Sec. 5.1.2). Section 5.2 discusses the distribution functions measured with angularly resolved Thomson scattering. A detailed description of the algorithm and thought process behind analysing this new kind of data is given in Section 5.2.1, while Section 5.2.2 focused on the physical interpretation of the measurements and comparisons to established theory. Section 5.3 reports measurements of reduced absorption in super-Gaussian plasmas, a result on the non-Maxwellian shape. A few outstanding questions are discussed in Section 5.4.
5.1 Measurements of non-Maxwellian distributions

Section 3.3 showed the sensitivity of collective Thomson-scattering spectrum to a super-Gaussian distribution function. While some complications to this were noted, single-angle collective Thomson scattering can still be a powerful tool in measuring the super-Gaussian order. Proving out the concept, and the existence of super-Gaussian distribution function was required before investing in the more complicated angularly resolved measurements.

Here, we present the first measurements of the interplay between inverse bremsstrahlung heating and ionization kinetics on the electron distribution function. An ultrafast Thomson-scattering system was used to collect the electron plasma wave spectrum, which enabled the picosecond evolution of the non-Maxwellian electron distribution function to be measured in a laser-produced plasma. The preferential heating of the slow electrons by a laser beam with an intensity of $2.5 \times 10^{14}$ W/cm$^2$, coupled with the redistribution of electron kinetic energy due to ionization, resulted in a non-Maxwellian electron distribution function. The shape of the electron distribution function, 60 ps into the plasma formation, was measured to be approximately a super-Gaussian of order 3.4. After the laser turned off, the electron density continued to increase by 15% over the next 40 ps ($\sim 25$ electron-ion collision times) due to collisional ionization. Over this time, the electron temperature decreased from 400 eV to 300 eV, which is consistent with the energy required for collisional ionization to increase the density. To determine the electron distribution functions consistent with the measured Thomson-scattered spectra in this rapidly evolving plasma, Vlasov-Fokker-Planck simulations that included laser heating, thermal transport and ionization were required. Laser heating was found to have the largest effect on the shape of the dis-
Figure 5.1: (a) Thomson-scattering spectrum measured from a plasma heated by an intensity of $2.5 \times 10^{14} \text{ W/cm}^2$. The heater beam begins at $t=0$ ps and the probe beam at $t=40$ ps. (b) The measured spectrum at 58 ps (gray points) plotted with spectrum calculated using Maxwellian (dashed green curve) and non-Maxwellian (orange curve) electron distribution functions. The best-fit spectra determined $T_e = 423 \text{ eV}$, $n_e = 2.03 \times 10^{19} \text{ cm}^{-3}$, $m=2$ (Maxwellian) and $T_e = 406 \text{ eV}$, $n_e = 2.04 \times 10^{19} \text{ cm}^{-3}$, $m=3.1$ (Non-Maxwellian).

Thomson-scattering experiments were performed using the Multi-Terawatt laser [93] at the Laboratory for Laser Energetics (Sec. 4.1.2) in the UDP chamber. The MTW laser provided a spatially averaged intensity of $2.5 \times 10^{14} \text{ W/cm}^2$. A supersonic Mach 3 gas jet [97] with an exit diameter of 1 mm was pressurized with argon (Ar) to 50 psi to achieve a neutral gas density of $2.4 \times 10^{18} \text{ cm}^{-3}$ at 1 mm above the nozzle. The Thomson-scattering probe beam, with a spatially averaged intensity of $\sim 10^{14} \text{ W/cm}^2$, arrived at the interaction volume 40 ps after the heater beam, resulting in a 20 ps temporal overlap of the beams. Thomson scattering was measured with the ultrafast time-resolved Thomson-scattering system (Sec. 4.3.3).
5.1.1 Evidence of non-Maxwellian distribution functions

Figure 5.1 shows a time resolved Thomson-scattering spectrum measured from an argon plasma. The spectrum [Fig. 5.1(b)] just prior to turning off the heater beam (∼58 ps) shows the effect of inverse bremsstrahlung heating through the modification of the electron distribution function and the need to include non-Maxwellian electron distribution functions to accurately reproduce the measured spectra. When limiting the spectral fit to Maxwellian electron distribution functions, the spectrum fails to reproduce three regions of the measurements: (1) the calculations underestimate the light scattered into the central region of the spectrum (∼500 nm to ∼550 nm), (2) the widths of the scattering features are too broad, and (3) the slope on the outermost edges of the scattering peaks are too shallow.

When calculating the spectra in Fig. 5.1, a super-Gaussian electron distribution function was assumed, \( f_m = C_m \exp\left[-\left(\frac{v}{v_m}\right)^m\right] \) (see Eq. 2.34). A super-Gaussian electron distribution function significantly improves the fit in all three regions and reduces the \( \chi^2 \) per degree of freedom from 3.75 to 1.33. This large improvement in \( \chi^2 \) per degree of freedom is a result of the model doing a better job matching the center of the variation, instead of the edge, removing any systematic trends from the residuals.

The calculated spectra were determined through a 3-dimensional \((m, T_e, n_e)\) \( \chi^2 = \sum_\lambda \frac{|P(\lambda) - P_s(\lambda)|^2}{1.15P_s(\lambda)} \) minimization of the measured spectrum \([P_s(\lambda)]\) and the calculated power (Eq. 3.1). The electron susceptibility (Eq. 3.5) was determined numerically and the calculated scattered power was adjusted to account for the finite aperture effects in the data [98] and the spectral response of the instrument (Sec. 4.3).
5.1.2 Temporal evolution and evidence of ionization kinetics

Figure 5.2 shows the evolution of the Thomson-scattering spectrum after the heating laser was turned off. Due to the rapidly evolving conditions, the Thomson-scattering spectra were calculated using electron distribution functions from the Vlasov-Fokker-Planck code K2 [99], which is 1 dimensional in space and 3 dimensional in velocity (1D3V). When including both inverse bremsstrahlung heating and thermal conduction in the simulations, the calculated Thomson-scattering spectra are in reasonable agreement with the measured features (Fig. 5.2 dashed blue curve). K2 includes elastic collision operators for electron-electron collisions in addition to terms describing inverse bremsstrahlung heating and thermal transport, including self-consistent electric fields and return currents [99].

Figure 5.3 shows that it is necessary to include ionization in the K2 calculations in order to match the measured plasma conditions. Including ionization also improves agreement with the measured spectra (Fig. 5.2). While Fig. 5.1 shows the need for non-Maxwellian distributions driven by inverse bremsstrahlung heating to reproduce
Figure 5.3: The measured (a) temperature and (b) density (black circles) are compared to K2 simulation results. The black error bars represent a 95% confidence interval in the given parameter. The results of a K2 simulation without atomic kinetics (dashed blue curves) and the results of a K2 simulation with atomic physics (red curve) are shown.

The spectra, the electron density and temperature reveal the need to include an atomic physics model. Without ionization, it is not possible to match the density, which rises by $\sim 15\%$ after the heating beam is turned off. During this time, the collisional ionization continues resulting in the rising density (Fig. 5.3). This process uses the kinetic energy of the free electrons to ionize the plasma further, thereby lowering the temperature. Without ionization, the simulations lack this energy loss mechanism and therefore overestimated the temperature by $\sim 15\%$ to 20%. In simulations without ionization, it is possible to alter the initial plasma conditions to achieve better agreement with the temperature (Fig. 5.2), but this results in distribution functions that generate spectra with poor agreement with the measured Thomson-scattering spectra. By coupling K2 with an atomic kinetics model, not only was it possible to account for the temperature and density evolution, the electron distribution functions were modified bringing the calculated Thomson-scattering spectra into better agreement with the measurements. The improved agreement can be seen (Fig. 5.2) in the slightly wider peak and lower central region of the model with ionization. Resulting in an improved minimum $\chi^2$,
which was 10% to 15% smaller than the model without atomic kinetics for each case. The improved spectral match in conjunction with the ability to reproduce the temperature and density evolution necessitate the inclusion of the atomic physics model.

In laser-produced plasmas, inverse bremsstrahlung heating [22, 23, 32], thermal transport [37, 38], laser-plasma instabilities [39], and atomic kinetic processes [40] all provide competing mechanisms that govern the shape of the electron distribution function. To determine the impact of ionization on the electron distribution function, an atomic physics model was coupled to K2. An inelastic collisional operator, sometimes called a Boltzmann operator, was used to model the changes to the distribution due to all atomic processes. The time evolution of the atomic states were determined through a set of coupled rate equations. The collisional rates that enter the rate matrix were obtained from direct integration of the actual distribution. The atomic data (energy levels and cross sections) was constructed based on a screened hydrogenic model using the code Cretin [100]. While the model used for these simulations includes different types of collisional and radiative processes (both bound-bound and bound-free), collisional ionization was identified as the main atomic process affecting the distribution function [40]. The simulations were performed using the experimental laser conditions. Simulations performed without the atomic physics model used a preionized plasma with an electron density of $2.2 \times 10^{19}$ cm$^{-3}$ (corresponding to an average ionization state of 9.1) and an electron temperature of 10 eV. When using the atomic physics model, ionization was self-consistently included and the simulations were initialized with a neutral density of $2.4 \times 10^{18}$ cm$^{-3}$.

Figure 5.4(a) shows the relative contributions of inverse bremsstrahlung heating, thermal transport, and atomic physics on the electron distribution function. The main deviation from Maxwellian was due to inverse bremsstrahlung heating and results in a reduction of slow electrons and an increase in electrons with a velocity of $\sim 1.5v_{th}$. 
Heat transport compounds with the effect of inverse bremsstrahlung heating by further increasing the number of bulk electrons. The electrons in the tail of the electron distribution function carry heat away from the Thomson-scattering volume while the slower electrons, that maintain charge neutrality, reinforce the bulk of the electron distribution function. Ionization suppresses the super-Gaussian shape by preferentially removing electrons from around \(2 - 3v_{th}\) and supplying electrons with little to no velocity.

Figure 5.4(b) shows how the distribution function evolves. The electron distribution at 58 ps, while the heater beam is on, was driven to a non-Maxwellian shape. It rapidly (< 10 ps) evolves to a new super-Gaussian distribution, but is still non-Maxwellian. Once this new electron distribution has been realized, the evolution slows and only small changes to the distribution continue through the end of the measurement (100 ps).

In summary, ultrafast Thomson-scattering measurements of the electron-plasma wave spectrum were used to determine the effects of laser heating and ionization on
the electron distribution function. While the argon plasma was being heated, super-Gaussian electron distribution functions were measured. After the heater beam was turned off, the electron density continued to rise and the electron temperature dropped as the free electrons continued to ionize the argon plasma.

Non-Maxwellian electron distribution functions from Vlasov-Fokker-Planck simulations that included both laser heating and ionization were required to reproduce the measured Thomson-scattering spectra, temperature evolution, and density evolution simultaneously. Thereby demonstrating the interplay between the effects of inverse bremsstrahlung and atomic kinetics on the shape of the electron distribution function, and the level of complexity needed to model the distribution function in simple experiments.

5.2 Measurements of arbitrary distribution functions

Angulary resolved Thomson scattering provides a method to measure the electron distribution function without assuming a functional form, such as super-Gaussian, or modeling the experiment with kinetic simulations. This allows the measured distribution function to capture all the relevant physics without needing to specify what that relevant physics is a priori. This technique combines the collective and non-collective regimes in order to directly measure the electron velocity distribution function over many orders of magnitude (see Section 3.4).

Here, we present the first measurements of complete electron distributions without any assumptions on their shape or the underlying physics that produced them. At these conditions the inverse bremsstrahlung heating dominated over thermalization by electron-electron collisions. To enable single-shot temporally and spatially resolved measurements of the electron distribution function over several orders of
magnitude, an optical diagnostic was invented that uses the angular dependence of scattering to simultaneously access the non-collective and collective nature of plasmas. This first-principles measurement showed that during significant heating by the laser beams, the distributions had a super-Gaussian shape in the bulk \( v < 3v_{th} \) with a Maxwellian tail \( v > 3v_{th} \). The super-Gaussian bulk is associated directly with the inverse bremsstrahlung heating and is well reproduced by the previous computational work\[23\]. The departure from super-Gaussian at high velocities was predicted by Fourkal et al.\[32\], but these measurements show this deviation at a higher velocity. Particle simulations show improved agreement and demonstrate the importance of isotropic heating in accurately predicting the high-velocity tail.

Angularly resolved Thomson scattering experiments that measured non-Maxwellian electron distribution functions resulting from laser heating were conducted on the OMEGA Laser System\[94\] at the Laboratory for Laser Energetics. Figure 5.5 shows the experimental configuration where five ultraviolet (351-nm UV) laser beams
were used to ionize and heat a gas. The uniform density neutral gas plume of argon, krypton, or hydrogen was produced using a supersonic Mach-3 gas jet\cite{97} with an exit diameter of 2 mm. To improve the uniformity of heating, each of the five UV heating beams used a distributed phase plate, polarization smoothing, and smoothing by spectral dispersion. This ultimately provided Gaussian focal spots, 2 mm above the center of the gas jet, with a $1/e^2$ diameter of 400 $\mu$m. The heating beams had a 500-ps pulse duration, full-width at half-maximum (FWHM), and 50 J to 200 J per beam. This resulted in a total overlapped UV intensity of $I_{UV}^{total} = 0.62 - 2.8 \times 10^{15}$ W/cm$^2$.

A 10-J green (526.5 nm) probe beam was focused to the same location as the heating beams. This probe beam had a shorter 200-ps FWHM duration and was delayed 300 ps or 600 ps from the rise of the heating beams. This allowed Thomson-scattering measurements at the end of the heating (300 to 500 ps) or shortly after the heating beams turned off (600 to 800 ps). The probe beam used a distributed phase plate to produce a flattop focal spot with a 100-$\mu$m FWHM diameter ($I_{200} = 6.5 \times 10^{14}$ W/cm$^2$). Unlike the heating beams, the probe did not employ smoothing by spectral dispersion or polarization smoothing. Instead, the linear polarization was rotated with a waveplate to maximize the Thomson-scattering efficiency and make it uniform as a function of angle.

