Convective Versus Absolute Two-Plasmon Decay in Inhomogenous Plasmas

Integrated gain

Intensity \( (10^{14} \text{ W/cm}^2) \)

Onset of absolute instability

\[ e^{\frac{2\pi \gamma_0^2}{|K'\nu_1\nu_2|}} \]
Accurate calculation of convective gain and absolute thresholds for TPD is greatly simplified in Fourier space.

- TPD is limited to a narrow range of densities. For such profiles, the eighth-order differential equation describing TPD becomes second-order in k-space.

- For small $k_\bot/k_0$, the convective spatial gain is roughly an order of magnitude larger than the Rosenbluth formula, and the absolute instability threshold is low.

- For larger $k_\bot/k_0$ the Rosenbluth formula is a better fit to the gain, and the absolute threshold is larger.
Outline

- Convective and absolute TPD in theory and experiment
- Advantages of the Fourier space approach
- Illustrative results
- Summary and conclusions
Both convective and absolute forms of the two-plasmon-decay (TPD) instability are expected to play a role in laser-fusion experiments

- Convective instability: Plasma waves arising from noise enter the interaction region, are amplified, and propagate out at an enhanced level.
  - Spatial growth $\rightarrow \infty$: absolute instability

- Absolute instability: Waves in the interaction region are amplified faster than they can propagate out; temporal growth continues until limited by nonlinear effects.

- Absolute instability predominates at small plasmon wave vectors; small group velocity, large phase velocity.

- Convective instability predominates at large wave vectors; large group velocity, smaller phase velocity (traps electrons more effectively).
The current TPD experiments allow for a rough estimate of the plasma-wave spectrum

The absolute instability would be just above threshold for \( k_p \lambda_{De} < 0.13 \).
The equations describing TPD are difficult to treat in configuration space.

- Using the velocity potential defined by $v = \nabla \psi$, the equations governing TPD can be written

$$\frac{\partial \psi}{\partial t} = \frac{e \phi}{m} - \frac{3 v_e^2 n_1}{n_0} - v_0 \cdot \nabla \psi; \quad \frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \nabla \psi) + v_0 \cdot \nabla n_1 = 0; \quad \nabla^2 \phi = 4 \pi e n_1.$$

- These lead to an eighth-order ODE. Simplifications are of questionable validity near the plasma wave turning points.

- Simple generic three-wave convective instability theory gives the spatial gain formula $G = \exp \left( \frac{2 \pi \gamma^2}{|K' v_1 v_2|} \right)$. Constant parameters, and the exponential function of intensity must break down at the absolute threshold ($G \to \infty$ for finite intensity).
For a linear density profile, a more sophisticated treatment is feasible using Fourier transforms

- TPD is confined to a narrow range of densities below quarter-critical, so a linear density profile should be a good approximation.

- For a linear density profile, Fourier transforming in space leads to two coupled first-order ODEs in k-space:

\[
\frac{dW_+}{d\kappa} = h(\kappa)W_-, \quad \frac{dW_-}{d\kappa} = -h^*(\kappa)W_+ \quad \text{for density profile} \quad \frac{n_1}{n_0} = 1 + \frac{x}{L};
\]

coupling coefficient \( h(\kappa) = \frac{\alpha \left( \frac{k_y}{k_0} \right) e^{i\alpha \sqrt{\kappa} \left( \kappa - 2\Omega \right)}}{\sqrt{\kappa^2 + \frac{1}{4} + \left( \frac{k_y}{k_0} \right)^2 - \kappa^2}}. \)

- Previous studies have employed this k-space formulation to treat the absolute instability (Liu and Rosenbluth, 1976; Simon et al., 1983).
Both absolute and convective forms of TPD can be studied using the k-space approach

- Absolute modes are found by searching for temporally growing modes localized in k-space (Simon et al., 1983). It can be difficult to obtain accurate results near the threshold.

- The convective instability can be studied using real $k$ and $\omega$; the absolute threshold can be identified with divergent spatial gain.

- $\begin{pmatrix} W_+ \\ W_- \end{pmatrix}$ represents the plasma wave amplitudes at $\begin{pmatrix} k + k_0, \omega + \omega_0 \\ k - k_0, \omega - \omega_0 \end{pmatrix}$.

- Incoming waves at a large negative $x$ are represented by $W_\pm (\kappa \rightarrow \pm \infty)$ and outgoing waves by $W_\pm (\kappa \rightarrow \mp \infty)$; numerical integration in k-space gives the gain factor.
At small values of $k_y/k_0$, spatial amplification is larger than predicted by the simple model and shows transition to absolute mode.
At larger values of $k_y/k_0$, spatial amplification is closer to the simple model; absolute threshold is higher.

\[ L_\mu = 400 \]
\[ T_{\text{keV}} = 2.0 \]
\[ k_y/k_0 = 0.1 \]
Off-resonance, instability remains convective, but gain may be large enough to make it effectively absolute.

\[ e^{\frac{2\pi\gamma_0^2}{K'\nu_1 \nu_2}} \]

- \( L_\mu = 400 \)
- \( T_{\text{keV}} = 2.0 \)
- \( k_y/k_0 = 0.1 \)
- Off-resonant \( \omega \)
Summary/Conclusions

Accurate calculation of convective gain and absolute thresholds for TPD is greatly simplified in Fourier space.

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