Comparisons Between Ray- and Wave-Based Calculations of Cross-Beam Energy Transfer

R. K. FOLLETT, D. H. EDGELL, D. H. FROULA, V. N. GONCHAROV, I. V. IGUMENSHCHEV, J. G. SHAW, and J. F. MYATT
University of Rochester, Laboratory for Laser Energetics
J. W. BATES, K. OBENSCHAIN, and J. L. WEAVER
Naval Research Laboratory

Summary

A 3-D wave-based model has been developed to understand the physics of cross-beam energy transfer (CBET) in an inhomogeneous plasma

- Detailed CBET calculations are used to test ray-based CBET models that are implemented in hydrodynamics codes
- The comparisons generally highlight the accuracy of ray-based models
- Discrepancies between the models are found related to beam speckle and polarization smoothing

LPSE solves the time-enveloped Maxwell's equations coupled to a linearized plasma response

- Maxwell's equations (time enveloped) $\vec{E} = \Re\{\vec{E}(\vec{x},t)\exp(-i\omega_0 t)\}$

$$\boxed{\frac{2i\omega_0}{c^2}\frac{\partial}{\partial t}\vec{E} + \nabla^2\vec{E} - \nabla(\nabla\cdot\vec{E}) + \frac{\omega_0^2}{c^2}\varepsilon(\omega;\vec{x},t)\vec{E} = 0}$$

$$E(\omega_0;\vec{x},t) = 1 - \frac{\omega_{pe}^2(\vec{x},t)}{\omega_0(\omega_0 + i\nu_{ei})}$$

$$\omega_{pe}^2 = \frac{4\pi e^2 n_e(\vec{x},t)}{m_e}$$

- Plasma response

$$\left[\partial_t + \vec{U}_0(\vec{x})\cdot\nabla\right]\left(\frac{\delta n}{n_0}\right) = -W$$

$$\left[\partial_t + \vec{U}_0(\vec{x})\cdot\nabla + 2\hat{\nu}_{IAW}\right]W = -\nabla^2\left[c_s^2\left(\frac{\delta n}{n_0}\right) + \frac{e^2}{4m_e\omega_0^2}\left|\vec{E}\right|^2\right]$$

$$W \equiv \nabla\cdot\delta\vec{U}$$

$$n_e \equiv n_0(\vec{x}) + \delta n(\vec{x},t)$$

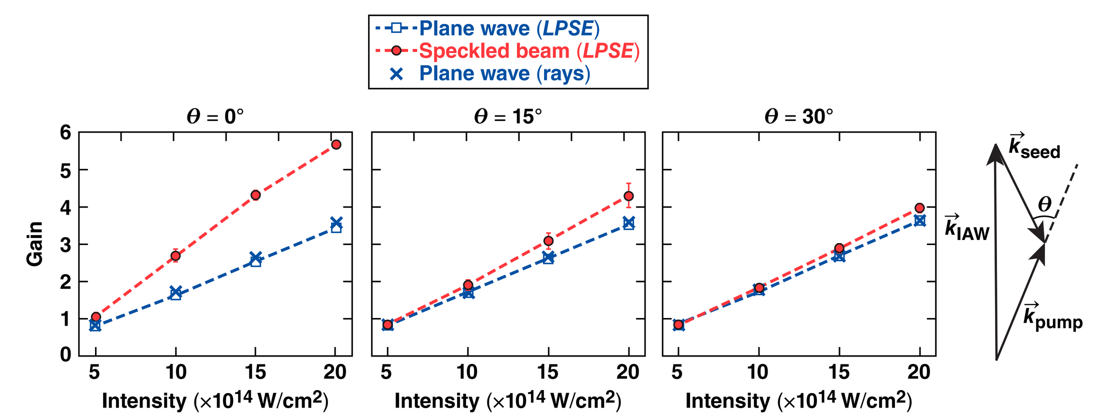
$$\vec{U} = \vec{U}_0(\vec{x}) + \delta\vec{U}(\vec{x},t)$$

E26192

The CBET gain is sensitive to beam speckle for gains greater than ~1 and relative beam angles of less than ~30°

$$\text{Gain} \equiv \log\left(\frac{\text{Seed energy out}}{\text{Seed energy in}}\right)$$

