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The effect of residual kinetic energy on apparent ion temperature in ICF implosions

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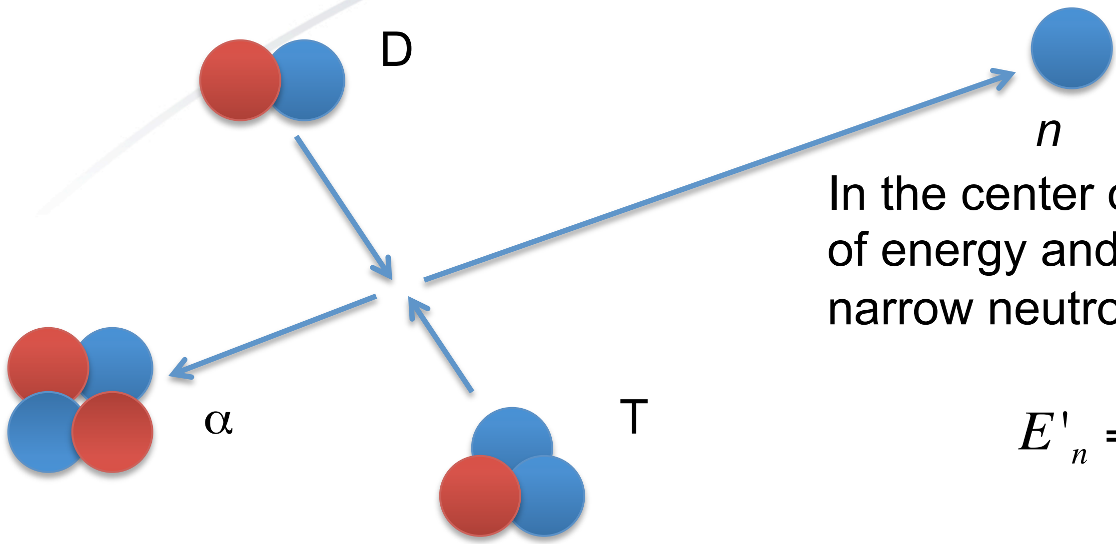
Outline

- Basic kinetics of fusion reactions
- Ion temperature effects on neutron spectrum energy width
- Effects of fluid velocity on neutron spectrum
  - Directed motion
  - Expanding or contracting spherical shells
  - Random distribution of energies
  - Comparison with past experiments
- DD vs DT temperatures
- Model for residual kinetic energy

Main points

- The energy distribution of fusion neutron is determined by the velocity of the center-of-mass of the reacting particles
- Fluid motion a fraction of the thermal velocity can lead to substantial broadening of the neutron distribution
- This broadening affects the DT and DD neutrons differently
- Energy going into fluid motion has several times the effect of thermal energy at broadening the spectrum

Fusion product energies are determined by kinetics of the reaction



In the center of mass frame, conservation of energy and momentum results in a narrow neutron energy spread.

$$E'_n = \frac{m_a}{m_a + m_n}\left(Q + K\right)$$

Q comes from the smaller mass of the alpha and n than for the two hydrogen isotopes. [$Q = (m_d + m_t - m_\alpha - m_n)c^2$]

K is the kinetic energy of the reacting particles and is determined from the overlap of the cross section with the distribution of relative velocities between the reacting species.

The distribution of values of K contributes a minimal amount to the neutron energy distribution.

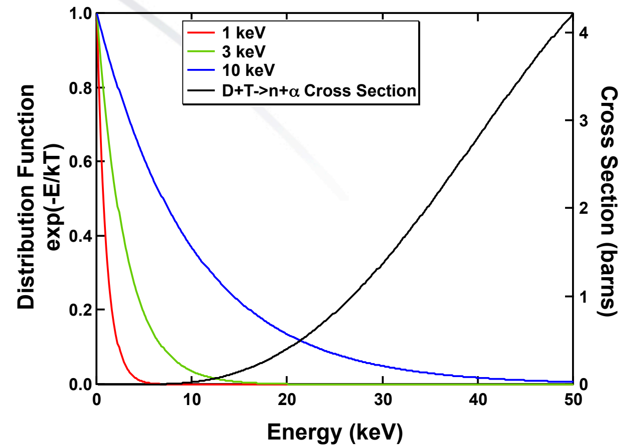
Some spread comes from the width of the Gamow peak, but not much

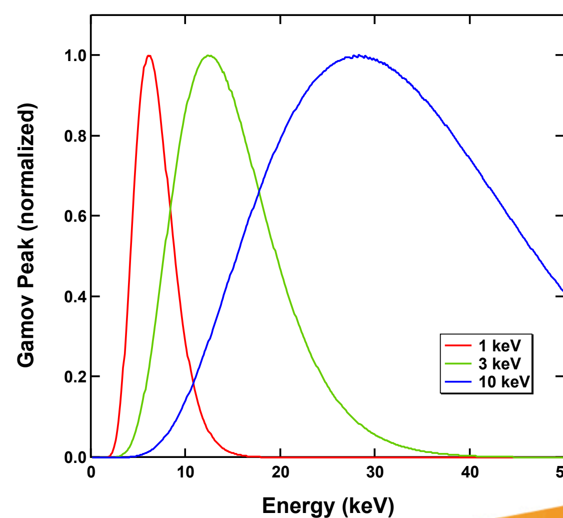
For a Maxwellian plasma, the ion energy falls as exp(-E/kT)

