Nonlinear Sound Waves in Two-Ion Plasma

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A plasma consisting of electrons and two types of ions supports two types of ion-sound waves (fast and slow), both of which can scatter incident laser radiation. At the high intensities characteristic of ICF experiments, sound waves driven by SBS are nonlinear. We study the dispersive and nonlinear characteristics of fast and slow waves, their tendency to steepen, and their ability to form solitons. Our analytic predictions are verified by two-fluid simulations. This work was supported by the U.S. Department of Energy Office of Inertial Confinement Fusion under Cooperative Agreement No. DE-FC03-92SF19460.

It is important to understand the physics of nonlinear sound waves in two-ion plasma

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- Many ICF experiments include multiple-species plasma.
- There are "fast" and "slow" modes of sound waves in two-ion plasma. Laser light can scatter from fast or slow sound waves.
- Depending on the plasma parameters, the growth rate of SBS on slow mode can be less than, comparable to, or greater than the growth rate on fast mode.[1] This remains true in the presence of Landau damping.[2,3]
- SBS can be saturated by nonlinear steepening,[4] nonlinear detuning,[5] or ion trapping.[2,3]

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[1] C. J. McKinstrie and M. V. Kozlov (this conference).
[2] Vu *et al.*, Phys. Plasmas 1, 3542 (1994).
[3] Williams *et al.*, Phys. Plasmas 2, 129 (1995).
[4] V. V. Kurin and G. V. Permitin, Sov. J. Plasma Phys. 8, 207 (1982).
[5] J. A. Heikkinen, S. J. Karttunen, and R. E. Salomaa, Phys. Fluids 27, 707 (1984).

Sound waves obey the IFP ion fluid, Poisson equations

- We nondimensionalize the equations in the standard way.*
- The Poisson, mass, and momentum equations are

$$\partial^2_{xx}\phi = e^{\phi} - \frac{n_l + \alpha n_h}{1 + \alpha}$$

$$\partial_t n_i + \partial_x (n_i U_i) = 0$$

$$(\partial_t + U_i \partial_x) U_i = -\beta_i \partial_x \phi - V_i^2 n_i \partial_x n_i ,$$

where

α = charge–density ratio $Z_h n_{h0}/Z_l n_{l0}$

β_i = charge-to-mass ratio $Z_i m_l / Z_l m_i$ ($\beta_l = 1$, $\beta_h = \beta$)

V_i = normalized thermal speed

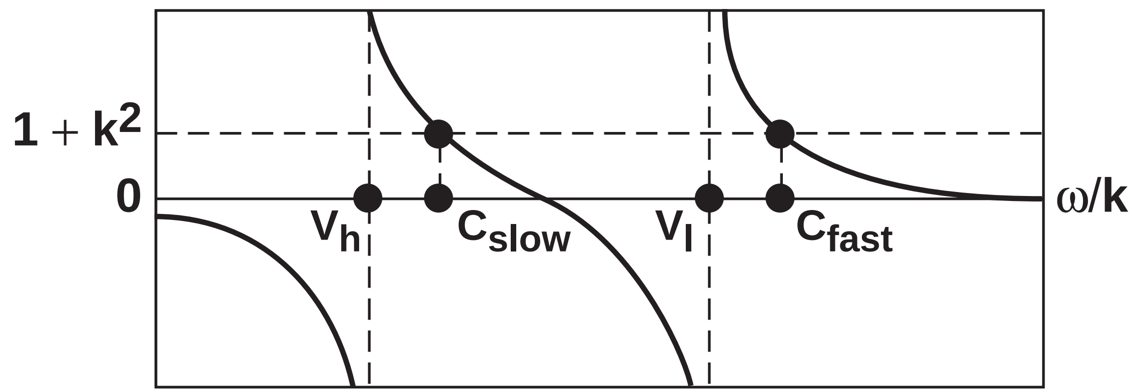
*Rozmus *et al*., Phys. Fluids B 4, 576 (1992).

Two linear sound waves exist

- **Dispersion relation**

$$1 + k^2 = \left[\frac{k^2}{\omega^2 - V_l^2 k^2} + \frac{\alpha\beta k^2}{\omega^2 - V_h^2 k^2} \right] \frac{1}{1+\alpha}$$

- **The phase velocities can be determined graphically.**



- **The charge-density perturbations**

$$q_h/q_l = \alpha\beta\left(c^2 - V_l^2\right)/\left(c^2 - V_h^2\right)$$

- **For the fast wave** $c_{fast} > V_c$ **and** $q_h/q_l > 0$.
- **For the slow wave** $V_h < c_{slow} < V_l$ **and** $q_h/q_l < 0$.