\subsection{5.2.1 Interpreting ARTS}

\textbf{Algorithm for determining the electron distribution}

Figure 4.10 shows the measured angularly resolved Thomson-scattering spectrum where the ensemble electron motion was encoded on the frequency of the scattered light. To decode the complete electron distribution function (Fig. 5.9), the total power
scattered was calculated across the range of scattering angles,

\[ P_s(\lambda_s, \theta) = C \left( \frac{2\lambda_0}{\lambda_s^3} - 1/\lambda_s^2 \right) n_e S(x), \tag{5.1} \]

and compared to the measured spectrum. Here, \( C \) is a scale factor. The dynamic form factor (Eq. 3.4) in this regime can be approximated by [42],

\[ S(x) \simeq \left| \frac{1}{1 + \chi_{Re}[x] + i\chi_{Im}[x]} \right|^2 f_e[x] \tag{5.2} \]

where \( x = \omega / kv_{th} \) is the normalized phase velocity. The frequency of the probed fluctuation is \( \omega = 2\pi c \left( \frac{1}{\lambda_s} - \frac{1}{\lambda_0} \right) \). This normalization removes temperature from the distribution function and it allows representing all Maxwellian distributions regardless of temperature with a single function. The ion dependence in the dynamic form factor was removed because the measurements were made only for \( \omega / k \gtrsim v_{th} \) where the ion contribution is negligible.

By assuming the distribution function is isotropic over the range in scattering directions probed, a single projection of the distribution function onto each of the probed vectors was used. To maintain the highest level of generality, the one-dimensional electron distribution function projected along the probed fluctuations was defined as a set of points, \( f_e[x] \), where the square brackets are used to denote the discrete domain, which consisted of 64 points. To increase the resolution of the spectral calculation, exponential interpolation of the electron distribution function between the points was used.

The electron velocity distribution function, measured in the spectra, and plasma conditions (Table 4.1) were determined simultaneously using an iterative gradient descent algorithm. A numerical distribution function \( (f_e[x]) \) was used to forward calculate a fluctuation spectrum (Eq. 5.2). This spectrum was used to calculate the synthetic
power scattered (Eq. 5.1), which was compared to the experimentally measured spectrum using,

\[
\chi^2 = \sum_{\lambda_s} \sum_{\theta < 40^\circ} \frac{\left[ P_m(\lambda_s, \theta) - P_s(\lambda_s, \theta) \right]^2}{\sigma^2}
\]

(5.3)

where \(\sigma\) is the variation of the data, which was \(\sim 5\%\) of the signal for all shots, found by examining the variation in the spectrum at large wavelengths. The region with scattering angles \(< 40^\circ\) was excluded because accurate calculation required orders of magnitude higher velocity space resolution. Above a scattering angle of \(110^\circ\) the spectra generally lacked sufficient signal and were dominated by noise.

A minimization of \(\chi^2\) over 69 free parameters defining the distribution function (64 points), plasma conditions \((n_e, T_e)\), and scale parameters (3 parameters) was performed on each shot. The most computationally expensive step was computing the susceptibility (Eq. 3.5). The real part of the susceptibility was solved using the method of rational integration [101]. Furthermore, to prevent numerical discontinuities in the susceptibility from the discrete distribution function, exponential interpolation was used to produce a continuous distribution function. Convergence was improved by performing a set of sequential fits with increasing number of distribution function points. Gradient descent minimizers often struggle to converge for high dimensional problems if the starting point is too far away from the true values. Fewer points provided significantly better convergence and starting points for the subsequent minimizations closer to the optimal values. The discrete domain, equally spaced points spanning \([0, 7v_{th}]\), was re-defined for each minimization requiring the distribution function to be remapped onto a new set of \(x_s\), since the spacing changes each time. This helped overcome local minima by preventing initialization in a local minima. It was found that \(\sim 9\) distribution points per thermal velocity (64 points spanning \(0 - 7v_{th}\)) was optimal for balancing computation time and resolving the distribution function.
Figure 5.6: (a) Measured and (b) calculated spectra from a krypton plasma used to determine the measured electron distribution shown in Fig. 5.9. (c) The measured (black curve) and calculated (red curve) spectra at four scattering angles (51°, 70°, 88°, and 107°) have been arbitrarily spaced along the y-axis.

For each measurement, the background spectrum was measured by repeating the experiment without the probe beam. The measured spectrum shown [Fig. 5.6 (a)] was obtained by subtracting the background data from the raw Thomson-scattering spectrum. Figure 5.6 (b) shows the final spectrum calculated using the measured distribution function for shot 94481. Comparing to the experimental spectrum [Fig. 5.6 (a)], the calculated spectrum qualitatively reproduces the measured spectrum in angle and wavelength across the entire spectral range [Fig. 5.6 (c)]. Some small discrepancies are seen at small scattering angle due to numerical artifacts and at high scattering angle where there is a small clip in the data. The spectrum was corrected for the spectral transmission of the system. A measurement of the transmission as a function of wavelength for the entire system was made using a calibrated tungsten lamp placed at the collection volume.

In order to match the measured spectrum, the effects of temporal gradients, finite angles, and instrument response were included in the calculation. The instrument re-
response function was included by convolving the scattered power with a 2-dimensional Gaussian with a spectral width of 1 nm and angular width of 1°, matching the measured instrument response function. The curvature of the primary mirror and finite solid angle of the probe beam result in a small change in the scattering angle as a function of height on the mirror. This was included in calculation by treating each angle at the CCD as a weighted sum of the true constituent scattering angles determined from an optical model of the instrument.

![Figure 5.7: (a) Temporally resolved electron plasma wave spectrum without background subtraction. (b) Spectrum integrated over the entire duration of the probe beam (black curve) is compared with calculated spectra using plasma conditions from the beginning (dashed orange curve) and end of the probe duration (dashed red curve). A calculated spectrum accounting for the temporal gradient in plasma conditions is shown in blue. (Swap (b) for an IAW spectrum and move (b) to arts section?)](image)

To account for the evolution of the plasma conditions, a range of plasma conditions were determined from the time-resolved spectrum. Figure 5.7 shows the evolution of the electron-plasma wave features and the effective broadening of the feature when time integrated over the duration of the probe. Simulated spectra that match the data at either the beginning or end of the probe [Fig. 5.7(b)] have the wrong peak wavelength and smaller width when compared to the measured spectrum integrated over the entire probe (black curve). By calculating the spectrum integrated over the range of plasma conditions the peak location and width were matched, while ensuring the spectrum
responds appropriately as the scattering angle was changed. The calculated angularly-resolved spectrum was integrated over this range of plasma conditions.

**Reduced model error analysis**

A 90% confidence interval was determined for the measured electron distribution function using the reduced model (Eq. 5.4). This confidence interval is shown as a gray region in Figure 5.9. Many of the same difficulties faced in calculating the numerical electron distribution function complicated the uncertainty analysis, preventing direct calculation of the uncertainties on the numerical distribution function. The high dimensionality led to saddle points that prevented the use of Hessian based uncertainty calculations. More complicated methods such a Monte Carlo or grid analysis were too computationally expensive.

In order to analyse the quality of the measured distribution functions, a reduced model comprised of super-Gaussian and Maxwellian functions was introduced,

\[ f_e(x) = A_1 \exp \left[ -\left( \frac{x}{x_1} \right)^m \right] + \exp[A_2] \exp \left[ -\left( \frac{x}{x_2} \right)^2 \right]. \]  

(5.4)

This model reproduced the distribution function both in the bulk and tail (Fig. 5.9), but tends to perform worse near the peak \((v < 0.5v_{th})\). The excellent agreement of the reduced model provided an alternative by defining the plasma conditions and distribution function in 7 parameters, (electron temperature, electron density, and the 5 parameters of the reduced model). In this lower dimensional space, it was computationally tractable to perform uncertainty analysis using Markov-chain Monte Carlo. Two additional parameters were added to determine the sensitivity to calibration of the wavelength axis. The calibration uncertainty was included to achieve a confidence region on the dis-
Figure 5.8: Marginal probability distributions of each parameter in the reduced model (Eq. 5.4) determined from Monte Carlo analysis.

...tribution function, which accounts for statistical (model) and instrumental sources of error.

Figure 5.8 shows the 2-parameter joint and single parameter marginal distributions from the Monte Carlo analysis. Numerous covariances can be seen between parameters as the contours are rotated ellipsoids. For two uncorrelated parameters, the contours of equal probability are circles, while two perfectly correlated parameters would result in elliptical contours with the semi-major axis aligned to $y = x$. The width parameter of the super-Gaussian ($x_1$) is inversely correlated with temperature, density, and amplitude. These are all related to maintaining the normalization of the distribution function. The positive correlation of the super-Gaussian order ($m$) and temperature is related to maintaining the width of the peak in the Thomson-scattering spectrum.
Table 5.1: Reduced-model parameters from Monte Carlo analysis for six shots. The uncertainty represents the standard deviation on these values.

<table>
<thead>
<tr>
<th>Shot Number</th>
<th>$A_1$</th>
<th>$x_1$</th>
<th>$m$</th>
<th>$A_2$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>95826</td>
<td>0.0413 ± 0.0068</td>
<td>1.91 ± 0.14</td>
<td>3.67 ± 0.57</td>
<td>−9.07 ± 0.60</td>
<td>3.02 ± 0.67</td>
</tr>
<tr>
<td>95830</td>
<td>0.0346 ± 0.0070</td>
<td>1.94 ± 0.20</td>
<td>3.57 ± 0.63</td>
<td>−6.54 ± 0.55</td>
<td>2.04 ± 0.55</td>
</tr>
<tr>
<td>95831</td>
<td>0.0510 ± 0.0055</td>
<td>1.58 ± 0.11</td>
<td>2.36 ± 0.35</td>
<td>−8.14 ± 0.42</td>
<td>2.70 ± 0.36</td>
</tr>
<tr>
<td>94475</td>
<td>0.0245 ± 0.0045</td>
<td>2.17 ± 0.17</td>
<td>4.22 ± 0.35</td>
<td>−6.74 ± 0.41</td>
<td>1.75 ± 0.22</td>
</tr>
<tr>
<td>94479</td>
<td>0.0360 ± 0.0044</td>
<td>1.82 ± 0.11</td>
<td>2.60 ± 0.37</td>
<td>−6.36 ± 0.43</td>
<td>1.60 ± 0.47</td>
</tr>
<tr>
<td>94481</td>
<td>0.0309 ± 0.0051</td>
<td>2.031 ± 0.051</td>
<td>3.92 ± 0.24</td>
<td>−9.15 ± 0.31</td>
<td>1.97 ± 0.31</td>
</tr>
</tbody>
</table>

The uncertainty analysis was performed using the Metropolis-Hastings algorithm with 72 walkers and 3,000 steps per walker. A burn-in period of 50 steps was found to sufficiently remove bias from the initial conditions of the walkers. A uniform prior was used on the parameters, but a lower reflective boundary at a temperature of 1 eV and a density of $10^{16}$ cm$^{-3}$ prevented nonphysical steps from causing algorithm-based errors.

Each of the single-parameter marginal distributions was fit with a Gaussian (Fig. 5.8) to determine the mean and variance. The standard deviation of the plasma conditions is given as an uncertainty on the conditions measured with the numerical distribution function analysis in Table 4.1. The mean and standard deviation for each of the reduced-model parameters is shown in Table 5.1. These uncertainties are dependent on two factors, how much the spectrum changes in response to the parameter and how much variation can be seen in the data. Using electron temperature as an example, changes can broaden the peak in wavelength. However, the spectral instrumental response can be thought of as setting a minimum on the broadening, since the spectral response also broadens the spectrum in wavelength. Noise in the data allows a range of parameter values to be contained within the bounds of the data, making it difficult to distinguish if a mismatch between a given data point and theoretical point is due to fluctuation or an incorrect parameter.

The 90% confidence interval on the distribution function (Fig. 5.9) was determined
with standard error propagation techniques of the uncertainties in Table 5.1. The uncertainty on the distribution function was,

$$\sigma_{fe} = \left[ \left( \frac{\partial f_e}{\partial A_1} \right)^2 \sigma_{A_1}^2 + \left( \frac{\partial f_e}{\partial x_1} \right)^2 \sigma_{x_1}^2 + \left( \frac{\partial f_e}{\partial m} \right)^2 \sigma_m^2 + \left( \frac{\partial f_e}{\partial A_2} \right)^2 \sigma_{A_2}^2 + \left( \frac{\partial f_e}{\partial x_2} \right)^2 \sigma_{x_2}^2 \right]^{1/2}.$$  \hspace{1cm} (5.5)

A multiplier on standard deviation for each shot was determined by integrating over the 9-dimensional probability distribution until 90% probability was achieved. Strictly, this is a confidence interval on the reduced model, but is reported on the numerical distribution.

The uncertainties in the measured distribution functions were due to the experimental limitations in the range of probed wave-vectors and instrumental response. Even with these uncertainties, the electron distribution function was measured to a level of precision more than sufficient to distinguish the distribution from Maxwellian or other functional forms.

### 5.2.2 Measured electron distribution functions

Figure 5.9 shows the measured numerical distribution function from a krypton plasma. This numerical distribution function was determined without making any assumptions about the shape or underlying physics. It shows a more flat-topped profile than the Maxwellian distribution and an elevated tail. These features are consistent with expectations based on inverse bremsstrahlung heating. For the results shown in Fig. 5.9, a super-Gaussian distribution function is in excellent agreement with the measurements for velocities less than $\sim 3v_{th}$ when using the overlapped intensity and the measured plasma conditions ($\alpha = 4.3^{+0.7}_{-0.6}$) to determine the super-Gaussian order.

