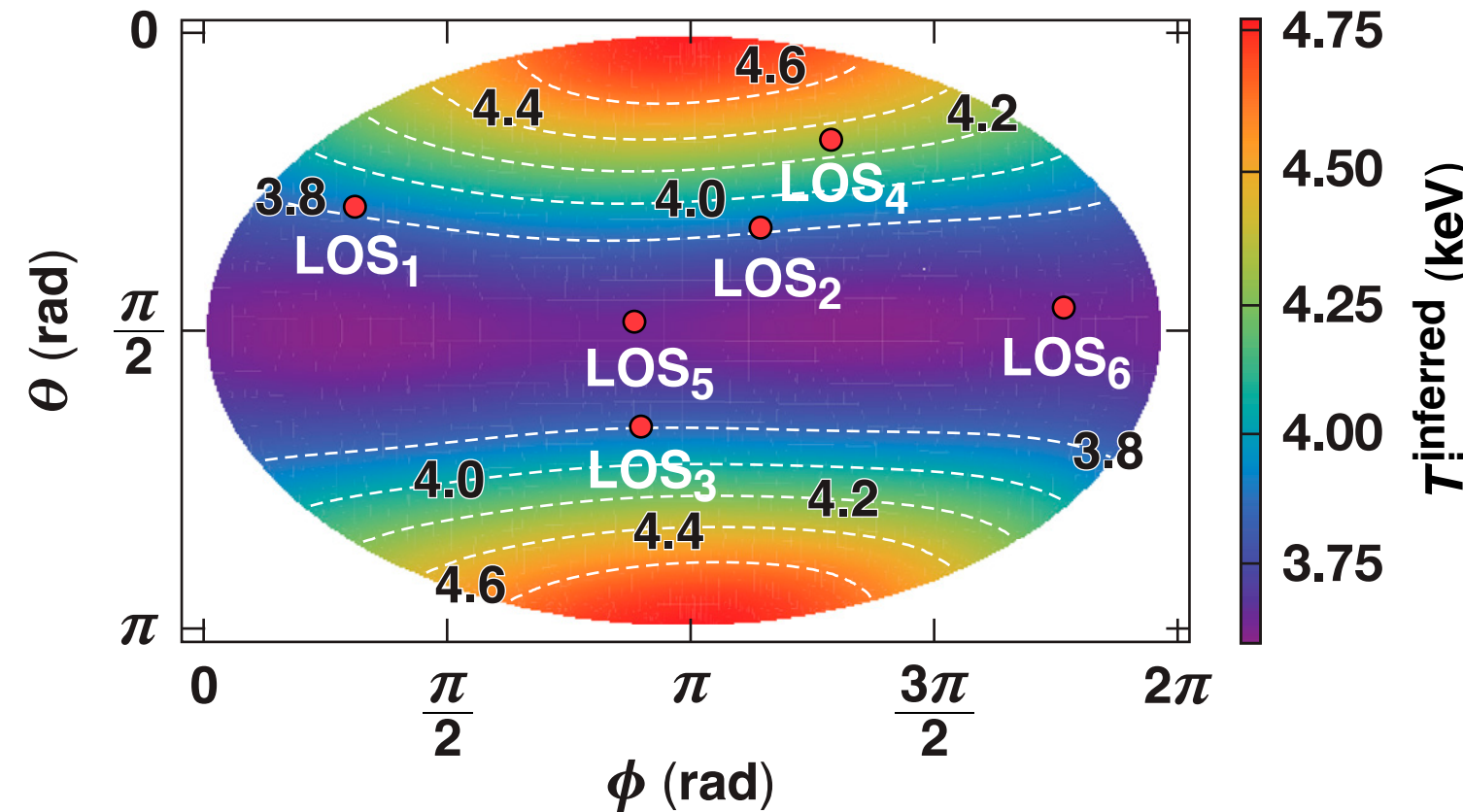


Impact of Three-Dimensional Hot-Spot Flow Asymmetry on Ion-Temperature Measurements in Inertial Confinement Fusion Experiments



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Summary

An analytical model has been developed to study three-dimensional flow effects on ion-temperature measurements



- The velocity variance in Brysk ion temperatures is shown to be uniquely determined by a complete set of six hot-spot flow parameters in terms of variance and covariance of the hot-spot flow velocity distribution
- An approximated solution to the minimum inferred ion temperature is derived and is shown to reproduce the thermal ion temperature for low-mode $\ell = 1$
- The isotropic velocity variance for low-mode $\ell = 2$ leads to minimum inferred ion temperatures well above the thermal ion temperature

Collaborators



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The ensembled average of fluid elements with a Gaussian distribution of neutron energy is applied to infer ion temperatures along different lines of sight (LOS)

Numerical method

- Ensemble averaging of fluid elements with Gaussian distribution of neutron energy

$$f_{\text{LOS}}(E_n) = \sum_{\text{cell}} \frac{Y_{\text{cell}}(t)}{Y_{\text{total}}(t)} \exp\left[-\frac{(E_n - \mu_{\text{LOS}})^2}{2\sigma^2}\right]$$