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Both sound waves are dispersive

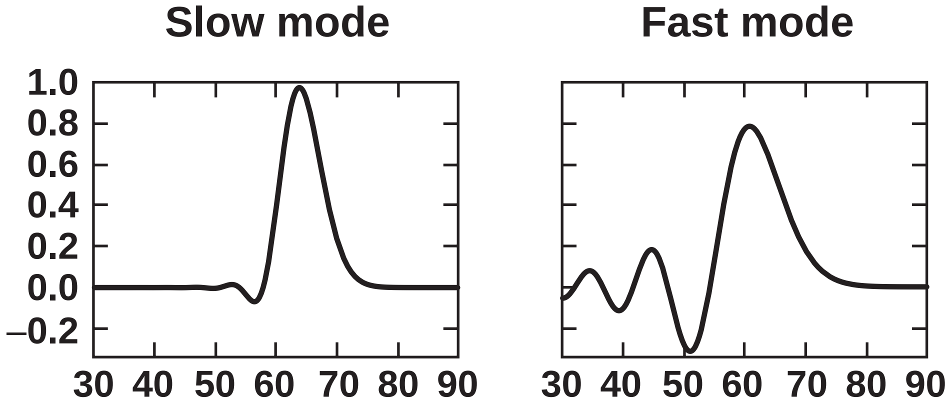
- If $\omega^2 \; . \; c^2 k^2 + dk^4$, then $\omega \; . \; ck + dk^3/2c$

$$d = -\left[1 + \frac{2\alpha\beta\left(c^2 - V_l^2\right)\left(c^2 - V_h^2\right)}{\left(c^2 - V_h^2\right)^2 + \alpha^2\beta^2\left(c^2 - V_l\right)^2}\right].$$

- $d_f < 0$ and $d_s < 0$; $|d_s| < |d_f|$.
- Wave packets shed energy as a dispersive wake.

Initial profile:
– **Gaussian**
(**width** $=$ **3, amplitude** $=$ **1**)

Distance of propagation: 60



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Nonlinearities steepen both sound waves

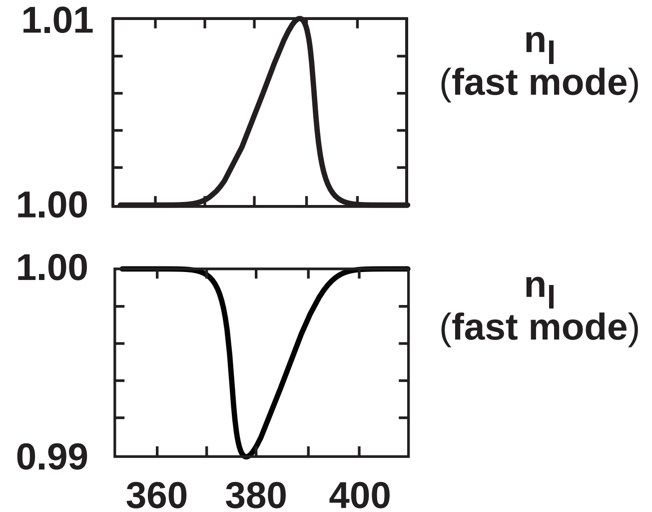
- In the absence of dispersion the Riemann variables satisfy

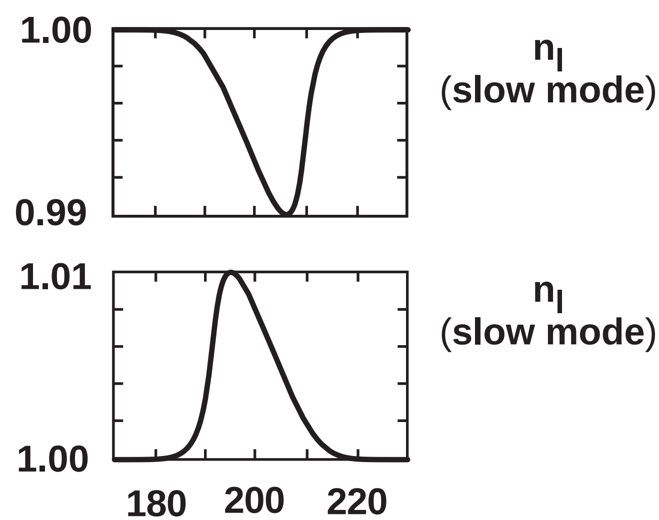
$$\frac{dR}{dt} = 0 \quad \text{or} \quad \frac{dx}{dt} = C,$$