CBET gain versus pump intensity for various beam-relative beam angles



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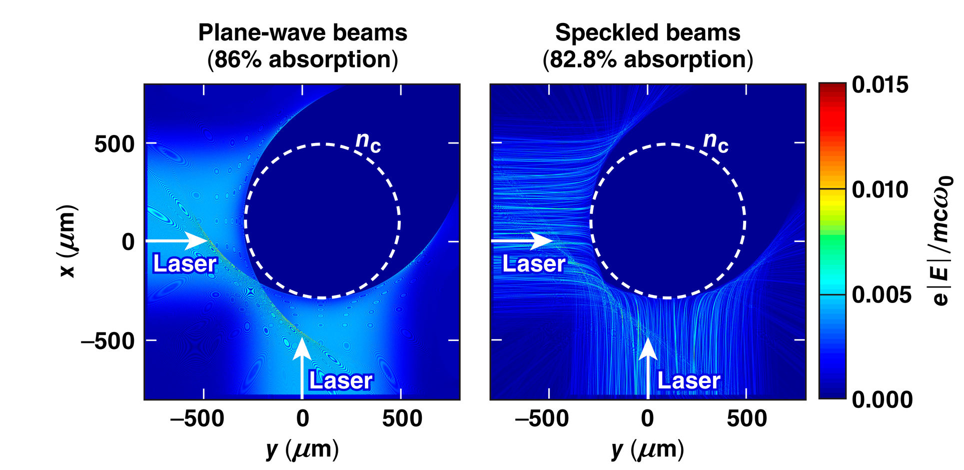
Speckled beams result in a modest decrease in laser absorption in OMEGA-scale, two-beam *LPSE* simulations at ICF-relevant plasma conditions

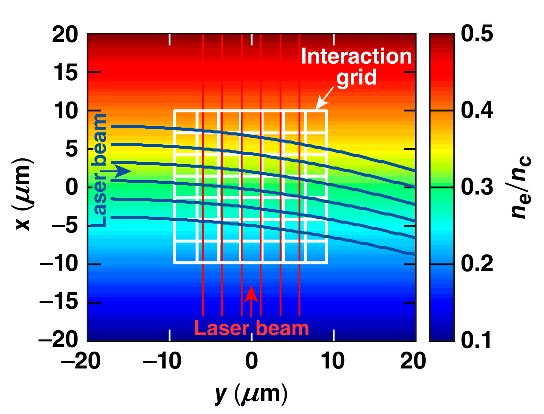
OMEGA-scale *LPSE* simulations ($I_{\text{single beam}} = 2 \times 10^{14}$ W/cm^2) in a *LILAC* plasma profile (1.6 billion grid cells)



E26199

Ray-based models calculate CBET by considering pairwise interactions between rays

Schematic of ray-interaction calculation



CBET interaction between plane waves in a homogeneous plasma

$$\frac{dI_i}{ds} = \sum_j I_i L_{ij}^{-1}$$

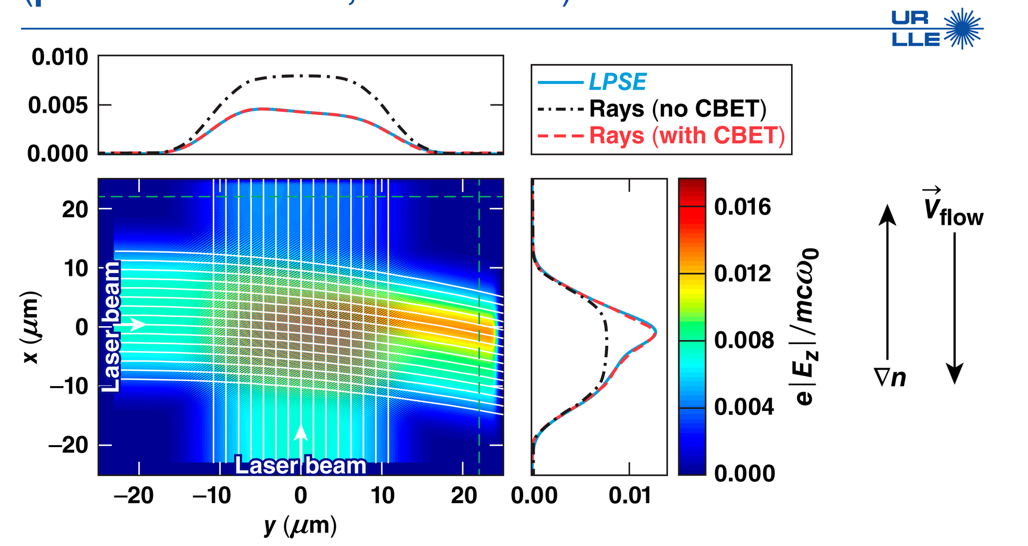
$$L_{ij}^{-1} = 5.88\times10^{-2}\frac{I_j\lambda}{T_e\left(1+3T_i/ZT_e\right)}\frac{n_e}{n_c}\frac{\omega_s}{\nu_i}P(\eta_{ij})$$

