A Measurable Three-Dimensional Ignition Criterion for Inertial Confinement Fusion

P. Y. Chang
University of Rochester
Fusion Science Center
for Extreme States of Matter and
Laboratory for Laser Energetics

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An analytic model of the hot-spot evolution provides a measurable 3-D ignition criterion for ICF.

- The one-dimensional (1-D) measurable Lawson criterion of Zhou and Betti* is extended to 3-D using the Yield-Over-Clean (YOC) as a measure of the implosion uniformity.

- The ignition parameter from the analytic theory depends on areal density, ion temperature, and yield-over-clean.

- The analytic model is in reasonable agreement with a simulation database yielding.

- Cryogenic implosions on OMEGA have achieved an ignition parameter $\chi^\text{fit} \approx 0.008$. Hydro-equivalent ignition on OMEGA requires $\chi \approx 0.04$.

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Collaborators

R. Betti, K. S. Anderson, and R. Nora
Laboratory for Laser Energetics and Fusion Science Center
University of Rochester

B. Spears
Lawrence Livermore National Laboratory

M. Fatenejad
University of Wisconsin–Madison

D. Shvarts
Nuclear Research Center, Negev, Israel
The one-dimensional ignition criterion depends on the total areal density and ion temperature (without calculated $\alpha$-particle deposition).

\begin{equation}
\langle \rho R_{\text{g/cm}^2} \rangle_n \left( \frac{T_{\text{keV}}^{\text{no-}\alpha}}{4.5} \right)^2 > 1 \text{ for } 3.5 < T_{\text{keV}}^{\text{no-}\alpha} < 7
\end{equation}

The effects of nonuniformities in the deceleration phase are added to the ignition model through a clean volume analysis.

- Nonuniformities reduce the volume where fusion reactions occur:
  \[ V_{\text{fusion}} \approx V_{\text{clean}} = \frac{4\pi}{3} R_{\text{clean}}^3 \]

- The hot spot energy balance is affected by a reduced “clean” volume
  \[ \frac{d}{dt}(PR^3) = -2PR^2 \frac{dR}{dt} + \frac{\varepsilon_\alpha}{8\pi} \int_0^{V_{\text{clean}}} \langle \sigma v \rangle n^2 dv \]
  \[ \varepsilon_\alpha = 3.5 \text{ MeV} \]

- \[ \langle \sigma v \rangle \approx C_\alpha T^3 \] is valid for \( T \sim 4 \) to 8 keV

Fusion reactions occur only within the clean volume.

*R. Kishony and D. Shvarts, Phys. Plasmas 8, 4925 (2001).*
Zhou* and Betti’s model is modified to include the clean radius in the hot-spot energy balance

*The hot-spot formation and ignition model is governed by three ODE’s

- **Hot-spot energy balance**
  \[
  \frac{d}{dt} (PR^3) = -2pR^2 \frac{dR}{dt} + \frac{E_\alpha}{8} \alpha C P^2 R^3_{clean} T
  \]
  Compression/expansion
  \[\alpha\text{-particles deposition}\]

- **Temperature equation (from hot-spot mass conservation)**
  \[
  \frac{d}{dt} \left( \frac{PR^3}{T} \right) = 0.87 \kappa_0 RT^{5/2} + C_{rad} \frac{P^2 R^3}{T^{5/2}}
  \]
  Heat conduction
  \[\kappa_{sp} = \kappa_0 T^{5/2}\]
  Radiation
  Neglected in this talk

- **Shell Newton’s law**
  \[
  M_{sh} \frac{d^2 R}{dt^2} = 4\pi PR^2
  \]
  Shell mass

The yield-over-clean (YOC) is used as a measure of the implosion uniformity.

\[
YOC = \left[ \frac{\text{Yield}_{3-D}}{\text{Yield}_{1-D}} \right] \sim \frac{R_{\text{clean}}^3}{R_{1-D}^3}
\]

- Develop 3-D model
- Compute yield-over-clean without \(\alpha\)-particle energy deposition
- Relate ignition criterion to YOC
- Compute ignition criterion (with \(\alpha\)-particle deposition) \(\rho R, T_i\)

\[
\text{Ignition criterion } f\left(\rho R, T_i, \text{YOC}\right) = 0
\]
Multiple models are used to assess the sensitivity of the ignition conditions to the hot-spot model.

Model I
The RT becomes nonlinear at different times, the spikes free fall

- Hot spot
- $R_{\text{clean}}$ (free fall)
- Shell rebound
- $R_{1-D}$
- $R$ vs. Time
- $t_{n\ell}$, $R_{\text{1-D Shell rebound}}$, RT spikes

Model II
The spikes grow at different rates (i.e. different slopes)

- Hot spot
- $R_{\text{clean}}$ (free fall)
- Shell rebound
- $R_{1-D}$
- $R$ vs. Time
- $t_{n\ell}$, $R_{\text{1-D Shell rebound}}$, RT spikes

Mathematical Expressions:

- $YOC^{\text{no-\alpha}} = YOC^{\text{no-\alpha}} (t_{n\ell})$
- $t_{n\ell} = t_{n\ell} (YOC^{\text{no-\alpha}})$
- $YOC^{\text{no-\alpha}} = YOC^{\text{no-\alpha}} \text{ (slope)}$
- slope = slope (YOC$^{\text{no-\alpha}}$)
- $R_{\text{clean}} = R_{\text{clean}} (YOC^{\text{no-\alpha}})$
The ignition model is cast in a dimensionless form using stagnation properties calculated without $\alpha$-particle deposition.

