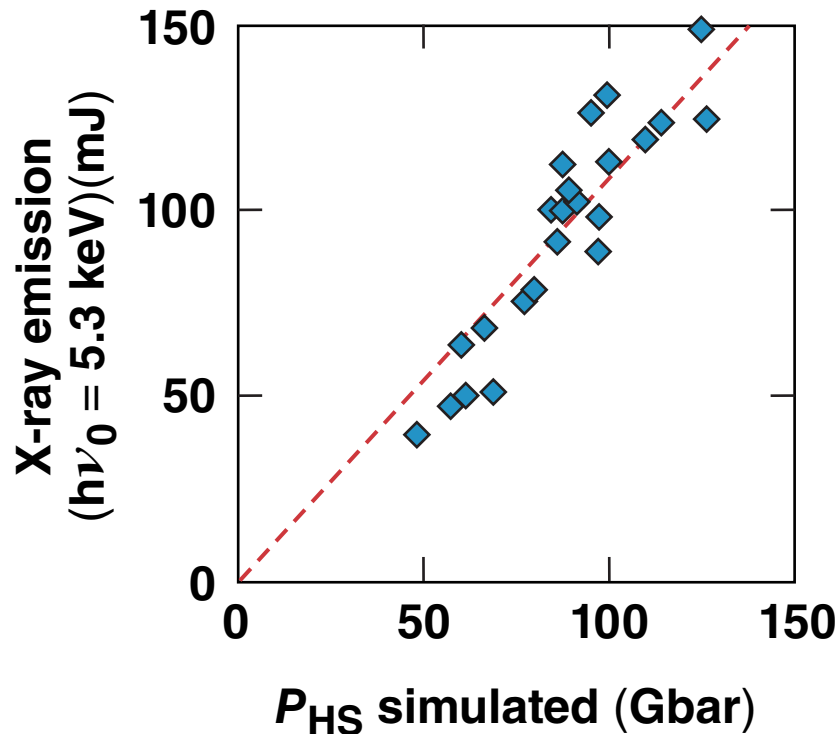


A Pressure Diagnostic Based on X-Ray Continuum Images of Compressed Isobaric Hydrogen Implosion Cores



Simulations of OMEGA cryo implosions



- Simulations represent the ranges $1.5 < \alpha_{\text{shell}} < 3.5$ and $1.3 \times 10^{13} < Y_n < 1.3 \times 10^{14}$

R. Epstein
University of Rochester
Laboratory for Laser Energetics

55th Annual Meeting of the
American Physical Society
Division of Plasma Physics
Denver, CO
11–15 November 2013

Summary

The x-ray emission of imploded cryogenic hot spots provides a diagnostic of the hot-spot pressure



- The x-ray emissivity of an imploded hydrogen hot spot, measured with an appropriate spectral response, scales as the square of the pressure and is nearly independent of temperature
- The hot-spot pressure profiles are directly related to the hot-spot emissivity profiles obtained from time-resolved implosion images
- The hot-spot stagnation pressure can be inferred from the total photon yield, image size, and neutron yield/rate measurements

Collaborators



F. J. Marshall and V. N. Goncharov*

**University of Rochester
Laboratory for Laser Energetics**

R. Betti, A. R. Christopherson, and R. Nora**

**University of Rochester
Fusion Science Center and Laboratory for Laser Energetics**

I. E. Golovkin and J. J. MacFarlane

Prism Computational Sciences

*V. N. Goncharov, G13.00001, this conference (invited).

**R. Nora, G13.00002, this conference (invited).

Measurements of x-ray emission from emissivity that is a function of pressure alone is potentially very important



- The hot-spot pressure, through the Lawson criterion,^{*} is a key measure of near-ignition implosion performance
- Pressures inferred from x-ray emission depend weakly on prior assumptions, such as temperature profiles, normalization to simulations, etc.
- Photon yield measurements of electron thermal parameters complement existing neutron yield diagnostics of ion thermal parameters
- The required spectral responses have been implemented on an existing instrument

^{*}J. D. Lawson, Proc. Phys. Soc. Lond. B 70, 6 (1957);
R. Betti *et al.*, Phys. Plasmas 17, 058102 (2010).