Using the distribution function and electron temperature from the angularly resolved Thomson-scattering instrument, the ion-acoustic wave spectrum was used to
Figure 5.9: Electron distributions on (a) logarithmic and (b) linear scale determined while the laser beams were heating the krypton plasma. The measured distribution (black points) is well reproduced in the bulk by a super-Gaussian function (orange curve) consistent with Matte et al. (Eq. 2.37, m=3.9). A formalism describing the Maxwellian tail from Fourkal et al.[32] (purple curve), a Maxwellian distribution (blue curve), results from particle simulation (green curve), and the super-Gaussian + Maxwellian result from the Monte Carlo analysis (red) are shown. The 90% confidence interval on the measured distribution function (gray region) is shown.

determine the average ionization state. Combining the average ionization state, electron temperature, and total laser intensity the ratio of inverse bremsstrahlung heating rate to electron-electron collision rate (α) was determined for each experiment (Fig.5.10).

Figure 5.10 shows the scaling of the super-Gaussian order (m) with the Langdon parameter (α) and super-Gaussian width (x_1). When using the super-Gaussian plus Maxwellian model, good agreement is found with the heuristic scaling from Matte et al.[23] for m (Eq. 2.37). The order of the electron distribution function increases from a Maxwellian (m = 2) towards a highly super-Gaussian shape (m = 5) as the inverse bremsstrahlung heating of the slow electrons dominates over the thermalization. The ratio of inverse bremsstrahlung heating to electron-electron collision rate was varied by changing the heater beam intensity (I_{UV}^{total} = 0.62 – 2.8 \times 10^{15} \text{ W/cm}^2) and the gas species (hydrogen, nitrogen, argon, and krypton), which resulted in a range of temper-
Figure 5.10: Scaling of the super-Gaussian order with relative strength of inverse bremsstrahlung heating to electron-electron collisions ($\alpha$) and super-Gaussian width ($x_1$). The values for each of the six shots are plotted with error-bars showing one standard deviation. The dashed lines are the expected scaling relations from Fokker-Planck simulations.

Figure 5.9 shows that the measured electron distribution transitions from a super-Gaussian to a Maxwellian shape at $\sim 3v_{th}$, whereas the theory from Fourkal et al.[32] predicts an earlier transition around $\sim 2.5v_{th}$ and more electrons in the tail. This departure of Fourkal from a super-Gaussian distribution was calculated considering a single plane-wave electromagnetic source, where electrons oscillating in the laser field collide with electrons in the tail, modifying the distribution function at high energies. By intro-
ducing five overlapped beams, consistent with the experimental configuration, particle simulations using the code Quartz (see Appendix A.2) show the number of electrons in the tail exceeds the super-Gaussian for velocities in the range $3.5 v_{th} \lesssim v \lesssim 4.5 v_{th}$, qualitatively consistent with the enhancement above super-Gaussian observed in the data. These results suggest that the increased uniformity due to multiple overlapped beams reduced the energy transferred to the high-velocity electrons.

### 5.3 Absorption

The most widely accepted and utilized component of the Langdon effect, is its predicted reduction in absorption. The diminished number of slow electrons, the electrons primarily responsible for heating, reduces the heating rate. With the ability to measure the distribution function, simultaneous measurements of the absorption enabled this effect to be shown definitively.

Experiments were performed on the OMEGA laser system[94] with a nearly identical platform to the arbitrary electron distribution function measurements. This time, eleven ultraviolet (UV, 351 nm) laser beams were focused to the center of the gas jet. Achieving a uniform overlapped intensity ranging from $I_{UV}^{total} = 0.35 - 2.8 \times 10^{15}$ W/cm$^2$. The oscillatory velocity was calculated from the total intensity, $v_{osc}^2 [cm^2/s^2] = 739 (I_{UV}^{total}[W/cm^2]/9 + I_{2\omega}[W/cm^2]/4)$.

#### 5.3.1 Absorption Measurement

To determine the absorption of the probe beam, two high-dynamic range scientific CCD cameras were installed to measure the incident and transmitted energy [102]. The system was capable of measuring 0.1% absorption. On all of the shots reported here, very
little beam spray outside of the original cone of the probe beam was observed and the transmitted beam was contained well within the measurement region.

To determine the effects of non-Maxwellian electron distribution functions on laser absorption, the incident and transmitted power of the probe beam were measured and compared with the absorption calculated assuming a Maxwellian electron distribution function. Spatially-resolved Thomson scattering (Section 4.3.1) was used to determine the plasma conditions along the beam path, which were required to compute the expected absorption for a plasma with a Maxwellian electron distribution function per unit length \((dL)[103]\),

\[
dA_{\text{Max}} = 1 - \exp \left[ - \left( \frac{\omega_{pe}}{\omega_0} \right)^2 \frac{V_{ei}}{V_g} dL \right] \tag{5.6}
\]

Integrating over the length of the plasma yielded the expected absorption for a Maxwellian plasma.

The ratio of the total counts measured by the CCD, in the illuminated region of interest \((ROI_{\text{out}})\), on the transmitted side, to the total counts on the input side CCD \((ROI_{\text{in}})\), \((\sum ROI_{\text{out}} / \sum ROI_{\text{in}})\) provided the transmission of the probe beam. Calibration of the transmission was measured when no plasma was present \((\sum ROI_{\text{cal,out}} / \sum ROI_{\text{cal,in}})\), and was determined to be stable over 12 hours with a 0.1% RMS variation. The absorption of the probe was calculated from this transmission measurement.

\[
A = 1 - \frac{\sum ROI_{\text{out}}}{\sum ROI_{\text{in}}} \times \frac{\sum ROI_{\text{cal.in}}}{\sum ROI_{\text{cal.out}}}. \tag{5.7}
\]

### 5.3.2 Comparison to Langdon theory

Figure 5.11 shows that the measured laser absorption was significantly less than the absorption calculated assuming a plasma with a Maxwellian electron distribution. The
Figure 5.11: The measured (red circles) and calculated (blue circles) absorption (Eq. 5.8), normalized to the absorption calculated assuming a Maxwellian electron distribution function, is plotted as a function of the ratio of the inverse bremsstrahlung heating rate to the electron-electron collision rate determined from the measured plasma conditions at the center of the plasma. Error-bars represent one standard deviation propagated from uncertainties in the measured plasma conditions. (remove part a)

absorption rapidly drops to $\sim 60\%$ of the Maxwellian expectation as the relative heating rate increases (large $Zv_{osc}^2/v_{th}^2$). When the inverse bremsstrahlung heating rate dominates over the electron-electron collision rate, the reduction in absorption is in reasonable agreement with the original predictions[22],

$$dA_{SG}(n_e, T_e) = \left[ 1 - \frac{0.553}{1 + (0.27v_{th}^2/Zv_{osc}^2)^{0.75}} \right] \times dA_{Max}(n_e, T_e). \quad (5.8)$$

where $dA_{Max}$ is the differential absorption calculated assuming a Maxwellian plasma and using the plasma conditions determined along the path of the probe beam using spatially-resolved Thomson scattering. The calculated absorption shown in Fig. 5.11 was computed by integrating Eq. 5.8 over the path of the probe beam.
5.4 Outstanding questions

The ability to measure the electron distribution function opens up many avenues for future work and further exploration. However, there are 4 questions that remain unanswered in this work. What is the source of the Maxwellian tails in the electron distribution function? Are the deviations in the distribution function at low velocity real? How much further can we extend the range of velocities over which the distribution function can be measured? Why is the measured absorption systematically high, more consistent with Z+1?

The Maxwellian tails in the distribution function were shown experimentally to exist, but not conform to current theories. Simulations indicated that the level of the tail could be altered based off the anisotropy of the heating. However, this does not prove that the tails are due to anisotropic heating or why a tail can be seen in the Maxwellian data [Fig. 5.12 (d)]. It is possible more could be learned by recovering the distribution function out to higher velocities. There is data at smaller scattering angles than was used in this analysis due to numerical issues. At smaller scattering angles the resonance shrinks rapidly requiring increasing numerical resolution.

Some of the measured distribution function show an anomalously steep slope for the first 3-4 points in the distribution function [Fig. 5.12 (a,c)]. The working theory is that the current algorithm does not accurately capture the behaviour of the distribution at these velocities. This information is present in the Thomson-scattering spectrum close to the ion-acoustic wave feature, which was blocked out of the data with a dielectric filter. However, this information also exists at larger scattering angles where the spectrum is less collective. Unfortunately the SNR in this region was insufficient, but this could be improved in future experiments.

The absorption was consistently higher than predicted by Langdon theory (Fig.
Figure 5.12: Measured electron distribution functions (black points) on a (a,c) linear and (b,d) logarithmic scale for (a,b) Shot 95830 and (c,d) Shot 95831. The super-Gaussian plus Maxwellian model (red curve), with parameters from the Monte Carlo analysis (Table 5.1), was compared to the data. A 90% confidence interval (gray) was computed from the super-Gaussian plus Maxwellian model. A Maxwellian distribution function (blue curve) is shown for reference. (a,b) Shot 95830 was the least constrained fit, highest error bars and (c,d) Shot 95831 was the closest to Maxwellian.

5.13). This was especially stark in hydrogen where the measured absorption doubled the Maxwellian expectations. If the nominal ionization state is used to calculate $A_{Max}$ and $Z + 1$ is used to calculate $A$, the agreement is improved for all the absorption data. While this might point to an issue with the way Maxwellian absorption is calculated there could also be other physics impacting the hydrogen data or an experimental issue. These measurements, especially in hydrogen, should be corroborated before any conclusions are drawn.
Figure 5.13: The Measured (red circles) and calculated (blue circles) absorption normalized to the absorption calculated assuming a Maxwellian electron distribution function, is plotted as a function of the ratio of the inverse bremsstrahlung heating rate to the electron-electron collision rate determined from the measured plasma conditions at the center of the plasma. The measured absorption is compared to the predicted scaling (blue curve) and an ad hoc scaling (dashed black curve).
Chapter 6

Conclusion

This work covers Thomson-scattering measurements with temporal, spatial, and angular resolution. These provided unprecedented levels of detail in laser-produced plasmas, measuring electron temperature, electron density, ionization state, and electron distribution function as a function of time or space.

Assuming Maxwellian electron distribution functions in Thomson-scattering analysis was shown to produce significant errors in the inferred plasma temperature and density [62]. Including the correct distribution function in accessible fit space, as done here with super-Gaussian order, can eliminate model-dependent errors with minimally increased statistical uncertainty. Including a super-Gaussian distribution function specifically can account for extra signal in the region between the electron plasma wave peaks. While not considered here, other distribution functions can correct other common problems in collective Thomson-scattering analysis such as peak signal asymmetry [37], relativistic effects [88], and apparent shift in fundamental frequency required to fit both peaks simultaneously.

In ultrafast Thomson-scattering measurements, the electron plasma wave spectrum was used to determine the effects of laser heating and ionization on the electron distribution function [67]. While the argon plasma was being heated, super-Gaussian
electron distribution functions were measured. After the heater beam was turned off, the electron density continued to rise and the electron temperature dropped as the free electrons continued to ionize the argon plasma. The non-Maxwellian electron distribution functions from Vlasov-Fokker-Planck simulations that included both laser heating and ionization were required to reproduce the measured Thomson-scattering spectra, temperature evolution, and density evolution simultaneously. Thereby demonstrating the interplay between the effects of inverse bremsstrahlung and atomic kinetics on the shape of the electron distribution function. These results reinforce the importance of using non-Maxwellian electron distribution functions in determining plasma conditions from Thomson-scattering measurements.

Angularly resolved Thomson scattering provided precise measurements of the distribution function over many orders of magnitude [69]. The distribution function was measured without a priori knowledge of its shape, and the technique will allow a range of applications to be studied beyond inverse bremsstrahlung heating. The measured distribution functions were determined to be significantly different from Maxwellian. These differences exist in the bulk, where the distribution function is super-Gaussian, and in the tail, where the distribution function is Maxwellian but with a different amplitude and width.

In inertial confinement fusion plasmas, it has long been assumed that these non-Maxwellian electron distribution functions lead to a reduction in laser heating and in this work this reduction was measured. Furthermore, it was predicted that non-Maxwellian distributions change the plasma wave damping, which results in a redistribution of the thermal energy in the fluctuation spectrum. This affects the atomic transition rates used to describe x-ray spectra [23] and plasma instabilities that grow from thermal noise, which could have a significant impact on predictive capabilities [8, 25, 38, 47].
In laser-plasma studies, the agreement between a super-Gaussian + Maxwellian reduced model and these measurements of the distribution function, in shape and scaling, provides confidence in using this closed form solution in hydrodynamic simulations as a more tractable option compared to Fokker-Planck simulations when the details of the electron distribution function are important [68]. For example, when using inline models to calculate the growth rates of cross-beam energy transfer[26] or stimulated Raman scattering[8, 25, 38, 47] in simulations of inertial confinement fusion experiments on the National Ignition Facility. More generally, the underlying electron distribution functions dictate the thermal transport in plasmas and deviation from Maxwellian will modify the transport coefficients used in hydrodynamic models of laboratory and astrophysical plasmas[3].

