Stopping of Fast Electrons in Dense Hydrogenic Plasmas

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Abstract

The energy deposition and penetration depth of fast electrons in dense hydrogenic fuel are of great concern in the contexts of fuel preheat of standard ICF targets as well as heating by relativistic electrons in the fast-ignition scheme. We use the recent theory of electron stopping in hydrogenic plasmas, including scattering effects [C. K. Li and R. D. Petrasso, Phys. Rev. E 70, 067401 (2004).], to determine the preheating effects on standard ICF targets, the penetration and blooming in fast-ignition fuel, and preheating of the recently proposed shock-ignition scheme (R. Betti and C. Zhou, “High-Density and High-$\rho$R Fuel Assembly for Fast-Ignition Inertial Confinement Fusion,” this conference). We also test the collisional electron transport model in the hybrid code LSP and compare it with the theoretical results of electron stopping and energy deposition.
Summary

The hot-electron range has been calculated for central, fast, and shock ICF ignition.

- Electron stopping lengths are found for the conditions of
  - NIF and OMEGA cryo direct drive,
  - fast ignition, and
  - shock ignition

- The theoretical predictions are compared with electron stopping simulations using the code LSP.

- Good agreement is found between theory and LSP for nonrelativistic electrons.

- LSP overestimates the stopping length for relativistic electrons.
Theory: key points

- Electrons lose energy in collisions with background plasma electrons, not ions.
- Electrons also lose energy when exciting Langmuir waves.
- The scattering off plasma electrons and ions deviates the electron trajectories, reducing the penetration length.
The scattering of nonrelativistic electrons is described by the Rutherford scattering cross section

\[
\frac{d\sigma}{d\Omega}^{ee,ei} = \frac{\rho_\perp^2}{4 \sin^4 (\theta'/2)}
\]

\[
\sigma_1 = \int (1 - \cos \theta') \frac{d\sigma}{d\Omega} d\Omega = 4\pi \rho_\perp^2 \ln \Lambda^{ee,ei}
\]

\[
\left( \frac{dE}{ds} \right)_b \approx -E n_e \sigma_1^{ee} / 2 \left( \nu \gg \nu_T \right)
\]

\[
\Lambda^{ei,ee} = \frac{b_{\text{max}}^{ei,ee}}{b_{\text{min}}^{ei,ee}}, \quad b_{\text{max}} = \lambda_D = \left[ 4\pi \left( \frac{n_i e_i^2}{kT_i} + \frac{n_e e^2}{kT_e} \right) \right]^{-1/2}
\]

\[
b_{\text{min}}^{ee,ei} = \max \left\{ \rho^{ee,ei}, \frac{\hbar}{2\mu^{ee,ei} \nu} \right\}, \quad \rho^{ee,ei} = \frac{ee, i}{\mu^{ee,ei} \nu^2}, \quad \mu_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}
\]
The stopping power in a linear direction is important for ICF applications.

\[
\frac{dE}{dx} = \langle \cos \theta \rangle^{-1} \frac{dE}{ds}
\]

\[
\langle \cos \theta \rangle = \exp \left( - \int_{0}^{s} k_1(s) \, ds \right) = \exp \left[ - \int_{E_0}^{E} k_1(E) \left( \frac{dE}{ds} \right)^{-1} \, dE \right],
\]

\[
k_1 = n_i \sigma_{ei} + n_e \sigma_{ee}, \quad \sigma_1 = \int (1 - \cos \theta) \frac{d\sigma}{d\Omega} \, d\Omega
\]

Electrons lose energy in binary collisions and exciting plasma waves

\[ \frac{dE}{ds} = \left( \frac{dE}{ds} \right)_b + \left( \frac{dE}{ds} \right)_p \]

\[ \left( \frac{dE}{ds} \right)_{b,p} \approx -\left( \frac{e\omega_p}{\nu} \right)^2 L_{b,p} \quad (\nu \gg \nu_T) \]

\[ L_b = \ln \Lambda^{ee} = \ln \left( \frac{\lambda_D}{b_{ee \text{min}}} \right), \quad L_p = \ln \left( \frac{1.123 \nu}{\lambda_D \omega_p} \right) \]

\[ \frac{dE}{ds} \approx -\left( \frac{e\omega_p}{\nu} \right)^2 L, \quad L = \ln \left( \frac{1.123 m \nu^2}{\hbar \omega_p} \right) \quad \left( \frac{\nu}{c} > \frac{2}{137}, \ E > 55 \text{ eV} \right) \]

(Bethe and Bloch formula)
ICF plasmas can be strongly coupled and degenerate

- Coupling constant: \( \Gamma_i = \frac{(Ze)^2}{ak_B T} \),
  \[
  \Gamma_i \approx 1.1 \times Z^2 \left( \frac{n}{10^{23} \text{ cm}^{-3}} \right)^{1/3} \left( \frac{T}{10 \text{ eV}} \right)^{-1}
  \]

- Electron degeneracy parameter:
  \[
  \Theta = \frac{k_B T}{E_F}, \quad E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}
  \]
  \[
  \Theta \approx 1.27 \times \left( \frac{T}{10 \text{ eV}} \right) \left( \frac{n}{10^{23} \text{ cm}^{-3}} \right)^{-2/3}
  \]
The coulomb logarithm should be modified in strongly coupled and degenerate plasmas

\[ \ln \Lambda_{ei,ee} = \ln \frac{b_{\text{max}}}{b_{\text{min}}} \]

\[ b_{\text{max}} = \max \{ \lambda_D, a \}, \quad \frac{1}{\lambda_D^2} = \frac{4\pi n_e e^2}{k_B (T_e^2 + T_F^2)^{1/2}} + \frac{4\pi n_i Z^2 e^2}{k_B T_i} \]

