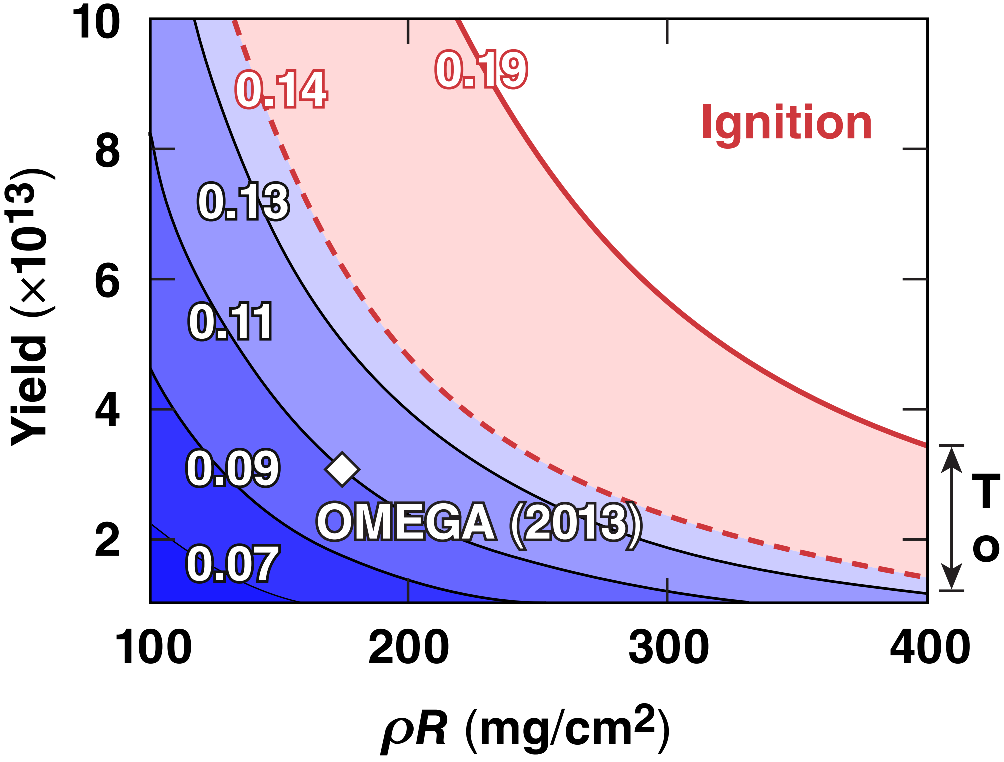
Theory of Hydro-Equivalent Ignition for Inertial Fusion and its Applications to OMEGA and the NIF



$$\chi \equiv \frac{P\tau}{P\tau_{\text{ig}}}$$

Threshold depends on imprinting

R. Nora
Fusion Science Center
University of Rochester
Laboratory for Laser Energetics

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Summary

Hydrodynamic equivalence and ignition theory allow for the comparison of OMEGA implosions to ignition-scale targets

- Hydrodynamically equivalent implosions are energetically scalable and have identical implosion velocities, laser intensities, and adiabats
- The measurable Lawson criterion can assess the performance of an implosion using experimental observables
- An OMEGA implosion with an areal density of 300 mg/cm^2 and neutron yield of 3 to 6 $\times$ 10^{13} would ignite on a hydrodynamically equivalent symmetric National Ignition Facility (NIF)-scale target (depending on the level of imprinting)

Collaborators

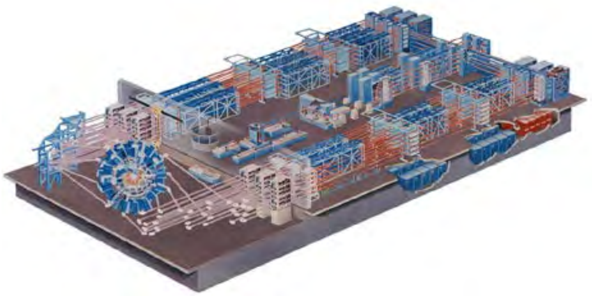
R. Betti*, K. S. Anderson, A. Shvydky, A. Bose*, K. M. Woo*,
A. R. Christopherson*, J. A. Marozas, T. J. B. Collins, P. B. Radha,
S. X. Hu, R. Epstein, F. J. Marshall, T. C. Sangster, and D. D. Meyerhofer*

University of Rochester
Laboratory for Laser Energetics
*also Fusion Science Center and also Department of Physics
and/or Mechanical Engineering

Motivation

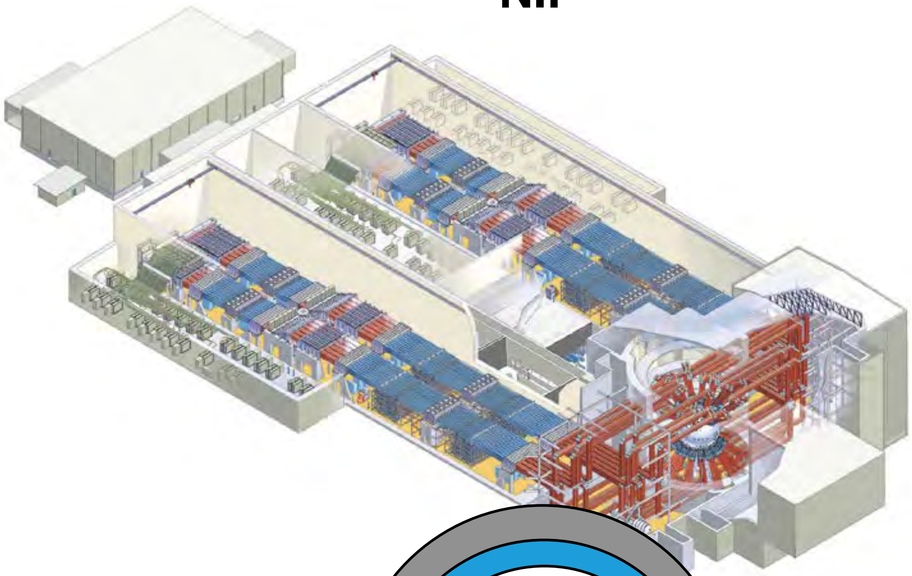
Hydrodynamic equivalence provides a tool to scale the performance of OMEGA implosions to NIF energies

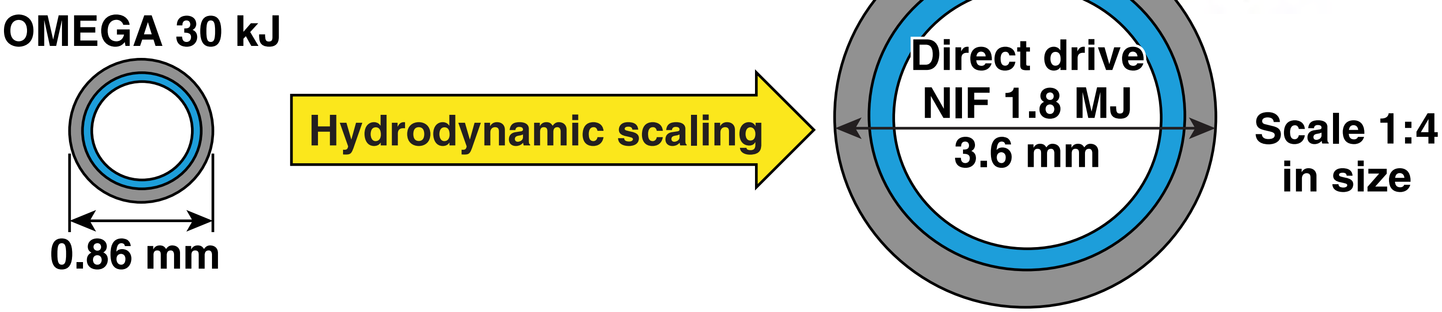
OMEGA



Scale 1:60
in energy

NIF





OMEGA 30 kJ

0.86 mm

Hydrodynamic scaling

Direct drive
NIF 1.8 MJ
3.6 mm

Scale 1:4
in size

The extrapolation is from 60-beam OMEGA to 192-beam symmetric NIF.