The uncertainties in the measured distribution functions were due to the experimental limitations in the range of probed wave-vectors and instrumental response. Even with these uncertainties, the electron distribution function was measured to a level of precision more than sufficient to distinguish the distribution from Maxwellian or other functional forms. Understanding the uncertainties in this distribution function analysis is important in determining problems where angularly resolved Thomson scattering can be applied. This is a tool with significant potential to elucidate the detailed physics occurring in plasmas, but an understanding of the uncertainties is required to know if the measured distribution function is distinguishable from other potential solutions.
Appendix A

Supplemental Information

A.1 List of symbols

\(A\) Absorption
\(A_{Max}\) Absorption calculated assuming a Maxwellian electron distribution function
\(c\) Speed of light in vacuum
\(e\) Charge on an electron
\(f_e\) Electron distribution function
\(I\) Intensity
\(k\) Wavevector of the probed Thomson-scattering fluctuation \(k_s - k_0\)
\(k_0\) Wavevector of the Thomson-scattering probe
\(k_s\) Wavevector of the collected Thomson-scattered light
\(m\) Super-Gaussian order
\(m_e\) Electron mass
$m_i$  Ion mass

$n_e$  Electron number density

$n_i$  Ion number density

$P_0$  Thomson-scattering probe power

$P_m$  Measured Thomson-scattering power

$P_s$  Calculated Thomson-scattering power

$r_0$  Classical electron radius

$ROI$  Region of interest

$S(k, \omega)$  Spectral density function

$SNR$  Signal to noise ratio

$T_e$  Electron temperature

$T_i$  Ion temperature

$v_g$  Group velocity

$v_{osc}$  Electron oscilatory velocity

$v_{th}$  Electron thermal velocity

$x$  Normalized phase velocity

$Z$  Charge state

$\alpha$  Langdon parameter

$\lambda$  Wavelength of the probed Thomson-scattering fluctuation

$\lambda_0$  Wavelength of the Thomson-scattering probe

$\lambda_D$  Electron Debye length

$\lambda_s$  Wavelength of the collected Thomson-scattered light

$\chi^2$  Chi squared fit metric

$\sqrt{T_e/m_e}$

$\left(1/\lambda_0 - 1/\lambda_s\right)^{-1}$

$\omega_{pe}/v_{th}$

$\sum \frac{(P_m - P_s)^2}{\sigma^2}$
χ_{e,i} \quad \text{Electron and ion susceptibilities}

\omega \quad \text{Frequency of the probed Thomson-scattering fluctuation} \quad \omega_k - \omega_0

\omega_0 \quad \text{Frequency of the Thomson-scattering probe}

\omega_{pe} \quad \text{Plasma frequency} \quad \sqrt{\frac{4\pi e^2 n_e}{m_e}}

\omega_s \quad \text{Frequency of the collected Thomson-scattered light}

\Omega \quad \text{Solid angle}

### A.2 Quartz particle simulation code

The Quartz particle simulation code integrates the electron equations of motion in multiple, speckled lasers using a fourth-order relativistic Runge-Kutta scheme, giving excellent energy conservation properties. The electromagnetic fields arising from the overlapping lasers are specified analytically to remove simulation grid effects. The electrostatic response field of the plasma (E_s) to the laser ponderomotive force is provided by a simple phenomenological model derived from complementary two-dimensional (2D) Vlasov-Fokker-Planck (VFP) calculations [99, 104] of field generation in laser speckles. In Quartz, the vector potential generated by \( N_b \) overlapping laser beams, each expressed in a Cartesian coordinate system \( x_b = x_b(x, \theta_b, \phi_b) \) in which the \( z_b \)-axis is aligned along the beam k-vector (\( k_b \)), generated with Gaussian speckle statistics and \( 4N^2 \) modes per beam, is given by [105]:

\[ A_L(x, t) = \sum_{b=1}^{N_b} \frac{A_b}{2N} \sum_{j,l=-N}^{N-1} \exp \left\{ i \Omega_{j,l,b}(x_b) \right\}, \]

where \( \Omega_{j,l,b}(x_b) = -\pi \frac{jx_b + ly_b}{N\lambda_L} - 2\frac{j^2 + l^2}{N^2} \frac{z_b}{L_z} + \phi_{j,l,b} + a_b A_b \exp \left[ i (k_b \cdot x_b - \omega t + \psi_b) \right] \), \( \omega \) the laser frequency, \( L_\perp \) is the mean transverse intensity correlation radius, \( L_z \) the mean speckle length, \( A_b \) the mean magnitude of the vector potential of the beam, \( a_b \) a unit vector specifying the polarization direction of the beam, \( \phi_{j,l,b} \) are variables randomly generated with value 0 or \( \pi \), and \( \psi_b \) is a random phase of the beam chosen...
between 0 and $2\pi$, and $N = 9$. The field at an arbitrary point $\mathbf{x}$ in the laboratory coordinate system was found by expressing the $\mathbf{x}_b$ in terms of $\mathbf{x}$ using the polar and azimuthal angles of the beams, $\theta_b$ and $\phi_b$. Note that each beam is generated with its own independent speckle statistics, and we also include smoothing by spectral dispersion. By solving the equations of motion $d\mathbf{p}/dt = -e \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right)$ and $\mathbf{v} = d\mathbf{x}/dt$ for each electron, where $\mathbf{p}$ is the momentum, $\mathbf{E} = \mathbf{E}_s - \partial \mathbf{A}_L / \partial t$ is the electric field and $\mathbf{B} = \nabla \times \mathbf{A}_L$ is the magnetic field, the evolution of the electron distribution function was studied.

The above laser parameters were chosen to match the experimental configuration, with a total of $N_b = 6$ beams (5 heaters and one probe). A Monte-Carlo method was used to include both electron-electron and electron-ion Coulomb collisions in the small-angle approximation [106]. A relatively sophisticated computational model of this sort was necessary because particle motion in multiple speckled beams becomes chaotic under the experimental conditions considered here.
Appendix B

Computation of Angularly resolved Thomson scattering

B.1 Mathematical Principles

B.1.1 From Book

To derive the equations used to calculate spectra for arbitrary distributions over a range of angle we will start from the equations for power scattered and spectral density given in Plasma scattering of electromagnetic radiation by D. H. Froula et al.

\[
P_s(R, \omega_s) = \frac{P_i r_0^2 L}{2 \pi} d\Omega \left( 1 + \frac{2 \omega_s}{\omega_i} \right) \left| \hat{s} \times (\hat{s} \times \hat{E}_{i0}) \right|^2 n_e S(k, \omega)
\]

Here \(d\Omega\) and \(d\omega\) are the solid angle about the scattering angle \(\theta\) and the frequency range about the scattering frequency \(\omega_s\). \(P_i\) is the input power or power of the probe beam, \(r_0\) is the classical electron radius, \(L\) is the length of the scattering volume in the direction of the probe’s propogation, \(\omega_i\) is the probe’s frequency. \(\hat{s}\) is the observation direction (direction of \(R\)) \(\hat{E}_{i0}\) is the polarization direction of the probe beam. \(n_e\) is the electron density.
\[ S(k, \omega) = \frac{2\pi}{k} \left| 1 - \frac{\chi_e}{\varepsilon} \right|^2 f_{e0} \left( \frac{\omega}{k} \right) + \frac{2\pi Z}{k} \left| \frac{\chi_e}{\varepsilon} \right|^2 f_{i0} \left( \frac{\omega}{k} \right) \quad (B.2) \]

Here \( k = |k| = |k_i - k_i| \), \( Z \) is the average ionization state, \( f_{e0} \) and \( f_{i0} \) are the one dimensional electron and ion distribution functions found by projecting the 3-dimensional distributions onto \( k \). \( \chi_e \) is the electron susceptibility and \( \varepsilon \) is the dielectric function.

Now we can make our first assumption

(1) scattered light will be measured for \( \left| \frac{\omega}{k} \right| \gtrsim v_{th} \)

With this assumption the second term in \( S(k, \omega) \) goes to 0 since \( f_{i0} \) goes to zero well before the \( v_{th} \) in most cases especially laser produced plasmas, as the laser heat the electrons.

\[ S(k, \omega) = \frac{2\pi}{k} \left| 1 - \frac{\chi_e}{\varepsilon} \right|^2 f_{e0} \left( \frac{\omega}{k} \right) \quad (B.3) \]

Where

\[ \chi_e(k, \omega) = \int_{-\infty}^{\infty} d\nu \frac{4\pi e^2 n_e}{m_e k^2} \frac{k \cdot \frac{\partial f_e}{\partial \nu}}{\omega - k \cdot \nu - i\gamma} \quad (B.4) \]

and

\[ \varepsilon = 1 + \chi_e + \chi_i \quad (B.5) \]

Then we will again use our first assumption to simplify

\[ \varepsilon = 1 + \chi_e \quad (B.6) \]
So we are left with

\[ S(k, \omega) = \frac{2\pi}{k} \left| \frac{1}{1 + \chi_e} \right|^2 f_{e0}\left( \frac{\omega}{k} \right) \] (B.7)

### B.1.2 Change to wavelength-angle

Our base equation B.1 is written in frequency-k space but for comparison to experiment it is useful to rewrite in wavelength-angle space.

\[ \frac{P_s(\lambda_s, \theta)}{d\omega_s} = \frac{P_s(R, \omega)}{2\pi c} \frac{\omega_s^2}{2\pi c} = \frac{P_i \rho_0^2 L}{4\pi^2 c} \left( \frac{1 + 2\omega}{\omega_i} \right) |\hat{s} \times (\hat{s} \times \hat{E}_i)|^2 n_e S(x, \theta) d\Omega \] (B.8)

Here we have left the R dependence unaltered. We will return to this as the method used depends on the diagnostic. \( \theta \) (scattering angle) is defined as the angle between the probe \( k_i \) and observation \( k_s \). We have also introduced the new normalized velocity,

\[ x \equiv \frac{v}{v_{th}} = \frac{\omega}{k v_{th}} \cdot \]

In order to write \( S(x, \theta) \) we have to make our second assumption

(2) the electron distribution is isotropic

The first thing we do with this assumption is give a definition for \( f_{e0} \)

\[ f_{e0}(k, \omega) = \int_{0}^{\infty} \int_{0}^{\infty} f_e\left( \frac{\omega}{k} \right) v_r d\nu_r d\nu_\theta \] (B.9)

We have used the freedom in defining cylindrical coordinates to set \( \hat{s} = \hat{k} \) making the projection onto the probed fluctuation simpler. We could write out the components
of the velocity in cylindrical but because of assumption (2) $f_e$ is only dependent on the magnitude of the velocity.

For super-Gaussian distributions the resulting distribution will be of the form

$$f_{e0}(v) = \frac{C_m}{v_{th}} \exp \left[ - \left( \frac{|v|}{\alpha_m v_{th}} \right)^m \right] = f_{e0}(x)/v_{th} \quad (B.10)$$

The vector dependence of the electron susceptibility ($\chi_e$) can be dealt with in a similar way. Again using cylindrical coordinates with $\hat{z} = \hat{k}$

$$\chi_e(k, \omega) = \int \int \int v_r dv_r dv_\theta dv_z \frac{4\pi e^2 n_e}{m_e k^2} \frac{k \cdot \frac{\partial f_e}{\partial v}}{\omega - k \cdot v - i\gamma} \quad (B.11)$$

evaluating the dot products

$$\chi_e(k, \omega) = \int \int \int v_r dv_r dv_\theta dv_z \frac{4\pi e^2 n_e}{m_e k^2} \frac{k \frac{\partial f_e}{\partial v_z}}{\omega - kv_z - i\gamma} \quad (B.12)$$
pulling everything outside the integrals

$$\chi_e(k, \omega) = \frac{4\pi e^2 n_e}{m_e k^2} \int dv_z \frac{k \frac{\partial}{\partial v_z} \int v_r dv_r dv_\theta f_e}{\omega - kv_z - i\gamma} \quad (B.13)$$

noting the definition of $f_{e0}$

$$\chi_e(k, \omega) = \frac{4\pi e^2 n_e}{m_e k^2} \int dv_z \frac{k \frac{\partial}{\partial v_z} f_{e0}(k, \omega)}{\omega - kv_z - i\gamma} \quad (B.14)$$

Now substituting $v = xv_{th}$ and omitting the directional notation since we are down to one dimension.
\[ \chi_e(x) = \frac{4\pi e^2 n_e}{m_e k^2} \int_{-\infty}^{\infty} v_{th} dx \frac{k/v_{th} \frac{\partial}{\partial x} f_{e0}(x) / v_{th}}{\omega - k x v_{th} - i\gamma} \]  

(B.15)

canceling \( k \) in the integral gives

\[ \chi_e(x) = \frac{4\pi e^2 n_e}{m_e k^2 v_{th}^2} \int_{-\infty}^{\infty} dx \frac{\omega}{k x v_{th}} \frac{\partial}{\partial x} f_{e0}(x) }{\omega - x - i\gamma} \]  

(B.16)

The factor of \( k v_{th} \) that multiplies \( i\gamma \) has be omitted since this integral is evaluated in the limit \( \gamma \rightarrow 0 \). We can make one more simplification

\[ \chi_e(x) = \frac{1}{k^2 \lambda_D^2} \int_{-\infty}^{\infty} dx \frac{\partial}{\partial x} f_{e0}(x) / {\omega - x - i\gamma} \]  

(B.17)

### B.1.3 Computation of \( \chi \) and \( f_{e0} \)

Equations B.8, B.17, and B.10 are sufficient to calculate the Thomson-scattering spectrum. However, in practice when the spectrum will be calculated millions of times as part of a fit to data, it is advantageous to use forms and simplifications which are most efficient, as far as computation time, while preserving accuracy.

First in order to maintain the highest possible level of generality the one dimensional distribution is defined as a set of points.

\[ f_{disc}: \{x_1, x_2, \ldots, x_n\} \rightarrow \mathbb{R}_{>0} \]  

(B.18)

i.e. \( f_{disc} \) is a mapping from a discrete domain onto the co-domain of the positive reals.
We will use the notational shorthand of $f_{\text{disc}}[x_i]$, where the square brackets are used to denote the discrete domain.

