Two-Dimensional MHD Simulations of Tokamak Plasmas with Poloidal Flows

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Summary

• The theory of poloidally rotating Tokamak plasmas predicts the creation of a series of shocks along the poloidal angle.

• Shocks cannot exist at equilibrium unless a source of momentum is present in the system.

• We have carried out 2-D compressible MHD simulations that confirm the theoretical predictions.
The relevant compressible MHD equations

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \]

Artificial viscosity

\[ \rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla P + \vec{J} \times \vec{B} + \nu \nabla \cdot \vec{\eta} \]

Viscous heating

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \]

\[ \nabla \cdot \vec{B} = 0 \]
The integration scheme

• Time-dependent equations are integrated using a predictor–corrector MacCormack scheme.

• Space derivatives are treated with standard finite-difference approximations.

• The scheme is second order accurate in both time and space.
Boundary conditions

• The following standard BC’s are implemented:
  – Normal flow is zero at the (non-viscous) boundary.
  – Normal magnetic field is zero at the boundary.
  – Normal pressure gradient is zero at the boundary.
  – Normal density gradient is zero at the boundary.
The following numerical smoothing techniques are implemented in the code:

- Artificial (numerical) viscosity is added to the system.
- Artificial (numerical) heat conduction is added to the system.
- In order to avoid numerical problems with the approximate BC’s, artificial terms are enhanced near the boundary.
- An additional diffusion equation is inserted to control the divergence of the magnetic field.
The predictions of theory:
formation and evolution shocks

- The magnetic field acts as a de Laval nozzle for any initial poloidal velocity profile.

- In a toroidal plasma, an equilibrium with a continuous, transonic (with respect to the poloidal sound speed) poloidal velocity profile cannot exist.

- Any initially transonic poloidal velocity profile will evolve into a series of shocks distributed along the poloidal angle.

- Shocks at equilibrium can only exist if a momentum source is present in the system.

Equilibrium profiles

• If no source of momentum is present in the system, the series of shocks will evolve into a radial discontinuity of the physical quantities at equilibrium.

Forbidden equilibrium profiles

Allowed equilibrium profiles
Velocity profile evolution

- The natural time scale for the system is the poloidal rotation period $\tau_{\text{rot}}$. 

![Velocity Profile Diagrams](image)
Velocity profile evolution (2)

- The time evolution of the system shows the formation of a radial discontinuity in the poloidal velocity profile.

\[ V_p \quad C_s \]

\[ R_{\text{major}} \]

\[ t = 2.9 \, \tau_{\text{rot}} \quad t = 3.7 \, \tau_{\text{rot}} \]
Conclusions

• A two-dimensional, time-dependent code for the analysis of the behavior of toroidal plasma with poloidal flow has been developed.

• In accordance with the predictions of the theory, the formation and evolution of a set of shocks has been observed in the poloidal direction.

• Shocks can be present in a steady-state system only if sustained by a momentum source.