Outline

- Theory of hydrodynamic equivalence
- OMEGA and NIF hydro-equivalent designs
- Hydro-equivalent ignition scaling from OMEGA to the NIF

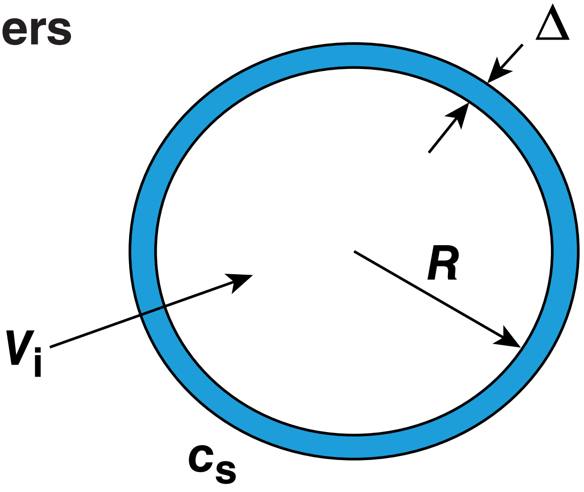
One-dimensional implosion equivalence requires equal Mach numbers

- The shell implodes with V_i and expands with c_s
- The Mach number $M \equiv \frac{V_i}{c_s}$ is the only independent dimensionless parameter*
- Using the isentropic implosion condition

$$P_a(I_L) \sim \alpha \rho^{5/3}$$

- 1-D similarity requires equal Mach numbers

$$M^2 \sim \frac{V_i^2}{\alpha^{3/5} P_a(I_L)^{2/5}}$$



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*J. D. Lindl, Phys. Plasmas 2, 3933 (1995).

Multidimensional implosion equivalence imposes additional requirements on entropy and velocity

- The multidimensional behavior is determined primarily by the Rayleigh–Taylor (RT) instability

- The number of e foldings of RT growth* is

$$N_e^{RT} = \int_0^{t_i} \gamma_{RT}\,dt = \int_0^{t_i} \left(\sqrt{kg} - 3kV_a\right) dt = \int_0^1 \left(\sqrt{\ell \frac{\ddot{\hat{R}}}{\hat{R}}} - 3\frac{\ell}{\hat{R}}\frac{V_a}{V_i}\right) d\tau$$

where $k \approx \frac{\ell}{R}$, $\hat{R} = \frac{R}{R_0}$, and $\tau = \frac{t \cdot V_i}{R_0}$

- 3-D similarity requires the same $\frac{V_a}{V_i}$

$$\frac{V_a}{V_i} = \frac{\dot{m}_a}{\rho V_i} \sim \frac{\dot{m}_a\left(I_L\right)}{P_a\left(I_L\right)^{3/5}} \frac{\alpha^{3/5}}{V_i}$$

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*H. Takabe *et al.*, Phys. Fluids 28, 3676 (1985).

Hydrodynamically equivalent targets geometrically scale with the total laser energy

- 1-D hydrodynamic equivalence: $M^2 \sim \dfrac{V_i^2}{\alpha^{3/5} P_a (I_L)^{2/5}}$

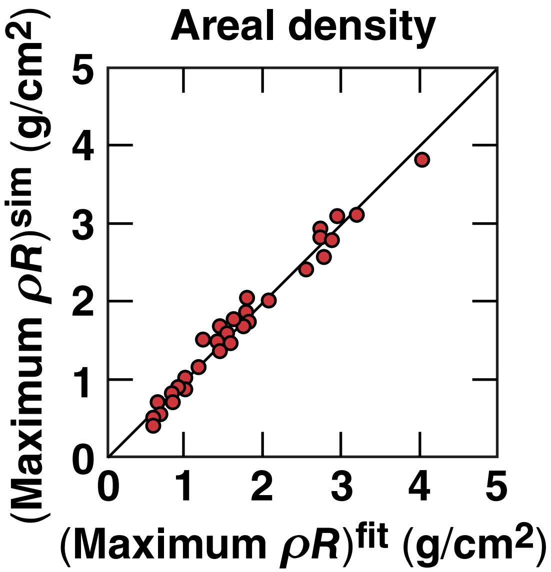
- 3-D hydrodynamic equivalence: $\dfrac{V_a}{V_i} \sim \dfrac{\dot{m}_a (I_L)}{P_a (I_L)^{3/5}} \dfrac{\alpha^{3/5}}{V_i}$

- Constant energy per unit volume: $E \approx 4\pi R^2 I_L \dfrac{R}{V_i} \longrightarrow \dfrac{E}{R^3} \sim \dfrac{I_L}{V_i}$

Hydrodynamic equivalence: fixed V_i, α, I_L

$$\boxed{R, \Delta, t \sim E^{1/3}, P_L \sim E^{2/3}, m_{sh} \sim E}$$

Hydrodynamic equivalence allows for laser-energy scaling of implosion performance



$$\rho R_{\text{g/cm}^2} \approx \frac{1.2}{\alpha^{0.54}}\left[\frac{E(\text{kJ})}{100}\right]^{1/3}\left[\frac{V_{\text{i}}(\text{cm/s})}{3\times10^7}\right]^{0.06}$$

$$\boxed{\rho R \sim E^{1/3}}$$

Hot-spot temperature

$$\boxed{T_{\text{hs}} \sim E^{0.07}}$$

1-D neutron yield

$$\boxed{Y_{\text{n}} \sim E^{3/2}}$$

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*C. D. Zhou and R. Betti, Phys. Plasmas 14, 072703 (2007).
**R. Betti *et al*., Phys. Plasmas 17, 058102 (2010).

Hydrodynamic equivalence breaks down when non-scalable physics have a significant impact on target performance

Non-scalable physics	Impact
Radiation transport in both the acceleration and deceleration phases $\left(\frac{\lambda^{\nu}_{\mathrm{mfp}}}{R}\right)_{\Omega} > \left(\frac{\lambda^{\nu}_{\mathrm{mfp}}}{R}\right)_{\mathrm{NIF}}$	Leads to radiation preheating on targets that are insufficiently shielded Small changes to the NIF-scale target geometry are made to compensate for this difference
Thermal transport in the hot spot	Affects mass ablation rates and RT growth factors
Fusion reactions	All hydrodynamic quantities must be calculated without alpha-particle deposition

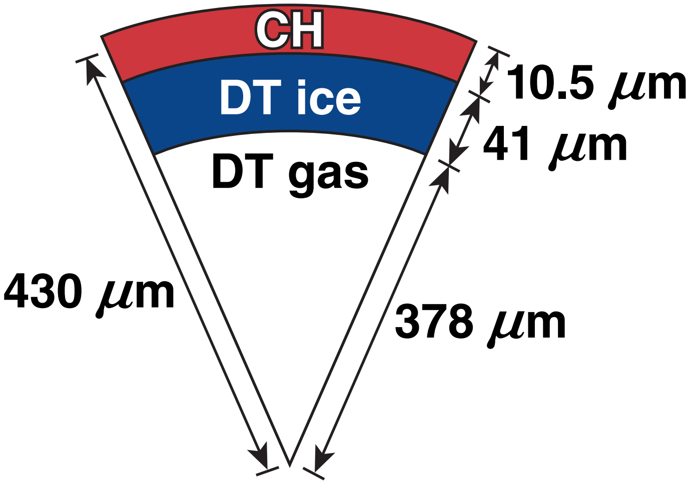
Laser–plasma instabilities are not considered in this work.