$$P(\eta) = \frac{\nu_i^2\eta}{(\eta^2-1)^2+\nu_i^2\eta^2}$$

$$\eta_{ij} = \frac{\omega_j - \omega_i - (\vec{k}_j - \vec{k}_i)\cdot\vec{u}}{\omega_s}$$

E25678a

Ray- and wave-based CBET models give the same result in simple interaction geometries (plane-wave beams, no caustics)

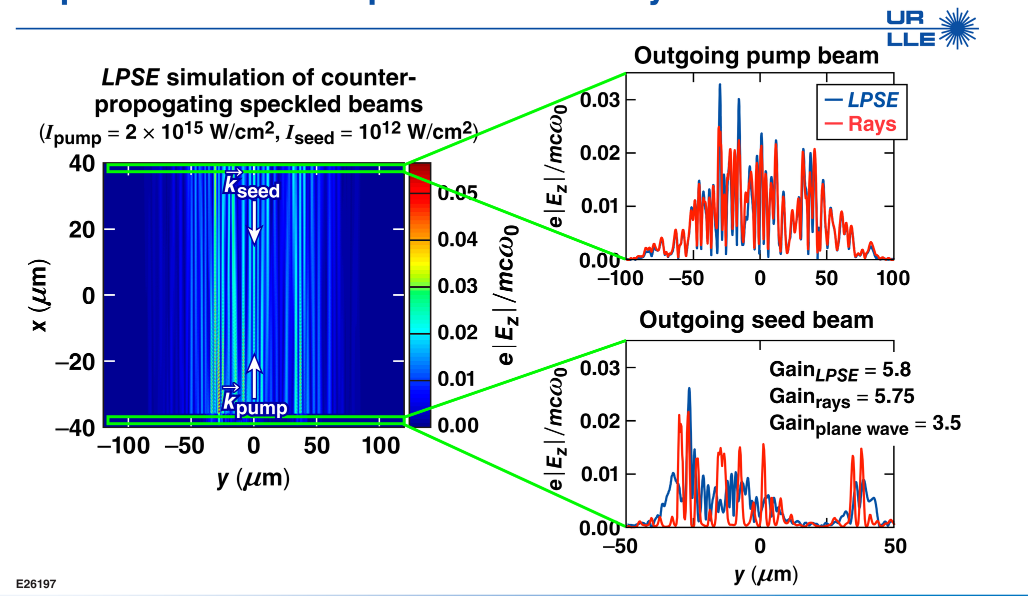


All of the approximations made in the ray model are satisfied in this configuration.

E26194

A good approximation to the average CBET between speckled beams can be obtained by using the linearity of Maxwell's equations to solve for the correct unperturbed field amplitudes in the ray-based calculation

LPSE simulation of counter-propagating speckled beams ($I_{\text{pump}} = 2\times10^{15}$ W/cm^2, $I_{\text{seed}} = 10^{12}$ W/cm^2)



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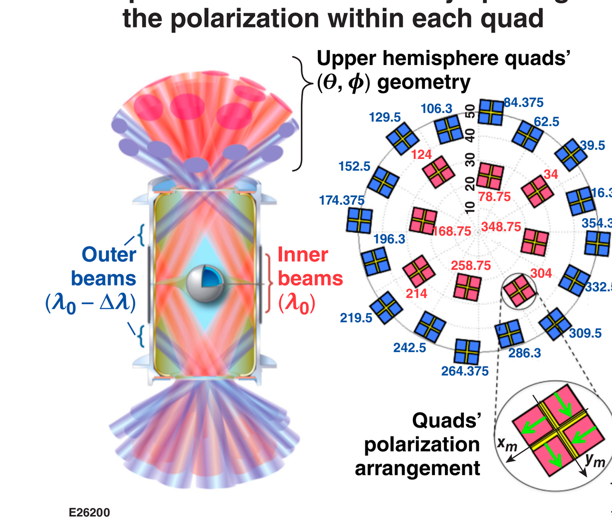
Polarization smoothing is accounted for in ray-based CBET models by multiplying the gain coefficient by a factor of $(1+\cos^2\theta)/4$*