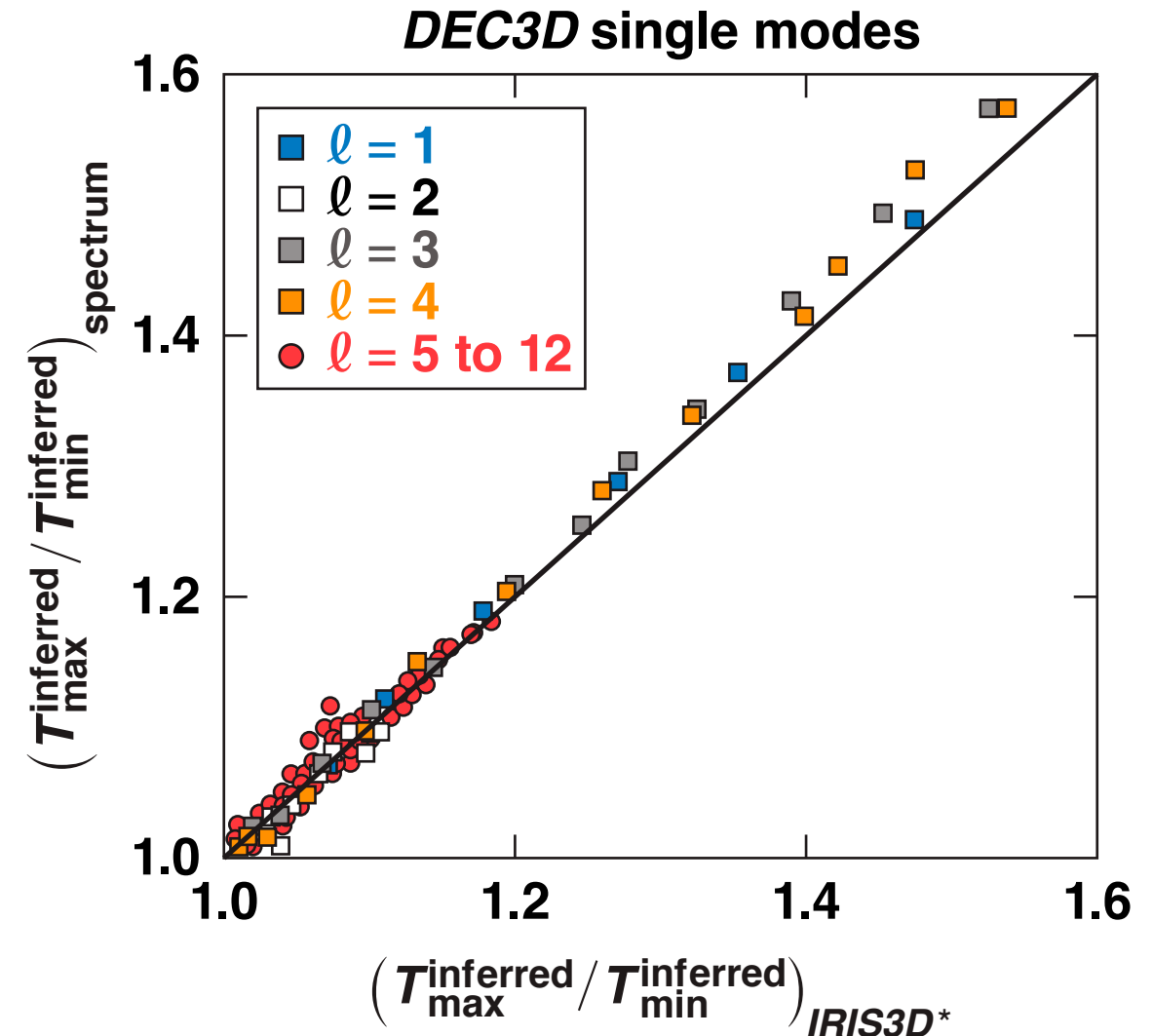
- Mean energy

$$\mu_{\text{LOS}} = E_0 + \vec{v} \cdot \hat{d} \sqrt{2m_n E_0}$$

Doppler shift term

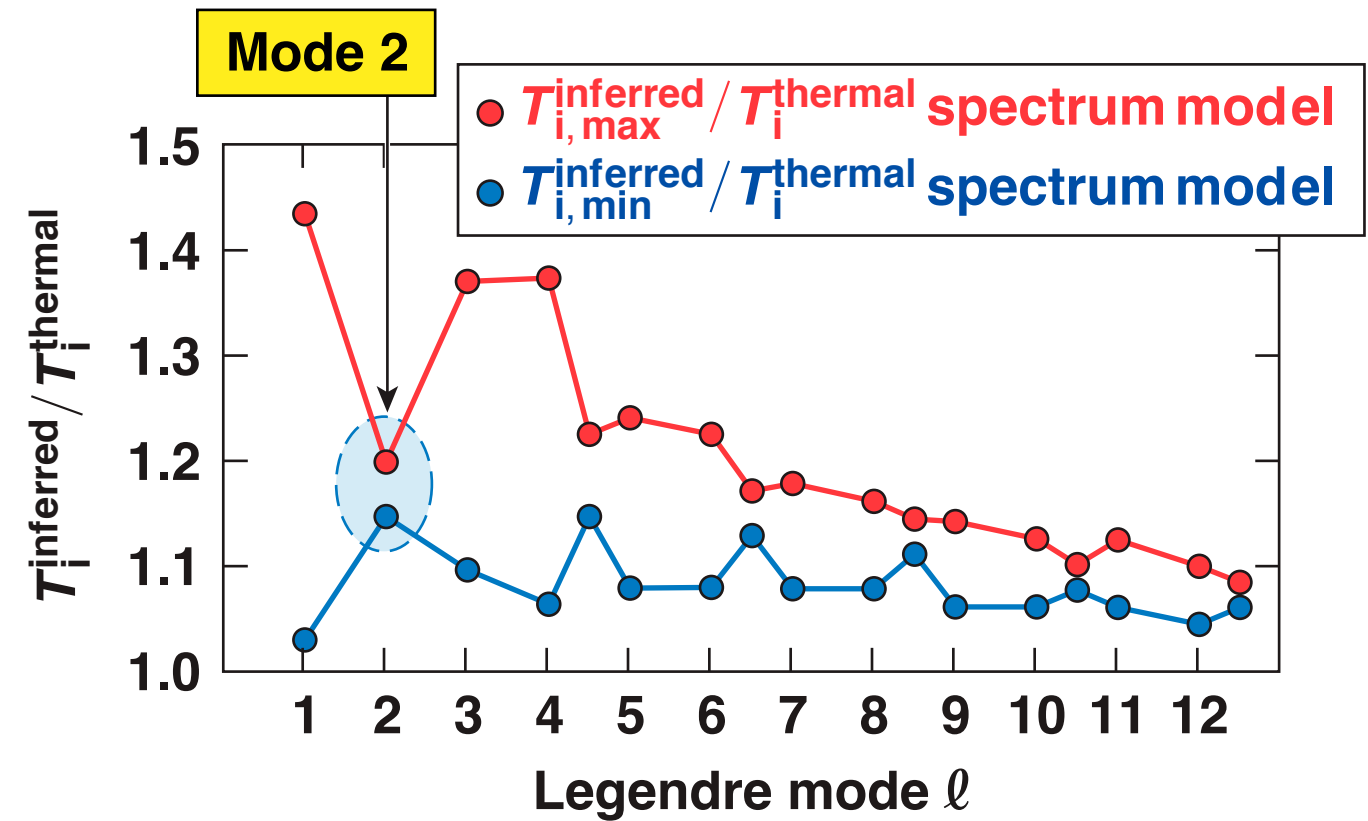
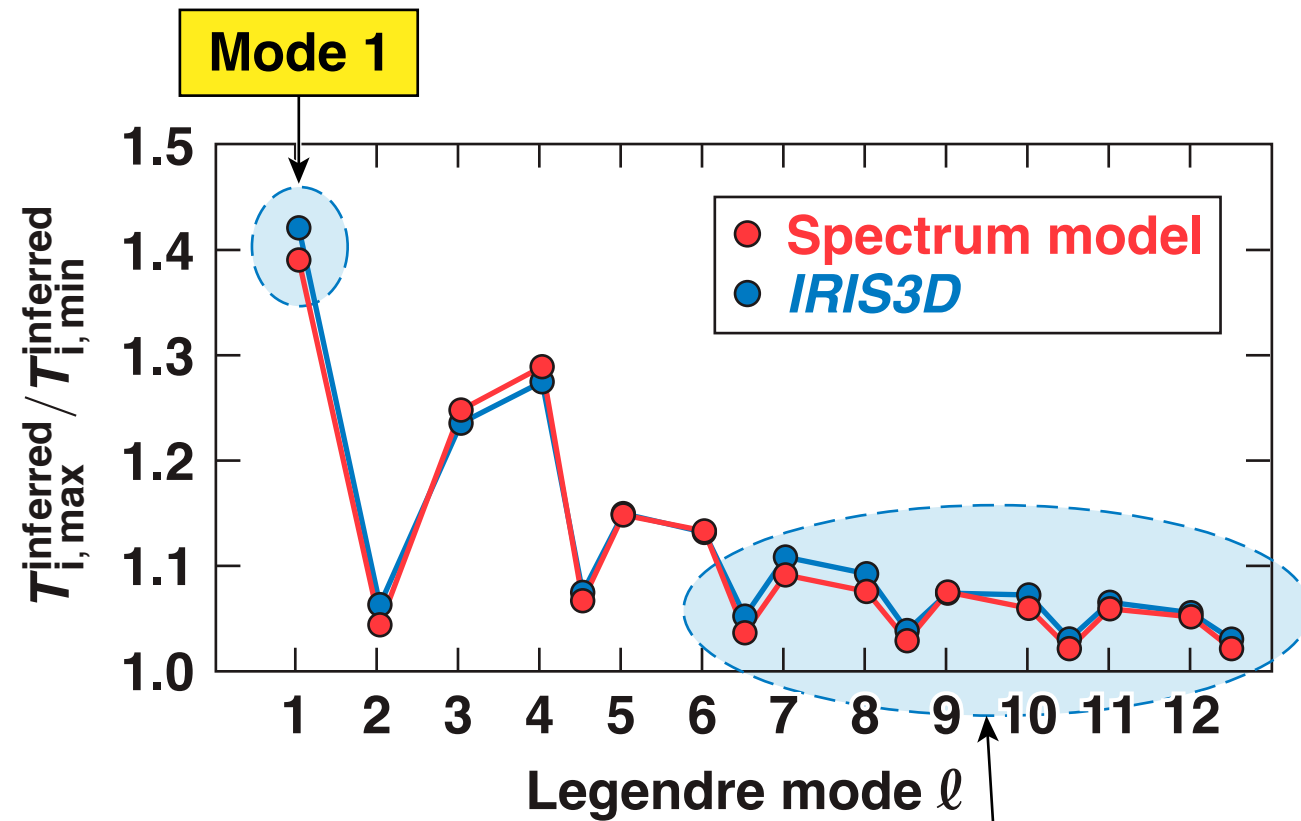
- Thermal velocity