While incredibly general the efficiency and accuracy of this form are based on the number of points used. To maintain the level of accuracy used in other computations of the Thomson-scattering spectrum $\sim 10,000$ points would be required. For efficient computation of the spectrum, most critically due to the number of free parameters (see Sec. B.1.5), it is desirable to use $< 100$ points. To regain some accuracy with the lower number of points we made another assumption,

1. The distribution in locally exponential.

This means if you look at a small portion of the distribution it will be well modeled by an exponential curve. We expect this assumption to be rather safe as many physical effects attempt to smooth out the distribution (collisions and damping) and most functional forms are built on exponentials (Gaussian, super-Gaussian, sum of Gaussian). This allows a far more accurate approximation of interstitial points using interpolation in logarithmic space.

$$f_{e0}(x) = \begin{cases} 
  f_{\text{disc}}[x] & x \in \{x_1, x_2, \ldots, x_n\} \\
  \exp \left[ \ln f_{\text{disc}}[x_a] + (\ln f_{\text{disc}}[x_b] - \ln f_{\text{disc}}[x_a]) \frac{x-x_a}{x_b-x_a} \right] & x \notin \{x_1, x_2, \ldots, x_n\}
\end{cases} \tag{B.19}$$

Integrals are generally expensive calculations and the pole in the $\chi_e$ integral (Eq. B.17) makes it especially tricky. The standard technique to get around this integral is use of the $Z$ function whose values can be found in Fried and Conte [107]. This requires the distribution function to be Maxwellian, a more general approach is to use
the method described by Palastro et al. [88] to create a lookup table for any given function. Since we wanted to change the distribution arbitrarily, lookup tables were too limiting. So we used a faster way to evaluate Eq. B.17 on the fly.

We preformed this in a method from Chapman and Williams [101]. To begin we will use the Sokhotsky-Plemelj theorem to address the pole.

\[
\lim_{\varepsilon \to 0} \int_{a}^{b} \frac{f(x)}{x \pm i\varepsilon} \, dx = \mp i\pi f(0) + \mathcal{P} \int_{a}^{b} \frac{f(x)}{x} \, dx \quad \text{(B.20)}
\]

Applying this to Eq. B.17

\[
\chi_{ei} = \frac{\pi}{k^2 \lambda_D^2} \frac{\partial f_{e0}}{\partial x} \bigg|_{\omega_{kvth}}
\]

\[
\chi_{er} = -\frac{1}{k^2 \lambda_D^2} \mathcal{P} \int_{-\infty}^{\infty} \frac{\partial f_{e0}/\partial x}{x - \omega_{kvth}} \, dx
\]

(B.21)

Here \( \mathcal{P} \) denotes the principle value and the subscripts \( r \) and \( i \) denote the real and imaginary components respectively. The imaginary part can be easily computed numerically leaving the principal value.

First we consider a simpler problem

\[
y = \int_{s_0}^{s_1} \frac{f}{g} \, ds
\]

(B.22)

Here, we assumed that \( f \) and \( g \) are piece-wise linear and we know their values at \( s_0 \) and \( s_1 \) are known. This allows us to define local linear approximations
\[ f_s = f_0 + u df \]  
(B.23)

\[ g_s = g_0 + u dg \]

where

\[ df = f_1 - f_0 \]

\[ dg = g_1 - g_0 \]  
(B.24)

\[ u \equiv \frac{s - s_0}{s_1 - s_0} \]

We take Eq. B.22, plug in Eq. B.23, and change variables to u

\[
\frac{y}{ds} = \int_0^1 \frac{f_0 + u df}{g_0 + u dg} du
\]

(B.25)

defining

\[ \bar{f} = \frac{1}{2}(f_1 + f_0) = f_0 + \frac{1}{2} df \]  
(B.26)

likewise, for \( \bar{g} \), Eq. B.25 becomes

\[
\frac{1}{\bar{g} - \frac{1}{2} dg + u dg} du = \int_0^1 \frac{\bar{f} + \left( u - \frac{1}{2} \right) df}{\bar{g} + \left( u - \frac{1}{2} \right) dg} du
\]

(B.27)

This integral can be solved in two regimes $|dg| \ll \bar{g}$ and $|dg| \ll \bar{g}$. The second case can be solved by Mathematica [108]
\[
\frac{df dg - (\bar{f} dg - \bar{g} df)(\ln[-dg + 2\bar{g}] - \ln[dg + 2\bar{g}])}{dg^2},
\]
\text{ (B.28)}

which can be rearranged to

\[
\frac{df}{dg} + \frac{\bar{f} dg - \bar{g} df}{dg^2} \ln \left( \frac{\bar{g} + dg/2}{\bar{g} - dg/2} \right).
\]
\text{ (B.29)}

If \(|dg| \ll \bar{g}\) we can Taylor expand Eq. B.27 about \(dg = 0\) out to second order in \(dg\)

\[
\frac{1}{\bar{g} + (u - \frac{1}{2}) dg} = \frac{1}{\bar{g}} - \frac{(u - \frac{1}{2}) dg}{\bar{g}^2} + \frac{(u - \frac{1}{2})^2 dg^2}{\bar{g}^3},
\]
\text{ (B.30)}

so Eq. B.27 becomes

\[
\int_0^1 du \frac{\bar{f}}{\bar{g}} + \frac{(u - \frac{1}{2}) df}{\bar{g}^2} - \frac{(u - \frac{1}{2}) \bar{f} dg}{\bar{g}^2} - \frac{(u - \frac{1}{2})^2 df dg}{\bar{g}^2} + \frac{(u - \frac{1}{2})^2 \bar{f} dg^2}{\bar{g}^3} + \frac{(u - \frac{1}{2})^3 df dg^2}{\bar{g}^3},
\]
\text{ (B.31)}

Each of these integrals is simple and can be solved trivially with the substitution \(v = u - 1/2\). With this substitution we see the \(v\) and \(v^3\) terms are odd so they will be zero. Solving the remaining three terms

\[
\frac{\bar{f}}{\bar{g}} - \frac{df dg}{12\bar{g}^2} + \frac{\bar{f} dg^2}{12\bar{g}^3} = \frac{\bar{f}}{\bar{g}} + \frac{\bar{f} dg^2 - df dg \bar{g}}{12\bar{g}^3} = \frac{\bar{f}}{\bar{g}} + [\bar{f} dg - \bar{g} df] \frac{dg}{12\bar{g}^3}.
\]
\text{ (B.32)}
Putting everything together, the final result is

$$\chi_e^r = -\frac{1}{k^2\chi_D^2}\chi_r^r$$

(B.33)

$$\chi_r^r = \sum \chi_{rat} dz$$  \hspace{1cm} (B.34)

$$\chi_{rat} = \begin{cases} \frac{\hat{f}}{\hat{g}} + [\hat{f}\hat{g} - \hat{g}\hat{f}] \frac{dg}{12\hat{g}^3} & \text{if } |dg| < \frac{\hat{g}}{10,000} \\ \frac{df}{dg} + [\hat{f}\hat{g} - \hat{g}\hat{f}] \frac{\ln(\frac{g+dg/2}{g-dg/2})}{dg} & \text{if } |dg| > \frac{\hat{g}}{10,000} \end{cases}$$ \hspace{1cm} (B.35)

**B.1.4 Instrumental Effects**

Through eqs. B.7, B.8 and B.10 it is possible to calculate the “true” scattering spectrum, as would be seen by a perfect detector. Physical measurement systems, consisting of collection, transport, spectrometers, and CCDs, have finite integration times, diffraction, aberrations, finite solid angle, and other physical limitations. The net effects of these physical limitation was considered in order to obtain precise and accurate modeling of the measurement.

To accomplish this, we modified Eq. B.8 as

$$\frac{P_s(\lambda_s, \theta)}{d\alpha_s} = R(\lambda) * S(\theta) * \left[ A \left( \frac{2\lambda_0}{\lambda_s^3} - \frac{1}{\lambda_s^2} \right) n_s W(\theta', \theta) \frac{1}{N}\sum_{\alpha} S(\lambda_s, \theta', \alpha) \right] d\Omega \hspace{1cm} (B.36)$$

here $R(\lambda)$ and $S(\theta)$ are instrument response functions to wavelength and angle respectively (Sec. B.1.4) and * denotes a linear convolution. $W(\theta', \theta)$ is a weighting function.
that converts from scattering angles to measured angles accounting for the finite aperture effect and the polarization efficiency (see Sec. B.1.4). Finally, $N_\alpha$ is the number of plasma conditions the spectrum is calculated for, so in combination with the summation, this gives an average spectrum over the range of plasma conditions integrated due to finite integration time of the detector.

**Transmission**

Optical elements and their coatings have wavelength dependent transmission. Similarly the quantum efficiency of photocathodes used in intensifiers are wavelength dependent. This change to the sensitivity of the instrument as a function of wavelength was dominated by the grating and photocathode for the angularly resolved Thomson-scattering instrument.

To calculate the transmission $T(\lambda)$ the spectrum from a well understood tungsten lamp was collected ($s_m(\lambda)$). So $T(\lambda) = s_m(\lambda) / s_k(\lambda)$ where $s_k(\lambda)$ is the known spectrum of the lamp. This transmission was used in Sec. B.1.5 to achieve the correct amplitudes.

**Gradients**

If the detector used for an experiment has an integration time larger than the evolution time of the plasma, it can be important to account for this in calculations. Changing plasma conditions (especially density) can move the peaks in wavelength as a function of time, resulting in a measured spectrum with broader peaks of a different shape than those calculated for a single set of plasma conditions. In our case, the integration time was $\sim 200$ ps and the characteristic evolution time was $\sim 1$ ps (based on electron-electron and electron-ion collision times).

The natural solution is to mimic this integration by calculating the spectra for the
progression of plasma conditions and then integrating over them. However, we are again faced with an issue of the number of spectra that must be calculated. The more spectra we use, the more accurate the result, but the scaling of computation time and memory requirements associated with this third dimension of calculation (wavelength and angle being the first two dimensions) is not favorable. It was found heuristically that seven spectra provided the appropriate compromise between efficiency and accuracy. Although this number is dependent on the plasma parameters in question and their range.

To this end, we modify the spectral density function with an added parameter $\alpha$, which represents the plasma conditions in the calculations.

$$S(\lambda_s, \theta, \alpha) = \frac{2\pi}{k(\lambda_s, \theta, \alpha) v_{th}(\alpha)} \left| \frac{1}{1 + \chi(x)} \right|^2 f_{e0}(x)$$  \hspace{1cm} (B.37)

Here, we have been explicit about the dependencies, writing $\chi$ and $f_{e0}$ as a function of $x$ which combines all three variables, and showing $k$ is dependent on all three variables but not through $x$, in fact recall $x = \omega / kv_{th}$.

**IRFs**

The measured spectrum is subject to blurring in the wavelength and angle directions due to optical effects such as diffraction, finite source size, and aberrations. These are treated by convolving the calculated spectrum with Gaussian instrument response function in the wavelength and angular directions.
The two instrument response functions \( R(\lambda) \) and \( S(\theta) \) are defined as

\[
R(\lambda) = \frac{1}{\sqrt{2\pi\sigma^2_\lambda}} \exp\left\{ \frac{-\lambda^2}{2\sigma^2_\lambda} \right\}
\]

\[
S(\theta) = \frac{1}{\sqrt{2\pi\sigma^2_\theta}} \exp\left\{ \frac{-\theta^2}{2\sigma^2_\theta} \right\}
\]

For these equations, the widths \( \sigma_\lambda \) and \( \sigma_\theta \) were determined to be 1 nm and 1°, respectively. The specular width was determined by fitting this Gaussian form to data of a 532 nm laser. The angular width was determined by tracing a collimated source through the system using the ray-tracing software FRED [109].

As written in Eq. B.37, applying the instrument response to the calculated spectrum was done with the standard linear convolution.

\[
f(x,t) \ast g(t) = \int_{-\infty}^{\infty} f(x,t-\tau)g(\tau)d\tau,
\]

**k-smearing**

Finite aperture effects, or k-smearing, is the result of the scattered power representing an infinitesimal solid angle (Eq. B.1). The first order approximation would be that the spectrum does not change significantly over the solid angle subtended by the collection optics. This is often the case for electron plasma wave spectra, as the resonant wavelength is dominated by the \( \omega_{pe} \). In this case, integrating over solid angle just yields another scale factor.

With the angularly resolved Thomson-scattering instrument, collecting \( f / 0.5 \times f / 5 \) the collection for a single scattering angle can have moderate k-smearing effects as the
solid angle, especially in the forward and reverse directions, can be significant. To account for this we follow the method of Follett et al. [98].

We start with a mapping from the optical modeling software FRED. In FRED a 1000 by 200 array of rays are launched from TCC covering a range of $\theta = 10^\circ \rightarrow 140^\circ$ and $\phi = 201.6^\circ \rightarrow 264.6^\circ$ relative to the propagation direction of the probe ($\hat{z}$). The mapping from FRED gives the (x,y)-coordinates there the ray intersects the final detector. Since FRED uses the accurate size of the optical elements, it accounts for obstructions and only rays which make it to the final image plane are included.

All the locations of rays on the final image plane are histogrammed into 1024 bins between -6.5 and 6.5 in x. This identifies the rays which hit each of the 1024 columns of the 1024 by 1024 13 mm square CCD detector. From this histogram identifiers which match a ray to a bin are kept.

For each of the bins, the list of $(\theta, \phi)$s for all rays in said bin, are converted to Cartesian coordinates $k_{si} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)$. These vectors are then rotated so the probe direction is aligned with the OMEGA P9 $\rightarrow$ P4 axis ($\hat{p}$).

$$k_{sf} = R \cdot k_{si} = k_{si} \cos \rho + \sin \rho (u \times k_{si}) + (1 - \cos \rho)(u \cdot k_{si})u \quad (B.40)$$

Here $k_{si}$ and $k_{sf}$ are the initial and final scattering vectors, $R$ is the rotation matrix which rotates $k_{si}$ by $\rho$ about $u$. As for any rotation of vector $a$ into $b$ we rotate by the angle between the vectors $\rho = \cos^{-1} \frac{a \cdot b}{|a||b|}$ about their mutually perpendicular vector $u = a \times b$. In our case $a = \hat{z}$ and $b = \hat{p}$.

A $15 \times 15$ grid of vectors are generated about the $\hat{z}$ axis. Those vectors outside a radius of $1/2$ are thrown out and the grid is resized to match the f-number of the probe.
beam (6.7) and renormalized so the vectors are unit vectors. The same rotation method is then used to align these probe vectors around $\hat{\rho}$.