Polarization smoothing is implemented on the NIF** by splitting the polarization within each quad



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$$\left\langle|\phi|^2\right\rangle_{PS} = \frac{1}{4}\left(1+\cos^2\theta\right)|\theta|_{\parallel}^2$$

E26200

*P. Michel *et al.*, Phys. Plasmas 20, 056308 (2013).
**NIF: National Ignition Facility

Ray-based CBET models make several approximations that are not always satisfied in inertial confinement fusion (ICF) applications

- Ion-acoustic waves (IAW's) ($\delta_n/n \ll 1$)
- Steady-state convective gain
- Polarization-averaged coupling constant
- Pairwise coupling between beams
- Local plane-wave approximation
  - not valid for speckled beams or at caustics
- Eikonal approximation [Wentzel–Kramers–Brillouin (WKB), envelope]
  - breaks down at caustics

Approximations that are not made in *LPSE*

E26191

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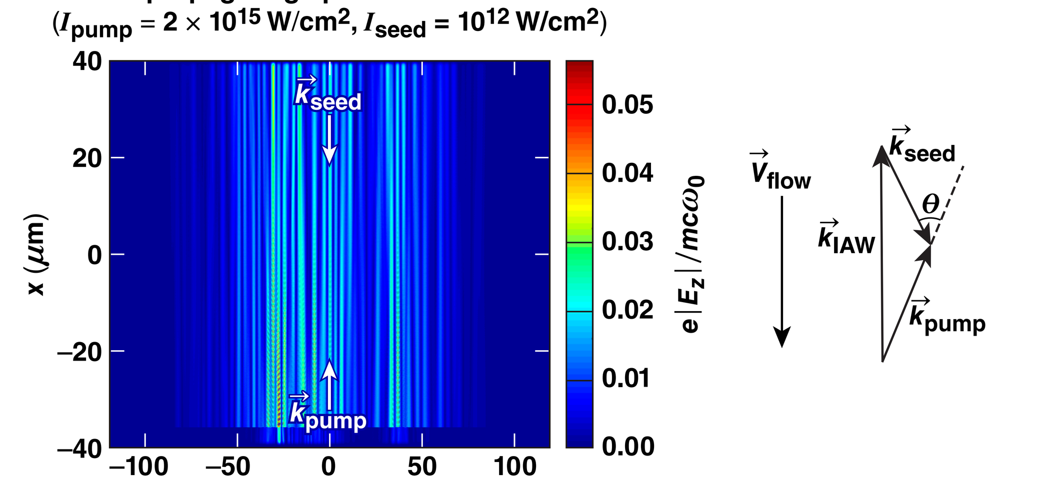

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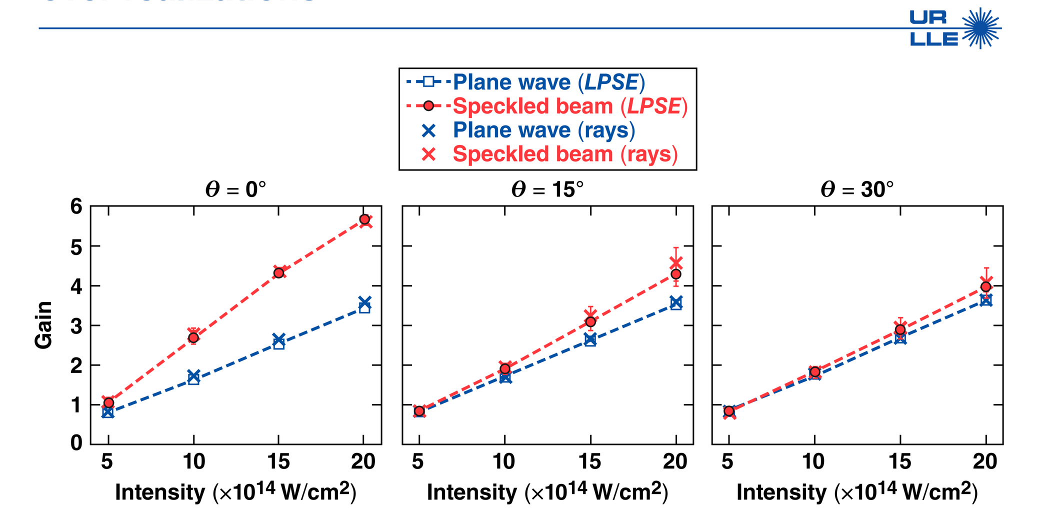
The speckled ray approach reproduces *LPSE* results to within one standard deviation of the average over realizations



