Comparisons Between Ray- and Wave-Based Calculations of Cross-Beam Energy Transfer

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Summary
A 3-D wave-based model has been developed to understand the physics of cross-beam energy transfer (CBET) in an inhomogeneous plasma.

- Detailed CBET calculations are used to test ray-based CBET models that are implemented in hydrodynamics codes.
- The comparisons generally highlight the accuracy of ray-based models.
- Discrepancies between the models are found related to beam-speckle and polarization smoothing.

LPSE solves the time-enveloped Maxwell's equations coupled to a linearized plasma response.

- Maxwell's equations (time enveloped) \( E_j \left( \frac{d}{dt} + \frac{c}{v_e} \right) \phi \left( \mathbf{x}, t \right) = \sum \mathbf{E}_e \cdot \mathbf{n}_e \),

- Plasma response \( \frac{d}{dt} \left( \frac{\mathbf{E}_e}{c} \right) - \nabla e^\phi = \sum \mathbf{E}_e \cdot \mathbf{n}_e \).

The CBET gain is sensitive to beam speckle for gains greater than -1 and relative beam angles of less than -30°.

Speculated beams result in a modest decrease in laser absorption in OMEGA-scale, two-beam LPSE simulations at ICF-relevant plasma conditions.

Ray-based models calculate CBET by considering pairwise interactions between rays.

- Ray-based CBET models make several approximations that are not always satisfied in inertial confinement fusion (ICF) applications:
  - Ion-acoustic waves (M8 wave < c):
  - Steady-state convection gain:
  - Polarization-averaged coupling constant:
  - Pairwise coupling between beams:
  - Local-plane approximation:
    - not valid for speckled beams or at caustics:
    - Elliptic approximation:
      - slow-top-fast, elliptical (WKB), envelope:
      - breaks down at caustics.

Ray- and wave-based CBET models give the same result in simple interaction geometries (plane-wave beams, no caustics).

- The factor of \((1 - \cos^2 \theta) / 4\) used to account for the modification of the CBET gain between beams with polarization smoothing is valid only when the speckle length is shorter than the interaction region.

A good approximation to the average CBET between speckled beams can be obtained by using the linearity of Maxwell's equations to solve for the correct unperturbed field amplitudes in the ray-based calculation.

Speculated beams can transfer more energy than plane-wave beams with the same average intensity.

- The speckled ray approach reproduces LPSE results to within one standard deviation of the average over realizations.

Polarization smoothing is accounted for in ray-based CBET models by multiplying the gain coefficient by a factor of \((1 - \cos^2 \theta) / 4\).

- The factor of \((1 - \cos^2 \theta) / 4\) comes from assuming that the interacting beams have random relative polarizations with uncorrelated speckle patterns and ensemble averaging over realizations.
- The factor \((1 - \cos^2 \theta) / 4\) is the result of the ponderomotive potential of the best wave order.

Speculated beams result in a modest decrease in laser absorption in OMEGA-scale, two-beam LPSE simulations at ICF-relevant plasma conditions.
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- Detailed CBET calculations are used to test ray-based CBET models that are implemented in hydrodynamics codes.
- The comparisons generally highlight the accuracy of ray-based models.
- Discrepancies between the models are found related to beam speckle and polarization smoothing.
Ray-based models calculate CBET by considering pairwise interactions between rays.

CBET interaction between plane waves in a homogeneous plasma:

\[
\frac{dI_i}{ds} = \sum_j I_j L_{ij}^{-1}
\]

\[
L_{ij}^{-1} = 5.88 \times 10^{-2} \frac{I_j \lambda}{T_e \left(1 + 3T_i/ZT_e\right)} \frac{n_e}{n_c} \frac{\omega_s}{\nu_i} P(\eta_{ij})
\]

\[
P(\eta) = \frac{\nu_i^2 \eta}{(\eta^2 - 1)^2 + \nu_i^2 \eta^2}
\]

\[
\eta_{ij} = \frac{\omega_j - \omega_i - (\vec{k}_j - \vec{k}_i) \cdot \vec{u}}{\omega_s}
\]
Ray-based CBET models make several approximations that are not always satisfied in inertial confinement fusion (ICF) applications

- Ion-acoustic waves (IAW’s) \((\delta_n/n \ll 1)\)
- Steady-state convective gain
- Polarization-averaged coupling constant
- Pairwise coupling between beams
- Local plane-wave approximation
  - not valid for speckled beams or at caustics
- Eikonal approximation
  [Wentzel–Kramers–Brillouin (WKB), envelope]
  - breaks down at caustics

\[\text{Approximations that are not made in } \text{LPSE}\]
LPSE solves the time-enveloped Maxwell’s equations coupled to a linearized plasma response