The separation angle ($\rho$) for each ray in the bin and each vector in the probe beam is calculated. These separation angles are histogramed and the normalized relative weights are interpolated onto an array of scattering angles from $19^\circ$ to $139^\circ$ with steps of half a degree. The result is relative weights of the contribution for each angle from $19^\circ$ to $139^\circ$ to the given bin, which represents a single column of pixel.

The entire process is repeated for each of the 1024 bins yielding a $1024 \times 241$ matrix $W(\theta, \theta')$. This matrix maps single scattering angles used to calculate spectra to the unique combination of scattering angles seen by each column of CCD pixels.

### B.1.5 Matching Data

With Eq. B.36 we can calculate spectra which can be matched to data. This process is broken into three sections, amplitude matching, axes definitions, and regression analysis.

#### Amplitude Matching

In order to model the amplitude of the Thomson scattered signal, assuming all other parameter are correct, the data must only include signal from Thomson scattering and the parameter $A$ (Eq. B.36) must be defined.

#### Background Subtraction

Light collected in the angularly resolved Thomson instrument can come from many sources and require multiple mitigation strategies. Since our model only calculates light from Thomson scattering, any other sources of light must be removed. Some of these sources are blocked by physical barriers, such as light not
originating from near TCC, or by time constraints, such as emission from the recapture of electrons.

The main sources of background light within the wavelength, temporal, and spacial constraints are bremsstrahlung emission and Thomson scattering from other beams. To remove these sources from angularly resolved Thomson data, a second shot at nearly the same plasma conditions is taken. If the plasma conditions were identical between a data and background shot we could subtract one from the other to obtain the Thomson-scattering signal. Variability in the laser and the neutral gas density can lead to variation in the backgrounds shape as well as amplitude.

To account for some of this change in amplitude and shape we define the data

\[
\text{Data} = (F_G - F_{G\text{dark}}) - [a_1(y - a_2)^2 + a_3(y - a_4) + a_5](B_G - B_{G\text{dark}})
\]

\[
a_i = \arg\min_a \sum_y [F_G(1000, y)] - [a_1(y - a_2)^2 + a_3(y - a_4) + a_5](B_G(1000, y))^2
\]

(B.41)

here \(F_G\) and \(B_G\) are the foreground and background shots, or the shots with and without the Thomson-scattering probe beam. The subscript dark denotes an image taken just prior to the shot with no exposure of the CCD, this measures the dark current on the CCD. The quadratic multiplying the background attempts to match the amplitude and shape, as a function of angle, using the 1000th spectral pixel as a reference. This location is chosen since there is no Thomson-scattering data at that large a wavelength (~ 653 nm). This does not take care of all the shot to shot shape changes, especially those caused by \(3\omega\) Thomson scattering, but it improves agreement.

The background data is also smoothed using a five by five kernel to prevent amplifying statistical fluctuations. This smoothing is done before the matching and sub-
traction. The signal is governed by Poisson statistics so without background smoothing these fluctuations would be amplified.

**Amplitude Scale Factors**  The amplitude factor $A$ (Eq. B.36) contains the remaining constants which inform the efficiency of scattering and the power input by the probe.

$$A = P_i r_0^2 L c |\hat{s} \times (\hat{s} \times \hat{E}_{i0})|^2$$  (B.42)

Instead of using this equation which introduces uncertainties from $P_i$ and $L$ we instead choose to define

$$A(\lambda) \equiv \begin{cases} 
A_1 \max_{\lambda \in [540\text{nm}, 631\text{nm}]} \frac{\text{Data}(\lambda, \theta)}{P_i(\lambda, \theta)} & \lambda \leq 510\text{nm} \\
0 & 510\text{nm} < \lambda < 540\text{nm} \\
A_2 \max_{\lambda \in [540\text{nm}, 631\text{nm}]} \frac{\text{Data}(\lambda, \theta)}{P_i(\lambda, \theta)} & \lambda \geq 540\text{nm} 
\end{cases}$$  (B.43)

This definition gives some added flexibility. The zero region models the spectral filter used to block the ion-acoustic wave Thomson scattering, while using two amplitudes ($A_1, A_2$) can account for peak asymmetries that are not currently well understood. The normalization to the red peak of each scattering angle removes remaining angular dependence of the amplitude and keeps the scale factors ($A_1, A_2$) near 1. Here we have made an assumption that the dipole efficiency term $|\hat{s} \times (\hat{s} \times \hat{E}_{i0})|^2$ does not vary over a single scattering angle. This assumption is based on the use of polarization rotation to minimize the variation. It is possible to relax this assumption and include it in the calculation of $W(\theta, \theta')$ if the polarization of the probe is measured.
Axes

Ensuring the axes of the computation match those of the data is always necessary. The wavelength axis, shown in plots as the x-axis, is calibrated each shot day using a mercury lamp. Three of mercury’s spectral lines are bright enough and within the wavelength range to be measured. A linear scaling is assumed to find the conversion from pixel to wavelength.

While calculating the spectrum (Eq. B.37) an order of magnitude extra wavelength points are used. 10250 linearly spaced wavelength points are used instead of the 1024 pixels to ensure accurate computations. The experimentally measure spectrum in a single pixel is integrated over the wavelength range collected by that pixel, conversely the calculated spectrum at a wavelength point is only the value at the associated wavelength. This difference can result in spectra calculated with too few points missing contributions of small features and being shifted slightly if a pixel is identified by one of its edges as opposed to its center of mass. The solution employed is to use an order of magnitude more points and report the signal for a pixel as the sum over 10.

\[ P_s(\lambda_i, \theta) = \sum_{j=10i-9}^{10i} P_s(\lambda_j, \theta) \]  

(B.44)

The Angular axis follows from the method in Sec.B.1.4. The weighting matrix is used to calculate the center of mass angle for each pixel, \( \theta_{cm} = W(\theta, \theta') \theta_{linear} \). Where \( \theta_{linear} \) is the vector of 241 linearly spaced angles used to calculate the spectra. This method for computing the angular axis captures the non-linearity where angles in the forward and backward direction are more compact.

The colorized plots showing power scattered as a function of angle and wavelength are displayed with two y-axes. This first axis is the center of mass scattering angle. The
second y-axis is a rough approximation of the normalized phase velocity being probed. This is calculated as

\[
\frac{v}{v_{th}} = \frac{\omega}{kv_{th}} \approx \sqrt{\frac{n_e}{T_e}} \frac{1}{1486k_0 \sin \frac{\theta}{2}}
\]  

(B.45)

This equation is a very rough approximation as it is found using \( \omega \approx \omega_{pe} \) and \( k_s \approx k_0 \). Using the first assumption directly and the second assumption in the law of cosines

\[
\frac{\omega}{kv_{th}} \approx \frac{\omega_{pe}/v_{th}}{\sqrt{2k_0^2(1 - \cos \theta)}}
\]  

(B.46)

This equation simplifies to the previous one by using the trig identity \( 1 - \cos \theta = 2 \sin^2 \theta / 2 \) and the equation for Debye length from the plasma formulary.

Regression

Regression analysis is the statistical process of identifying the most likely relation between variables. In this case amplitude, wavelength and angle. Since we have built a detailed relation between these variables (Eq. B.36,B.37,B.43), we look for the most likely values of each free parameter.

We use the approach of least squares, where a functional form is used to relate the free parameters to the data. In this approach the most probable set of parameters are those which minimize the sum of the squared residuals \( \sum_i [D_i - M_i]^2 \). Here the residuals are defined as the difference between the data \( D \) and the model \( M \). We call the sum of residuals...
\[
\chi^2 = \sum_{i=8}^{66} \sum_{k=30}^{90} [D(i, k) - M(i, k)]^2 + \sum_{i=94}^{200} \sum_{k=30}^{90} [D(i, k) - M(i, k)]^2 \tag{B.47}
\]

where

\[M(i, k) = \sum_{j=5i-4}^{5i} \sum_{l=10k-9}^{10l} P_s(\lambda_j, \theta_l)\]

\[D(i, k) = \sum_{j=5i-4}^{5i} \sum_{l=10k-9}^{10l} Data(\lambda_j, \theta_l)\]  \tag{B.48}

here we have summed the data and theory over one resolution unit, 5 pixels in wavelength and 10 pixels in angle, and only used a sub-domain for the calculation. This sub-domain is 448 nm to 509 nm, 539 nm to 652 nm, and 58° to 115°. The wavelength restrictions eliminate areas with no signal (large wavelength and filter location) and areas with 3\omega signal (small wavelengths). While the angle restrictions eliminate an area at large angle with poor signal and a region at small angle that is poorly modeled.

The free parameters whose likelihood we seek to maximize are: \(A_1, A_2, n_e, T_e\), and \(f_{disc}[x]\). Since these parameters are not linearly separable, the problem is one of non-linear least squares. Non-linear least squares problem do not necessarily have a closed form solution so we employ a gradient descent algorithm to find parameters that minimize \(\chi^2\). We used the constrained minimization algorithm \texttt{fmincon} from MATLAB [90].

## B.1.6 Post-processing

During minimization the normalization of the distribution function is not maintained. This is because the shape and amplitude of the distribution function is allowed to vary simultaneously with the density and temperature. This is advantageous algorithmicly because a small step in \(n_e\) or \(T_e\) correlates to a much larger multidimensional step in
Therefore the resulting density temperature and distribution function must be renormalized in order to have physically meaningful parameters.

This is achieved in a 5 step process:

\[
n_{e,\text{true}} = n_e \times \int_{-\infty}^{\infty} f_e(x) \, dx \quad \text{(B.49)}
\]

\[
f_e = f_e \nabla \cdot \int_{-\infty}^{\infty} f_e(x) \, dx \quad \text{(B.50)}
\]

\[
T_{e,\text{true}} = T_e \times \int_{-\infty}^{\infty} x^2 f_e(x) \, dx \quad \text{(B.51)}
\]

\[
f_e = f_e \sqrt{\frac{T_{e,\text{true}}}{T_e}} \quad \text{(B.52)}
\]

\[
x = x \sqrt{\frac{T_e}{T_{e,\text{true}}}} \quad \text{(B.53)}
\]

### B.2 ARTS code

Computation of angularly resolved Thomson scattering spectra and fitting of angularly resolved Thomson scattering data was accomplished with a suite of codes collectively referred to as the angularly resolved Thomson-scattering code. The code is given for the 4 main functions. AngTSDataFitter takes inputs from a GUI, collects the required information for other codes, and handles the plotting of the initial and final states of the minimizer as well as calling the minimizer. The minimizer fmincon minimizes \( \chi^2 \) defined using the difference between the data and the theoretical model from the function ArtemisModel. The function ArtemisModel handles diagnostic corrections to
the computed scattered spectrum including, instrument response function, k-smearing, and gradients. The function ApproxThomson3 is called to compute the scattered spectrum as a function of wavelength, angle, and plasma conditions. ApproxThomson3 is a slightly generalized version of the standard Thomson scattering code [42], which allows for the fast computation of the susceptibility using the function ratint. A new class called ThomsonInputs is also defined, this class tracks the values and fit status of all relevant parameters enabling fitting of individual parameters to easily be turned on or off without rewriting the minimizer statement for each possible case.

```matlab
function x=AngTSDataFitter(Fshotnum,Bshotnum,lineLocs,TSinputs)
% This function takes the inputs from the ANGTSDATAFITTERGUI ... and preforms the data corrections then fits the data ... returning the fit result
%The inputs from the GUI are Shot number, lineout locations, ... background shot number, probe wavelength, electron ... temperature, electron density, m, amp1 and amp2, ionization ... state, starting distribution function type and the number ... of distribution function points to use in numerical ... distribution function fitting.

%% Hardcoded inputs
% These are the detector info and fitting options
D.Detector='ideal';
D.BinWidth=10;
D.NumBinInRng=0;
D.TotalNumBin=1023;
spectralFWHM=.9;
angularFWHM=1;
norm2B=0; %0 no normalization
%1 norm to blue
%2 norm to red
options = optimoptions(@fmincon,'Display','iter','PlotFcns',[],...
'UseParallel',true,'MaxIter',300,'MaxFunEval',10000,'TolX',...
1e-10);
scaterangs=19:.5:139;
feDecreaseStrict=1; %forces the result to have a decreasing ... distribution function (no bumps)

% Read Data
```
FG = hdfread(['ATS-s num2str(Fshotnum) .hdf'], 'Streak_array');
BG = hdfread(['ATS-s num2str(Bshotnum) .hdf'], 'Streak_array');
load ('MyColormaps_TS','TS_mycmap');
FG=squeeze(FG(1,:,:)-FG(2,:,:));
BG=squeeze(BG(1,:,:)-BG(2,:,:));
FG=double(FG);
BG=conv2(double(BG),1/25*ones(5),'same');

%Fit a line of the background times a polynomial to a line of ... the data
xx=1:1024;
fun=@(x) sum(((FG(1000,:) -((x(1) *(xx-x(4)).^2 + x(2) *(xx-x(4)) + ... + x(3)))*BG(1000,:))).^2);
corrfactor=fmincon(fun, [.1 .1 1.15 300], [], []);
newBG=(corrfactor(1) *(xx-corrfactor(4)).^2 + ... + corrfactor(2) *(xx-corrfactor(4)) + corrfactor(3)).*BG;
data=FG-newBG;

load('angsFRED.mat','angsFRED')
xax=angsFRED;

if Fshotnum<95000
  yax=(0:1023)*.214116+449.5272;
else
  yax=(0:1023)*.2129+439.8;
end

% Correct Thruput
load('spectral_sensitivity.mat','speccal');
if Fshotnum<95000
  data=data./speccal;
else
  specax=(0:1023)*.214116+449.5272;
  speccalshift=interp1(specax,speccal,yax,'linear',speccal(1));
  data=data./speccalshift;
end

if norm2B==1
  %normalize to blue peak
  data=data./max(data);
  D.PhysParams={Inf,max(max(double(FG-BG)))};
elseif norm2B==2
  %normalize to red peak
  data=data./max(data(470:900,:));
  D.PhysParams={Inf,max(max(double(FG-BG)))};
else
  D.PhysParams={Inf,0};
end
ttl=['Artemis data: Shot ' num2str(Fshotnum)];
ColorPlots(yax, xax, rot90(data), 'Title', ttl, 'Kaxis', ...
[Tinputs.ne.Value*1E20, Tinputs.Te.Value, 526.5], 'Name', ...
'Data')
curlims=caxis;
caxis([0 curlims(2)])
line([yax(1) yax(end)], [xax(end-lineLocs) xax(end-lineLocs)])