Outline

- Theory of hydrodynamic equivalence
- **OMEGA and NIF hydro-equivalent designs**
- Hydro-equivalent ignition scaling from OMEGA to the NIF

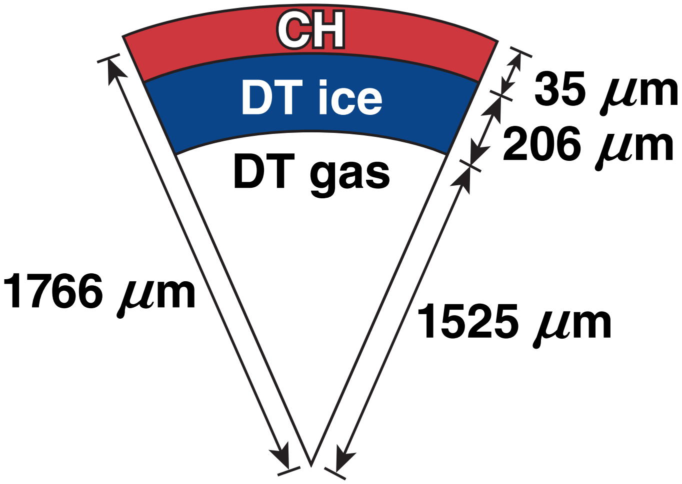
Hydro-equivalent implosions are designed from current OMEGA targets

OMEGA



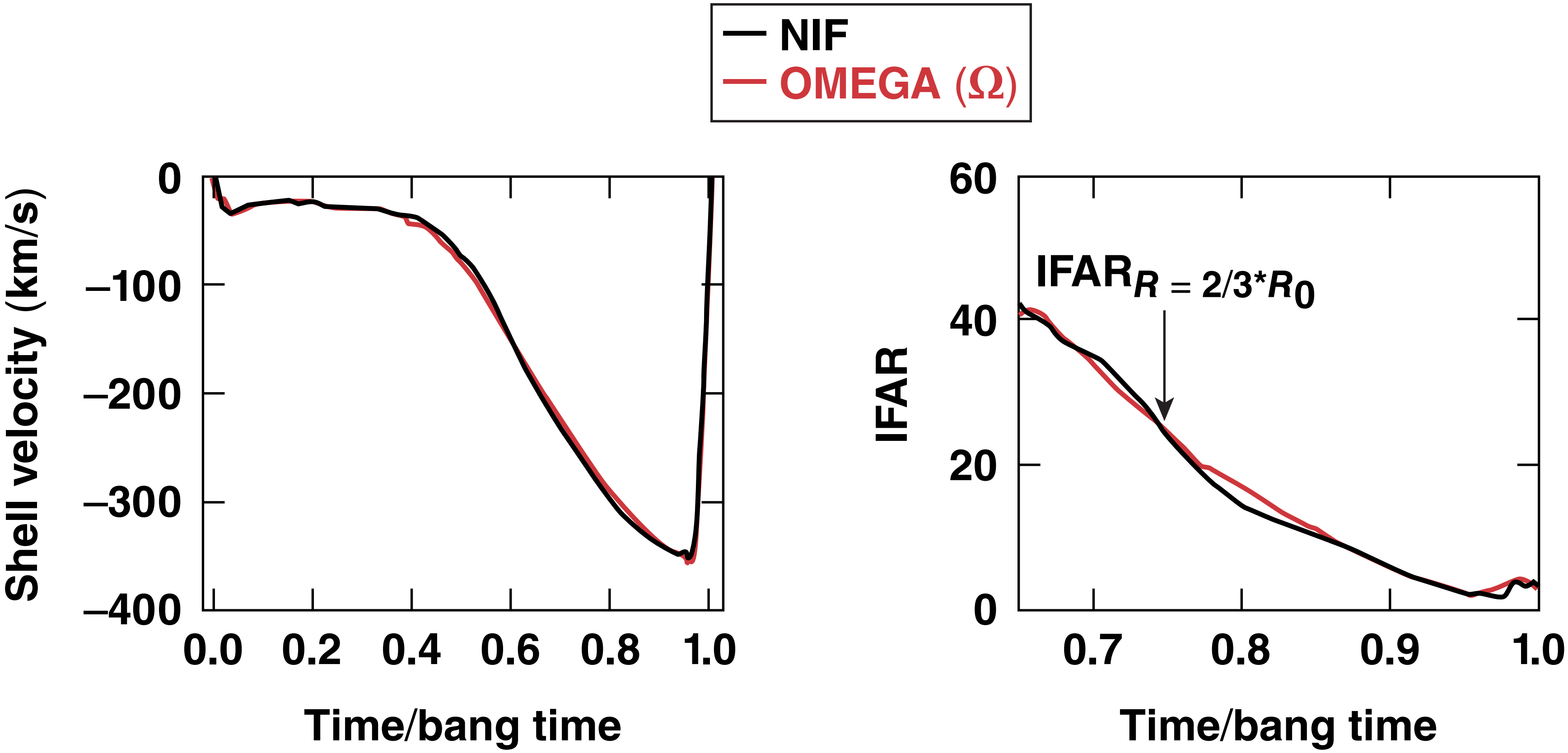
E_L (kJ)	27
V_i (km/s)	350
$\langle\alpha\rangle$	3.0
IFAR	26
$\langle\rho R\rangle_n$ (g/cm^2)	0.3
Y_n (1-D)	1.6×10^{14}

NIF



E_L (kJ)	1840
V_i (km/s)	350
$\langle\alpha\rangle$	3.0
IFAR	24
$\langle\rho R\rangle_n$ (g/cm^2)	1.2
Y_n (1-D)	$3.3 \times 10^{19}/49$

The normalized time evolution of the implosion velocity and in-flight aspect ratio (IFAR) are the same for the NIF and OMEGA

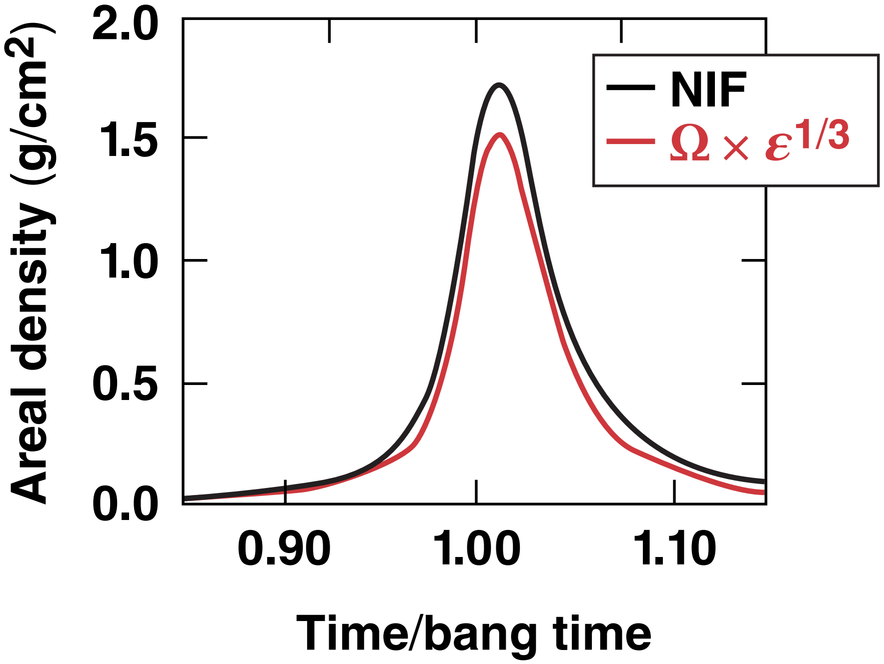


- Same 1-D hydro
- Same RT growth (3-D)