Hydrogen free-free (FF) emission is described accurately by simple pressure and temperature scaling



- The Kramers “free-free” emissivity expression

$$\epsilon_{\nu}^{\text{FF}} = \frac{32}{3} \sqrt{\frac{\pi}{3}} \chi_{\text{H}} (\alpha a_0)^3 \langle Z^2 \rangle n_e n_i \left(\frac{\chi_{\text{H}}}{kT} \right)^{1/2} g_{\text{FF}} e^{-h\nu/kT}$$

- The Kulsrud* approximation for the Gaunt factor applies at high energy and high temperature

$$g_{\text{FF}} \approx \frac{2\sqrt{3}}{\pi} \beta \left(\frac{kT}{h\nu} \right)^{1/2}, \quad \beta = 0.87, \quad h\nu > kT, \quad kT \gg \chi_{\text{H}}$$

- For ideal gas: $P = (n_e + n_i)kT$

$$\epsilon_{\nu}^{\text{FF}} = \text{const} \times P^2 (h\nu)^{-1/2} \times \frac{e^{-h\nu/kT}}{T^2}$$

*R. M. Kulsrud, *Astrophys. J.* **119**, 386 (1954).

W. J. Karzas and R. Latter, *Astrophys. J. Suppl. Ser.* **6, 167 (1961).

The emissivity is almost entirely a function of pressure when measured with an appropriate spectral response



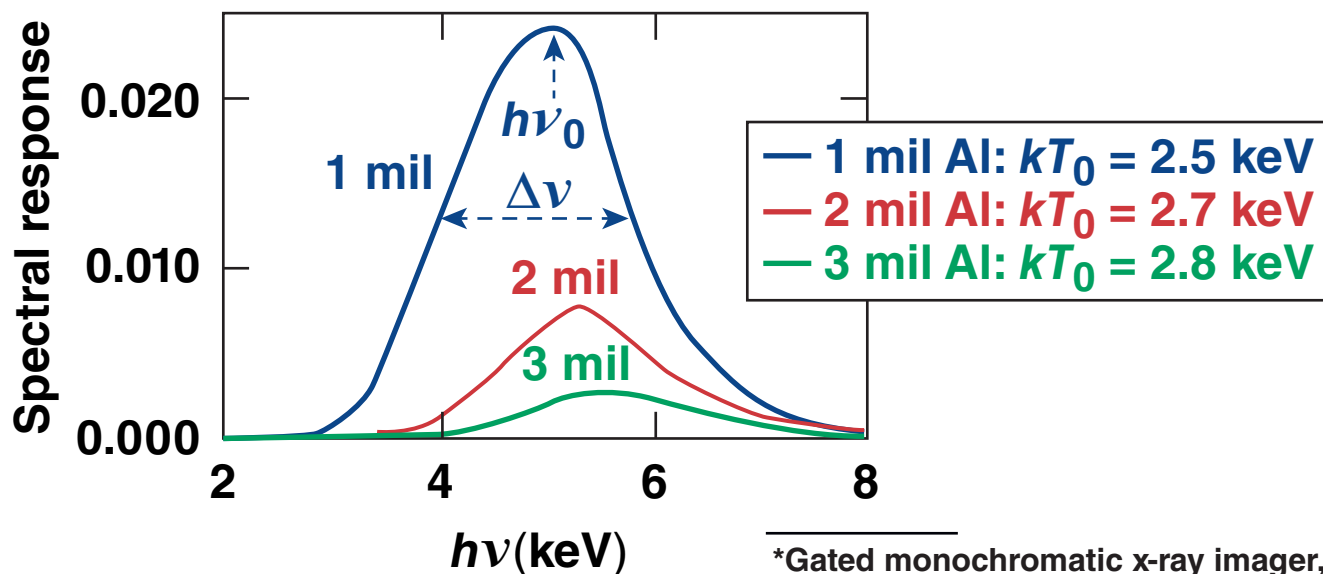
- Integrate the FF emissivity over a width $\Delta\nu$ at $\nu = \nu_0$

$$E_{\nu_0} = \int \varepsilon_{\nu}^{\text{FF}} F(\nu) d\nu = \text{const} \times P^2 \times (h\nu_0)^{-1/2} \Delta\nu \times \frac{e^{-h\nu_0/kT}}{T^2}$$

