Self-Similar Multimode Bubble-Front Evolution of the Ablative Rayleigh–Taylor Instability in Two and Three Dimensions

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Summary

The self-similar nonlinear evolution of the multimode ablative Rayleigh–Taylor instability (ARTI) is studied numerically in both two and three dimensions

- The nonlinear multimode bubble-front penetration follows the $\alpha_b gt^2$ scaling law with $\alpha_b$ dependent on the initial conditions and ablation velocity
- Nonlinear ARTI is dominated by bubble competition, indicating that mass ablation reduces $\alpha_b$ with respect to the classical value for the same initial perturbation amplitude
- Ablation-driven vorticity accelerates the bubble velocity and prevents the transition from the bubble competition to the bubble merger regime at large initial amplitudes, leading to higher $\alpha_b$ than in the classical case
Collaborators

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The nonlinear multimode bubble-front penetration of the classical RTI follows the $\alpha_b A_T g t^2$ scaling law.

Self-similarity of nonlinear multimode RTI can be achieved in two ways:*  
1. Bubble merger: saturation of shorter wavelength modes leading to a universal $\alpha_b$  
2. Bubble competition: exponential growth and saturation of long wavelength modes, $\alpha_b$ increases logarithmically with initial perturbation

3-D simulations:
Bubble competition:
$$\alpha_b = \frac{C \sqrt{\pi}}{4} \left[ \ln \left( \frac{2C \sqrt{\pi}}{k \langle h_{0k} \rangle} \right) - 1 \right]^{-1} \quad (C \sim 0.56)$$

Bubble merger: $\alpha_b \sim 0.02$ to 0.04

3-D experiments:
$\alpha_b \sim 0.04$ to 0.08, $C = 0.95$

*G. Dimonte, Phys. Rev. E 69, 056305 (2004);  
The ablation effect on the nonlinear multimode evolution is not well understood

- Bubble merger theory shows mass ablation reduces $\alpha_b$: $\alpha_b = (1 - b \dot{V}_a) \alpha_C$

- ARTI experiments on OMEGA show that $\alpha_b = 0.04$ is slightly lower than CRTI experiments** and spectrum shifts to longer wavelengths****

- Recent experiments on the NIF show that nonlinear ARTI can grow faster than Haan’s model***

Ablation effect on single RTI mode:
- Suppress linear growth rate†
  $$\gamma = \sqrt{A r_k g - bk V_a}$$

- Enhances nonlinear bubble velocity‡
  $$U_{b, \text{rot}} = \sqrt{g(1 - r_d) / C_g k + r_d \omega_0^2 / 4k^2}$$

- Nonlinearly destabilize small scale RTI‡‡

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Both 2-D and 3-D planar simulations are used to investigate the multimode ARTI

- Simulation setup corresponds to a typical acceleration phase of a direct-drive target
- 2-D simulations: $L_x = 100 \ \mu m$, 3-D simulation: $L_x = L_y = 50 \ \mu m$, Grid size: 0.1 $\mu m$, Linear cutoff: $k_{cl} = 1 \ \mu m^{-1}$

Bubble-front penetration: 
$$h_b = IT_{lead}^{bub} - IT_{t=0}^{eq}$$

Time-varied acceleration:*
$$S = \left[ \int \sqrt{g(t)} \right]^2$$

Assuming $A_T \sim 1$:
$$\alpha_b = \frac{\partial h_b}{\partial S}$$

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The nonlinear multimode bubble-front penetration follows the $\alpha_b gt^2$ scaling law

- Nonlinear ARTI is dominated by bubble competition and $\alpha_b$ depends on initial perturbation
- Mass ablation reduces $\alpha_b$ with respect to the classical value for the same initial perturbation amplitude
- $\alpha_b$ in ARTI can be higher than CRTI when initial perturbation is large
The dependence of $\alpha_b$ on initial perturbation and ablation is derived from the bubble competition model* modified by ablation.

CRTI bubble competition: 
\[ \alpha_c = \frac{C\sqrt{\pi}}{4} \left[ \ln \left( \frac{2C\sqrt{\pi}}{k\langle h_{0k} \rangle} \right) - 1 \right]^{-1} \]

Eq. (1)

ARTI linear growth: 
\[ \gamma \approx \sqrt{gk} - bkV_a = \gamma_{cl}(1 - b\hat{V}_a) \]
\[ \hat{V}_a = \sqrt{k / gV_a} \]

Linear phase: 
\[ h_b = A_b + V_a t = h_0 \exp(\gamma t) + V_a t \]

Nonlinear bubble penetration: 
\[ h_b = U_b (t - t_{NL}) + h_b^{NL} \]
\[ U_b = C\sqrt{g\lambda} / 2 \]

Apply self-similar condition: 
\[ \frac{\partial h_b}{\partial \lambda} = 0 \implies \alpha_b \sim \frac{(1 - b\hat{V}_a)C\sqrt{\pi}}{4} \left( \ln \frac{2C\sqrt{\pi}}{k_0 h_0} - 1 \right)^{-1} = (1 - b\hat{V}_a)\alpha_c \]

Eq. (2)

Mass ablation suppresses nonlinear bubble growth by reducing $\gamma$

*G. Dimonte, Phys. Rev. E 69, 056305 (2004);
Simulations are used to quantify the dependence of $\alpha_b$ on $h_0$ and $V_a$

- $C_{3-D}/C_{2-D} \sim 1.6$: 3-D bubble velocity is $1.7 \times$ larger than 2-D
- $b = 4.2$ for both 2-D and 3-D: the same linear-dispersion relation
- $P_j(m,n)$ = initial mode spectrum that decays $\sim k^j$ with modes $m$ through $n$ with $k=m \times 2\pi/L$

$P(0,4-16)$

- $k_0 = 0.063 \mu m^{-1}$
- $b = 4.2$
- $C_{2-D} = 0.6$
- $C_{3-D} = 0.95$
The mode-structure comparison between classical and ablative RTI shows larger bubbles dominate the asymptotic behavior.
Nonlinear ARTI is still in the bubble-competition regime even for large-amplitude small-scale initial perturbations

- Ablation-generated vorticity can keep the nonlinear ARTI in the bubble-competition regime
- $\alpha_b$ in ARTI can reach higher values than in CRTI for sufficiently large initial perturbations

Linear cutoff: $m \sim 16$
P0(20-40): small-scale initial perturbation
P2(5-20): large-scale initial perturbation
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The bubble-competition theory may be used to explain the hydrodynamic stability boundary observed in laser-fusion implosion experiments

- The allowed IFAR depends on the initial perturbation
- The Omega experiments indicate that $h_{0} \sim 0.01 \mu m$

\[ V_a = \frac{\dot{m} \alpha_F^{1/\gamma}}{P_a^{1/\gamma}} \Rightarrow \alpha_F = 3.4 \left( \frac{V_a}{3.5} \right)^{5/3} \]

\[ \Rightarrow \text{IFAR} = 20 \left( \frac{\alpha_F}{1.1} \right)^{1.1} = 23 \left( \frac{V_a}{3.5} \right)^{1.83} \]

Stability cliff

In OMEGA experiments: low-adiabat ($\alpha_F < 3.5$) implosions are degraded mainly by small-scale RTI*

\[ \text{IFAR}_0 = \frac{R_0}{\Delta_0}, \]

$R_0$: in-flight capsule radius
$\Delta_0$: in-flight shell thickness

Assume same initial perturbation for RT (does not account for RM)

\[ \text{IFAR}_0 = \frac{R_0}{\Delta_0}, \]