The 1-D areal density and neutron rate scale as predicted

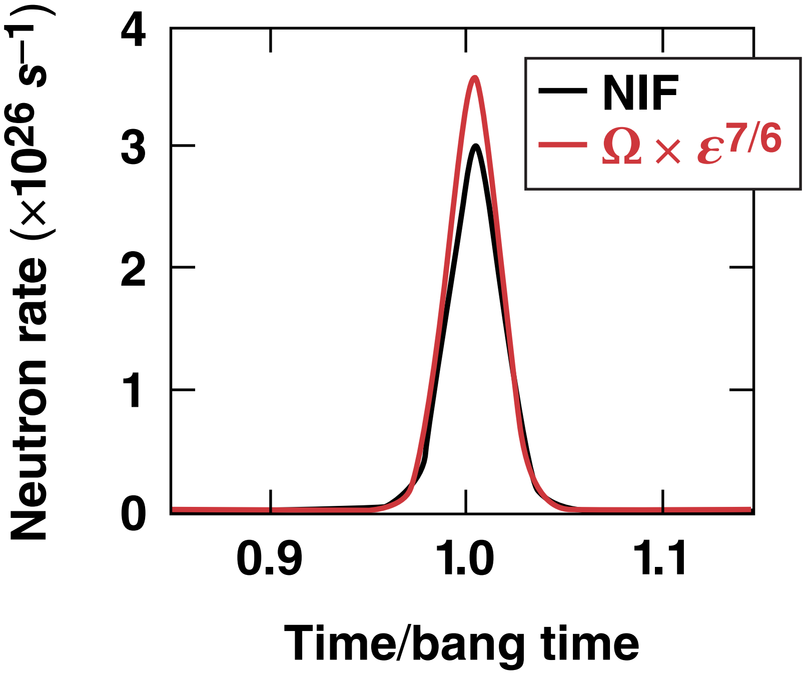
$$\varepsilon \equiv \frac{E_{\text{NIF}}}{E_{\Omega}}$$

Comparison of scaled ρR



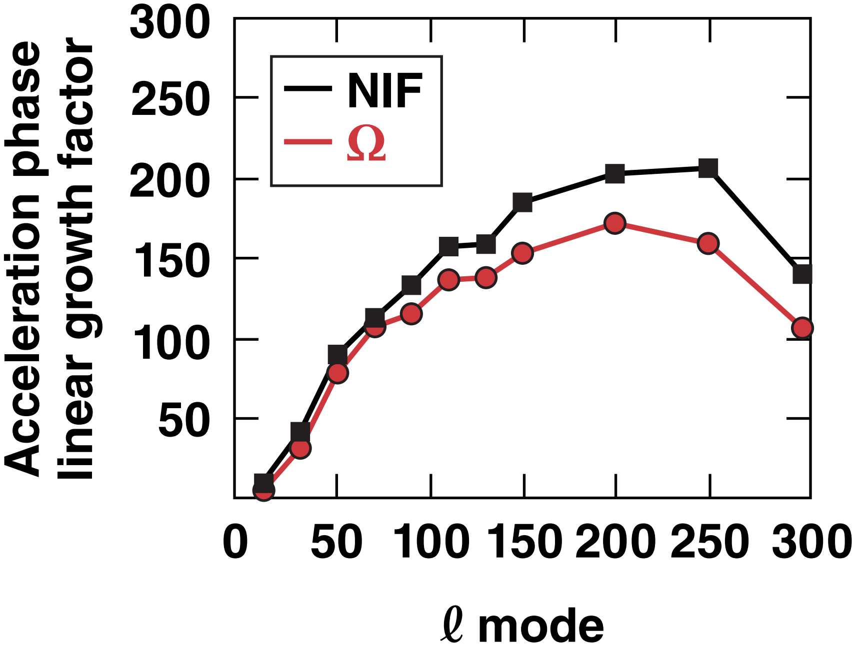
$\rho R \sim E^{1/3}$

Comparison of scaled $\dot{N}_n$

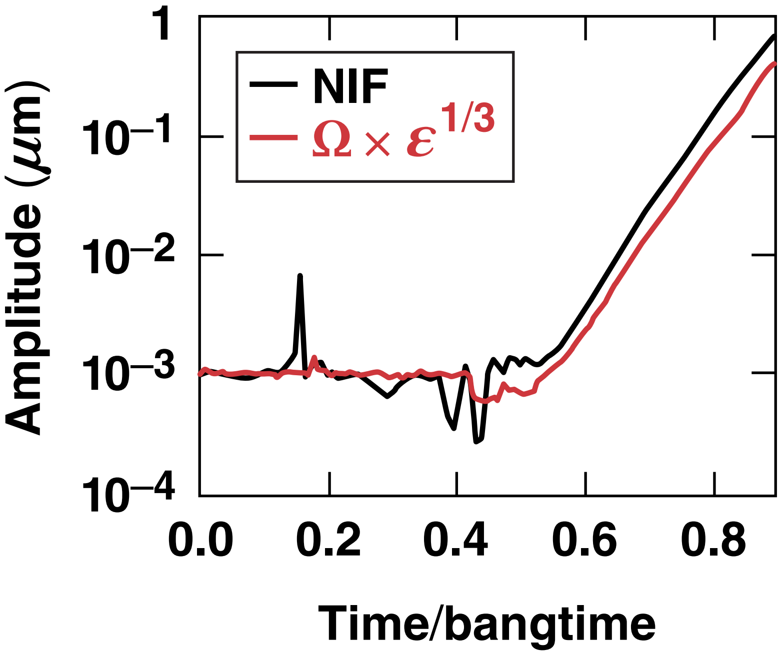


$\dot{N}_n \sim E^{7/6}$

Two-dimensional *DRACO*[*] single-mode growth-factor simulations confirm that the acceleration-phase RT scales hydrodynamically



Time-dependent evolution of single-mode RT growth ($\ell = 130$)



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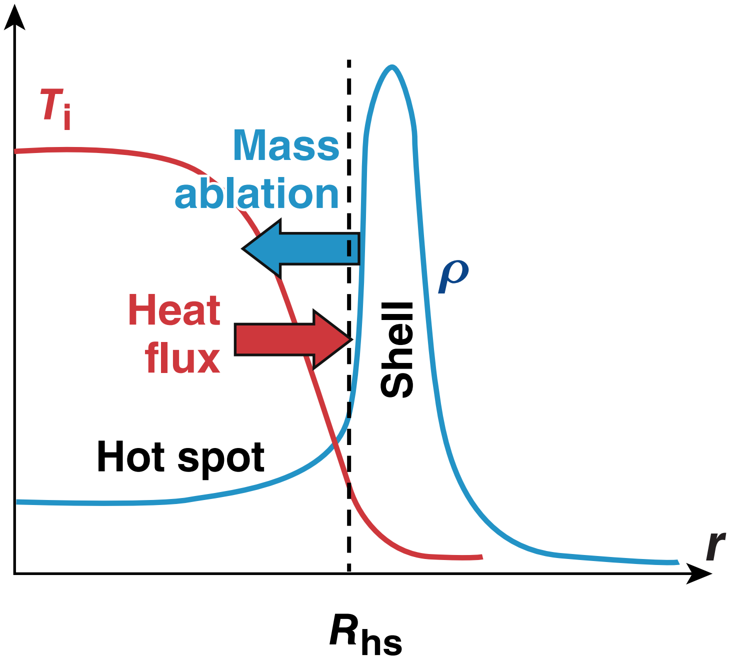
[*] P. B. Radha *et al*., Phys. Plasmas 12, 032702 (2005).