- The criterion for temperature independence is $\frac{d}{dT} \left(\frac{e^{-h\nu_0/kT}}{T^2} \right) = 0$

or $h\nu_0 = 2kT$

GMXI* filtered responses $F(\nu)$



*Gated monochromatic x-ray imager,
F. J. Marshall and J. A. Oertel, Rev. Sci. Instrum. **68**, 735 (1997);
F. J. Marshall et al., Phys. Rev. E **49**, 4381 (1994).

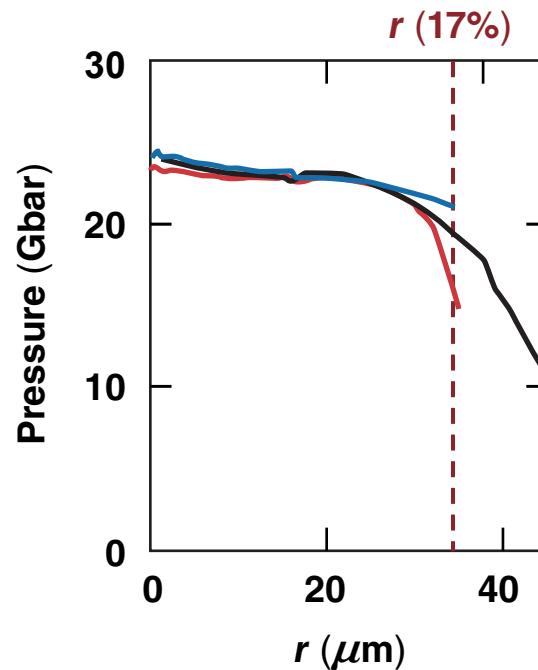
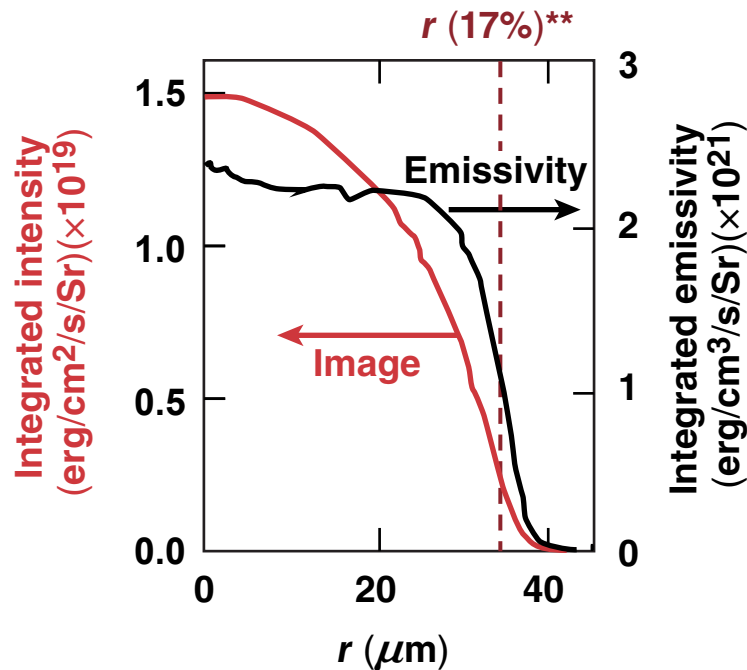
A simulated time-resolved image provides an emissivity profile that accurately measures the hot-spot pressure



- Emissivity profile from an Abel-inverted LILAC/Spect3D* image at 5.3 keV
- Pressure profiles obtained from the emissivity profile

$$E_{\nu_0} = \text{const} \times P^2$$

Shot 68791 at stagnation



Simulated pressure:
 — LILAC

Inferred pressure:
 — $kT = 2.65 \text{ keV}$
 $= h\nu_0/2$
 — $kT \text{ (LILAC)}$

*Prism Computational Sciences, Inc., Madison, WI 53711.
 G. A. Kyrala et al., Rev. Sci. Instrum. **81, 10E316 (2010);
 M. A. Barrios et al., Phys. Plasmas **20**, 072706 (2013).