%% f/5 angular smearing
load('angleWghtsFredfine.mat','weightMatrix')
%
xie=linspace(0,7,Tinputs.fe.Length);
[~, lamAxis, =]=lamParse([min(yax) ...
max(yax)], Tinputs.lam.Value, 10250);

%% Setup x0
if isempty(Tinputs.fe.Value)
    Tinputs=Tinputs.initFe(xie);
end
[Tinputs, x0, lb, ub]=genX(Tinputs);

%% Plot starting point
Thryinit=ArtemisModel(Tinputs, xie, scaterangs, x0, weightMatrix, ...
spectralFWHM, angularFWHM, lamAxis, xax, D, norm2B);
if ~norm2B
    Thryinit=Thryinit./max(Thryinit(470:900,:));
    Thryinit=Thryinit.*max(data(470:900,:));
    Thryinit=Tinputs.amp1.Value*Thryinit;
end
chisq = sum(sum((data([40:330 ...
470:900], 90:1015)−Thryinit([40:330 470:900], 90:1015)).ˆ2));
Thryinit(330:470,:)=0;

ColorPlots(yax, xax, rot90(Thryinit), 'Kaxis', ...
[Tinputs.ne.Value*1E20, Tinputs.Te.Value, 526.5], 'Title', ...
'Starting point', 'Name', 'Initial Spectrum');
ColorPlots(yax, xax, rot90(data-Thryinit), 'Title', ...
'Initial difference: \chi^2 = ' ...
num2str(chisq)), 'Name', 'Initial Difference');
load('diffcmap.mat', 'diffcmap');
colormap(diffcmap);

if norm2B
caxis([-1 1]);
else
caxis([-8000 8000]);
end
%% Perform fit

tic
if ~isempty(x0)
    if feDecreaseStrict && TSinputs.fe.Active
        A=zeros(length(x0));
        inds=sub2ind(size(A), ...
            TSinputs.fe.Location-TSinputs.fe.Length+1: ...
            TSinputs.fe.Location-1, ...
            TSinputs.fe.Location-TSinputs.fe.Length+1: ...
            TSinputs.fe.Location-1);
        A(inds)=-1;
        A(inds+length(x0))=1;
        b=zeros(length(x0),1);
    else
        A=[];
        b=[];
    end
end

n=10;
dataResunit=arrayfun(@(i) ...
    sum(data(:,i:i+n-1),2),1:n:1024-n+1,'UniformOutput',0);
dataResunit=cell2mat(dataResunit);

n=5;
dataResunit=arrayfun(@(i) ...
    sum(dataResunit(i:i+n-1,:),1),1:n:1024-n+1, ...
    'UniformOutput',0);
dataResunit=cell2mat(dataResunit);

[x,~,~,~,grad,hess]=fmincon(@(x)chiSqArtemis(x, ...
    TSinputs, xie, scaterangs, weightMatrix, spectralFWHM, ...
    angularFWHM, lamAxis, xax, D, ...
    dataResunit,norm2B),x0,A,b,[],[],lb,ub,[],options);
else
    x=chisq
    for i=1:length(lineLocs)
        figure('Name','Lineout ' num2str(i))
        plot(yax,sum(data(:,lineLocs(i):lineLocs(i)+25),2))
        hold on
        plot(yax,sum(Thryinit(:,lineLocs(i):lineLocs(i)+25),2))
        title(['Lineout at pixel ' num2str(lineLocs(i)) ' \circ'])
    end
end

figure('Units','normalized','position', [.1 .1 .25 ... .6], 'Name', ['Lineouts'])
hold on
for i=1:length(lineLocs)
    plot(yax,50000*i+sum(data(:,lineLocs(i): ...
        lineLocs(i)+25),2))
    plot(yax,50000*i+sum(Thryinit(:,lineLocs(i): ...
        lineLocs(i)+25),2))
end
xlabel('Wavelength (nm)')
xlim([450 650])
set(gca,'YColor','none','fontsize',16,'fontweight','bold')
title(['Lineouts ' num2str(xax(end-lineLocs)) '\circ'])
return
end
toc

function csq=chiSqArtemis(x, TSinputs, xie, sas, wghts, ...
spectralFWHM, angularFWHM, lamAxis, xax, D, data, norm2B)
Thry=ArtemisModel(TSinputs, xie, sas, x, wghts, ...
spectralFWHM, angularFWHM, lamAxis, xax, D, norm2B);

n2=10;
Thry=arrayfun(@(i) ...
    sum(Thry(:,i:i+n2-1),2),1:n2:1024-n2+1, ...
    'UniformOutput',0);
Thry=cell2mat(Thry);

n2=5;
Thry=arrayfun(@(i) ...
    sum(Thry(i:i+n2-1,:),1),1:n2:1024-n2+1, ...
    'UniformOutput',0);
Thry=cell2mat(Thry');
if ¬norm2B
    [amp1,amp2]=genRest(TSinputs,x);
    Thry=Thry./max(Thry(94:180,:));
    Thry=Thry.*max(data(94:180,:));
    Thry(1:90,:)=amp1*Thry(1:90,:);
    Thry(90:end,:)=amp2*Thry(90:end,:);
end

uncert=(0.01*(data([8:66 94:200],30:end-12)./500)).^2;
uncert(uncert<200)=200;

\[
    \text{csq} = \sum\left(\frac{(\text{data}([8:66 \, 94:200],30:\text{end-12})/500)-\text{Thry}([8:66 \, 94:200],30:\text{end-12})/500)^2}{\text{uncert}}\right)
\]
end

%% Plot Result
Thryfin=ArtemisModel(TSinputs,xie,scaterangs,...
x,weightMatrix,spectralFWHM,angularFWHM,lambdaAxis,xax,D,norm2B);
n2=10;
ThryfinRes=arrayfun(@(i) ...
    sum(Thryfin(:,i:i+n2-1),2),1:n2:1024-n2+1, 'UniformOutput',0);
ThryfinRes=cell2mat(ThryfinRes);
xaxRes=arrayfun(@(i) mean(xax(i:i+n2-1)),1:n2:1024-n2+1, ...
    'UniformOutput',0);
xaxRes=cell2mat(xaxRes);
n2=5;
ThryfinRes = arrayfun(@(i) ... 
    sum(ThryfinRes(i:i+n2-1,:),1),1:n2:1024-n2+1, ... 
    'UniformOutput',0);
ThryfinRes = cell2mat(ThryfinRes');
yaxRes = arrayfun(@(i) mean(yax(i:i+n2-1)),1:n2:1024-n2+1, ... 
    'UniformOutput',0);
yaxRes = cell2mat(yaxRes);

if ~norm2B 
    [amp1,amp2]=genRest(TSinputs,x);
    ThryfinRes=ThryfinRes./max(ThryfinRes(94:180,:));
    ThryfinRes=ThryfinRes.*max(dataResunit(94:180,:));
    ThryfinRes(1:90,:) = amp1*ThryfinRes(1:90,:);
    ThryfinRes(90:end,:) = amp2*ThryfinRes(90:end,:);
end
ThryfinRes(66:94,:) = 0;
chisqfin = sum(sum((dataResunit([8:66 ... 
    94:200],30:end-12)-ThryfinRes([8:66 94:200],30:end-12)).^2));

if ~norm2B 
    [amp1,amp2]=genRest(TSinputs,x);
    Thryfin=Thryfin./max(Thryfin(470:900,:));
    Thryfin=Thryfin.*max(data(470:900,:));
    Thryfin(1:450,:) = amp1*Thryfin(1:450,:);
    Thryfin(450:end,:) = amp2*Thryfin(450:end,:);
end
Thryfin(330:470,:) = 0;
chisqfin = sum(sum((data([40:330 ... 
    470:900],90:1015)-Thryfin([40:330 470:900],90:1015)).^2));

[Tefin,nefin, e, e] = genTS(TSinputs,x);
ColorPlots(yaxRes,xaxRes,rot90(ThryfinRes),'Kaxis', ... 
    [nefin*1E20,Tefin,526.5],'Title','Final ... 
    Spectrum','Name','Final Spectrum');
ColorPlots(yax,xax,rot90(data-Thryfin),'Title', ['Final ... 
    difference: \chi^2 = ' num2str(chisqfin)],'Name','Final ... 
    Difference');
load('diffcmap.mat','diffcmap');
colormap(diffcmap);
if norm2B 
    caxis([-1 1]);
else 
    caxis([-8000 8000]);
end

for i=1:length(lineLocs) 
    figure('Name',['Lineout ' num2str(i)]) 
    plot(yaxRes,sum(dataResunit(:,lineLocs(i)/10: ... 
    lineLocs(i)/10+1),2)) 
    hold on
```
plot(yaxRes,sum(ThryfinRes(:,lineLocs(i)/10: ... 
    lineLocs(i)/10+1),2))
title(['Lineout at pixel ' num2str(lineLocs(i)) ': ' ... 
    num2str(xax(end-lineLocs(i))) '\circ'])

figure('Units','normalized','position',[.1 .1 .25 ... 
    .6], 'Name', ['Lineouts'])
hold on
for i=1:length(lineLocs)
    plot(yaxRes,100000*i+sum(dataResunit(:,lineLocs(i)/10: ... 
        lineLocs(i)/10+1),2))
    plot(yaxRes,100000*i+sum(ThryfinRes(:,lineLocs(i)/10: ... 
        lineLocs(i)/10+1),2))
end
xlabel('Wavelength (nm)')
xlim([450 650])
set(gca,'YColor','none','fontsize',16,'fontweight','bold')
title(['Lineouts ' num2str(xax(end-lineLocs)) '\circ'])

x=setTo(TSinputs,x);
x.fitprops.grad=grad;
x.fitprops.hess=hess;
end

function modl=ArtemisModel(TSins, xie, sas, x, wghts, ... 
    spectralFWHM, angularFWHM, lamAxis, xax, D, norm2B)
    if TSins.m.Active && ~TSins.fe.Active
        [Te,ne,lam,fecur]=genTS(TSins,x,xie);
    else
        [Te,ne,lam,fecur]=genTS(TSins,x);
    end
    [Te,ne]=genGradients(TSins,Te,ne,7);
    fecur=exp(fecur);
    xiecur=[-flip(xie) xie(2:end)];
    fecur=[flip(fecur) fecur(2:end)];
    Thry=ApproxThomson3(Te,ne*1E20,lamAxis([1 ... 
        end]),lam,sas,fecur,xiecur);
    Thry=mean(Thry,1);
    Thry=squeeze(Thry);
    Thry=permute(Thry,[2 1]);
    [amp1,amp2,blur]=genRest(TSins,x);
    modl=wghts*Thry;
```
[mod1, lamAx] = S2Signal(mod1, lamAxis, D); % rebins the spectrum into pixels, similar to rebinning the resolution units in AngTSDataFitter

mod1 = addIRF2D(spectralFWHM + blur, angularFWHM, lamAx, xax, mod1);
mod1 = rot90(mod1, 3);

% normalize to blue peak
if norm2B
    mod1 = mod1 ./ max(mod1(470:900,:));
    mod1 = ampl * mod1;
end

if ~isempty(TSins.specCurvature) && TSins.specCurvature.Active
    invfunc = @(xy) [xy(:,1), xy(:,2) - ... (TSins.specCurvature.Value/(512^2)) * (xy(:,1) - 512).^2];
    mod1 = imwarp(mod1, geometricTransform2d(invfunc), ...
                 'OutputView', imref2d(size(mod1)));
end
end

function [Thry] = addIRF2D(width1, width2, ax1, ax2, data)
% ADDIRF adds a gaussian instrument response function to the data with a
% FWHM given the the input parameter width
% ax1 is the xaxis or the axis corresponding to a row lineout
% ax2 is the yaxis or the axis corresponding to a column lineout

stddev1 = width1 / 2.3548;
origin1 = (max(ax1) + min(ax1)) / 2; % Conceptual origin so the convolution doesn't shift the signal

inst_func1 = (1/(stddev1 * sqrt(2*pi))) * ... exp(-((ax1 - origin1).^2 / (2 * (stddev1)^2)));

stddev2 = width2 / 2.3548;
origin2 = (max(ax2) + min(ax2)) / 2; % Conceptual origin so the convolution doesn't shift the signal

inst_func2 = (1/(stddev2 * sqrt(2*pi))) * ... exp(-((ax2 - origin2).^2 / (2 * (stddev2)^2)));

Thry = conv2(inst_func2, inst_func1, data, 'same');
Thry = repmat(sum(data, 2) ./ sum(Thry, 2), 1, size(data, 2)) .* Thry;
end
function [formfactor] = ApproxThomson3(Te, ne, lamrang, lam, ... 
    sa, DF, x)
% APPROXTHOMSON3 calculates the Thomson spectrum using ...
% ratint and is capable of taking a vector for sa,Te,ne. ...
% All are expected as row vectors and are reshaped as needed

% hard coded inputs
Va=0;
interpAlg='spline';

% basic quantities
C=2.99792458e10;
Me=510.9896/Cˆ2; % electron mass KeV/C^2
re=2.8179e−13; % classical electron radius cm
Esq = Me*Cˆ2*re; % sq of the electron charge KeV−cm
constants = sqrt(4*pi*Esq/Me); % sqrt(4*pi*eˆ2/Me)
sarad=sa*2*pi/360;
sarad=reshape(sarad,1,1,[]);% scattering angle in radians
npts=10250;