where the characteristic speeds C satisfy

$$n_l + \alpha n_h = \frac{n_l}{(c - u_l)^2 - V_l^2 n_l^2} + \frac{\alpha \beta n_h}{(c - u_h)^2 - V_h^2 n_h^2}.$$

- Fast mode $(q_h/q_l > 0)$.
  - If $n_l > 1$ (< 1) then $n_h > 1$ (< 1); wave steepens at front (back)
- Slow mode $(q_h/q_l < 0)$.
  - Heavy ions play a more important role as we could expect.
  - If $n_h > 1$ (< 1) then $n_l < 1$ (> 1); wave steepens at front (back)





Fast and slow solitary waves exist

- Nonlinearity and dispersion cancel each other in solitary waves.
- Solitary waves satisfy potential equation

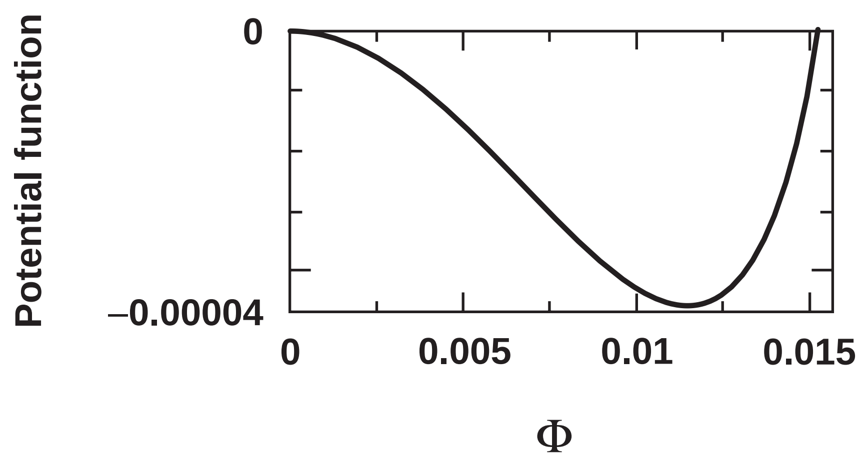
$$(d\phi/d\xi)^2 + P(\phi) = 0,$$

where $\xi = x - ct$. Potential function $P(\phi)$ is a complicated function of ϕ,c and plasma parameters.

- ϕ_{max} is an increasing function of c determined by $P(\phi) = 0$.
- Solitary wave ceases to exist when ϕ_{max} reaches either of the critical values

$$(V_l - c)^2 / 2 \text{ or } (c - V_h)^2 / 2\beta.$$

(Thermal particles resonate with the wave.)



If nonlinearity and dispersion are small, IFP equations can be approximated by the KDV equation

- Both nonlinearity and dispersion are included in the KDV equation

$$\partial_t \phi + C\partial_x \phi + N\phi\partial_x\phi + D\partial^3_{xxx} \phi = 0,$$

which is a good approximation of IFP equations in the case of small amplitudes and large length scales.

- Coefficients C, N, and D are complicated functions of plasma parameters and differ for fast and slow modes.