The fusion cross section rises rapidly with energy

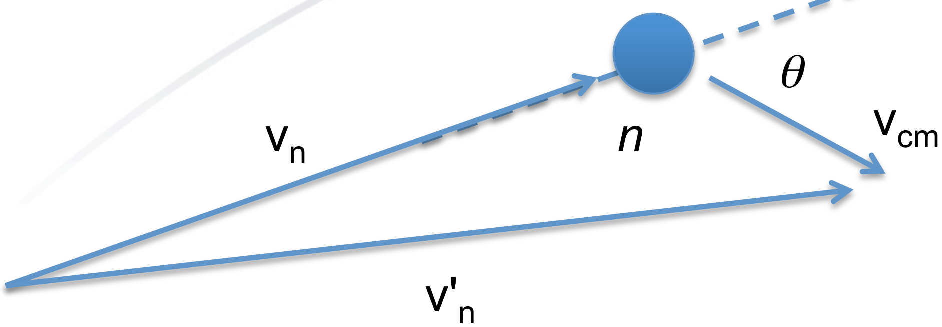
The Gamow peak is the product, showing the peak value of K and the spread

For DT	FWHM of K	Neutron portion
1 keV	4.7 keV	3.8 keV
3 keV	11.9 keV	9.5 keV
10 keV	31.8 keV	25.4 keV





Motion of the center-of-mass changes the neutron energy in the lab frame



$$E_n = \frac{1}{2} m_n \left(\mathbf{v_n} + \mathbf{v_{cm}}\right)^2$$
$$= \frac{1}{2} m_n \left(v_n^2 + 2 v_n v_{cm} \cos\theta + v_{cm}^2\right)$$
$$= \frac{1}{2} m_n v_n^2 \left[1 + 2\frac{v_{cm}}{v_n}\cos\theta + \left(\frac{v_{cm}}{v_n}\right)^2\right]$$

So the change in the neutron energy due to the center-of-mass motion is:

$$\Delta E_n \approx E_n' \left(2\frac{v_{cm}}{v_n}\cos\theta\right)$$
$$= m_n v_n' v_{cm} \cos\theta$$

ΔE_n is proportional to v_{cm}
(NOT v_{cm}^2)

Two Maxwellians can be expressed in terms of relative and center-of-mass velocities

For Maxwellians of a given temperature:

$$f_D \propto \exp\left(-\frac{m_D v_D^2}{2kT}\right)$$

$$f_T \propto \exp\left(-\frac{m_T v_T^2}{2kT}\right)$$

The product is given by:

$$f_D f_T \propto \exp\left(-\frac{m_D v_D^2 + m_T v_T^2}{2kT}\right)$$

$$\propto \exp\left(-\frac{\mu v^2 + M V^2}{2kT}\right)$$

Where:

$$\mu = \frac{m_D m_T}{m_D + m_T}$$

$$M = m_D + m_T$$

$$v = \left|\mathbf{v_D} - \mathbf{v_T}\right|$$

$$\mathbf{V} = \frac{m_D \mathbf{v_D} + m_T \mathbf{v_T}}{m_D + m_T}$$

Thus, two Maxwellians have a center-of-mass distribution with a width:

$$\sigma_V = \sqrt{\frac{kT}{M}}$$

$$\exp\left(-\frac{MV^2}{2kT}\right) = \exp\left(-\frac{M\left(V_x^2 + V_y^2 + V_z^2\right)}{2kT}\right)$$

From the width of the center-of-mass distribution, the neutron width can be found

$$\Delta E_n = E_n'\left(2\frac{v_{cm}}{v_n}\cos\theta\right)$$

$$= m_n v_n' v_{cm}\cos\theta$$

$$\sigma_V = \sqrt{\frac{kT}{M}}$$

Setting using $v_{cm,z} = v_{cm}\cos\theta$, and the width of the distribution of $v_{cm,z}$, $\sigma_{v,z}$=kT/M, gives:

$$\sigma_n = E_n'\left(\frac{2}{v_n'}\sqrt{\frac{kT}{M}}\right)$$

Since:

$$v_n' = \sqrt{\frac{2E_n'}{m_n}}$$

We get:

$$\sigma_n = \sqrt{\frac{2m_n kT\, E_n'}{M}}$$

See, for example, H. Brysk, *Plasma Phys.* **15**, 611 (1973).