$$\sigma^2 = 2m_n E_0 T_i^{\text{thermal}} / (m_n + m_\alpha)$$



*F. Weilacher, P. B. Radha, and C. Forrest, Phys. Plasmas **25**, 042704 (2018).

Mode 1 exhibits a large ion-temperature ratio while mode 2 exhibits a large minimum inferred ion temperature well above the thermal ion temperature



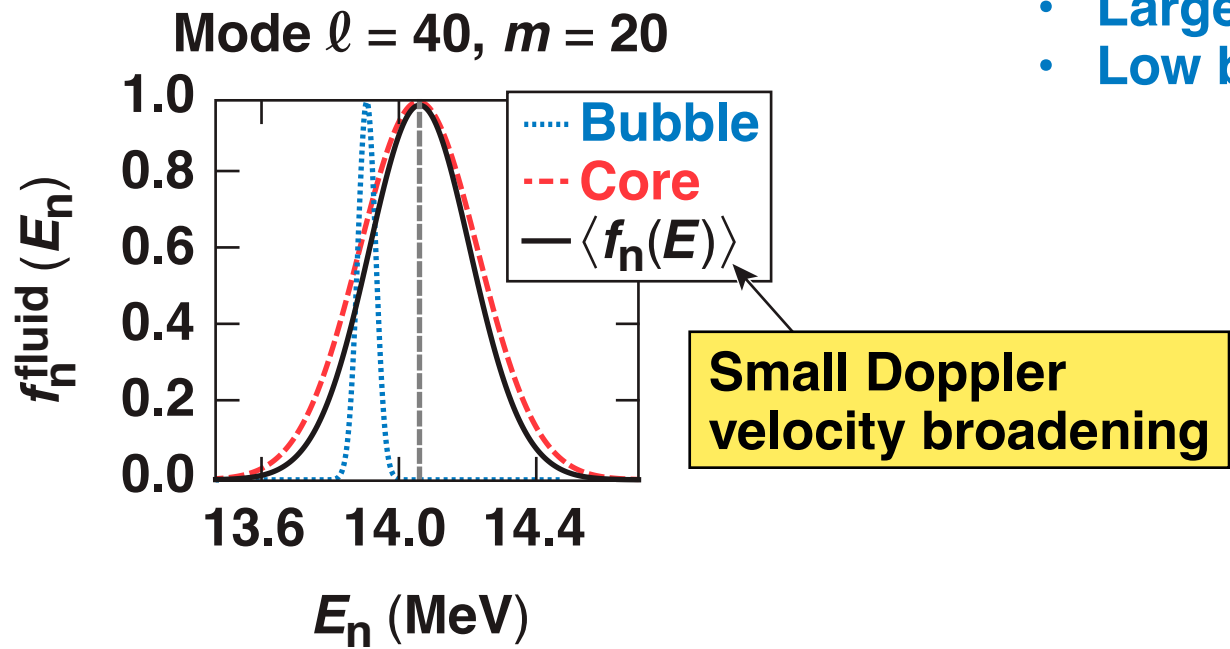
Ion-temperature ratio decreases with Legendre mode number.

