Application of the Low-Frequency Energy Principle to Wall Modes

Normalized growth rate \( \left( \gamma \tau W \right) \)

Fluid theory

Kinetic theory

Normalized beta \( (\beta_N) \)

Centra toroidal rotation

Maximum of \( \omega_i \)

\[ \Omega = 0 \]
\[ \Omega = 0.25 \omega_i \]
\[ \Omega = 0.5 \omega_i \]
\[ \Omega = 0.75 \omega_i \]
\[ \Omega = 1.0 \omega_i \]
Collaborators

Bo Hu
University of Rochester

Janardhan Manickam
Princeton Plasma Physics Laboratory

Jeffrey Freidberg
Massachusetts Institute of Technology
Summary

Marshall’s contribution to the kinetic-energy principle has been instrumental in our understanding of the interaction between particles and MHD modes.

- The low-frequency energy principle\(^1\),\(^2\) is applied to resistive wall modes.

- When all the kinetic species (alphas, ions, and electrons) are included, the RWM growth is strongly reduced or fully suppressed in the low-rotation regime for ITER-like plasmas.

Outline

• Marshall’s contribution to the low-frequency energy principle.

• The low-frequency energy principle application to wall modes.

• The qualitative picture of RWM interaction with trapped particles.

• The PEST kinetic postprocessor and the RWM stability in ITER-like plasmas.
The trapped particle contribution to the macroscopic plasma stability is included in the kinetic energy principle (KEP); Marshall’s contribution to the KEP dates as early as 1959.


\[
\delta W = \delta W_F + \delta W_K
\]

\[
\delta W_K = -\frac{1}{2} \int dV \left\{ \left( \delta P_\parallel - \delta P_\perp \right) \kappa \cdot \xi + \delta P_\perp \nabla \cdot \xi_\perp \right\}
\]

\[
\delta P_\parallel,\perp = \frac{1}{2} m \int_{\text{trapped}} d\tilde{v} \left( v_\parallel,\perp^2 \delta f \right)
\]

From the solution of the Vlasov equation in the limit of small Larmor radius and \( \omega >> \omega_* \).

\[
\delta W_{Kj} \sim \int dV \beta_j \int_{\Lambda_{\text{trap}}} d\Lambda \hat{t}_{\text{bounce}} \left| \langle \kappa \cdot \xi \rangle \right|^2
\]

Positive-definite (stabilizing)
The “slow” trapped-particle orbit-drift motion was not included in the early formulations of the KEP

\[ \omega_D = \langle [V_{NB} + V_k] \cdot \nabla S \rangle \]

\[ \omega_D \sim \frac{V_{th} r_L}{R r} \sim \omega_\ast \]

Precession or magnetic-drift frequency

Precession motion of the trapped-particle banana orbits
Van Dam, Rosenbluth, and Lee\textsuperscript{3} (1982) generalized the energy principle to include the precession motion of trapped particles

- The Rosenbluth–Rostoker (and Kruskal–Oberman) KEP was equivalent to requiring that both the trapped-particle magnetic moment and longitudinal action are conserved.

- Van Dam, Rosenbluth, and Lee\textsuperscript{3} recognized that if the mode frequency $\omega \ll \omega_D$, then the “magnetic flux passing through the precessional drift orbit is adiabatically conserved.”

- The condition $\omega \ll \omega_D$ was easily satisfied for energetic particle species, such as in the microwave-heated poloidal ring of 100 to 500 keV electrons in the Elmo Bumpy Torus.

Using the third invariant connected with the fast precession motion, Van Dam, Rosenbluth, and Lee derive the generalized kinetic energy principle

\[ \delta W_{Kh} \sim \int dV \beta_h \int_{\Lambda_{\text{trap}}} d\Lambda \hat{t}_{\text{bounce}} \left| \langle \kappa \cdot \xi \rangle \right|^2 \frac{\omega \star h}{\omega_{Dh}} \]

- A similar form of the KEP was derived independently (submitted five months earlier) by T. M. Antonsen, Jr., B. Lane, and J. J. Ramos [Phys. Fluids 24, 1465 (1981)].