The spreading due to the center-of-mass is the dominant contribution

Ion Temperature (keV)	Gamow Peak FWHM (keV)	"Brysk" FWHM (keV)
1	3.8	177
3	9.5	306
10	25.4	559

From the measured neutron width, an ion temperature is inferred

From the previous slide:

$$\sigma_n = \sqrt{\frac{2m_n kT E'_n}{M}}$$

Thus:

$$\boxed{kT = \frac{M}{2m_n E'_n}\sigma_n^2}$$

σ_n is often measured using the time-of-flight broadening of the neutron signal.

$$\sigma_{tof} = \frac{dt}{dE_n}\sigma_n$$

$$t = \frac{d}{v_n} = d\sqrt{\frac{m_n}{2E_n}}$$

$$\frac{dt}{dE_n} = \frac{d}{2}\sqrt{\frac{m_n}{2E_n^3}}$$

$$\sigma_{tof} = \frac{d\,m_n}{2E_n}\sqrt{\frac{kT}{M}}$$

$$FWHM_{tof} = \left[\sqrt{\frac{2\ln 2}{M}}\frac{m_n}{E_n}\right]d\sqrt{kT}$$

See, for example, Murphy, Chrien, and Klare, *RSI* **68**, 610 (1997).

Collective motion of the source leads to upshift in one direction, down shift in the other

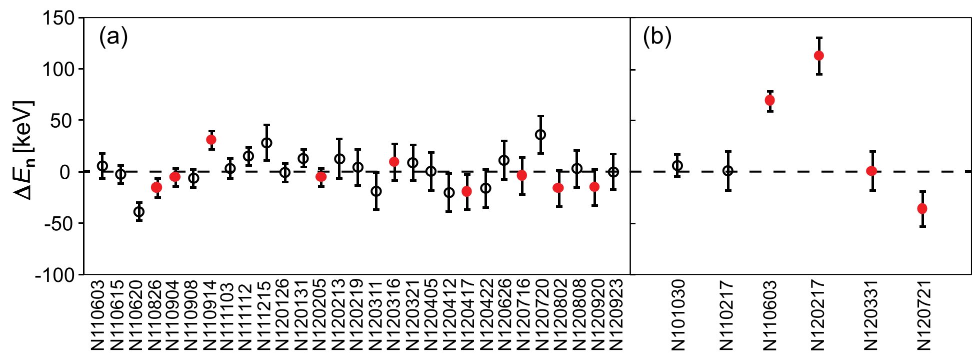
Our previous expression, which was used for the center of mass velocity of two particles, can be applied to the plasma as a whole:

$$\Delta E_n = E_n' \left(2 \frac{v_{cm}}{v_n} \cos\theta \right)$$

$$= m_n v_n' v_{cm} \cos\theta$$

M Gatu-Johnson *et al* report measurements of this effect on NIF shots using the MRS.

Cryo-layered implosions PDD Exploding pushers



M. Gatu-Johnson et al, Phys Plasmas 20, 042707 (2013).

Now consider an ensemble of fluid elements with a Gaussian distribution of velocities

Why Gaussian?

- **Continuous distribution of velocities**
- **Characterized by a single number**
- **Simplifies the math**

Using:

$$\Delta E_n = E'_n \left(2 \frac{v_{f,z}}{v_n} \right)$$

$$\sigma_{n,f} = m_n v'_n \sigma_{v,f} = \sqrt{2 m_n E'_n} \sigma_{v,f}$$

Convolving the contributions of thermal CM and fluid CM, we can show:

$$\sigma_n^2 = \sigma_{n,th}^2 + \sigma_{n,f}^2 = 2 m_n E' \frac{kT}{M} + 2 m_n E'_n \sigma_{v,f}^2 = 2 m_n E' \left(\frac{kT}{M} + \sigma_{v,f}^2 \right)$$

Interpreting the measured neutron width as due entirely to ion temperature over estimates T_i

Using the relationship:

$$kT = \frac{M}{2m_n E'_n}\sigma_n^2$$

We can show:

$$kT_{app} = \frac{M}{2m_n E'_n} \times 2m_n E'\left(\frac{kT}{M} + \sigma_{v,f}^2\right)$$

$$= kT + M\sigma_{v,f}^2$$

Thus, a Gaussian fluid velocity increases the apparent ion temperature an amount proportional to the sum of the masses of the reactants.

$$\sigma_{v,f} = \sqrt{\frac{kT_{app} - kT}{M}}$$

How much energy is contained in the fluid velocity? Thermal energy?