For high modes, the fast moving cold bubbles do not significantly contribute to ion-temperature measurements

Effect of cold bubbles for high modes

$$f_{\text{LOS}}(E_n) = \sum_{\text{cell}} \frac{Y_{\text{cell}}(t)}{Y_{\text{total}}(t)} \exp\left[-\frac{(E_n - \mu_{\text{LOS}})^2}{2\sigma^2}\right]$$

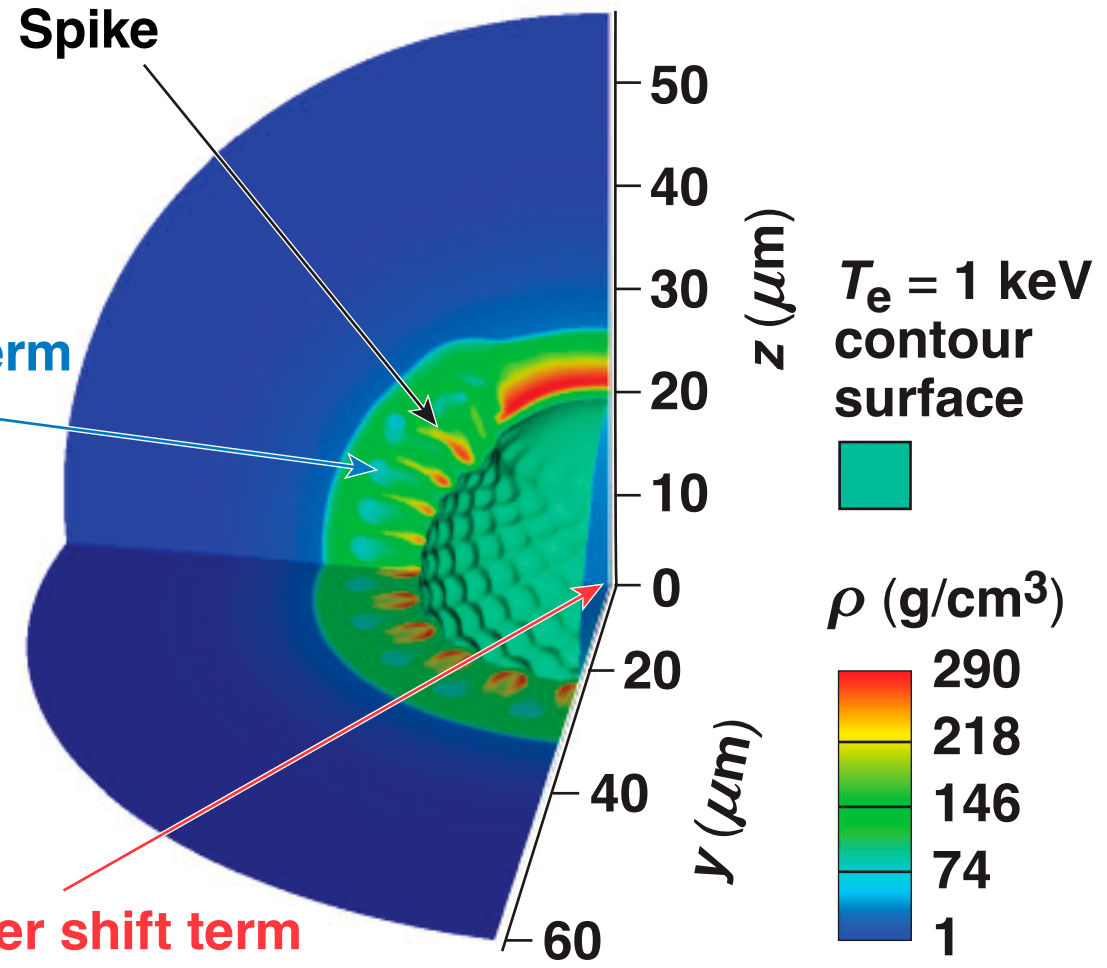
Burn weight



- Bubble**
- Large Doppler shift term
 - Low burn weight

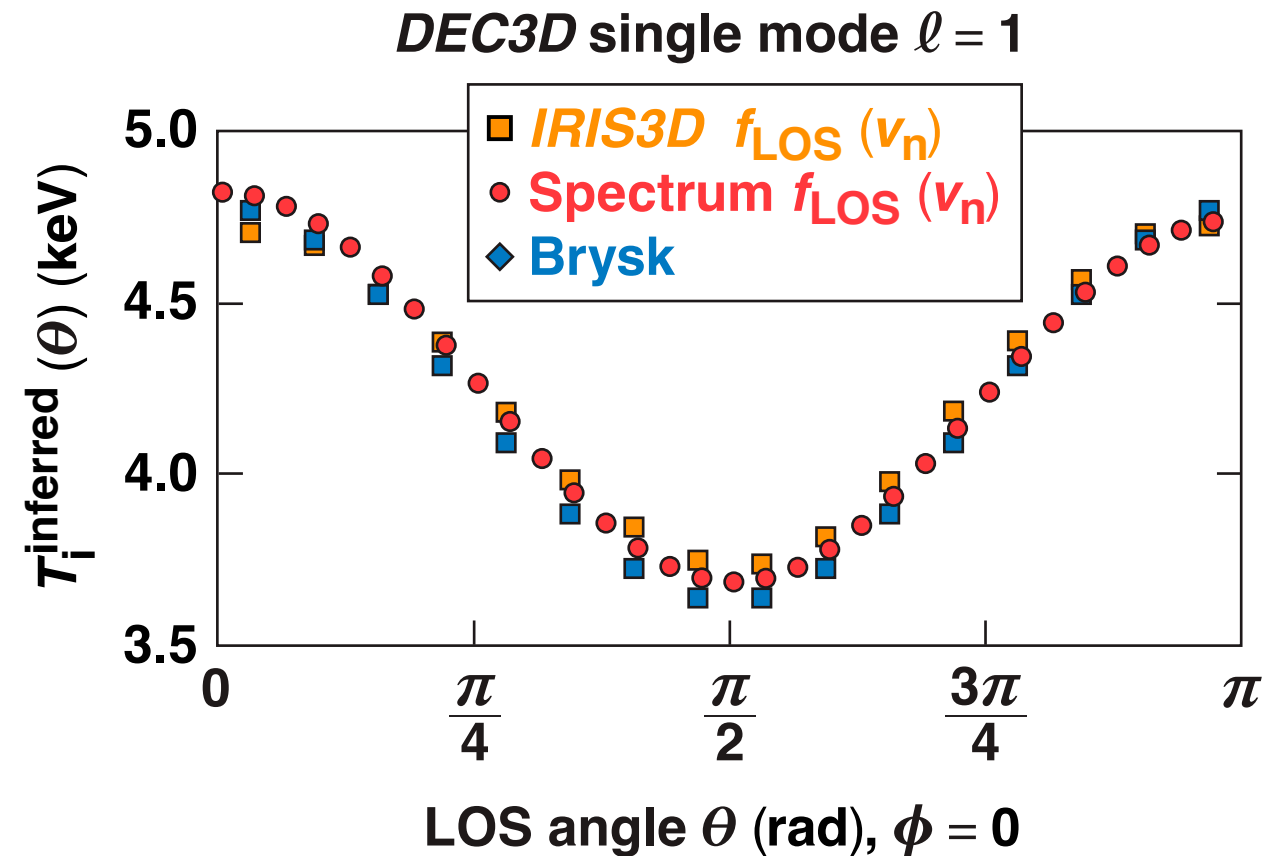
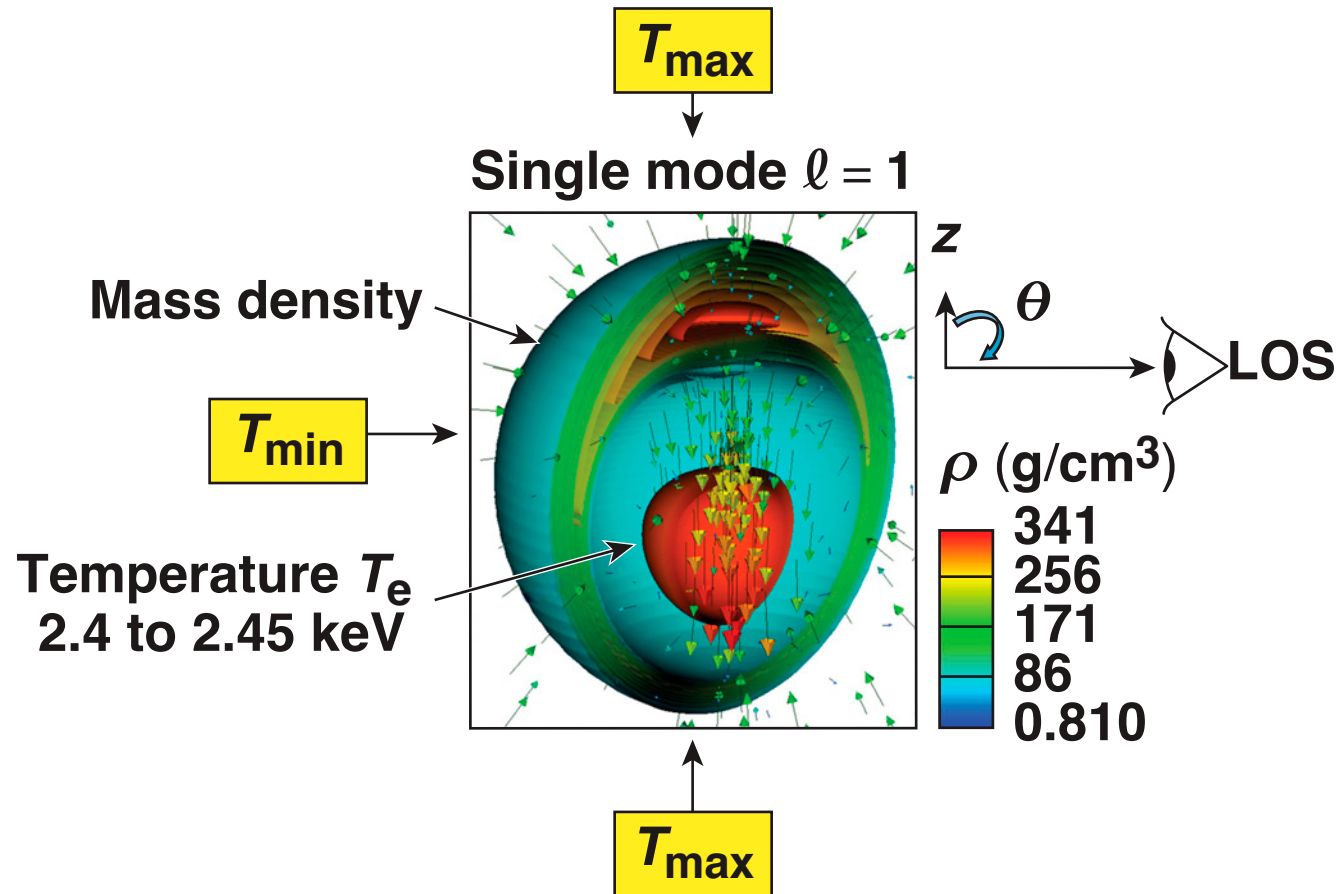
- Core**
- Small Doppler shift term
 - High burn weight

DEC3D* single mode
 $L = 40, m = 20$



*K. M. Woo *et al.*, Phys. Plasmas **25**, 052704 (2018).

Ion temperatures along different lines of sights are affected by the velocity variance of the hot-spot fluid velocity distribution



Brysk ion temperature*

$$T_i^{\text{inferred}}(\theta, \varphi) = T_i^{\text{thermal}} + (m_n + m_\alpha) \text{var} [\vec{v} \cdot \hat{d}]$$

*H. Brysk, Plasma Phys. **15**, 611 (1973).
T. J. Murphy, Phys. of Plasma **21**, 072701 (2014).
D. H. Munro, Nucl. Fusion **56**, 036001 (2016).