This could be a viable approach for including speckle effects in ray-based CBET models.

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The factor of $(1+\cos^2\theta)/4$ used to account for the modification of the CBET gain between beams with polarization smoothing is valid only when the speckle length is shorter than the interaction region*

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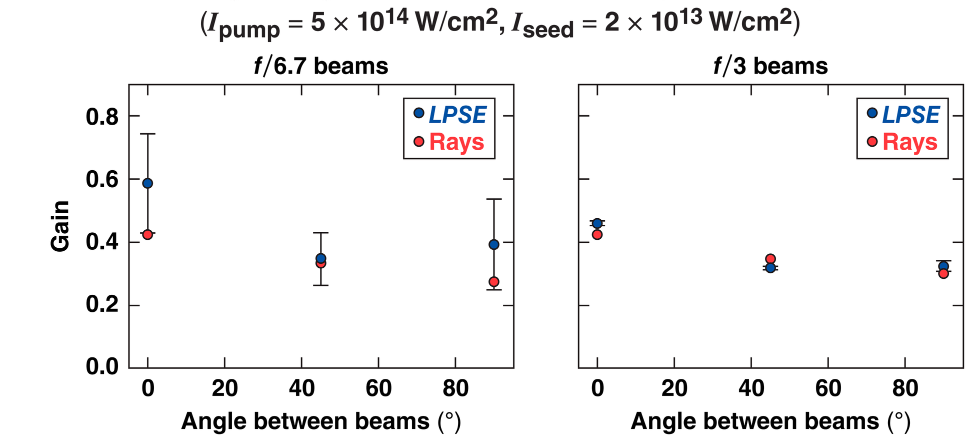

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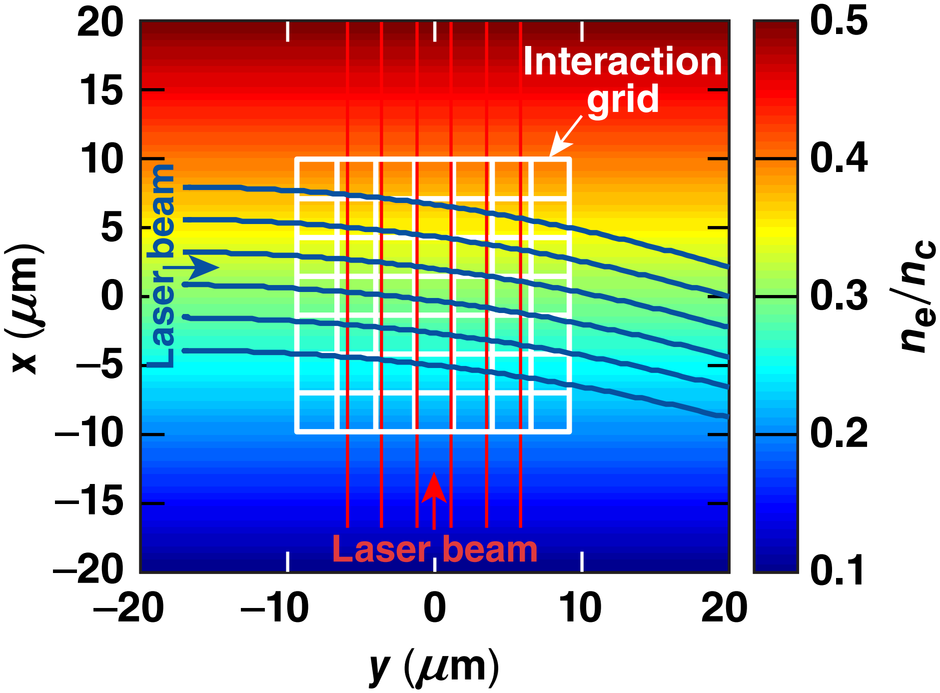