The kinetic energy can be calculated:

$$E_k = V \int \frac{1}{2} \rho v^2 f_k(v)\, d^3 v$$

Using a Gaussian distribution:

$$f_k(v_f) = \frac{1}{\left(2\pi\sigma_v^2\right)^{3/2}} v_f^2 e^{-v_f^2/2\sigma_v^2}$$

Which integrates to:

$$E_k = \frac{3}{2} \rho \sigma_v^2 V$$

The thermal energy is given by:

$$E_{th} = V \frac{3}{2} (n_i + n_e) kT$$

For hydrogenic species, $n_i = n_e$, so:

$$E_{th} = 3kT \frac{\rho V N_A}{\langle A \rangle}$$

The fraction of energy in fluid velocity affects the apparent ion temperature

$$\sigma_n^2 = 2m_n E'\left(\frac{kT}{M} + \sigma_{v,f}^2\right)$$

$$= 2m_n E'\left(\frac{E_{th}}{3M}\frac{\langle A\rangle}{\rho V N_A} + \frac{2}{3}\frac{E_k}{\rho V}\right)$$

$$= \frac{m_n^2 v_n'^2}{\rho V}\left[\frac{\langle A\rangle}{3(A_1 + A_2)}E_{th} + \frac{2}{3}E_k\right]$$

$$= \frac{m_n^2 v_n'^2}{\rho V}E_{tot}\left[\frac{\langle A\rangle}{3(A_1 + A_2)}(1 - f_{rke}) + \frac{2}{3}f_{rke}\right]$$

Note:

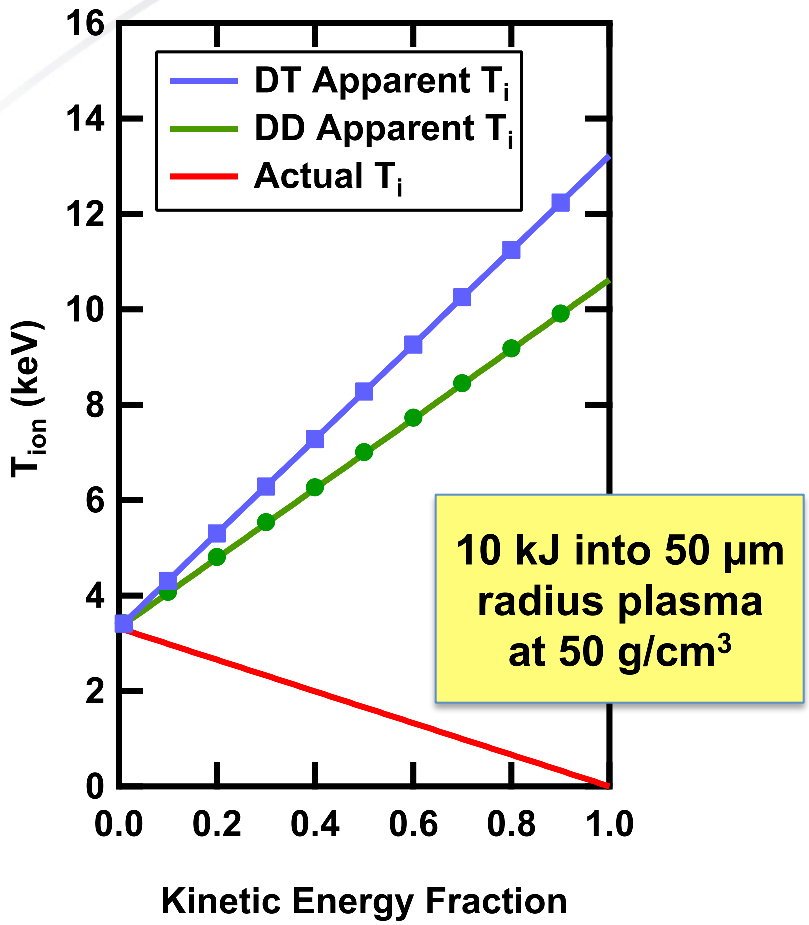
The factor in front of E_k is 2/3

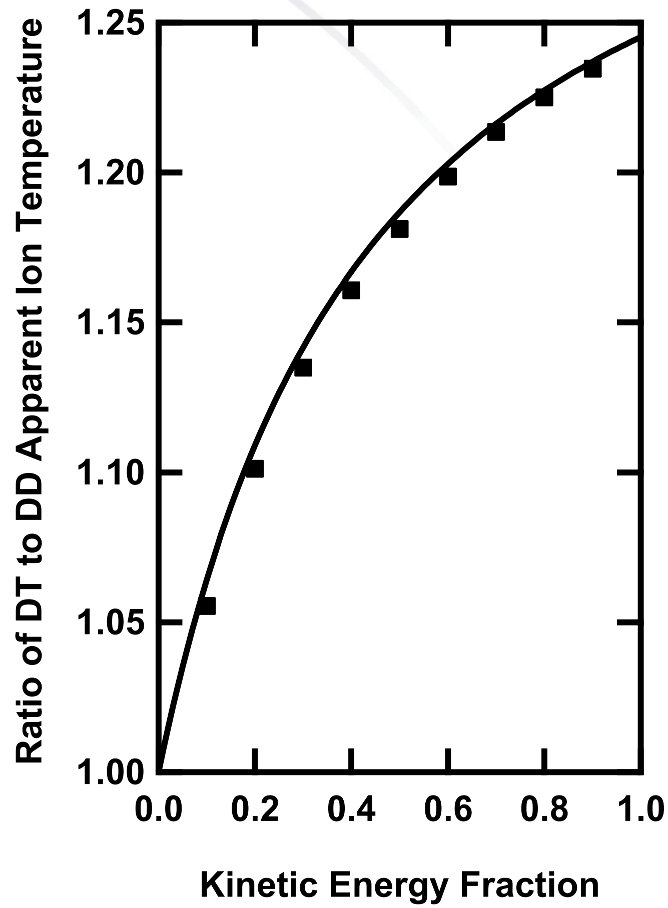
The factor in front of E_{th} is:
2.5/[3(5)] for DT
2.5/[3(4)] for DD