- This form of the KEP had extremely important applications in Tokamak physics.
The low-frequency KEP has improved our understanding of Tokamak physics; fishbones, sawtooth suppression by fast ions, and many resonant interactions with MHD modes.

- When applied to the $m = 1$ internal kink, the KEP including fast ions show the existence of a new branch with the frequency $\sim \omega_{Dh}$, which is destabilized by the hot-ion pressure (fishbones) [L. Chen, R. B. White, and M. N. Rosenbluth, Phys. Rev. Lett. 52, 1122 (1984)].

- The same KEP also revealed the existence of stable regimes for the $m = 1$ mode (sawtooth suppression by fast ions) [Coppi et al., Phys. Rev. Lett. 63, 2733 (1989)].

- The KEP had many more applications, mostly in the area of interaction between trapped particles and MHD modes [TAE interaction: G. Y. Fu and C. Z. Chang, Phys. Fluids B 4, 3722 (1992); Energetic Particle Modes: S.-T. Tsai and L. Chen, Phys. Fluids B 5, 3284 (1993)].
The low-frequency energy principle can also be applied to wall modes

\[ \gamma W = - \frac{\delta W_{\infty}^{\text{MHD}} + \delta W_K}{\delta W_b^{\text{MHD}} + \delta W_K} \]

\[ \delta W_K = \delta W_K^i + \delta W_K^e + \delta W_K^\alpha \]

\[ \delta W_K = \Re[\delta W_K] + \Im[\delta W_K] \]

Instability condition

\[ -\delta W_b^{\text{MHD}} \delta W_{\infty}^{\text{MHD}} > |\delta W_K|^2 + \Re[\delta W_K] \left( \delta W_b^{\text{MHD}} + \delta W_{\infty}^{\text{MHD}} \right) \]

Without kinetic effects, the RWM is unstable between the wall and no-wall beta limits.

- Normalized RWM growth rate without kinetic effects.

\[
\gamma = \frac{\beta}{\beta_{\infty}} = \frac{\beta_{b}}{\beta_{N}}
\]

**Normalized growth rate**

**Fluid theory**

\(\gamma_{W}\)

\(\beta_{N} = \text{no-wall limits}\)

\(\beta_{b} = \text{wall limits}\)
Qualitative analysis of the instability condition with kinetic effects

\[ \delta W_{\text{MHD}}^\infty \sim \beta_\infty - \beta \]
\[ \delta W_{\text{MHD}}^b \sim \beta_b - \beta \]
\[ \delta W_K \sim \beta (X + iY) \]

Instability drive
\[ \frac{1}{4} (\beta_b - \beta_\infty)^2 \]

Stabilizing

Destabilizing

\[ -\delta W_{\text{MHD}}^b \delta W_{\text{MHD}}^\infty \]
\[ > |\delta W_K|^2 \]
\[ + \Re [\delta W_K] \left( \delta W_{\text{MHD}}^b + \delta W_{\text{MHD}}^\infty \right) \]
Five regimes of RWM stability/instability

\[
X = \text{Re}(\delta W_k)/\beta
\]

\[
Y = \text{Im}(\delta W_k)/\beta
\]

- **I**: Full stabilization
- **II**: Ideal kink mode
- **III**: Ideal kink mode
- **IV**: Ideal kink mode
- **V**: Ideal kink mode
Necessary condition for stabilization requires relatively small $\delta W_K$

- Kinetic stabilization enhanced for strong dissipation. Full suppression for $\beta_b \sim 2 \beta_\infty$ requires

$$|\delta W_K| > 0.29 \delta W_{\text{Drive}}^{\text{MHD}} \Rightarrow \delta W_{K}^{\text{Minimum}} = \beta (0.08 + i 0.28)$$

- Kinetic stabilization in the absence of dissipation ($Y = 0$) requires $X > 0.5$: $\delta W_K > 0.5 \beta \Rightarrow |\delta W_K| > 0.5 \delta W_{\text{Drive}}^{\text{MHD}}$
Low-frequency kinetic theory of the RWM: approximations

• RWM frequency: \( \omega \sim 1/\tau_W - 50/\tau_W \) (\( \tau_W \) = wall time)

• \( \omega << \omega_D, \omega_i \) \(\leftarrow\) zero-mode frequency
  – \( \omega_D \) magnetic-drift frequency
  – \( \omega_i \) ion-diamagnetic-drift frequency

• \( \nu_{\text{eff}} << \omega_D, \omega_i \) \(\leftarrow\) collisionless ions (and electrons?)

