Stability of Self-Focused Filaments in Laser-Produced Plasmas

R. W. Short

Laboratory for Laser Energetics, U. of Rochester

The stability of self-focused light filaments in laser-produced plasmas is investigated using a self-consistent cylindrical density and intensity filament model and a full wave-equation treatment for the light. It is found that, if the filament radius is small enough that only one electromagnetic waveguide mode propagates, modulational (“sausage”) perturbations are convectively unstable but the spatial growth rate is very small. In larger filaments, supporting two or more modes, the instability is much stronger and can be absolute. Consequences for laser–plasma interactions are discussed. This work was supported by the U.S. Department of Energy Office of Inertial Confinement Fusion under Cooperative Agreement No. DE-FC03-92SF19460, the University of Rochester, and the New York State Energy Research and Development Authority.
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41st Annual Meeting of the American Physical Society
Division of Plasma Physics
Seattle, WA
15–19 November 1999
Summary

Small, single-mode filaments are essentially stable

- In small, self-consistent filaments only a single waveguide will propagate.

- Modulations propagate at the group velocity (near $c >> c_s$) and interact weakly with surrounding plasma.

- Larger filaments support two or more modes; beat moves slowly ($\sim c_s$), leading to stronger instabilities, which can be absolute.

- Convective spatial growth rates for single-mode filaments lead to little growth over typical scale lengths in laser-produced plasmas.
Outline

- Equilibrium filament model
- Linearized perturbation
- Calculation of growth rates
- Summary and conclusions
The filament is modeled by a self-consistent equilibrium:

- The laser light amplitude satisfies the wave equation in cylindrical geometry:

\[
\left[ c^2 \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) + \frac{\omega_0^2 - \omega_0^2 p_0(r) - c^2 k_0^2}{r} \right] \Psi_0(r) = 0, \quad \Psi_0 \equiv eE_{\text{max}} / m\omega_0 v_T
\]

- The density is determined by pressure balance with the ponderomotive force of the pump:

\[
\frac{\omega_0^2 p_0(r)}{\omega_0^2} = \frac{n_0(r)}{n_c} = \frac{N_0}{n_c} e^{-\frac{1}{4} \Psi_0^2(r)}
\]

- Together with the boundary conditions

\[
\Psi_0(r)_{r \to \infty} \to 0; \left( d\Psi_0 / dr \right)_{r=0} = 0,
\]

this gives nonlinear eigenvalue problem determining \( k_0, \Psi_0, \) and the density profile \( n_0(r) \).
Intense filaments can support more than one waveguide mode

- The pump wave propagates in the fundamental mode.
- Higher-order modes at the pump frequency have smaller axial wave numbers.
Multimode modulations move more slowly than single-mode modulations

- Modulations of a single mode move at the group velocity $\Delta \omega / \Delta k$, near the speed of light.

- Perturbations of the filament density profile tend to move at the sound speed $c_s$, which is much slower.

- Different modes have different dispersion relations, so they can have $\omega_1 > \omega_0$ but $k_1 < k_0$, so that $\Delta \omega / \Delta k = (\omega_0 - \omega_1)/(k_0 - k_1)$ is much smaller than $c$ and comparable to $c_s$.

- The slower density perturbations resulting from multimode modulations are more nearly resonant with ion-acoustic waves, resulting in stronger coupling.
Multimode perturbations have faster growth rates

- From the slope of the real frequency curve the group velocity for the modulation is $\sim 900 \, c_s$ ($c = 1000 \, c_s$).

- The peak spatial growth rate is $\sim 10^{-4} \, \omega_0/c$; such a filament propagates for thousands of microns before substantial growth occurs.
Growth rates for single-mode perturbations are small

• Note that the group velocity vanishes near $\kappa > 0.2$, suggesting the possibility of absolute instability.
Multimode perturbations can be absolutely unstable

- Let $\kappa_{\text{max}}$ be the wave number for maximum growth rate, and $\Omega_{\text{max}}, \Omega'_{\text{max}}, \Omega''_{\text{max}}$ the corresponding frequency and its derivatives. Then the criterion for absolute instability is

$$\Im(\Omega_{\text{max}}) > \frac{1}{2}(\Omega'_{\text{max}})^2 \Im\left(\frac{1}{\Omega''_{\text{max}}}ight)$$

- For the case on the previous slide this inequality is $0.164 > 0.047$, so for this wave number the instability is absolute.

- Consequently this filament will be disrupted within a few picoseconds.
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