Impact of Three-Dimensional Hot-Spot Flow Asymmetry on Ion-Temperature Measurements in Inertial Confinement Fusion Experiments

K. M. Woo
University of Rochester
Laboratory for Laser Energetics

60th Annual Meeting of the American Physical Society
Division of Plasma Physics
Portland, OR
5–9 November 2018
An analytical model has been developed to study three-dimensional flow effects on ion-temperature measurements

- The velocity variance in Brysk ion temperatures is shown to be uniquely determined by a complete set of six hot-spot flow parameters in terms of variance and covariance of the hot-spot flow velocity distribution.
- An approximated solution to the minimum inferred ion temperature is derived and is shown to reproduce the thermal ion temperature for low-mode $\ell = 1$.
- The isotropic velocity variance for low-mode $\ell = 2$ leads to minimum inferred ion temperatures well above the thermal ion temperature.
Collaborators


University of Rochester
Laboratory for Laser Energetics

D. Shvarts
Ben Gurion University of the Negev

J. Sanz
Universidad Politecnica de Madrid

A. Bose
Massachusetts Institute of Technology
The ensembled average of fluid elements with a Gaussian distribution of neutron energy is applied to infer ion temperatures along different lines of sight (LOS)

**Numerical method**

- Ensemble averaging of fluid elements with Gaussian distribution of neutron energy
  
  \[ f_{\text{LOS}}(E_n) = \sum_{\text{cell}} \frac{Y_{\text{cell}}(t)}{Y_{\text{total}}(t)} \exp \left[ -\frac{(E_n - \mu_{\text{LOS}})^2}{2\sigma^2} \right] \]

- Mean energy
  
  \[ \mu_{\text{LOS}} = E_0 + \mathbf{v} \cdot \mathbf{d} \sqrt{\frac{2m_n}{E_0}} \]

- Thermal velocity
  
  \[ \sigma^2 = 2m_n E_0 T_i^{\text{thermal}} / (m_n + m_\alpha) \]

\[ \frac{T_{\text{inferred}}}{T_{\text{inferred}}} \]

**DEC3D single modes**

Mode 1 exhibits a large ion-temperature ratio while mode 2 exhibits a large minimum inferred ion temperature well above the thermal ion temperature.
For high modes, the fast moving cold bubbles do not significantly contribute to ion-temperature measurements.

Effect of cold bubbles for high modes:

$$f_{\text{LOS}}(E_n) = \sum_{\text{cell}} \frac{Y_{\text{cell}}(t)}{Y_{\text{total}}(t)} \exp \left[ -\frac{(E_n - \mu_{\text{LOS}})^2}{2\sigma^2} \right]$$

For high modes, the fast moving cold bubbles do not significantly contribute to ion-temperature measurements.

- **Bubble**
  - Large Doppler shift term
  - Low burn weight

- **Core**
  - Small Doppler shift term
  - High burn weight

DEC3D* single mode

$L = 40, \ m = 20$

**Burn weight**

Small Doppler velocity broadening

- **Bubble**
- **Core**

Ion temperatures along different lines of sights are affected by the velocity variance of the hot-spot fluid velocity distribution.

Brysk ion temperature:

\[ T_{i,\text{inferred}}(\theta, \varphi) = T_{i,\text{thermal}} + (m_n + m_\alpha) \text{var}[\vec{v} \cdot \hat{d}] \]

T. J. Murphy, Phys. of Plasma 21, 072701 (2014).
The velocity variance is decomposed into a complete set of six hot-spot flow parameters to characterize the hot-spot flow asymmetry.

**Analytic model**

Decomposition of the velocity variance into variance and covariance

\[ \mathbf{v} = v_i \hat{x}_i \quad \text{and} \quad \mathbf{d} = g_i \hat{x}_i \]

\[ \text{var}[\mathbf{v} \cdot \mathbf{d}] = \langle (\mathbf{v} \cdot \mathbf{d})^2 \rangle - \langle \mathbf{v} \cdot \mathbf{d} \rangle^2 = \langle v_i g_i (v_j g_j) \rangle - \langle v_i g_i \rangle \langle v_j g_j \rangle \]

\[ \sigma_{ij} = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle \quad \text{and} \quad \hat{T} = T / (m_n + m_\alpha) \]

\[ \hat{T}_{\text{inferred}} = \hat{T}_{\text{thermal}} + \sigma_{\text{iso}}^2 + g_i g_j \Delta \sigma_{ij} \delta_{ij} + g_i g_j \sigma_{ij} (1 - \delta_{ij}) \]

3 var \((i = j)\) \quad 3 cov \((i \neq j)\)

\[ \text{var} = \sigma_{ii} = \sigma_{\text{iso}}^2 + \Delta \sigma_{ii} \quad \text{where} \quad \sigma_{\text{iso}}^2 = \min[\sigma_{11}, \sigma_{22}, \sigma_{33}] \]
Six $T_i$ measurements form a linear system with an invertible LOS matrix

**Analytic model**

State vector $\vec{\sigma} = (\sigma_{11}, \sigma_{22}, \sigma_{33}, 2\sigma_{12}, 2\sigma_{23}, 2\sigma_{31})$

Ion temperatures away from six LOS's are given by

$$\vec{T}_{\text{new}} = \left( \vec{I} - \hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1} \right) \cdot \vec{T}_{\text{th}} + \hat{M}_{\text{new}} \cdot \hat{M}_{\text{LOS}}^{-1} \cdot \vec{T}_6$$

Departure matrix that has small values of matrix elements for current six LOS's on OMEGA
The full map of inferred ion temperatures and its minimum can be extrapolated from six ion-temperature measurements.

\[ T_i^{\text{avg}} = 3.55, \min \left[ \dot{M}_{\text{new}} \cdot M_{\text{LOS}}^{-1} \cdot T_6 \right]_{\text{keV}} = 3.52 \]

Single mode \( \ell = 1 \)

Ion temperatures away from six LOS’s are approximated by

\[ \dot{T}_{\text{new}} = \dot{M}_{\text{new}} \cdot M_{\text{LOS}}^{-1} \cdot T_6 \]
Mode $\ell = 2$ has a large neutron-averaged weight for the radial flow within the hot bubble producing a large isotropic velocity variance.
Summary/Conclusions

An analytical model has been developed to study three-dimensional flow effects on ion-temperature measurements

- The velocity variance in Brysk ion temperatures is shown to be uniquely determined by a complete set of six hot-spot flow parameters in terms of variance and covariance of the hot-spot flow velocity distribution.
- An approximated solution to the minimum inferred ion temperature is derived and is shown to reproduce the thermal ion temperature for low-mode $\ell = 1$.
- The isotropic velocity variance for low-mode $\ell = 2$ leads to minimum inferred ion temperatures well above the thermal ion temperature.