Factor of 4.0 for DT
Factor of 3.2 for DD

Energy into kinetic energy is 3.2 to 4.0 times as important in spreading the neutron energy than is thermal energy.

Increased kinetic energy fraction raises apparent ion temperature while lowering actual ion temperature





Conclusions: Stagnation in ICF implosions is often incomplete

- Ion temperature determination from neutron spectra usually assumes a stationary plasma
- Residual motion can significantly affect the inference of an ion temperature
- Comparison of DD and DT ion temperatures in DT plasmas can provide a measure of the fluid velocities

Imploding/exploding shell of fusing material

A uniformly expanding or contracting shell of fusing material will have a single velocity, but all directions. In this case, cos θ is uniformly distributed between -1 and 1, so based on the expression:

$$\Delta E_n = E'_n\left(2\frac{v_{cm}}{v_n}\cos\theta\right)$$

ΔE_n is uniformly distributed between

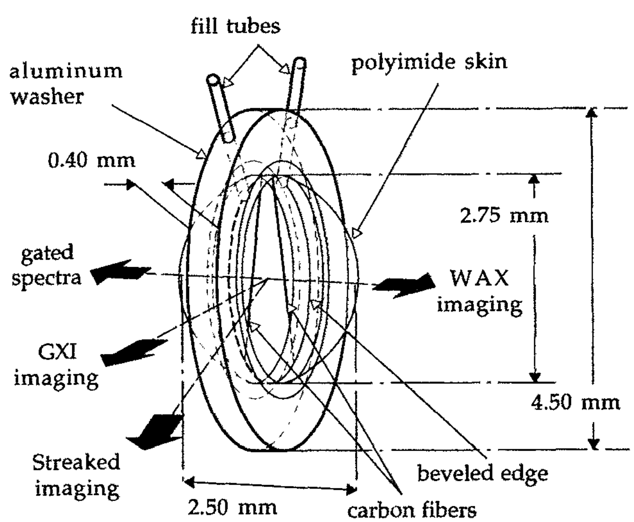
$$-E'_n\left(2\frac{v_{cm}}{v_n}\right) < \Delta E_n < E'_n\left(2\frac{v_{cm}}{v_n}\right)$$

If the shell has a finite temperature, then the neutron energy distribution is a convolution of the Gaussian with the square function described above.

Gas bag targets

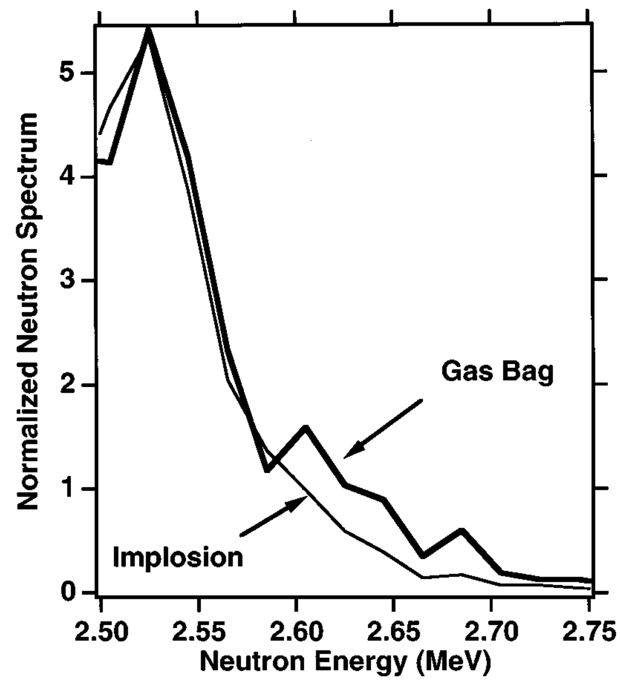
In the early 1990's, LLNL and LANL performed "long scale length plasma" experiments to understand LPI effects relevant to NIF.

A subset of these were filled with deuterated hydrocarbon gas.



Kalantar et al, Phys Plasmas 2, 3161 (1995).

Neutron spectra had a broad component not seen in spectra from implosions.