The velocity variance is decomposed into a complete set of six hot-spot flow parameters to characterize the hot-spot flow asymmetry



Analytic model

Decomposition of the velocity variance into variance and covariance

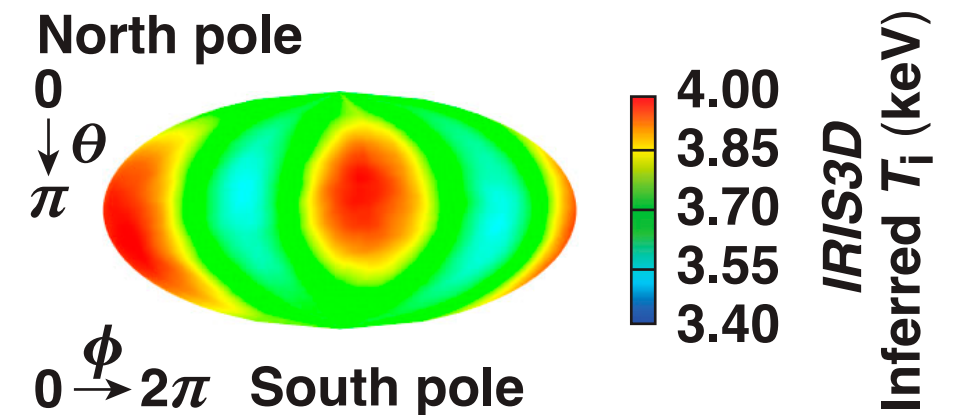
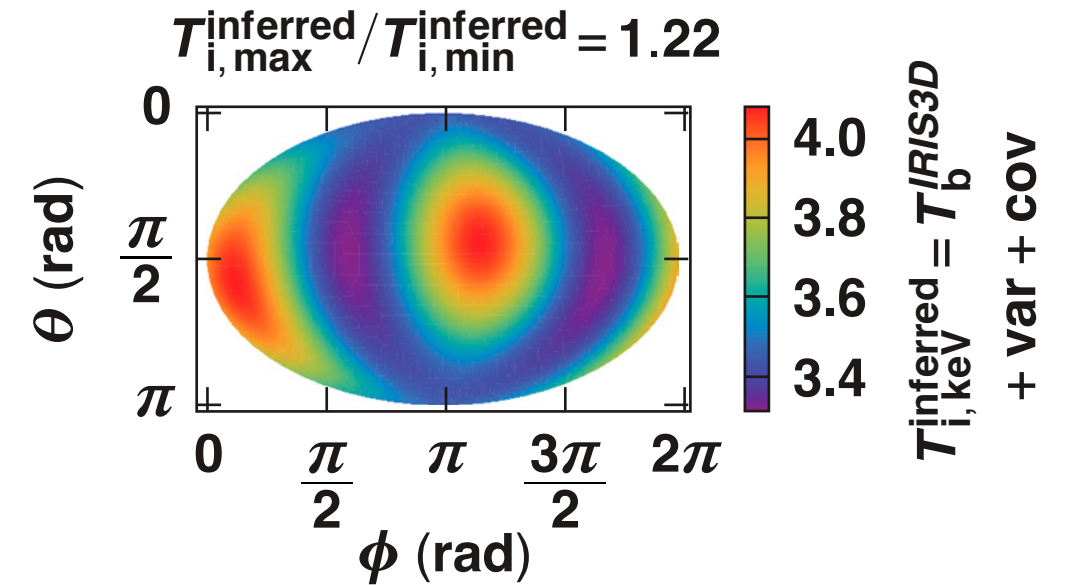
$$\vec{v} = v_i \hat{x}_i \quad \text{and} \quad \hat{d} = g_i \hat{x}_i$$

$$\text{var}[\vec{v} \cdot \hat{d}] = \langle (\vec{v} \cdot \hat{d})^2 \rangle - \langle \vec{v} \cdot \hat{d} \rangle^2 = \langle (v_i g_i)(v_j g_j) \rangle - \langle v_i g_i \rangle \langle v_j g_j \rangle$$

$$\sigma_{ij} \equiv \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle \quad \text{and} \quad \hat{T} \equiv T / (m_n + m_\alpha)$$

$$\hat{T}_i^{\text{inferred}} = \hat{T}_i^{\text{thermal}} + \underbrace{\sigma_{\text{iso}}^2 + g_i g_j \Delta \sigma_{ij} \delta_{ij}}_{3 \text{ var } (i=j)} + \underbrace{g_i g_j \sigma_{ij} (1 - \delta_{ij})}_{3 \text{ cov } (i \neq j)}$$

$$\text{var} = \sigma_{ii} = \sigma_{\text{iso}}^2 + \Delta \sigma_{ii}, \quad \text{where} \quad \sigma_{\text{iso}}^2 = \min[\sigma_{11}, \sigma_{22}, \sigma_{33}]$$



Six T_i measurements form a linear system with an invertible LOS matrix

Analytic model

$$\underbrace{\hat{T}_i^{\text{inferred}}}_{\vec{T}_6} = \underbrace{\hat{T}_i^{\text{thermal}}}_{\vec{T}_{\text{th}}} + g_i g_j \sigma_{ij} \rightarrow \vec{T}_6 = \vec{T}_{\text{th}} + \underbrace{\hat{M}_{\text{LOS}}}_{\text{LOS matrix}} \cdot \underbrace{\vec{\sigma}}_{\text{State vector}} \rightarrow \vec{\sigma} = \hat{M}_{\text{LOS}}^{-1} \cdot (\vec{T}_6 - \vec{T}_{\text{th}})$$

$\vec{\sigma} = (\sigma_{11}, \sigma_{22}, \sigma_{33}, 2\sigma_{12}, 2\sigma_{23}, 2\sigma_{31})$

