Measurable Lawson Criterion and Hydro-Equivalent Curves for Inertial Confinement Fusion

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49th Annual Meeting of the
American Physical Society
Division of Plasma Physics
Orlando, FL
12–16 November 2007
The ignition condition is derived in terms of two parameters ($\langle T_i \rangle_n$ and total $\langle \rho R \rangle_n$) that can be directly measured in ICF implosions.

- This new form of the Lawson’s criterion is used to determine how close to ignition OMEGA’s cryogenic implosions are currently performing.

- Hydro-equivalent curves in the $(\rho R, T_i)$ plane show how current OMEGA implosions perform when scaled up to the NIF.

- The conventional relation hot-spot $\rho R \sim 0.3$ g/cm$^2$, $T_i \sim 10$ keV does not correctly reproduce the ignition conditions.
The hot-spot ignition condition is given by the balance of alpha heating with the energy losses, including the expansion losses.

\[ \frac{\dot{E}_{\text{hs}}}{E_{\text{hs}}} = \frac{1}{\tau_{\alpha}} - \frac{1}{\tau_{\text{rad}}} - \frac{1}{\tau_{\text{exp}}} > 0 \]

\[ 1/\tau_{\alpha} \sim n_{\text{hs}}^2 \langle \sigma v \rangle / P_{\text{hs}} \quad \text{Alpha heating} \]

\[ 1/\tau_{\text{rad}} \sim n_{\text{hs}}^2 \sqrt{T_{\text{hs}}} / P_{\text{hs}} \quad \text{Radiation cooling} \]

\[ 1/\tau_{\text{exp}} \sim \sqrt{\dot{R}_{\text{hs}} / R_{\text{hs}}} \quad \text{Expansion} \]

\[ M_s \dot{R}_{\text{hs}} = 4\pi P_{\text{hs}} R_{\text{hs}}^2 \quad \text{shell Newton's law} \]
The expansion losses represent the internal energy lost by the hot spot and transferred to the surrounding dense shell as kinetic energy.

\[
\frac{1}{\tau_{\text{exp}}} \sim \sqrt{\frac{\ddot{R}_{\text{hs}}}{R_{\text{hs}}}} \quad \text{Expansion}
\]

\[
M_s \ddot{R}_{\text{hs}} = 4\pi P_{\text{hs}} R_{\text{hs}}^2 \quad \text{Shell Newton’s law}
\]

\[
M_s \sim \rho_s \Delta_s R_s^2 \quad \Delta_s = \text{shell thickness}
\]

\[
\frac{1}{\tau_{\text{exp}}} \sim \sqrt{\frac{P_{\text{hs}} R_{\text{hs}}}{M_s}} \quad \text{Shell mass}
\]
The ignition condition depends on shell areal density, implosion velocity, and hot-spot ion temperature.

Hot-spot pressure and temperature

\[ \frac{P_{hs}}{R_{hs}} M_s f(T_i)^2 > \text{const} \]

Hot-spot radius \quad Shell mass

- \( P_{hs} \sim \left( \frac{P_{hs} R_s^3}{R_s^3} \right) \sim \left( \frac{M_s V^2}{R_s^3} \right) \)
- \( M_s \sim \rho_s \Delta_s R_s^2 \)
- \( R_s \sim R_{hs} \)

\[ f(T) \sim T_i \left[ 1 - \left( \frac{T_{\text{brem}}}{T_i} \right)^{2.5} \right] \]

for \( 4 < T_i < 13 \) \( T_{\text{brem}} \leq 4.4 \) keV

Shell areal density \quad Implosion velocity
Eliminating the velocity and energy leads to an ignition condition depending on shell areal density and hot-spot ion temperature:

\[ V \sim T_i^{0.8} \alpha_{if}^{0.15} E_L^{0.056} \]

\[ E_L \sim (\rho_s \Delta_s)^3 \alpha_{if}^{1.75} V^{-0.18} \]

\[ (\rho \Delta) T_i^{2.1} \alpha_{if}^{0.03} \left[ 1 - (T_{brem}/T_i)^{2.5} \right]^{1.2} > \text{const} \]

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\(^1\text{C. Zhou and R. Betti, Phys. Plasmas 14, 072703 (2007).}\)
The simple scaling relation compares favorably with a set of simulations of marginally ignited capsules.

Scaling

$$(\rho_s \Delta_s) T_i^{2.1} \left[ 1 - \left( \frac{T_{\text{brem}}}{T_i} \right)^{2.5} \right]^{1.2} > \text{const}$$

1-D LILAC → simulations

Gain = 1

$$\langle \rho R \rangle_n \left( \frac{\langle T_i \rangle_n}{4} \right)^{1.4} \left[ 1 - \left( \frac{2.6}{\langle T_i \rangle_n} \right)^{2.5} \right]^{0.37} > 1 \text{ g/cm}^2$$

- Simulations are carried out for $1 < \alpha_{if} < 5$ and $2 \times 10^7 < V_j < 5.5 \times 10^7 \text{ cm/s.}$
The $T_i$ and total $\rho R$ from simulations of marginally ignited capsules lay on a single curve; a measurable Lawson’s criterion depends on burn average $T_i$ and total $\rho R$. 

The graph shows the relationship between the neutron-average ion temperature without alpha deposition $\langle T_i \rangle_n^{\text{no alpha}}$ and the neutron-average areal density $\langle \rho R \rangle_n$. The ignition domain is highlighted, with sub-ignited $T \approx T$ (no alpha) and ignition and gain $\langle T_i \rangle_n = 1$ indicated. The graph includes 1-D simulations data points.
Hydro-equivalent curves on the \((\rho R, T_i)\) show how close current OMEGA cryogenic implosions are to ignition and how they perform when scaled up to the NIF.

Using the scaling relations for \(\rho R\) and \(T_i\), hydro-equivalent curves are plotted versus laser energy for fixed values of the implosion velocity and in-flight adiabat.

\[
(\rho_s \Delta_s) \sim E_L^{0.33} \alpha_{if}^{-0.55} V^{0.06} \\
T_i \sim V^{1.25} \alpha_{if}^{-0.19} E_L^{0.07}
\]

- **Ignition and Gain**
- **NIF point design**
- **1-D marginal ignition**
- **OMEGA DD, 16 kJ 2007**
- \(\alpha_{if} = 2.5\)
- \(V = 2.5 \times 10^7\) cm/s
- **OMEGA DT equivalent of the NIF point design**
- **Hydro-equivalent curves**
  - \(V_i = 4 \times 10^7\) cm/s, \(\alpha_{if} = 2.5\)

TC8026
The condition for alpha particle confinement (hot-spot $\rho R \geq 0.3 \text{ g/cm}^2$) is always satisfied.

At low $T_i$, ignition requires hot-spot $\rho R$ well above 0.3 g/cm$^2$. 

Gain = 1
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