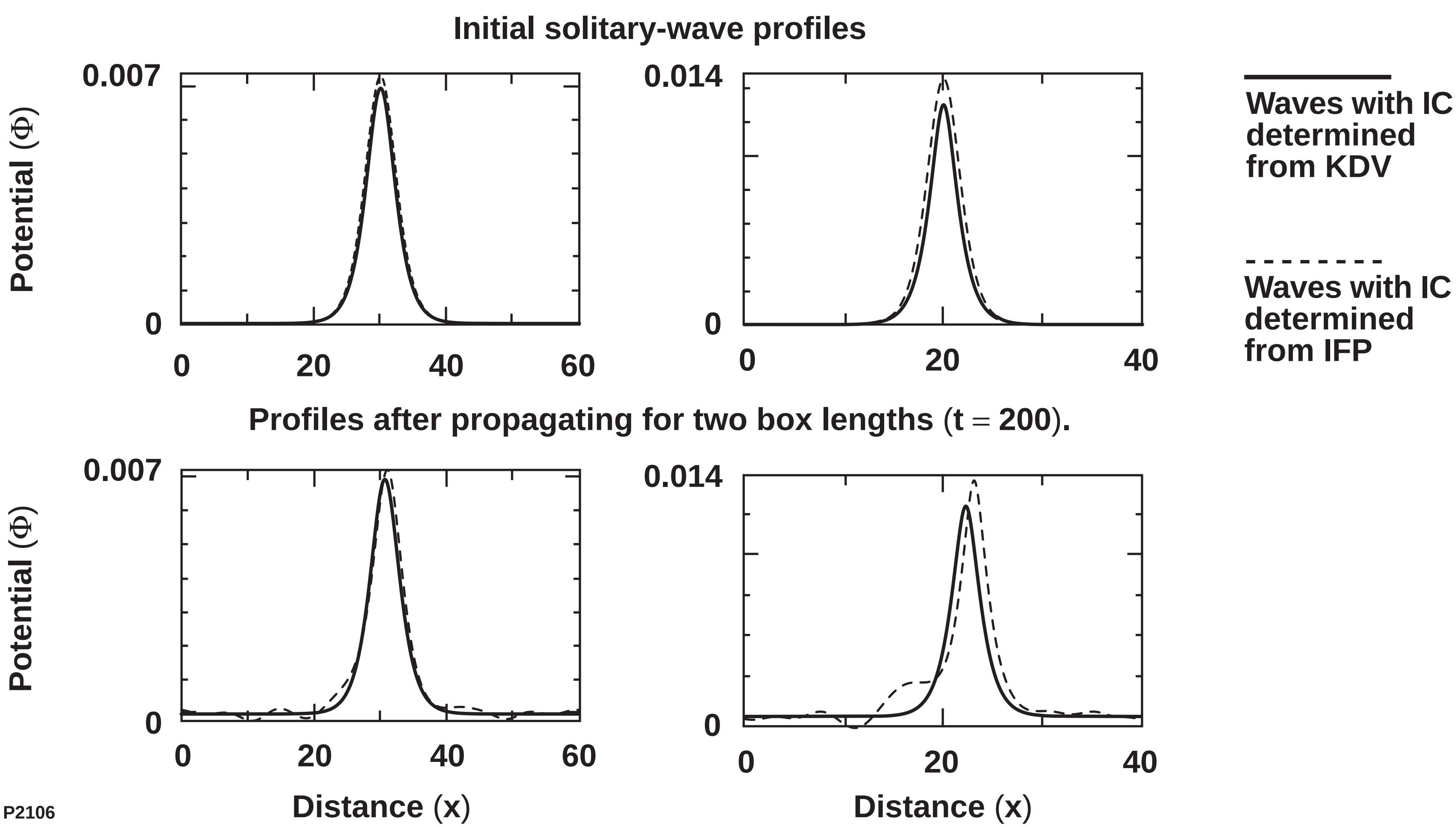
- We can readily derive solitary wave profile for the KDV equation:

$$\phi(\xi) = \frac{3\varepsilon}{N}\,\mathrm{sech}^2\left[\sqrt{\frac{\varepsilon}{4D}} \bullet \xi\right],$$

where $\xi = X - M \bullet t$ and $M = C + \varepsilon$.

Numerical simulations confirmed features of soliton

- “Fast” soliton is similar to soliton in 1-ion plasma.
- In “slow” soliton $n_h > 1$, but $n_l < 1$.
- Condition $\phi_{max} < \phi_{crit}$ limits amplitude and speed of soliton.
- In contrast to IFP soliton amplitude of KDV soliton is not limited.



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Summary

Physics of nonlinear sound waves in two-ion plasma is well understood

- Nonlinear and dispersive effects of the fast and slow modes of sound waves in two-ion plasma were studied extensively, making possible the future analysis of nonlinear saturation of SBS.
- Low-amplitude sound waves in two-ion plasma generated by SBS can be modeled by the set of two KDV equations.*

*Rozmus *et al*., Phys. Fluids B 4, 576 (1992).