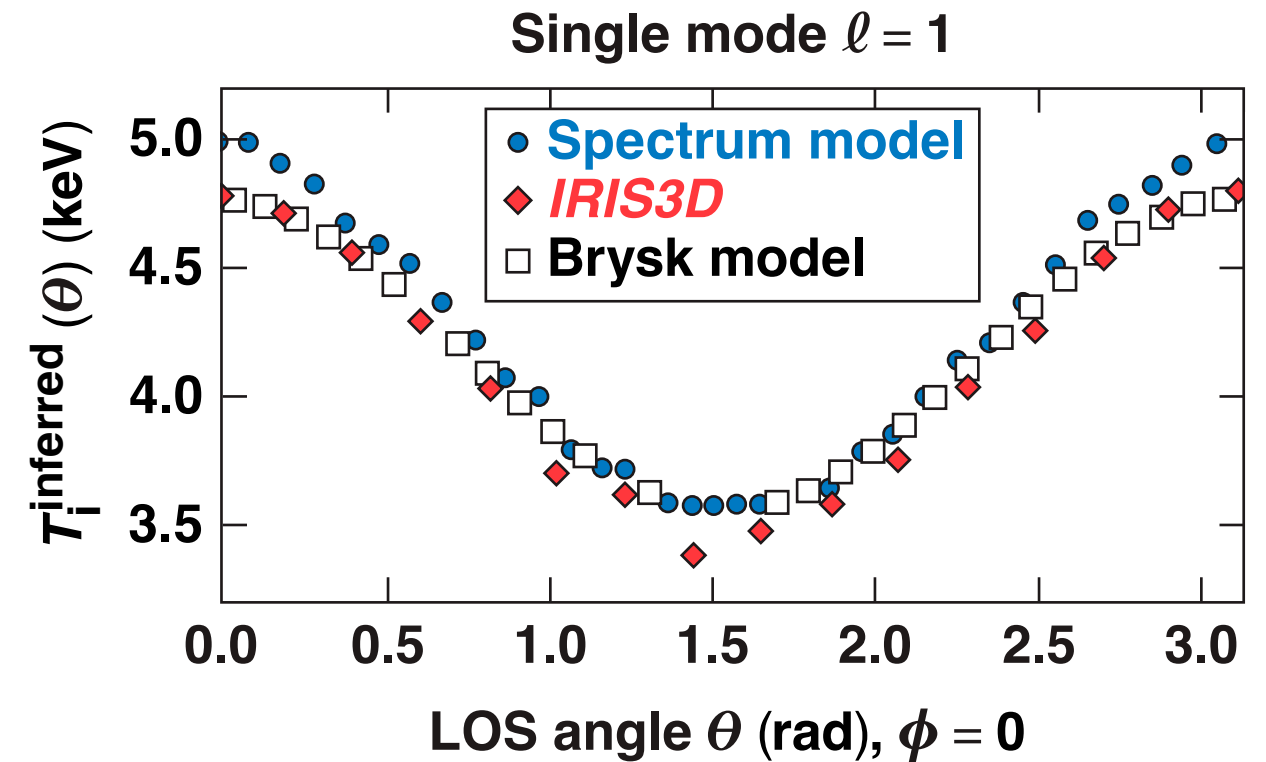
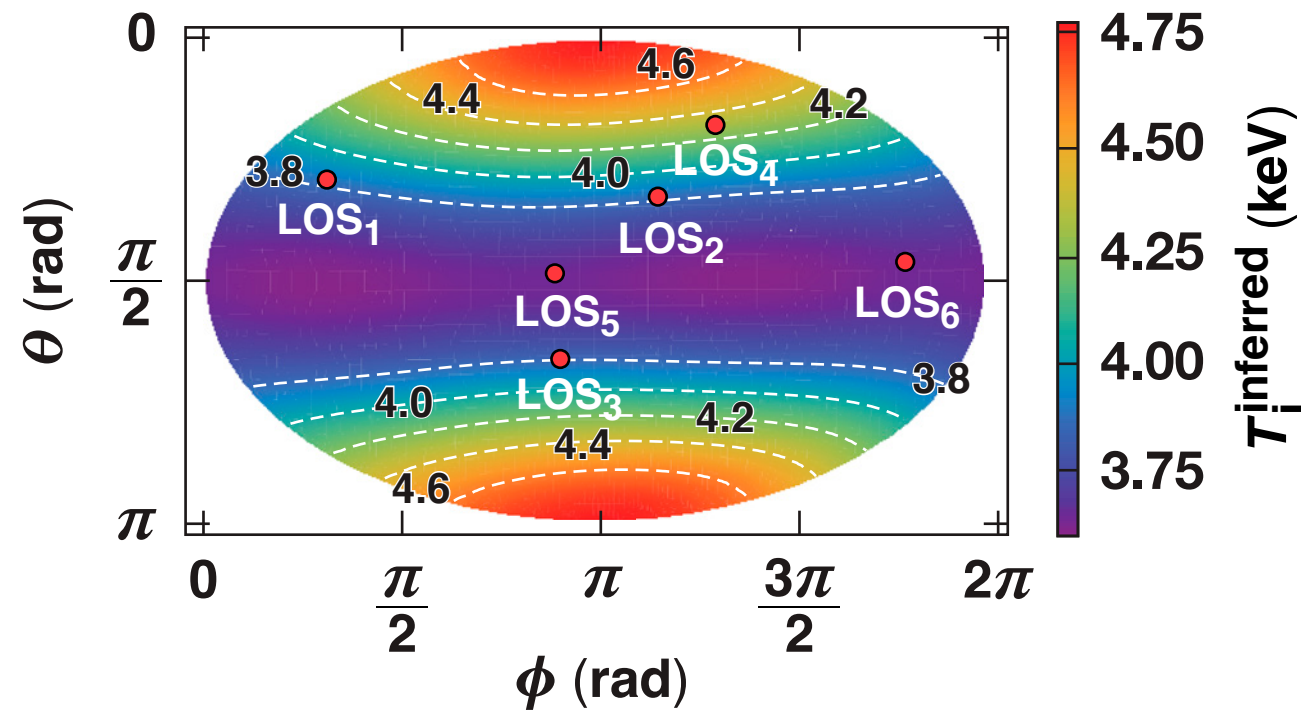
Ion temperatures away from six LOS's are given by

$$\vec{T}_{\text{new}} = (\hat{I} - \hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1}) \cdot \vec{T}_{\text{th}} + \hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1} \cdot \vec{T}_6$$

Departure matrix that has small values of matrix elements for current six LOS's on OMEGA

The full map of inferred ion temperatures and its minimum can be extrapolated from six ion-temperature measurements

$$[T_i]_{\text{keV}}^{n-\text{avg}} = 3.55, \min [\hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1} \cdot \vec{T}_6]_{\text{keV}} = 3.52$$