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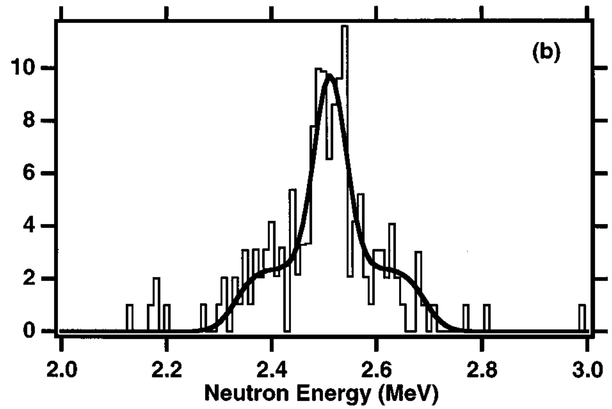
Gas bag neutron spectra could be fit with a temperature and an expansion velocity

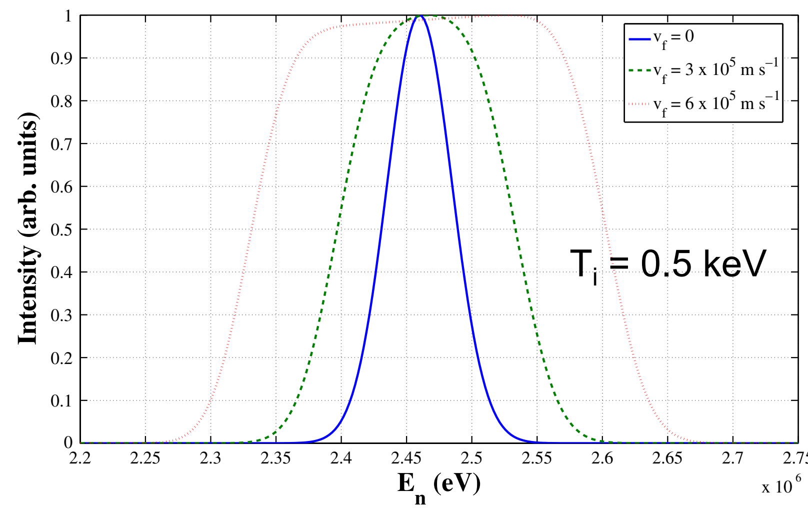
The convolution of a Gaussian and a square function can be written:

$$f(E) = \frac{1}{4m_n v_n' v_f}\left[\mathrm{erf}\left(\frac{E - E_n' + m_n v_n v_f}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{E - E_n' - m_n v_n v_f}{\sqrt{2}\sigma}\right)\right]$$



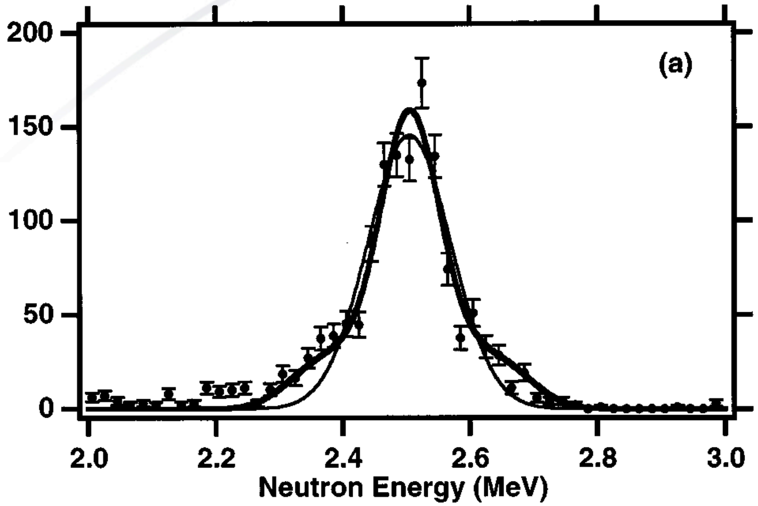
This provided a good fit to GASBAG spectra: 0.75 keV, 7.8 × 10^7 cm/s expansion.

Appelbe and Chittenden provide a formal, accurate derivation of this distribution.



Appelbe and Chittenden, Plasma Phys Control. Fusion 53, 045002 (2011).