- Maxwell’s equations (time enveloped) \( \vec{E} = \Re \{ \vec{E}(\vec{x}, t) \exp(-i\omega_0 t) \} \)

\[
\frac{2i\omega_0}{c^2} \frac{\partial}{\partial t} \vec{E} + \nabla^2 \vec{E} - \nabla (\nabla \cdot \vec{E}) + \frac{\omega_0^2}{c^2} \varepsilon(\omega; \vec{x}, t) \vec{E} = 0
\]

\[
E(\omega_0; \vec{x}, t) = 1 - \frac{\omega_{pe}^2(\vec{x}, t)}{\omega_0(\omega_0 + i\nu_{ei})}
\]

\[
\omega_{pe}^2 = \frac{4\pi e^2 n_e(\vec{x}, t)}{m_e}
\]

- Plasma response

\[
\left[ \partial_t + \vec{U}_0(\vec{x}) \cdot \nabla \right] \left( \frac{\delta n}{n_0} \right) = -W
\]

\[
\left[ \partial_t + \vec{U}_0(\vec{x}) \cdot \nabla + 2\vec{V}_{law} \right] W = -\nabla^2 \left[ c_s^2 \left( \frac{\delta n}{n_0} \right) + \frac{e^2}{4m_e \omega_0^2} |\vec{E}|^2 \right]
\]

\[
W \equiv \nabla \cdot \delta \vec{U}
\]

\[
n_e \equiv n_0(\vec{x}) + \delta n(\vec{x}, t)
\]

\[
\vec{U} = \vec{U}_0(\vec{x}) + \delta \vec{U}(\vec{x}, t)
\]
Ray- and wave-based CBET models give the same result in simple interaction geometries (plane-wave beams, no caustics)

All of the approximations made in the ray model are satisfied in this configuration.
Speckled beams can transfer more energy than plane-wave beams with the same average intensity.

LPSE simulation of counter-propagating speckled beams

\( I_{\text{pump}} = 2 \times 10^{15} \text{ W/cm}^2, \ I_{\text{seed}} = 10^{12} \text{ W/cm}^2 \)
The CBET gain is sensitive to beam speckle for gains greater than \(~1\) and relative beam angles of less than \(~30^\circ\)

\[
\text{Gain} \equiv \log\left(\frac{\text{Seed energy out}}{\text{Seed energy in}}\right)
\]

CBET gain versus pump intensity for various beam-relative beam angles

- Plane wave (LPSE)
- Speckled beam (LPSE)
- Plane wave (rays)

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A good approximation to the average CBET between speckled beams can be obtained by using the linearity of Maxwell’s equations to solve for the correct unperturbed field amplitudes in the ray-based calculation.

LPSE simulation of counter-propogating speckled beams

\( I_{\text{pump}} = 2 \times 10^{15} \text{ W/cm}^2, \ I_{\text{seed}} = 10^{12} \text{ W/cm}^2 \)

Outgoing pump beam

\( \frac{e|E_z|}{mc\omega_0} \)

Outgoing seed beam

\( \frac{e|E_z|}{mc\omega_0} \)

Gain

\( \text{Gain}_{\text{LPSE}} = 5.8 \)

\( \text{Gain}_{\text{rays}} = 5.75 \)

\( \text{Gain}_{\text{plane wave}} = 3.5 \)
The speckled ray approach reproduces *LPSE* results to within one standard deviation of the average over realizations.

This could be a viable approach for including speckle effects in ray-based CBET models.
Speckled beams result in a modest decrease in laser absorption in OMEGA-scale, two-beam LPSE simulations at ICF-relevant plasma conditions.

OMEGA-scale LPSE simulations ($I_{\text{single beam}} = 2 \times 10^{14} \text{ W/cm}^2$) in a LILAC plasma profile (1.6 billion grid cells)

Plane-wave beams (86% absorption)

Speckled beams (82.8% absorption)
Polarization smoothing is accounted for in ray-based CBET models by multiplying the gain coefficient by a factor of $(1 + \cos^2 \theta)/4$.

The factor of $(1 + \cos^2 \theta)/4$ comes from assuming that the interacting beams have random relative polarizations with uncorrelated speckle patterns and ensemble averaging over realizations when deriving the ponderomotive potential of the beat wave.

\[
\langle \phi^2 \rangle_{PS} = \frac{1}{4}(1 + \cos^2 \theta) |\theta|^2
\]

Polarization smoothing is implemented on the NIF** by splitting the polarization within each quad.

**NIF: National Ignition Facility

The factor of \((1+\cos^2\theta)/4\) used to account for the modification of the CBET gain between beams with polarization smoothing is valid only when the speckle length is shorter than the interaction region.*

CBET gain versus relative beam angle for beams with polarization smoothing averaged over 12 realizations of polarization/phase

\((I_{\text{pump}} = 5 \times 10^{14} \text{ W/cm}^2, I_{\text{seed}} = 2 \times 10^{13} \text{ W/cm}^2)\)