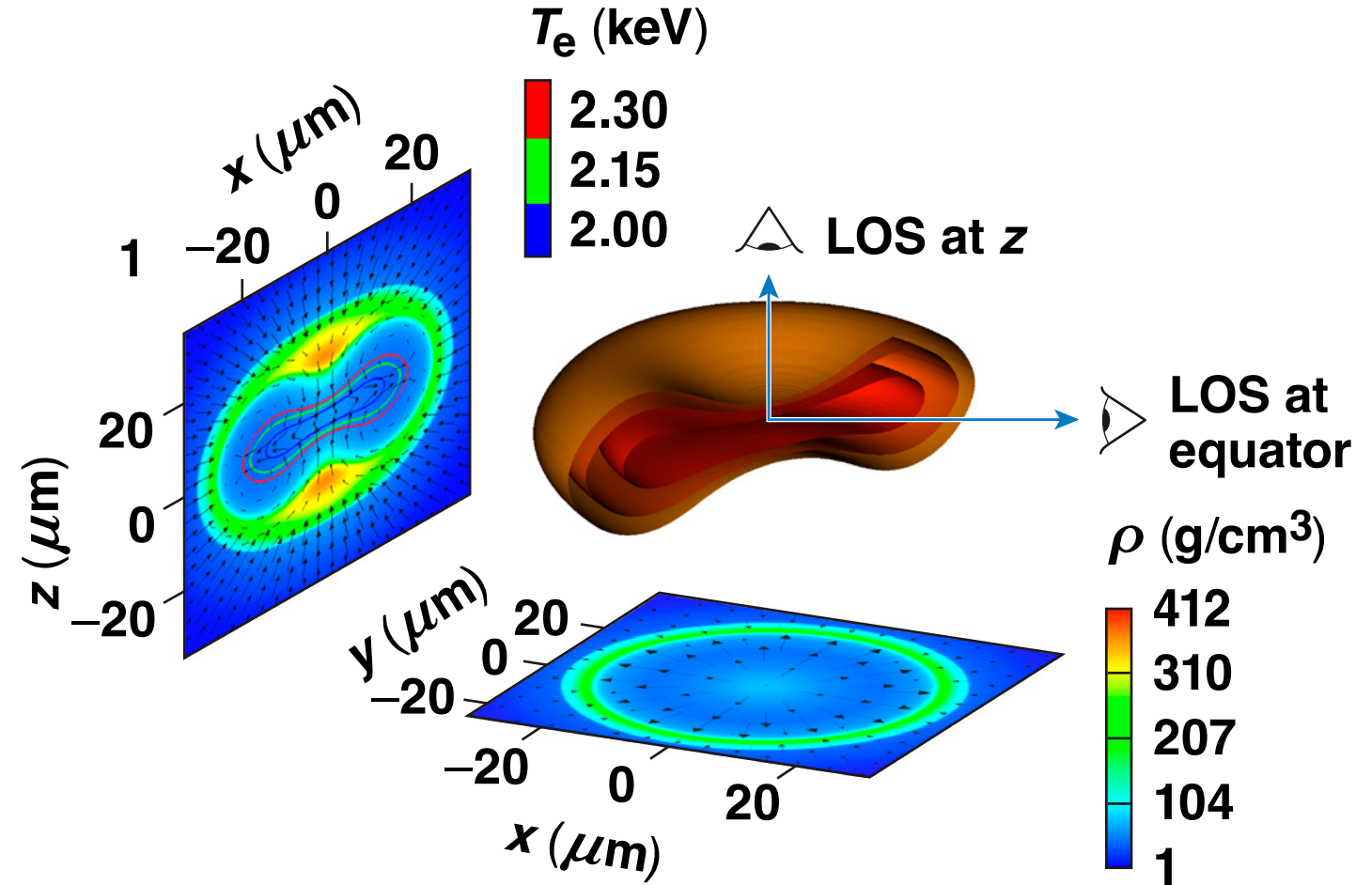
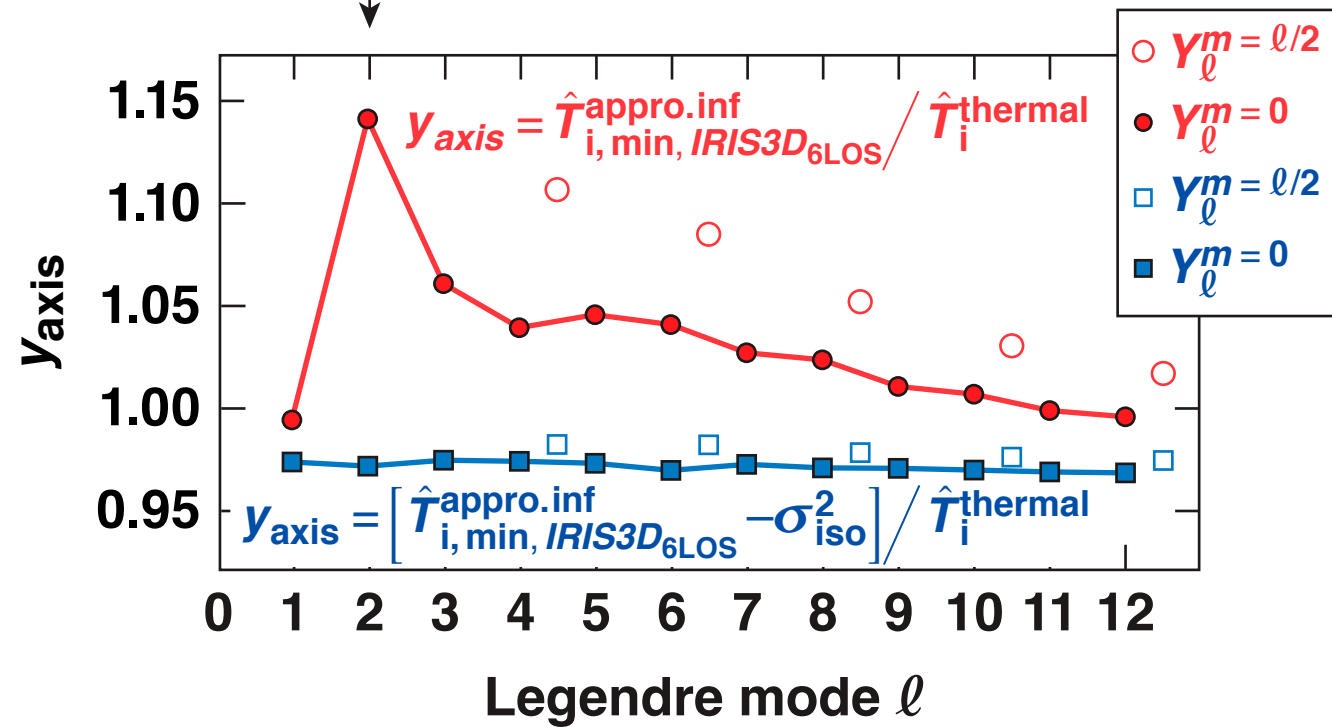


Ion temperatures away from six LOS's are approximated by

$$\vec{T}_{\text{new}} = \hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1} \cdot \vec{T}_6$$

Mode $\ell = 2$ has a large neutron-averaged weight for the radial flow within the hot bubble producing a large isotropic velocity variance

Large T_{\min} for mode 2



$$\hat{T}_{i, \min}^{\text{inferred}} = \min \left[\hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1} \cdot \vec{T}_6 \right]_{4\pi}$$

$$\hat{T}_{i, \text{mode } 2}^{\text{inferred}} = \hat{T}_i^{\text{thermal}} + \min [\sigma_{11}, \sigma_{33}]$$

An analytical model has been developed to study three-dimensional flow effects on ion-temperature measurements

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