• \( \Omega_{\text{rot}} \sim \omega_i \) \(\leftarrow\) quasi-stationary plasma

• For large rotation frequencies (\( \Omega_{\text{rot}} >> \omega_i \)), the resonance with the trapped-particle bounce motion becomes important.\(^4,5\) (Not included here)

• In the presence of sufficient dissipation, the wall mode can be suppressed by fast rotation.\(^6–9\) (Not included here)

\(^7\)A. Bondeson and D. J. Ward, Phys. Rev. Lett. 72, 2809 (1994).
A resonance occurs between the precession frequency and the $E \times B$ Doppler-shifted mode frequency.

$$\delta W_{Kj} \sim \int dV \beta_j \int \Lambda_{\text{trap}} d\Lambda \hat{t}_{\text{bounce}} |\langle \kappa \cdot \xi \rangle|^2 I_\varepsilon (\Lambda, \bar{r})$$

$$\hat{\varepsilon} = \frac{\varepsilon}{T_j}$$

$$I_\varepsilon = \int_0^\infty \hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} \omega_j \left( \hat{\varepsilon} - \frac{3}{2} \right) + \omega_{\text{Doppler}} d\hat{\varepsilon}$$

$$\omega_{\text{Doppler}} \equiv \omega_{\text{Lab}} - \omega_E$$

$$\omega_{\text{Doppler}} \equiv \omega_{Dj} \hat{\varepsilon}$$

Resonance condition
Ions can resonate in the absence of plasma rotation

Mode-particle resonance:
Doppler-shifted frequency = magnetic-drift frequency

\[ \omega_{\text{doppler}} = \omega_{\text{Di}} \]

Zero-frequency approximation

\[ \omega \times B = \Omega_{\text{rot}} - \omega \times B \]

Ion-force balance equation

\[ \omega \times B = \Omega_{\text{rot}} - \omega \times B \]

\[ \omega_{\text{Di}} = \omega \times i - \Omega_{\text{rot}} \]

Ions can resonate in the absence of plasma rotation
The strongest resonance comes from suprathermal particles ($\Omega_{\text{rot}} = 0$ case)

$$\hat{\varepsilon} = \frac{\varepsilon}{T_j}$$

Large-aspect-ratio approximation

$$\hat{\varepsilon}_\text{res} = r_\omega \equiv \frac{\omega_{\text{th}}}{\Omega_{\text{Di}}} \sim 5 - 8$$
Large-aspect-ratio ordering underestimates the size of ion-kinetic terms

\[ \frac{\delta W_{Ki} \left( \omega_E \ll \omega_D \right)}{\delta W_{Fluid}} \sim \sqrt{\varepsilon} \]

\[ \frac{\delta W_{Ki} \left( \omega_E \gg \omega_D \right)}{\delta W_{Fluid}} \sim \varepsilon^{3/2} \]

\[ \frac{|\delta W_{Ki}|}{\varepsilon^{3/2} \delta W_F} > \sqrt{\varepsilon} \]

\[ \omega_E = 0 \]
\[ \Omega_{\text{rot}} = \omega \* i \]

\[ \omega_E = -\omega \* i \]
\[ \Omega_{\text{rot}} = 0 \]

\[ |\omega_E| \]
\[ \omega \* i \]

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Electrons can resonate for $\Omega_{\text{rot}} > \omega_i$; collisionality reduces the resonant interaction but does not eliminate it.

