Effect of Temporal Density Variation and Convergent Geometry on Nonlinear Bubble Evolution in the Classical Rayleigh–Taylor Instability

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\[ \rho = \text{const} \]

\[ \rho = \rho_0 e^{-0.4t} \]

Spherical

Planar

\[ U_b (\mu m/ns) \]

Time (ns)

0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
Summary
Layzer’s model has been extended to include temporal variation in density and convergence effect

- Layzer’s model describes the nonlinear bubble evolution in planar geometry with constant density.

- Temporal density variation and convergent effect are important in ICF implosions.

- Density variation modifies the asymptotic bubble growth to
  \[ \eta_0 = U_L \int \frac{\rho(t')}{\rho(t)} \, dt', \quad U_L = \sqrt{\frac{g}{3k}}. \]

- Extension of Layzer’s model to spherical geometry leads to
  \[ \eta_0 \rightarrow r_0(t) \int \frac{\bar{U}_L(t')}{r_0(t')} \, dt', \quad \bar{U}_L(t) = \sqrt{\frac{gr_0(t)}{\ell}} \text{ for a solid sphere.} \]
Layzer’s nonlinear RT model is only valid for planar geometry

\[ \Phi = a(t) \cos(kx)e^{-k(y-\eta_0)} \]

- \( U = \nabla \Phi \)
- \( \nabla^2 \Phi = 0 \)

\[ \sqrt{\frac{g}{3k}} \]

Expansion near bubble tip

\[ \eta = \eta_0 + \eta_2 x^2 \]

Bubble curvature \( R = \frac{1}{2n_2} \)

Bubble amplitude \( \eta_0 \)

Bubble velocity \( U_b = \dot{\eta}_0 \)

Bubble tip velocity saturates at \( \sqrt{g/3k} \)

- \( g = 100 \) (\( \mu m/\text{ns}^2 \))
- \( k = 0.5 \) (\( \mu m^{-1} \))
Density variation can be easily included in the model

\[ \nabla^2 \Phi = -\frac{\dot{\rho}}{\rho} \]

\[ \Phi = a(t) \cos(kx) e^{-k(y-\eta_0)} - \frac{\dot{\rho}}{\rho} \frac{y^2}{2} \]

Asymptotic solution

\[ \eta_0 \lim_{t \to \infty} \frac{\sqrt{g}}{3k} \int \rho(t') dt' \]
The Layzer’s model is extended to include the spherical convergence effect

- **Solid sphere:** \( \rho r_0^3 = \text{const} \)

- \( \Phi = a(t) r^{\ell} P_\ell(\cos \theta) - \left( \frac{\ddot{r}_0}{3} + \frac{\rho \dot{\rho}}{\rho} \right) \frac{r_0^2}{r} - \frac{r_0^2}{r} \frac{\dot{\rho}}{\rho} \)

- \( \frac{\partial \Phi}{\partial r} = \dot{r}_0 \) at equilibrium

- \( \frac{d}{dt} \left( \frac{\eta_2}{r_0} \right) = \frac{d}{dt} \left( \frac{\eta_0}{r_0} \right) \left( 2 \ell \frac{\eta_2}{r_0} - \frac{\ell(\ell+1)}{4} \right) \)

- \( \frac{\eta_2}{r_0} \xrightarrow{t \to \infty} \frac{\ell+1}{8} \)

- \( -\frac{1}{r_0^2} \frac{d}{dt} \left( r_0^2 \xi \right) + \ell \xi^2 = -\frac{\ddot{r}_0}{r_0}, \quad \xi = \frac{\eta_0}{r_0} \)

- \( \dot{\xi} = -\sqrt{-\frac{\ddot{r}_0}{\ell r_0}}, \quad \ell >> 1 \)
Asymptotic analysis agrees with an exact solution of the model

- $\ell = 200$, $g = 100 \left( \frac{\mu m}{ns^2} \right)$, $r_0(0) = 400 \left( \mu m \right)$
- $r_0(t) = r_0(0) - \frac{gt^2}{2}$, $\eta_0(0) = -10^{-3} \left( \mu m \right)$

\[ \eta_0 \rightarrow r_0(t) \int \frac{\tilde{U}_L(t')}{{r_0(t')}} \, dt', \quad \tilde{U}_L(t) = \sqrt{\frac{gr_0(t)}{\ell}} \]
Even though bubble amplitude decreases in solid sphere, $\frac{\eta_0}{r_0}$ always increases.

\[
\ell \frac{\eta_0}{r_0} > \frac{\eta_0(t_1)}{r_0(t_1)} < \frac{\eta_0(t_2)}{r_0(t_2)}
\]
If $\rho r_0^3 \neq \text{const (shell)}$, asymptotic amplitude is determined by a first-order nonlinear differential equation

$$\ell \ddot{\xi}^2 - \dot{m}\left[\xi^2 \left(1 + 2(\ell + 1)\frac{\ddot{\xi}}{m}\right)^2 - 2\frac{\ddot{\xi}}{r_0} + \frac{\ddot{r}_0}{r_0} m^2\right] = 0$$

$$\xi = \rho r_0^2 \eta_0, \quad m(t) = \rho(t) r_0^3(t),$$

$$\eta_2 \rightarrow \frac{\ell + 1}{8}$$

$$r_0(t) = 400 - \frac{100t^2}{2}, \quad \rho(t) = \rho_0 e^{\epsilon t}$$
Bubble amplitude in spherical geometry does not decrease with radius

\[ U_b^{\text{planar}} = \sqrt{\frac{g}{k}} \quad ? \quad U_b^{\text{spherical}} = \sqrt{g \frac{r_0(t)}{\ell}} \]

\[ \rho = \text{constant} \]

• Similar results are derived for cylindrical geometry in Reference 1.

\[ Y. \text{Yedvab et al., presented at the 6th IWPCTM, Marseille, France, 18–21 June 1997, p. 528.} \]
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• Layzer’s model describes the nonlinear bubble evolution in planar geometry with constant density.

• Temporal density variation and convergent effect are important in ICF implosions.

• Density variation modifies the asymptotic bubble growth to

\[ \eta_0 = U_L \int \frac{\rho'(t')}{\rho(t)} \, dt' / \rho(t), \quad U_L = \sqrt{\frac{g}{3k}}. \]

• Extension of Layzer’s model to spherical geometry leads to

\[ \eta_0 \rightarrow r_0(t) \int \frac{\bar{U}_L(t')}{r_0(t')} \, dt', \quad \bar{U}_L(t) = \sqrt{\frac{g r_0(t)}{\ell}} \quad \text{for a solid sphere.} \]