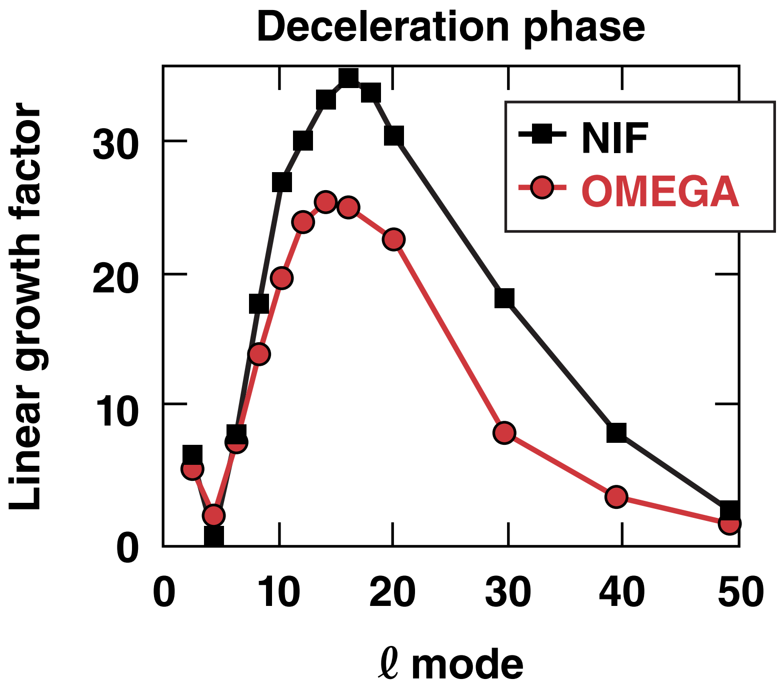
RT growth in the *deceleration* phase of inertial confinement fustion (ICF) implosions does not scale hydro-equivalently*

- Larger targets are more unstable during the deceleration phase (less ablative stabilization)

$$\gamma_{\mathrm{RT}}^{**} = 0.9\sqrt{\frac{k\langle g\rangle}{1+k\langle L_m\rangle}} - 1.4\,k\langle V_{\mathrm{a}}\rangle$$

$$\langle V_{\mathrm{a}}\rangle \sim \frac{T_0^{5/2}}{R_{\mathrm{hs}}\rho_{\mathrm{sh}}} \longrightarrow \langle V_{\mathrm{a}}\rangle \sim E^{-1/6}$$




Deceleration phase

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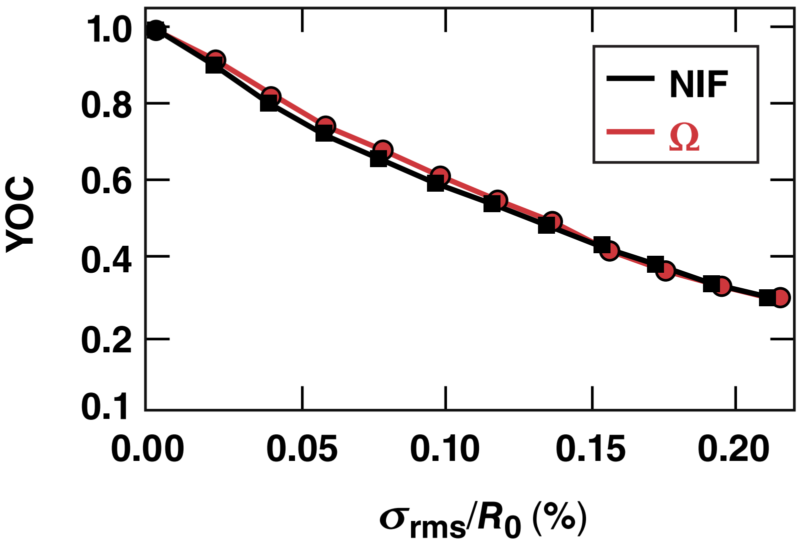
*See A. R. Bose *et al*., YO4.00008, this conference.

**V. Lobatchev and R. Betti, Phys. Rev. Lett. 85, 4522 (2000).

Two-dimensional simulations show that slight differences in the deceleration-phase of hydro-equivalent implosions have little impact on the yield-over-clean (YOC)

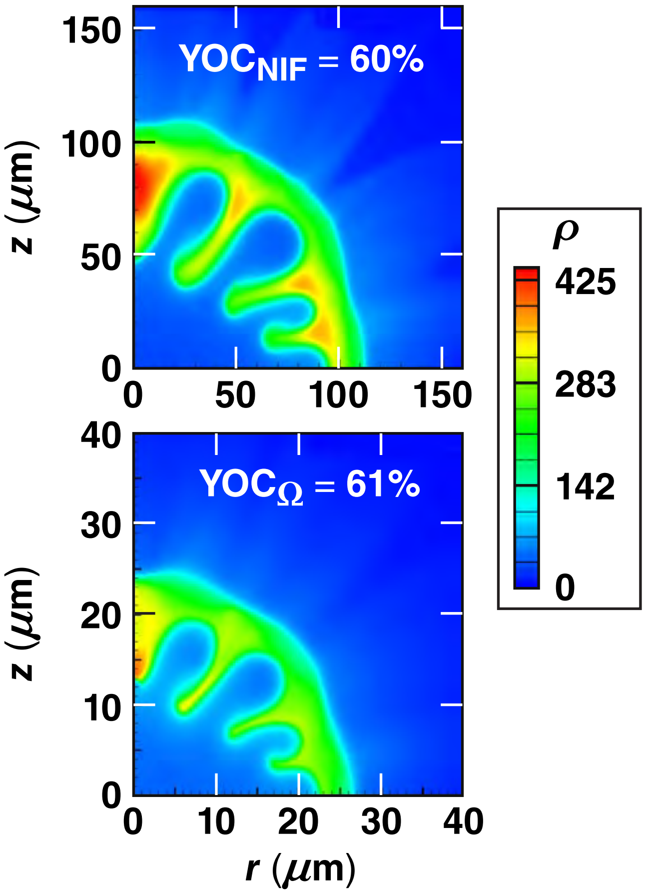
$$\text{YOC} \equiv \frac{Y_n\,(\text{3-D})}{Y_n\,(\text{1-D})}$$

Ice spectrum 10 < ℓ < 50



Density profiles shown at time of peak neutron rate

σ_{rms}/R_0 (%) = 0.098



- Similar results are obtained for dominant low-ℓ modes
- Differences in YOC are expected at implosion velocities >400 km/s

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See A. R. Bose *et al*., YO4.00008, this conference.

Hydro-equivalence provides a means of comparison for implosions at different energies

- **1-D hydro**
  - **identical: V_i, I_L, α, IFAR**
  - **energetically scaled: target geometry, performance metrics (ρR, yield, etc…)**
- **3-D hydro**
  - **similar acceleration-phase RT growth**
  - **negligible difference in the deceleration-phase RT growth**

The OMEGA and NIF-scale designs presented are hydro-equivalent.

Outline

- Theory of hydrodynamic equivalence
- OMEGA and NIF hydro-equivalent designs
- **Hydro-equivalent ignition scaling from OMEGA to the NIF**

The generalized Lawson criterion scales as $\text{YOC}^{0.4}$

$$\chi \equiv \frac{P\tau}{P\tau(T)_{\text{ig}}} \approx \rho R_{\text{g/cm}^2}^{0.61} \left[\frac{0.24\,Y_{16}}{m_{\text{mg}}^{\text{DT}}}\right]^{0.34} \text{YOC}^{0.06}$$

- χ has been tuned such that when $\chi = 1$, gain = 1
- All hydrodynamic quantities are calculated without alpha-particle deposition
- The YOC is used as a measure of the impact of 3-D nonuniformities so that

$$Y_n = \text{YOC} \times Y_n^{\text{1-D}} \longrightarrow \chi \sim \text{YOC}^{0.4}$$

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R. Betti *et al*., Phys. Plasmas 17, 058102 (2010);
P. Y. Chang *et al*., Phys. Rev. Lett. 104, 135002 (2010).