Expansion/contraction can greatly affect the apparent ion temperature



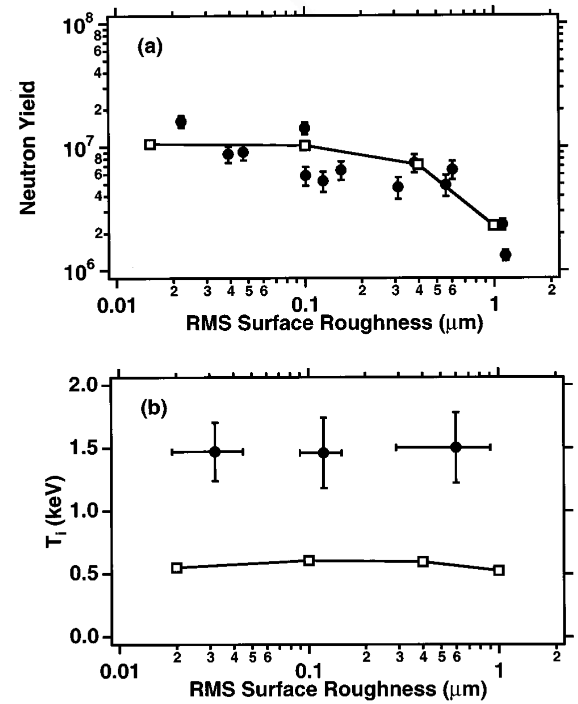
Summed spectra from all deuterated GASBAG targets:

Gaussian fit gives 3.5 keV

Shell fit gives 1.6 keV, 8.1×10^7 cm/s

Murphy, Chrien, and Klare, RSI **68**, 614 (1997);
Chrien et al, Phys Plasmas **5**, 768 (1998).

Chrien's CD shell implosions match the simulated neutron yield, but apparent T_{ion} is off by a factor of three.



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Examples: Landen Ge-doped, high-convergence Nova implosions

- O. L. Landen, et al [Phys. Plasmas 3, 2094 (1996)] reported:
  - Temperatures 30% greater than calculated for indirectly driven capsule experiments performed on the Nova laser
  - Simulated yield was 3–4 times the observed yield.
  - Attribute the effects to a smaller emitting core than calculated with less burn in the cooler regions outside the core so that the burn-averaged ion temperature is higher than calculated.

Examples: Chrien CD shell implosions

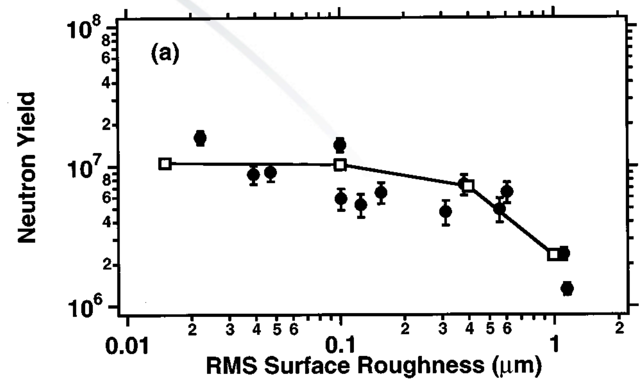
Chrien et al imploded H-filled CD shells with varying surface roughness.

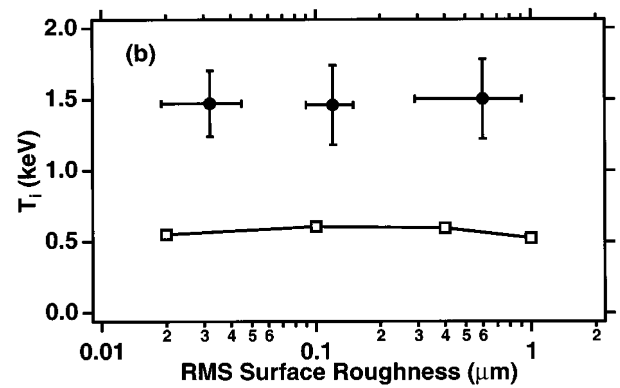
Yield agreed with simulations; T_i did not.

Assuming simulated T_i (0.6 keV) is correct and measured T_i (1.4 keV) is affected by motion,

$$\sigma_{v,f} = \sqrt{\frac{kT_{app} - kT}{M}}$$

Gives a velocity of 1.4 × 10^7 cm/s.