- Ignition depends on two parameters: $\gamma_\alpha$ and $\text{YOC}^{\text{no-}\alpha}$

\[
\begin{align*}
\frac{d}{d\tau} \left( \hat{\rho} \hat{R}^3 \right) &= -2 \hat{\rho} \hat{R}^2 \frac{d\hat{R}}{d\tau} + \gamma_\alpha \hat{T} \hat{p}^2 \left[ \hat{R}_{\text{clean}} \left( \text{YOC}^{\text{no-}\alpha} \right) \right]^3 \\
\frac{d}{d\tau} \left( \hat{\rho} \frac{\hat{R}^3}{\hat{T}} \right) &= \hat{R}^5/2 \\
\frac{d^2 \hat{R}}{d\tau^2} &= \hat{\rho} \hat{R}^2 \\
\tau &= \frac{t V_i}{R_{\text{no-}\alpha}^{\text{stag}}} \\
\hat{R} &= \frac{R}{R_{\text{no-}\alpha}^{\text{stag}}} \\
\hat{\rho} &= \frac{\rho}{\rho_{\text{stag}}} \\
\hat{T} &= \frac{T}{T_{\text{stag}}} \\
\hat{p} &= \frac{P}{P_{\text{stag}}} \\
\end{align*}
\]

Stagnation properties calculated without $\alpha$'s

Ignition condition: for a fixed $\text{YOC}^{\text{no-}\alpha}$, find critical $\gamma_\alpha$ leading to singular solutions.

$\gamma_\alpha = 0.07 \left( \rho R \right)^{3/4} \left( T_{\text{no-}\alpha}^{\text{stag}} \right)^{15/18}$

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*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).*
The 3-D ignition condition is approximately independent of the hot-spot models.

Model independent ignition criterion: 

\[ \gamma_\alpha (YOC^{no-\alpha})^{4/5} > 1.2 \]

Ignition parameter: 

\[ \chi = \rho R_{tot}^{no-\alpha} \left( \frac{T^{no-\alpha}}{4.6} \right)^{5/2} (YOC^{no-\alpha})^{16/15} > 1 \]
The clean volume model is implemented in *LILAC* and is used to generate a database of gain curves. The ignition condition is tuned with the simulation database.

- In the 1-D code *LILAC*, $\langle \sigma \nu \rangle$ is replaced by $\langle \sigma \nu \rangle_{\text{YOC}} \approx \langle \sigma \nu \rangle \frac{V_{\text{clean}}}{V_{1-D}}$

Find exponents and constant ($\mu, \delta, T_0$) in ignition conditions to minimize the spread at $1/2 \ G_{1-D}$

$$\chi_{\text{fit}} = (\rho R)^{\alpha-\alpha} \left( \frac{T_{\text{no-}\alpha}}{T_0} \right)^{\mu} (\text{YOC}^{\alpha-\alpha})^\delta$$

**Ignition parameter:**

$$\chi_{\text{fit}} = (\rho R)^{\alpha-\alpha} \left( \frac{T_{\text{no-}\alpha}}{4.7} \right)^2 (\text{YOC}^{\alpha-\alpha})^{0.7} > 1$$

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K. Anderson (U05.00004).
Cryogenic implosions on OMEGA have achieved $\chi^{\text{fit}} \approx 0.008$; hydro-equivalent ignition on OMEGA requires $\chi \approx 0.04$

- For cryogenic implosions on OMEGA (have achieved)

$$\left( \rho R \right)_{\text{stag}}^{\text{no-}\alpha} \approx 0.2 \text{ g/cm}^2, \quad T_{\text{stag}}^{\text{no-}\alpha} \approx 2.1 \text{ keV}, \quad \text{YOC} = 0.1^*$$

$$\chi^{\text{fit}} = 0.008$$

- Hydro-equivalent ignition on OMEGA requires

$$\left( \rho R \right)_{\text{stag}}^{\text{no-}\alpha} \approx 0.3 \text{ g/cm}^2, \quad T_{\text{stag}}^{\text{no-}\alpha} \approx 3.4 \text{ keV}, \quad \text{YOC} = 0.15$$

$$\chi^{\text{fit}} = 0.04$$

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*T. C. Sangster (NI2.00002).

**R. Betti (PT3.00001).
An analytic model of the hot-spot evolution provides a measurable 3-D ignition criterion for ICF and is in good agreement with simulation results.

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