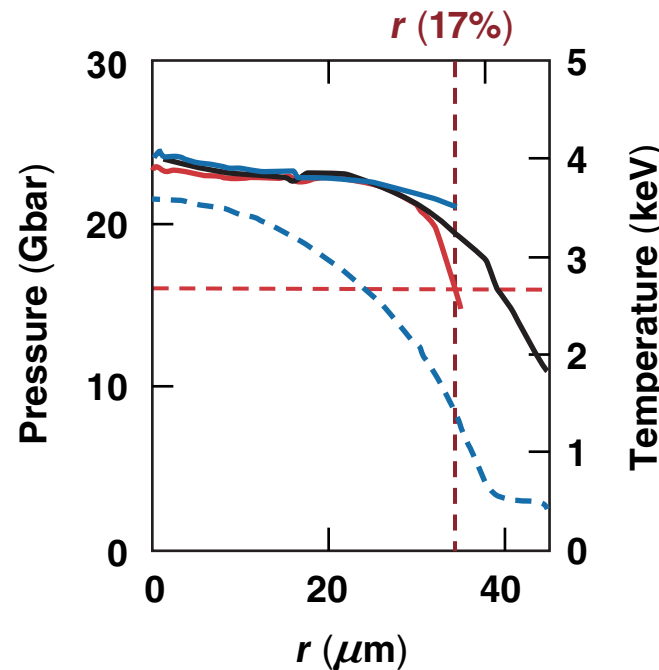
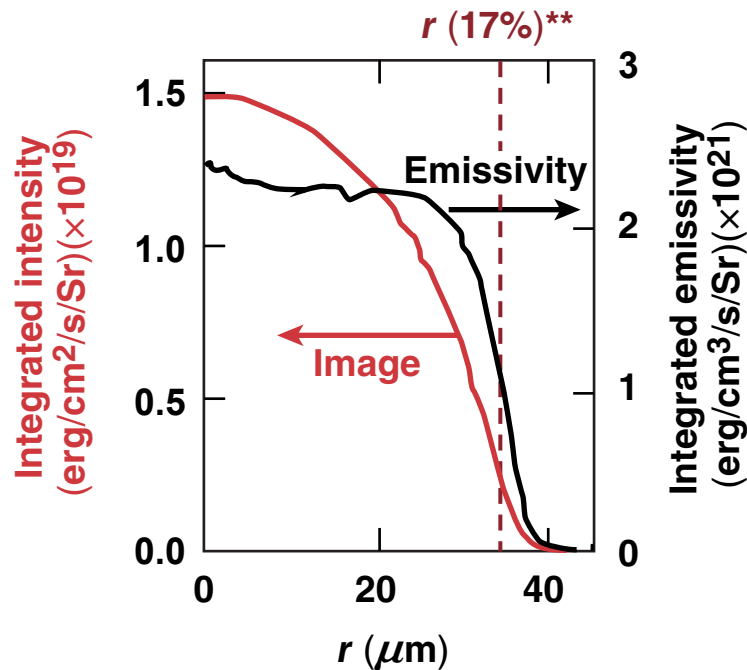
A simulated time-resolved image provides an emissivity profile that accurately measures the hot-spot pressure



- Emissivity profile from an Abel-inverted LILAC/Spect3D* image at 5.3 keV
- Pressure profiles obtained from the emissivity profile

$$E_{\nu_0} = \text{const} \times P^2$$

Shot 68791 at stagnation



Simulated pressure:
 — LILAC

Inferred pressure:
 — $kT = 2.65 \text{ keV}$
 $= h\nu_0/2$
 — kT (LILAC)

Temperature:
 - - - $kT = 2.65 \text{ keV}$
 - - - kT (LILAC)

*Prism Computational Sciences, Inc., Madison, WI 53711.
 G. A. Kyrala et al., Rev. Sci. Instrum. **81, 10E316 (2010);
 M. A. Barrios et al., Phys. Plasmas **20**, 072706 (2013).