The Lawson parameter is scaled from OMEGA to the NIF using the hydrodynamic-equivalence scaling laws

$$\chi \sim \rho R_{\mathrm{g/cm^2}}^{0.61} \left[\frac{0.24\, Y_{16}^{\mathrm{1\text{-}D}}}{m_{\mathrm{mg}}^{\mathrm{DT}}} \right]^{0.34} \mathrm{YOC}^{0.4}$$

$$\Downarrow$$

Hydrodynamic scaling relations

$$\rho R_{\mathrm{g/cm^2}} \sim E^{1/3},\ Y_{\mathrm{1\text{-}D}} \sim E^{3/2},\ m_{\mathrm{mg}}^{\mathrm{DT}} \sim E$$

$$\Downarrow$$

$$\chi \sim E^{0.37}\ \mathrm{YOC}^{0.4}$$

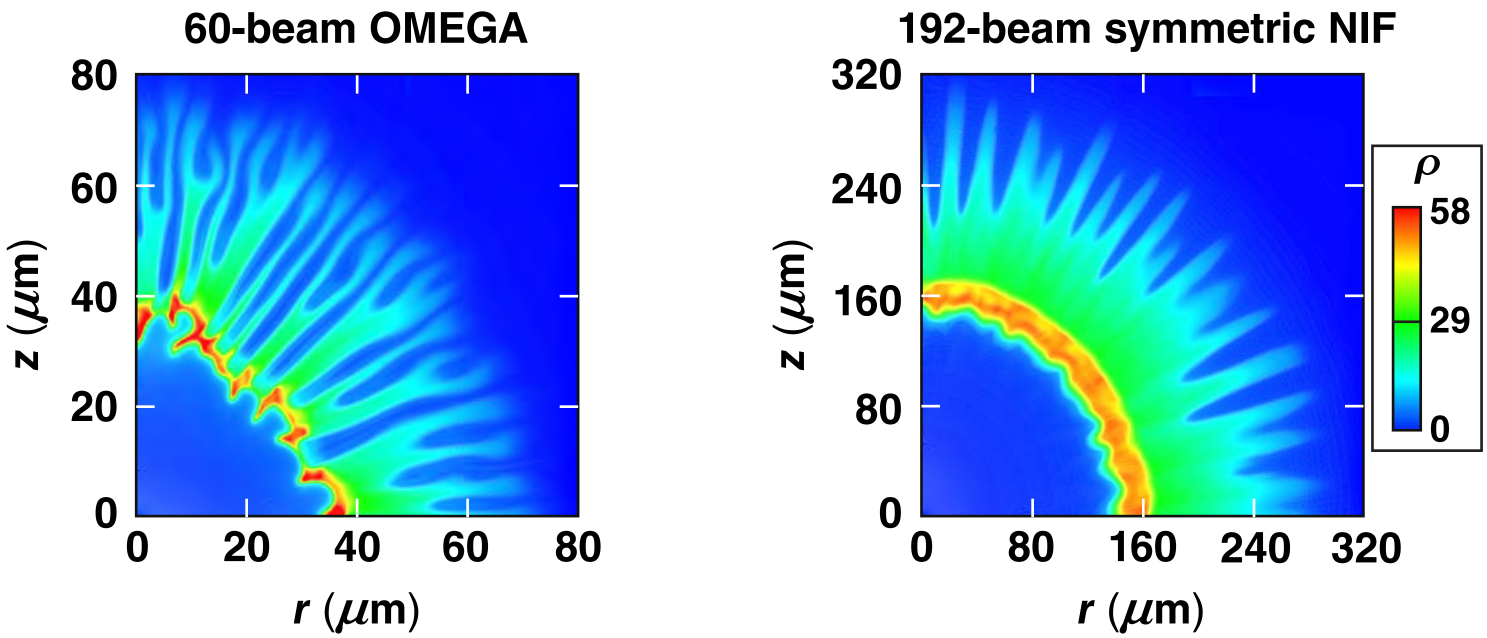
$$\Downarrow$$

$$\boxed{\chi^{\mathrm{NIF}} = \chi^{\Omega} \left(\frac{E_{\mathrm{NIF}}}{E_{\Omega}} \right)^{0.37} \left(\frac{\mathrm{YOC}_{\mathrm{NIF}}}{\mathrm{YOC}_{\Omega}} \right)^{0.4}}$$

Two-dimensional multimode ice and imprinting simulations are used to determine the YOC scaling ratio from OMEGA to the NIF

2-D smoothing by spectral dispersion (SSD),
Imprint spectrum $2 < \ell < 100$,
1-μm-rms ice roughness

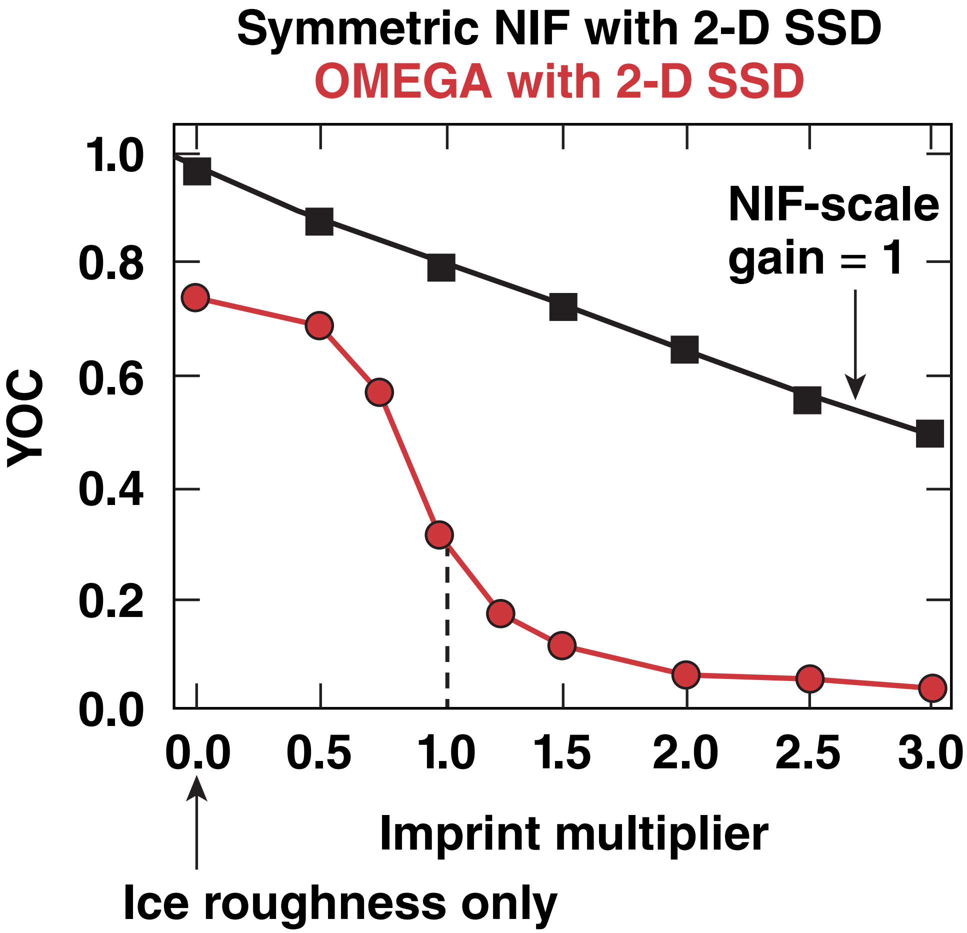
Density profiles shown at the end of the acceleration phase