\[ \omega_E = \omega_i \]
\[ \Omega_{\text{rot}} = 2\omega_i \]

\[
\begin{align*}
\Omega_{\text{rot}} &
\equiv \frac{\omega_i}{\bar{\omega}_{\text{th}}} \sim 5-8, \\
\hat{\varepsilon} &
\equiv \frac{\varepsilon}{T_e}
\end{align*}
\]
In a burning plasma, nonresonant and resonant $\alpha$ particles contribute to the stabilization.

$$\delta W_{K\alpha} \sim \int dV \int_{\Lambda_{\text{trap}}} d\Lambda \hat{t}_{\text{bounce}} \left| \langle \mathbf{k} \cdot \xi \rangle \right|^2 \beta_{\alpha} \frac{\omega^*_{\alpha}}{\omega_{D\alpha}} I_\varepsilon (\Lambda, \bar{r})$$

$$I_\varepsilon = \int_0^1 \frac{\hat{\varepsilon}}{\hat{\varepsilon} - \left( \omega_E / \omega_{D\alpha} \right)} d\hat{\varepsilon} \quad \hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_{\alpha}}.$$

- In high-$\beta$ plasmas, $\omega_D$ can be significantly smaller than large-aspect-ratio prediction $\rightarrow$ strong resonance can occur between $\omega_E$ and $\omega_{D\alpha}$

- Nonresonant contribution always stabilizing and enhanced by ratio $\omega^*_{\alpha} / \omega_{D\alpha} >> 1$

- $\alpha$ contribution is significant since $\nabla p_{\alpha}$ is large where the RWM eigenfunction is large
Analytic predictions are not reliable based on the variability of $\omega_D$, with respect to $\omega_E$ (drift reversal, rotation profile, eigenfunction shape); quantitative assessment requires a stability code.

$\delta W^\infty_{\text{MHD}}$

$\delta W^b_{\text{MHD}}$

$\xi$

\[ \delta W_K = \delta W^i_K + \delta W^e_K + \delta W^\alpha_K \]

\[ \gamma^\tau_w = -\frac{\delta W^\infty_{\text{MHD}} + \delta W_K}{\delta W^b_{\text{MHD}} + \delta W_K} \]


The RWM plasma-eigenfunction $\xi$ is approximated by the ideal-MHD $\tilde{\xi}$ from PEST for marginally stable wall position.

- For each equilibrium with $\beta_\infty < \beta < \beta_b$, the wall is moved closer to the plasma until marginal stability is reached.

- Since the RWM is essentially zero frequency, the corresponding plasma eigenfunction computed with PEST is used to approximate the RWM eigenfunction.

- Such an eigenfunction is used to compute fluid and kinetic contributions to $\delta W$. 
The RWM stability of an ITER-advanced Tokamak scenario is studied using a symmetrized plasma and conforming wall.
Most of the kinetic contribution is produced within the $q = 3$ surface

- Alpha contribution is significant since $\nabla p_\alpha$ is large where the RWM eigenfunction is large.
- Alpha contribution can be comparable to ions and electrons for $\langle \beta_\alpha \rangle = 0.15 \langle \beta \rangle$. 
The growth of the RWM is strongly reduced or fully suppressed by the kinetic effects in slow-rotating ITER-like plasmas.

- RWM normalized growth rate with and without kinetic effects and varying plasma rotation frequencies $\Omega(0)$.

![Diagram showing normalized growth rate vs. normalized beta]
Summary/Conclusions

The low-frequency energy principle\(^1,2\) applied to wall modes shows the possibility of RWM growth reduction/suppression at low rotation frequencies.

- Marshall's contribution to the kinetic-energy principle has been instrumental in our understanding of the interaction between particles and MHD modes.
- The low-frequency energy principle is applied to wall modes.
- When all the kinetic species (alphas, ions, and electrons) are included, the RWM growth is strongly reduced or fully suppressed in the low-rotation regime for ITER-like plasmas.

\(^1\)M. N. Rosenbluth and N. Rostoker, Phys. Fluids 2, 23 (1959).