The time-integrated x-ray yield can be used to infer the hot-spot pressure



- Photon yield infers a hot-spot pressure P using

$$Y_{\nu_0} = \iint E_{\nu_0}(r) dV dt = \text{const} \times \langle P^2 \rangle_t \times Vt$$

- The inertial force of shell deceleration balanced by core pressure at the time of stagnation gives* $M_{SH} R/t^2 = 4\pi R^2 P$

- Postulate a hot-spot adiabat** $P \propto \alpha_{HS} V^{-5/3}$

- Obtain $Vt \propto (\alpha_{HS} M_{SH})^{1/2} P^{-1}$

- Obtain the scaling expression $Y_{\nu_0} \propto (\alpha_{HS} M_{SH})^{1/2} P$

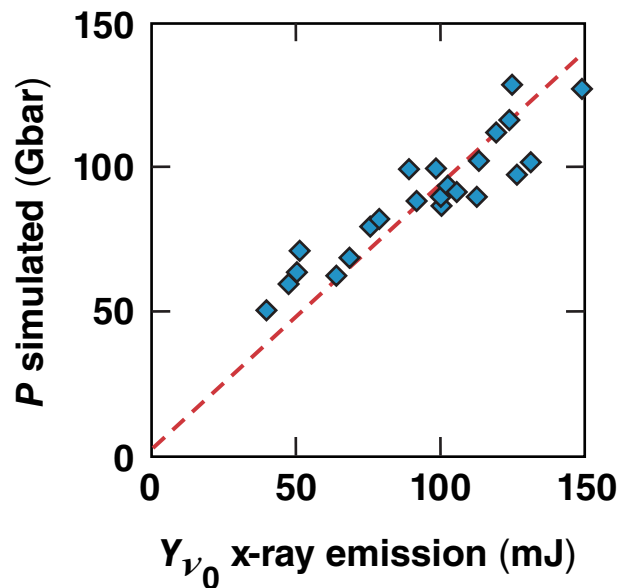
*R. Betti *et al.*, Phys. Plasmas 17, 058102 (2010).

**R. Betti *et al.*, Phys. Plasmas 9, 2277 (2002).

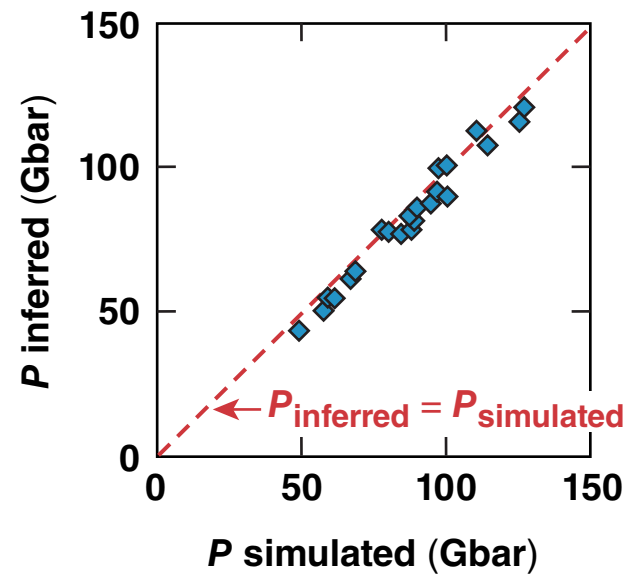
In a large sample of simulated cryo implosions, hot-spot pressure is correctly inferred from the simulated integrated x-ray emission



Using: $P \propto Y_{\nu_0}$, $\nu_0 = 5.3 \text{ keV}$
 Y_{ν_0} from *LILAC/Spect3D*
 P from *LILAC*



Using: $Y_{\nu_0} = \text{const} \times P^2 \times Vt$
 V from 17% image contour radius
 t from (neutron yield)/(neutron rate)



- Simulations represent the ranges $1.5 < \alpha_{\text{shell}} < 3.5$, $1.3 \times 10^{13} < Y_n < 1.3 \times 10^{14}$
- Excess x-ray emission would indicate ablator mix into the hot spot*

*T. Ma *et al.*, *Phys. Rev. Lett.* **111**, 085004 (2013);
 T. C. Sangster *et al.*, *Phys. Plasmas* **20**, 056317 (2013).