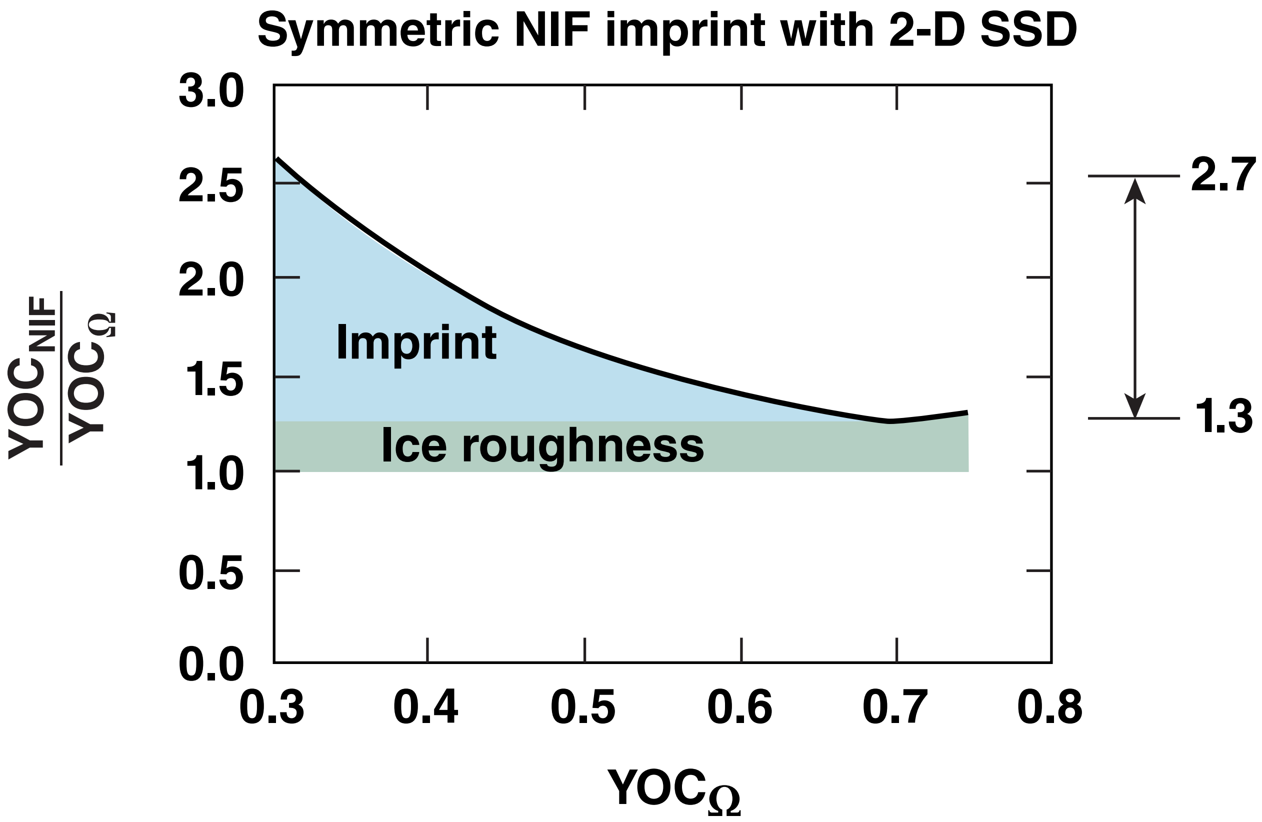
$\text{YOC}_{\Omega} = 0.5$

$\text{YOC}_{\text{NIF}} = 0.8$

Varying the imprint amplitude shows the YOC ratio is not unique to all implosions

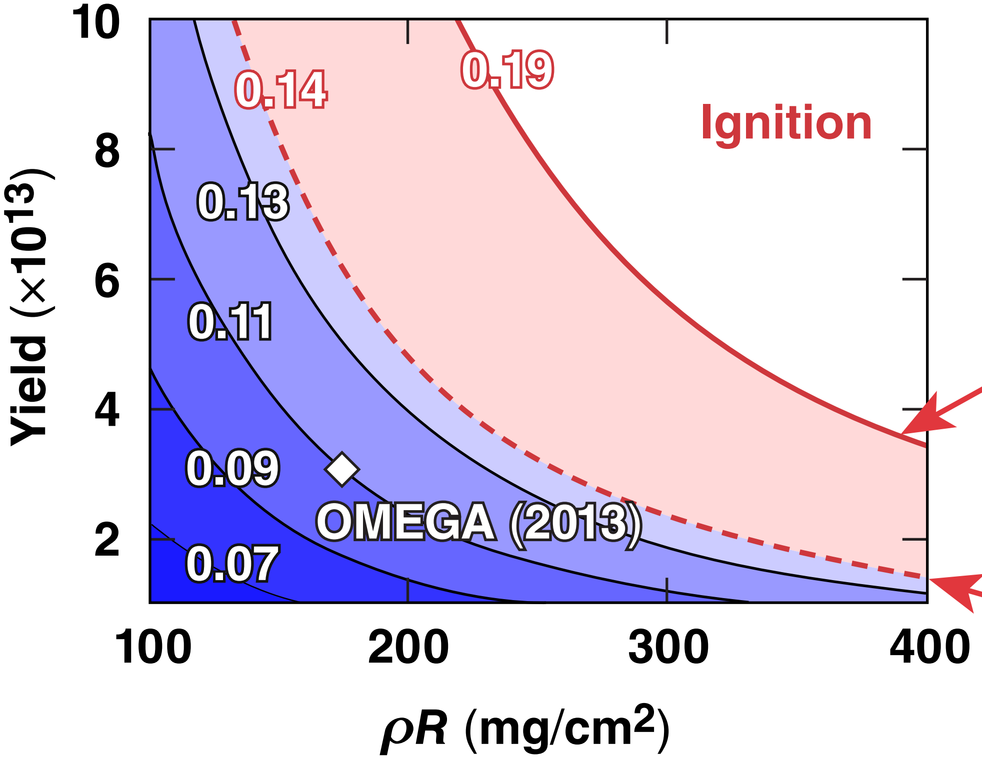


The YOC ratio is between 1.3 and 2.7 depending on the relative level of imprinting



$$\boxed{1.3 \leq \frac{YOC_{NIF}}{YOC_{\Omega}} \leq 2.7}$$

Areal densities and neutron yields required for hydro-equivalent ignition on OMEGA follow the 3-D Lawson criterion



$$\chi \equiv \frac{P\tau}{P\tau_{ig}}$$

Ice only

$$\frac{YOC_{NIF}}{YOC_{\Omega}} = 1.3$$

Ice and max imprint

$$\frac{YOC_{NIF}}{YOC_{\Omega}} = 2.7$$

Hydro-equivalent ignition threshold

$$0.6 \leq \left(\frac{\rho R_{g/cm^2}}{0.3}\right)^{1.8} \left(\frac{Y_{3\text{-}D}}{4 \times 10^{13}}\right) \leq 1.4$$

Hydro-equivalently scaling the Lawson criterion provides a guiding path for future OMEGA implosions

	Yield ($\times 10^{13}$)	Areal density (mg/cm^2)	$\langle P \rangle$ (Gbar)	χ_Ω
OMEGA's current record (shot 69514)	3.0	173	32*	0.11
Hydro-equivalent ignition (2.7 × YOC improvement)	3.0	270	60	0.14
Hydro-equivalent ignition (1.3 × YOC improvement)	6.0	300	100	0.19

An OMEGA implosion with a $\rho R \approx 300$ mg/cm^2 and neutron yield of 3 to 6 $\times 10^{13}$ extrapolates to hydro-equivalent ignition on a symmetric NIF-scale target.