(Y. T. Lee and R. M. More)\(^1\)

\[ L = \ln \left( \frac{1.123 \, m \nu^2}{\hbar \omega_p} \right) + O \left( \frac{\nu_F^2}{\nu^2} \right), \quad \nu_F = \left( \frac{kT_F}{m} \right)^{1/2} \quad (\nu \gg \nu_F) \]

(X.-Z. Yan et al.)\(^2\)

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We modify the logarithmic term in the stopping power of Li and Petrasso (2004) for relativistic electrons

\[ L_b \approx \ln \left[ mc \sqrt{\frac{(\gamma - 1)/2 \lambda_D}{\hbar}} \right] + \frac{1}{2} + \frac{1}{16} \left( \frac{\gamma - 1}{\gamma} \right)^2 - \left( \frac{2\gamma - 1}{\gamma^2} \right) \ln 2 \]

\[ \left( \frac{d\sigma}{d\Omega} \right)^{ei,ee} \approx \frac{\left( \rho^e_{ei,ee} \right)^2}{4 \sin^4 (\theta/2)}, \quad \rho^e_{ei} = \frac{Zr_0}{\gamma \beta^2}, \quad \rho^e_{ee} = \frac{2(\gamma + 1)r_0}{(2\sqrt{\gamma + 1} \gamma)^2 \gamma \beta^2}, \quad r_0 = \frac{e^2}{mc^2} \]

\[ b_{min}^{ei} = \frac{\hbar}{2\gamma m v}, \quad b_{min}^{ee} = \frac{\hbar}{2 mc} \sqrt{\frac{2}{\gamma - 1}} \]

An approximate expression for the electron stopping length is obtained

- Hydrogenic plasma ($Z = 1$) nonrelativistic case

$$\frac{dE}{dx} = \langle \cos \theta \rangle^{-1} \frac{dE}{ds},$$

$$\langle \cos \theta \rangle^{-1} = \exp \left[ \int_{E_0}^{E} k_1(E) \left( \frac{dE}{ds} \right)^{-1} dE \right] = \exp \left( \int_{E}^{E_0} \frac{\alpha}{E} dE \right) \approx \left( \frac{E_0}{E} \right)^{\alpha}$$

$$\alpha = \frac{\ln \Lambda^{ee} + (\ln 2 - 1/2)/2}{\ln \Lambda^{ee} + \ln \Lambda^p},$$

$$E \approx E_0 \left( 1 - x/x_{\text{max}} \right)^{1/2+\alpha},$$

$$x_{\text{max}} = \frac{E_0}{(2 + \alpha)} \left( \frac{dE}{ds} \right)^{-1}_0 = \frac{E_0^2}{(2 + \alpha) 2\pi e^4 n \ln (2E_0/\hbar \omega_p)}.$$
40- to 100-keV electrons can preheat NIF targets

- NIF direct-drive acceleration phase

- DT
  \[ \rho = 5 \text{ g/cm}^3 \]
  \[ n_e = 1.2 \times 10^{24} \text{ cm}^{-3} \]
  \[ T_e = T_i = 25 \text{ eV}, \]
  Target thickness = 25 \( \mu \text{m} \)

- \( E_0 = 10, 40, 100 \text{ keV} \)
20- to 100-keV electrons can preheat OMEGA targets

- OMEGA direct-drive acceleration phase

- DT
  \[ \rho = 4 \text{ g/cm}^3 \]
  \[ n_e = 0.96 \times 10^{24} \text{ cm}^{-3} \]
  \[ T_e = T_i = 20 \text{ eV}, \]
  Target thickness = 10 \( \mu \text{m} \)

- \( E_0 = 10, 30, 100 \text{ keV} \)
A range of 1- to 5-MeV electrons is computed for fast-ignition targets

- DT
  \[ \rho = 300 \text{ g/cm}^3 \]
  \[ n_e = 7 \times 10^{25} \text{ cm}^{-3} \]
  \[ T_e = T_i = 5 \text{ keV} \]
- \( E_0 = 0.5, 1, 5 \text{ MeV} \)
Shock-ignition targets can provide a higher energy gain at a lower pulse energy*

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*C. Zhou and R. Betti, G01.00008.
Shock-ignition targets can be affected by 80+ (60+) keV electrons

- **DT**
  - $\rho = 13 \text{ g/cm}^3$
  - $n_e = 3.1 \times 10^{24} \text{ cm}^{-3}$
  - $T_e = T_i = 20 \text{ eV}$

- **Target thickness**
  - $\rho R = 0.07 \div 0.4 \text{ g/cm}^2$
  - $\rho R = 0.03 \div 0.06 \text{ g/cm}^2$

- **$E_0 = 10, 30, 70, 100 \text{ keV}$**
The simulations using LSP show a good agreement with theory for the range of 10–100 keV electrons.

LSP: hybrid PIC code*

Plasma–fluid, electron beam–particle

theory

Energy (keV) vs. 
\( \rho r \) (g/cm\(^2\)) (x 10\(^{-3}\))

\( \rho x \) (g/cm\(^2\)) (x 10\(^{-3}\))

\( \rho y \) (g/cm\(^2\))

\( t = 1 \) fs

\( t = 0.02 \) ps

\( t = 0.04 \) ps

\( \langle E \rangle \) (keV)

\( \rho \langle x \rangle \) (g/cm\(^2\)) (x 10\(^{-3}\))

*Developed by Mission Research Corporation, Albuquerque, NM.
LSP simulations overestimate the range of 1-MeV electrons in fast ignition by a factor of 2

- DT
  \( \rho = 300 \text{ g/cm}^3 \)
  \( T_e = T_i = 5 \text{ keV} \)
Summary/Conclusions

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