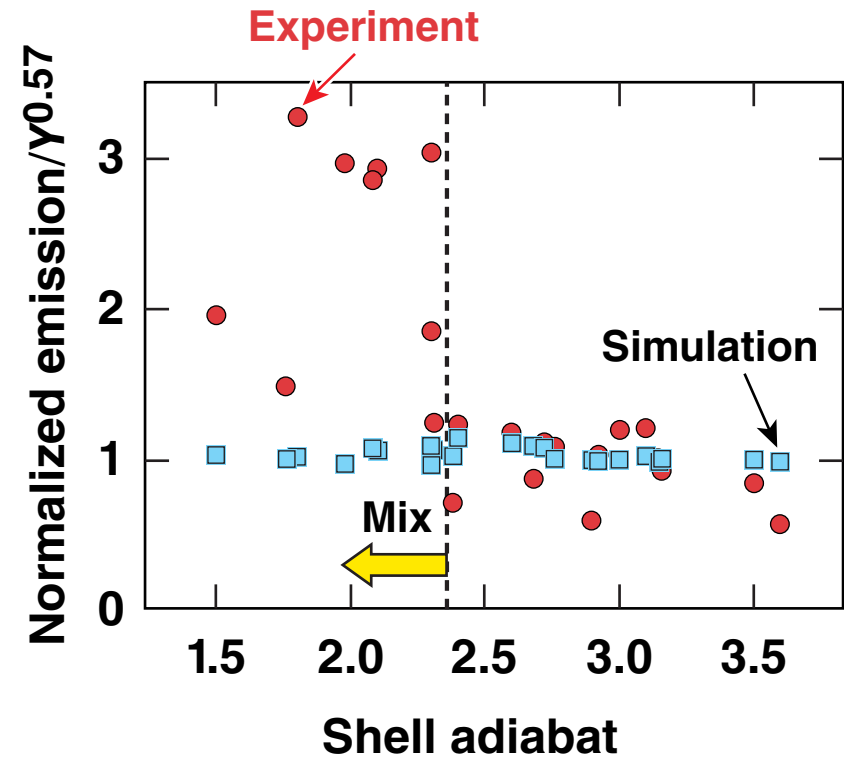
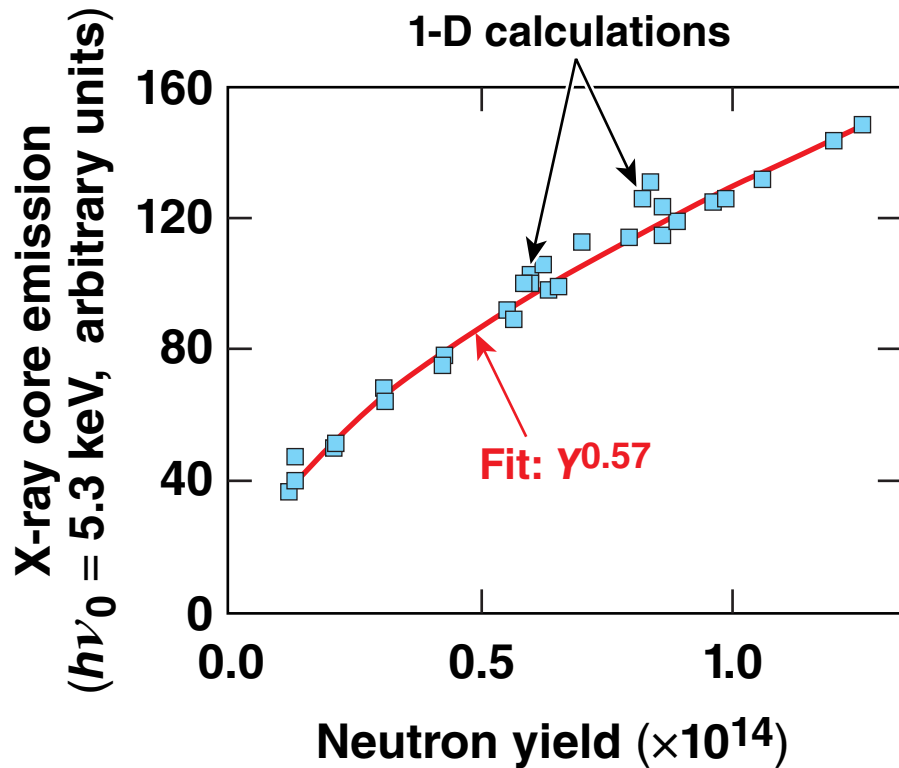
Summary/Conclusions