% [omgL,omgs,¬,¬]=lamParse(lamrang,lam,npts);
% calculating k and omega vectors
omgpe=constants*sqrt(ne)'; % plasma frequency Rad/s
omg = omgs - omgL;
ks=sqrt (omgs.ˆ2−omgpe.ˆ2)/C; % laser wavenumber in Rad/cm
kL=sqrt (omgLˆ2−omgpe.ˆ2)/C; % laser wavenumber in Rad/cm
k=sqrt(ks.ˆ2+kL.ˆ2−2*ks.*kL.*cos(sarad));
kdotv = k*Va;
omgdop=omg − kdotv;

% plasma parameters
vTe=sqrt(Te/Me)'; % electron thermal velocity
klde=(vTe./omgpe).*k;

% electron susceptibility
% calculating normilized phase velcoity (xi's) for electrons
xie=omgdop./(k.*vTe) ;
fe_vphi=exp(interp1(x,log(DF),xie,interpAlg,−Inf));
fe_vphi(isnan(fe_vphi))=0;
df=diff(fe_vphi,1,2)./diff(xie,1,2);
df(:,end+1,:)=zeros(length(ne),1,length(sa));
chiEI = pi./(klde.ˆ2).*sqrt(-1).*df;

h=0.01;
minmax=8.2;
h1=1000;
xi1 = linspace(-minmax-sqrt(2.)/h1,minmax+sqrt(2.)/h1,h1);
xi2 = (-minmax:h:minmax);
ratdf = gradient(exp(interp1(x,log(DF),xi1,interpAlg,-Inf)), ...
    xi1);
ratdf(isnan(ratdf))=0;

for iw=1:length(xi2)
    chiERratprim(iw)=real(ratint(ratdf,xi1-xi2(iw),xi1));
end

chiERrat=interp1(xi2,chiERratprim,xie,'spline');

chiERrat=- 1./(klde.^2).*chiERrat;

chiE=chiERrat+chiEI;
epsilon=1+(chiE);
ele_comp=double(fevphi)./vTe;

SKW_ele_omg = ...
    2*pi*1./klde.*(ele_comp)./((abs(epsilon)).^2) .*vTe./omgpe;

PsOmg = (SKW_ele_omg).*((1+2*omgdop/omgL).*re.^2).*ne';
lams=2*pi*C./omgs;
PsLam = PsOmg*2*pi*C./lams.^2;
%PsLam = bsxfun(@rdivide,PsOmg *2*pi*C,lams.^2);

formfactor = PsLam;

function out=ratint(f,g,z)

% Integrate f/g dz taking each to be piecewise linear. This is
% more accurate when f/g has a near-pole in an interval
% f,g and z are 1D complex arrays.
% Based on newlip routine by Ed Williams.

zdif = z(2:end)-z(1:end-1);
out = sum(ratcen(f,g).*zdif);
end

function out=ratcen(f,g)

% Return "rationally centered" f/g
% such that int.s(1)^s(0) ds f(s)/g(s) = ...
% sum(ratcen(f,g)*s(dif)) when
% f and g are linear functions of s.
% This allows accurate integration through near poles of f/g
% Based on newlip routine by Ed Williams.

def = f(2:end) - f(1:end-1);
gdf = g(2:end) - g(1:end-1);
fav = 0.5*(f(2:end)+f(1:end-1));
gav = 0.5*(g(2:end)+g(1:end-1));
out = 0.*fdif;
iflat = abs(gdif) < 1.e-4*abs(gav);
tmp = (fav.*gdif - gav.*fdif);
rf = fav./gav + tmp.*gdif./(12.*gav.^3);
rfn = fdif./gdif + tmp.*log((gav+0.5*gdif)./...
      (gav-0.5*gdif))./gdif.^2;
out(iflat) = rf(iflat);
out(~iflat) = rfn(~iflat);
end

classdef ThomsonInputs
    properties
        amp1
        amp2
        lam
        Te
        Z
        ne
        m
        fe
        blur
        specCurvature
        fitprops
    end
    methods
        function obj = ThomsonInputs(varargin)
            % creates the ThomsonInputs object. standard inputs ...
            % have 4 properties. "Active" is 1 if the ...
            % variable it being fit or 0 otherwise. "Value" ...
            % is the initial value. "Location" is the index ...
            % where the variable can be found in the ...
            % minimized array x. "Bound" are the set of ...
            % default lower and upper bounds for the minimizer.
obj.amp1.Location=0;
obj.amp1.Bounds=[0; 10];

obj.amp2.Location=0;
obj.amp2.Bounds=[0; 10];

obj.lam.Location=0;
obj.lam.Bounds=[525; 528];

obj.Te.Location=0;
obj.Te.Bounds=[0.01; 3];

obj.Z.Location=0;
obj.Z.Bounds=[1; 25];

obj.ne.Location=0;
obj.ne.Bounds=[.01; 2];

obj.m.Location=0;
obj.m.Bounds=[2; 5];

obj.fe.Value=[];
obj.fe.Location=0;

%fit props is given a dummy Active properties to ...

 switch nargin
     case 0

         obj.amp1.Active=0;
         obj.amp1.Value= []; 

         obj.amp2.Active=0;
         obj.amp2.Value= []; 

         obj.lam.Active=0;
         obj.lam.Value= []; 

         obj.Te.Active=0;
         obj.Te.Value= []; 

         obj.Z.Active=0;
         obj.Z.Value= []; 

         obj.ne.Active=0;
         obj.ne.Value= []; 

         obj.m.Active=0;
         obj.m.Value= [];
obj.fe.Active=0;
obj.fe.Length=64;
obj.fe.Type='DLM';
obj.fe.Bounds=repmat([-100;-.5],1,...
    obj.fe.Length);

    case 8
    obj.amp1.Active=0;
    obj.amp1.Value=varargin{5};

    obj.amp2.Active=0;
    obj.amp2.Value=varargin{6};

    obj.lam.Active=0;
    obj.lam.Value=varargin{1};

    obj.Te.Active=0;
    obj.Te.Value=varargin{3};

    obj.Z.Active=0;
    obj.Z.Value=varargin{7};

    obj.ne.Active=0;
    obj.ne.Value=varargin{2};

    obj.m.Active=0;
    obj.m.Value=varargin{4};

    obj.fe.Active=0;
    obj.fe.Length=64;
    obj.fe.Type=varargin{8};
    obj.fe.Bounds=repmat([-100;-.5],1,...
        obj.fe.Length);

    case 17
    %The expected order is toglam, boxlam, ...
        togne, boxne, togTe, boxTe, togm, boxm, ...
        togamp1, boxamp1, togamp2, boxamp2, ...
        togZ, boxZ, togfe, boxfe, boxfetype
    obj.amp1.Active=varargin{9}.Value;
    obj.amp1.Value=str2double( ...
        varargin{10}.String));

    obj.amp2.Active=varargin{11}.Value;
    obj.amp2.Value=str2double( ...
        varargin{12}.String));

    obj.lam.Active=varargin{1}.Value;
    obj.lam.Value=str2double( varargin{2}.String);
obj.Te.Active=varargin{5}.Value;
obj.Te.Value=str2double( varargin{6}.String);

obj.Z.Active=varargin{13}.Value;
obj.Z.Value=str2double( varargin{14}.String);

obj.ne.Active=varargin{3}.Value;
obj.ne.Value=str2double( varargin{4}.String);

obj.m.Active=varargin{7}.Value;
obj.m.Value=str2double( varargin{8}.String);

obj.fe.Active=varargin{15}.Value;
obj.fe.Length=str2double( ... 
    varargin{16}.String);
obj.fe.Type=varargin{17}.String;
obj.fe.Bounds=repmat([-100;-.5],1, ... 
    obj.fe.Length);

end

end

%these 2 methods are functions to add the blur and ...
%curvature properties which are special use ...
%properties and therefore non-standard
function obj = addBlur(obj,blurActive,blurValue)
    obj.blur.Active=blurActive;
    obj.blur.Value=blurValue;
    obj.blur.Location=0;
    obj.blur.Bounds=[0; 10];
end

function obj = addCurvature(obj,curvActive,curvValue)
    obj.specCurvature.Active=curvActive;
    obj.specCurvature.Value=curvValue;
    obj.specCurvature.Location=0;
    obj.specCurvature.Bounds=[.1; 10];
end

function obj = ...
    addGradients(obj,TeActive,TeValue,neActive,neValue)
    %add gradient percentage values to ne and Te
    if TeActive
        obj.Te.gradient=TeValue;
    end
    if neActive
        obj.ne.gradient=neValue;
    end
end

function [obj,x,lb,ub] = genX(obj)
% This method returns the minimizer array x and ...
% assigns all the location values to the location property

props = properties(obj);
x = [];
lb = [];
ub = [];

for i = 1:length(props)
    if obj.(props{i}).Active
        x = [x obj.(props{i}).Value];
        lb = [lb obj.(props{i}).Bounds(1,:)];
        ub = [ub obj.(props{i}).Bounds(2,:)];
        obj.(props{i}).Location = length(x);
    end
end

function obj = initFe(obj, xie)
% populate the Value field of fe from the other fields
if strcmp(obj.fe.Type, 'DLM')
    obj.fe.Value = log(NumDistFunc({obj.fe.Type, ...
                                 obj.m.Value}, xie, ...
                                 obj.fe.Type)');
elseif strcmp(obj.fe.Type, 'Fourkal')
    obj.fe.Value = log(NumDistFunc({obj.fe.Type, ...
                                 obj.m.Value, ...
                                 obj.Z.Value}, xie, obj.fe.Type)');
end

obj.fe.Value(obj.fe.Value ≤ −100) = −99;
end

function [Te, ne, lam, fecur] = genTS(obj, x, varargin)
% This method returns the inputs for ...
% approxThomson3, i.e. Te ne % lam and fecur
if obj.Te.Active
    Te = x(obj.Te.Location);
else
    Te = obj.Te.Value;
end
if obj.ne.Active
    ne = x(obj.ne.Location);
else
    ne = obj.ne.Value;
end
if obj.lam.Active
    lam = x(obj.lam.Location);
else
    lam = obj.lam.Value;
end
if obj.fe.Active
fecur = x(obj.fe.Location - obj.fe.Length + 1: ... 
    obj.fe.Location);
elseif ~isempty(varargin) && obj.m.Active
    obj.m.Value = x(obj.m.Location);
    obj = obj.initFe(varargin{1});
    fecur = obj.fe.Value;
else
    fecur = obj.fe.Value;
end

function [Te, ne] = genGradients(obj, Te, ne, varargin)
%converts the Te and ne values into vectors ... 
    distributed within the relevant ranges
if ~isempty(varargin)
    arlen = varargin{1};
else
    arlen = 10;
end
if isfield(obj.Te, 'gradient') && ...
    isfield(obj.ne, 'gradient')
    Te = linspace((1 - obj.Te.gradient/200)*Te, ...
        (1 + obj.Te.gradient/200)*Te, arlen);
    ne = linspace((1 - obj.ne.gradient/200)*ne, ...
        (1 + obj.ne.gradient/200)*ne, arlen);
elseif isfield(obj.Te, 'gradient')
    Te = linspace((1 - obj.Te.gradient/200)*Te, ...
        (1 + obj.Te.gradient/200)*Te, arlen);
    ne = repmat(ne, 1, arlen);
elseif isfield(obj.ne, 'gradient')
    ne = linspace((1 - obj.ne.gradient/200)*ne, ...
        (1 + obj.ne.gradient/200)*ne, arlen);
    Te = repmat(Te, 1, arlen);
end

function [amp1, amp2, blur] = genRest(obj, x)
%This method returns the rest of the parameters ... 
    required for the remainder of the fitter, i.e ... 
    amp1 amp2 blur. Future properties can be added here
if obj.amp1.Active
    amp1 = x(obj.amp1.Location);
else
    amp1 = obj.amp1.Value;
end
if obj.amp2.Active
    amp2 = x(obj.amp2.Location);
elseif obj.amp1.Active
    amp2 = x(obj.amp1.Location);
else
    amp2 = obj.amp2.Value;
end
if obj.blur.Active
    blur=x(obj.blur.Location);
else
    blur=obj.blur.Value;
end
end

function obj = setTo(obj,x)
%This method returns the object with all active ... 
    fit parameters reset to the values given
    props=properties(obj);

    for i=1:length(props)
        if obj.(props{i}).Active
            obj.(props{i}).Value=x(obj.( ... 
                props{i}).Location);
        end
    end
    if obj.fe.Active
        obj.fe.Value=x(obj.fe.Location- ... 
            obj.fe.Length+1:obj.fe.Location);
    end
end

function obj = genErrors(obj)
%This method adds Error fields to each of the ... 
    active fit parameters
    props=properties(obj);
    if isfield(obj.fitprops,'hess')
        errormat=real(diag(sqrt(inv(obj.fitprops.hess))));
        for i=1:length(props)
            if obj.(props{i}).Active
                obj.(props{i}).Error=errormat( ... 
                    obj.(props{i}).Location);
        end
        if obj.fe.Active
            obj.fe.Error=errormat(obj.fe.Location- ... 
                obj.fe.Length+1:obj.fe.Location)';
        end
        else
            disp('Errors can not be calculated. No Hessian ... present');
        end
end

function tf = sameActiveState(obj1,obj2)
%compares the states of each properties in the two ... 
    instances of the ThomsonInputs object as well ... 
    as a the length of the distribution function. ...
Return a true-false value saying if they have the same states

```matlab
props = properties(obj1);
c = 0;
for i = 1:length(props)
    if ~isempty(obj1.(props{i})) && ~...
        isempty(obj2.(props{i})) && ...
        obj1.(props{i}).Active == obj2.(props{i}).Active
        c = c + 1;
    end
end
if c == length(props)
    if obj1.fe.Active
        if obj1.fe.Length == obj2.fe.Length
            tf = true;
        else
            tf = false;
        end
    else
        tf = true;
    end
else
    tf = true;
end
else
    tf = false;
end
end
```
Bibliography


[90] MATLAB. fmincon.