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*See R. Epstein *et al.*, NO4.00010, this conference.

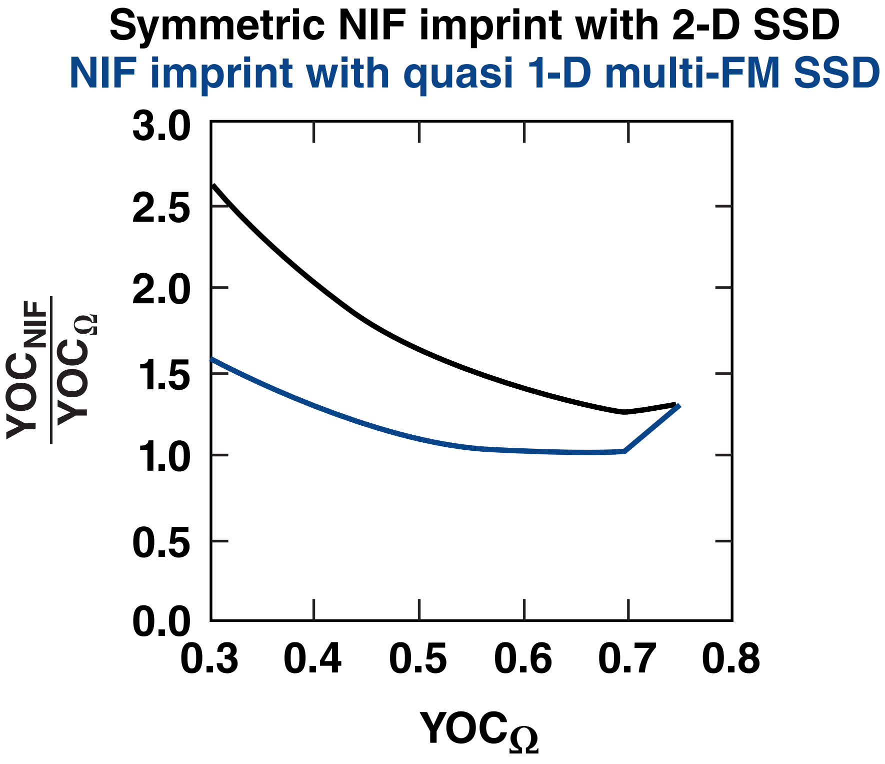
Summary/Conclusions

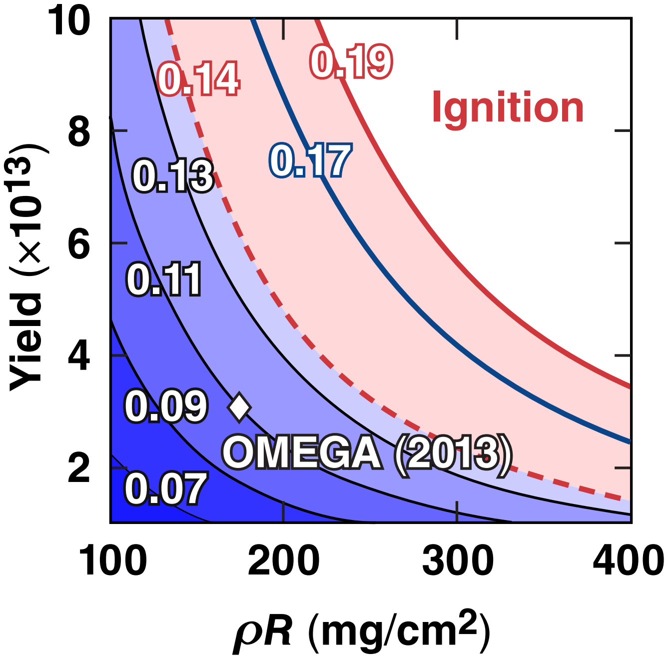
Hydrodynamic equivalence and ignition theory allow for the comparison of OMEGA implosions to ignition-scale targets

- Hydrodynamically equivalent implosions are energetically scalable and have identical implosion velocities, laser intensities, and adiabats
- The measurable Lawson criterion can assess the performance of an implosion using experimental observables
- An OMEGA implosion with an areal density of 300 mg/cm^2 and neutron yield of 3 to 6 $\times$ 10^{13} would ignite on a hydrodynamically equivalent symmetric National Ignition Facility (NIF)-scale target (depending on the level of imprinting)

NIF imprint simulations with quasi 1-D multi-FM* smoothing can also be used to extrapolate the hydro-equivalence ignition threshold

- 1-D multi-FM has 3× more initial imprint modulation when compared to 2-D SSD**





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*See A. Shvydky *et al.*, YO4.00006;
and M. Hohenberger *et al.*, YO4.00007 this conference.
**Courtesy J. A. Marozas, private communication.

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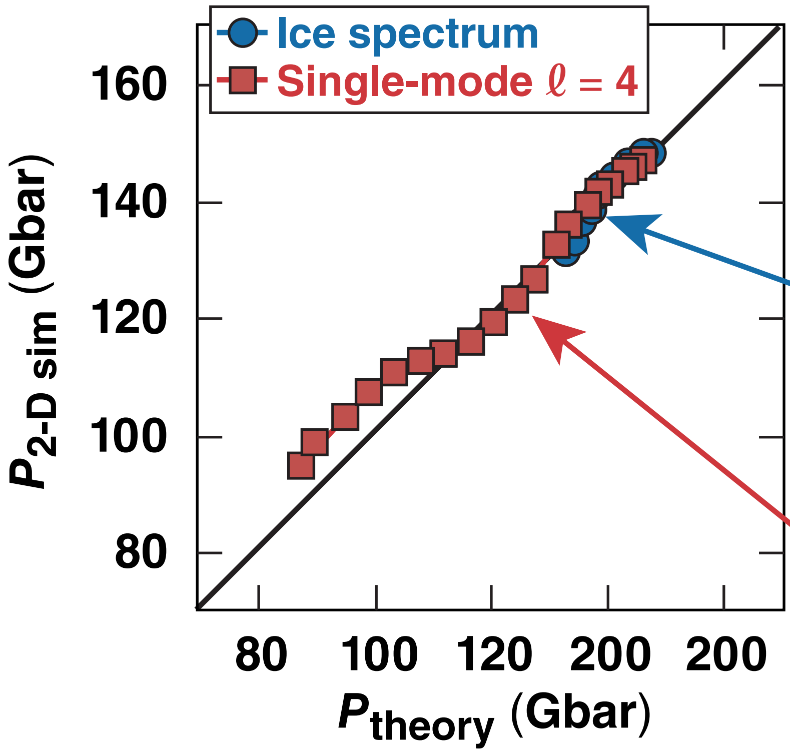
The pressure required for hydro-equivalent ignition can be inferred from the Lawson criterion

$$P(\text{Gbar}) \approx \frac{27}{\tau_{\text{burn}}^{\text{ns}}} \left\langle \rho R_{\text{g/cm}^2} \right\rangle_{\text{n}}^{0.61} \left(\frac{0.24\, Y_{\text{n}}^{16}}{M_{\text{mg}}^{\text{DT}}}\right)^{0.34} \left(\frac{4.7}{\langle T \rangle_{\text{n}}^{\text{keV}}}\right)^{0.8} \text{YOC}^{0.06}$$



- **Benchmark of pressure formula with 2-D simulations**

Ice spectrum
Gain = 1.34
No burn $P(0)$ = 133 Gbar
No burn $\langle P \rangle$ = 100 Gbar

I = 4
Gain = 1.04
No burn $P(0)$ = 123 Gbar
No burn $\langle P \rangle$ = 97 Gbar

About 130-Gbar central pressure (~100-Gbar average pressure) is required for hydro-equivalent ignition at V_{imp} ~ 350 km/s, α = 2.