The x-ray emission of imploded cryogenic hot spots provides a diagnostic of the hot-spot pressure



- The x-ray emissivity of an imploded hydrogen hot spot, measured with an appropriate spectral response, scales as the square of the pressure and is nearly independent of temperature
- The hot-spot pressure profiles are directly related to the hot-spot emissivity profiles obtained from time-resolved implosion images
- The hot-spot stagnation pressure can be inferred from the total photon yield, image size, and neutron yield/rate measurements

Core x-ray emission exhibits simple scaling with neutron yield in cryogenic implosion simulations*



- Excess x-ray emission for low adiabats suggests ablator mix into the hot spot
- Measured yields are consistent with $Y_{\nu} \propto Y_n^{0.57}$ scaling for higher adiabats

The neutron yield scaling of photon yield is obtained for the isentropic compression of isobaric hot spots



- Neutron yield from the Bosch–Hale* reaction rate:

$$Y_n = \iint n_D n_T \langle \sigma v \rangle dV dt \propto P^2 T^{2+\varepsilon} Vt \quad \langle \sigma v \rangle \propto T^{4+\varepsilon}$$

- Photon yield

$$Y_{\nu_0} = \iint E_{\nu_0}^{FF}(r) dV dT \propto P^2 T^\eta Vt \quad e^{-h\nu_0/kT} / T^2 \propto T^\eta$$

- The inertial force of shell deceleration is balanced by core pressure at the time of stagnation gives**

$$Vt \propto \alpha_{HS}^{1/2} M_{SH} P^{-1}$$

- Postulate a hot-spot adiabat $P \propto \alpha_{HS} V^{5/3}$

- Obtain scaling $Y_{\nu_0} \propto (Y_n)^q$, where $q = \frac{5+2\eta}{9+2\varepsilon} \approx 0.56$

*H.-S. Bosch and G. M. Hale, Nucl. Fusion **32**, 611 (1992).

R. Betti *et al.*, Phys. Plasmas **17, 058102 (2010).

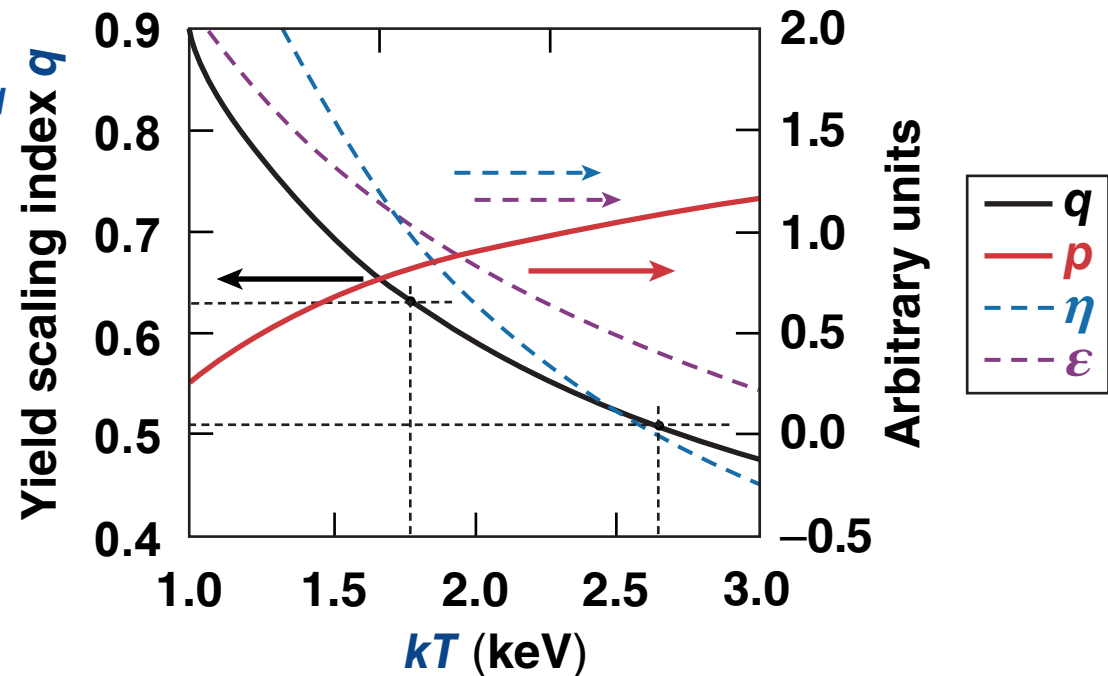
The yield scaling index obtained from scaling arguments agrees with 1-D *LILAC* results over a broad temperature range