R. E. Chrien, N. M. Hoffman, J. D. Colvin, C. J. Keane, O. L. Landen, and B. A. Hammel, Phys. Plasmas 5, 768 (1998).

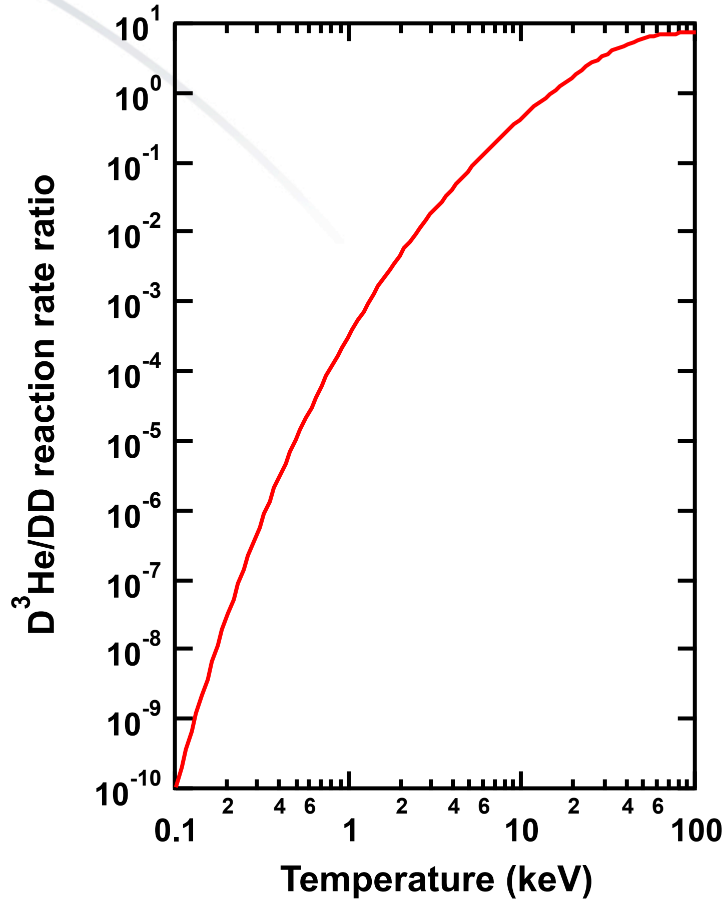
Examples: Rigg implosions with ^{3}He

Rigg et al imploded capsules with deuterium and ^{3}He fill. They measured the yield of D(d,n)^{3}He and ^{3}He(d,p)^{4}He. The ratio of the rates of these reactions is temperature dependent.

Temperature from neutron spectroscopy exceeded that from yield ratios by 0.7 to 1.7 keV. Using

$$\sigma_{v,f} = \sqrt{\frac{kT_{app} - kT}{M}}$$

Gives a velocity of $1.7 \pm 0.4 \times 10^7$ cm/s.



J. R. Rygg, J. A. Frenje, C. K. Li, F. H. Seguin, R. D. Petrasso, J. A. Delettrez, V. Y. Glebov, V. N. Goncharov, D. D. Meyerhofer, S. P. Regan, T. C. Sangster, and C. Stoeckl, Phys. Plasmas 13, 052702 (2006).

Examples: Cyro-layered implosions on NIF

Jones et al describe detailed modeling of cryogenic layered experiments on NIF.

Simulations gave 3.4 keV expected ion temperature

Measurements gave 4.43 keV.

Assuming the simulation is correct, using:

$$\sigma_{v,f} = \sqrt{\frac{kT_{app} - kT}{M}}$$

Gives a velocity of 1.4 × 10^7 cm/s.

O. S. Jones et al, Phys. Plasmas 19, 056315 (2012).

Examples: High-foot shots on NIF

O. A. Hurricane et al report both DD and DT ion temperatures in their Nature paper. The ratio of T_{iDT} to T_{iDD} is 1.16 for one shot and 1.23 for a second. 1.23 is near the value (1.25) when fluid velocity dominates the width.

Park et al report DT ion temperatures that exceed simulated by 1.25±0.16 keV (4.2 vs 2.95), and DD ion temperatures that exceed simulated by 0.9±0.2 keV (3.7 vs 2.8).

Using $$\sigma_{v,f} = \sqrt{\frac{kT_{app} - kT}{M}}$$

Gives a velocity of

DT: $1.55 \pm 0.10 \times 10^7$ cm/s

DD: $1.46 \pm 0.16 \times 10^7$ cm/s

Thus, a single velocity of ~1.5×10^7 cm/s brings the temperatures into agreement with each other and with simulations.

Shell velocity was 3.12×10^7 cm/s

O. A. Hurricane *et al*, Nature 506, 0343 (2014).

H.-S. Park *et al*, Phys. Rev. Lett. 112, 055001 (2014).

Things not considered but possibly relevant

- Kinetic effects
  - Separation of D and T, increasing one relative to the other in hotter regions
  - Long mean free paths relative to ion temperature gradient, hot ions escaping into cold plasma resulting in net outward motion
- Different temperatures for D and T
- Inaccuracies in the response function of current-mode neutron detectors

LLNL believes that they see apparent ion temperature anisotropies

- Different apparent ion temperatures are seen on different detectors at different locations
- Temperatures ranged from, for example, 2 to 4 keV on a single shot
- They believe that this may be an indication of anisotropic fluid motion in the burning plasma
  - Working to develop models to determine fluid motions from observed ion temperatures
  - Limited to low-mode by the analysis techniques
  - Limited views prevent detailed analysis

Can experiments be performed to test this?

- Perform implosions with ~1% T and ~1% ^{3}He in deuterium
  - Use yield ratios of D^{3}He to DD to get actual ion temperatures
  - Compare DD and DT apparent ion temperatures to obtain fluid motion, and therefore corrected nToF ion temperatures
- Vary the fluid motion through:
  - Drive asymmetry [Thomas & Kares, PRL 109, 075004 (2012)]
  - Capsule roughness [Landen et al, JQSRT 54, 245 (1995)]
  - Capsule shape [Dodd et al, LA-UR-13-20274]