Variational Principles and Self-Organization in Two-Fluid Plasmas

Swadesh Mahajan[1], Zensho Yoshida[2]

[1] Institute for Fusion Studies, University of Texas at Austin
Austin, Texas, USA

[2] Graduate School of Frontier Sciences, University of Tokyo
Tokyo, Japan

We investigate the phenomenon of the self-organization of plasmas into “relaxed states” of varying complexity.

Although many of the relaxed states in recent literature follow from apparently standard variational principles, self-organization is not a general tendency of plasmas. We have found that self-organization towards an ordered structure occurs only under rather restrictive conditions. Not all variational principles are well posed; even when they are, not all solutions to variational principles lead to stable equilibria—an essential minimum qualification for relaxed states.

We have derived a new theoretical framework [1] for a well-posed variational principle, which invokes a target functional whose form is mathematically “coercive” (i.e., involving higher-order derivatives than in the associated constraint functions). The coercive target functional is the measure of turbulence (and, simultaneously, of the dissipation), and its minimization is exactly the characterization of self-organization. During the approach towards equilibrium, the constraints (viz., constants of motion of the ideal model) are adjusted, through a weakly dissipative process, so that the relaxed state is a stable equilibrium independent of the direct effects of dissipation.

Using the generalized ion enstrophy as such a target functional, we have derived a criterion for the self-organizing relaxation of a two-fluid plasma into so-called magneto-fluid states. These states of stable equilibria, also known as double Beltrami states [2], are formed by a strong interaction between the plasma flow and the magnetic field. The sheared velocity field is essential to determine the existence, as well as the properties, of these equilibria. The generalized ion enstrophy can, in fact, be viewed as the Hamiltonian associated with the magneto-fluid field, which unifies the flow and the magnetic fields.

In addition, this new formalism provides a fresh interpretation for Taylor relaxation, to which it reduces in the limit of zero flow. An important difference with the Woltjer-Taylor force-free relaxed states, however, is that these new magnetofluid states are able to confine a plasma by means of the generalized Bernoulli mechanism.

These new states, which do not require spatial symmetry (i.e., flux surfaces) for equilibria, may be important for understanding plasma structures found in astrophysics (e.g., coronal mass ejections and solar smoke rings) and laboratory plasmas with strong flow (e.g., field-reversed configuration and possibly the H-mode). Preliminary results from a small experiment to realize these magnetofluid states [3] will also be presented.

[1] Z. Yoshida and S. M. Mahajan, to appear in Phys. Rev. Lett. (March, 2002); available as IFS Report No. 938 at w3fusion.ph.utexas.edu/ifs/reports2002.html.

[2] S. M. Mahajan and Z. Yoshida, Phys. Rev. Lett. **81**, 4863 (1998).

[3] H. J. Quevado, R. Bengtson, S. M. Mahajan, and P. M. Valanju, Bull. Am. Phys. Soc. **46** (8), 275 (2001) QP1-93.