$$Y_{\nu_0} \propto \left[\frac{(M_{SH})^{2/9} (M_{HS})^{10/9}}{(\alpha_{HS})^{4/9}} \right]^p (Y_n)^q$$

$$q = \frac{5 + 2\eta}{9 + 2\varepsilon} \approx 0.56$$

$$p = \frac{1 + (\varepsilon - \eta)/2}{1 + 2\varepsilon/9} \approx 1.0$$



- The scaling $q = 0.57 \pm 0.06$ is obtained for $kT = 2.21 \pm 0.43$ keV and is to be compared with $q = 0.57$ from *LILAC* results

Hot-spot photon and neutron yields scale with parameters specific to the particular implosions



- Neutron and photon yields

$$Y_n \propto PT^{2+\varepsilon} (\alpha_{HS})^{1/2} (M_{SH})^{1/2}$$

$$Y_{\nu_0} \propto PT^\eta (\alpha_{HS})^{1/2} (M_{SH})^{1/2}$$

- Postulate a hot-spot adiabat $P \propto \alpha_{HS} V^{-5/3}$

- Obtain scaling $Y_{\nu_0} \propto \left[\frac{(M_{SH})^{2/9} (M_{HS})^{10/9}}{(\alpha_{HS})^{4/9}} \right]^p (Y_n)^q$, where

$$p = \frac{1 + (\varepsilon - \eta)/2}{1 + 2\varepsilon/9} \approx 1.0$$

$$q = \frac{5 + 2\eta}{9 + 2\varepsilon} \approx 0.56$$

Shell mass, hot-spot mass, and adiabat variations obscure the hot-spot neutron and photon yield scaling.

Excess hot-spot x-ray emission above the expected clean DT level gives the mix fraction of shell C in the hot spot¹



- Free-free (FF) plus bound-free (BF) x-ray “yield”

$$Y_\nu \propto n_i n_e \left[\langle Z^2 \rangle_I \left(\frac{\chi_H}{kT} \right)^{1/2} + 2 \langle Z^4 e^{\chi/kT} \rangle_I \left(\frac{\chi_H}{kT} \right)^{3/2} \right] \langle e^{-h\nu/kT} \rangle_{\text{det}} Vt$$

- The composition is DT, $n_D = n_T$ with a trace $x = n_Z/(n_D + n_T)$ of a contaminant, e.g., C ($Z = 6$)

Using $n_e = (n_D + n_T)(1 + xZ)$ $n_i \langle Z^n \rangle = (n_D + n_T)(1 + xZ^n)$
and the neutron yield:

$$Y_n \approx n_D n_T \langle \sigma \nu \rangle Vt$$

Obtain

$$Y_\nu / Y_n \approx f(T)(1 + xZ)(1 + x j_Z / j_{DT})$$

The shell-mix fraction x is obtained¹ from Y_ν / Y_n ratio measurements and emissivity j_Z, j_{DT} values from OPAL² and DCA³ tables.

¹T. Ma *et al.*, Phys. Rev. Lett. **111**, 085004 (2013).

²F. J. Rogers, F. J. Swenson, and C. A. Iglesias, Astrophys. J. **456**, 902 (1996).

³H. A. Scott and S. B. Hansen, High Energy Density Phys. **6**, 39 (2010).