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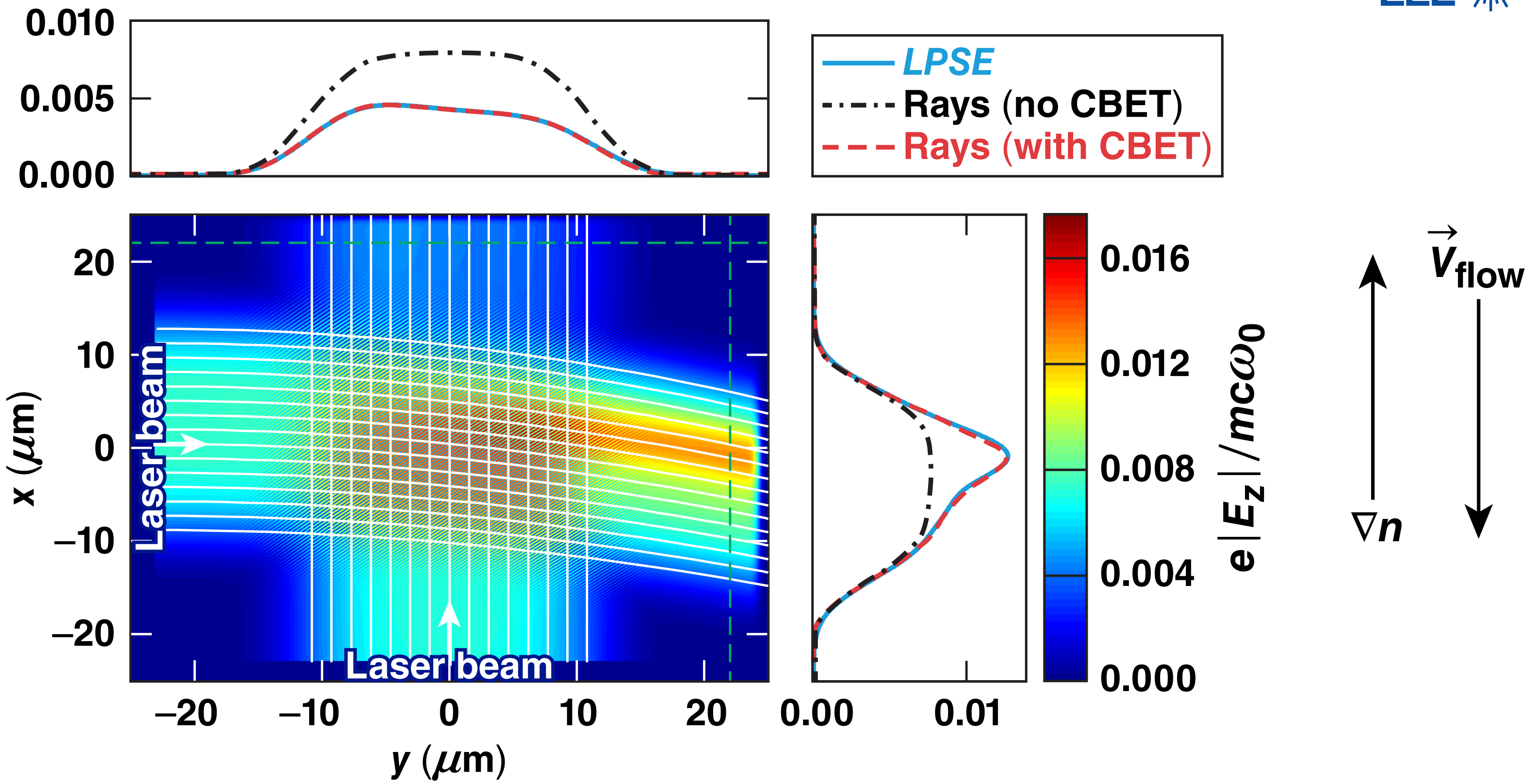

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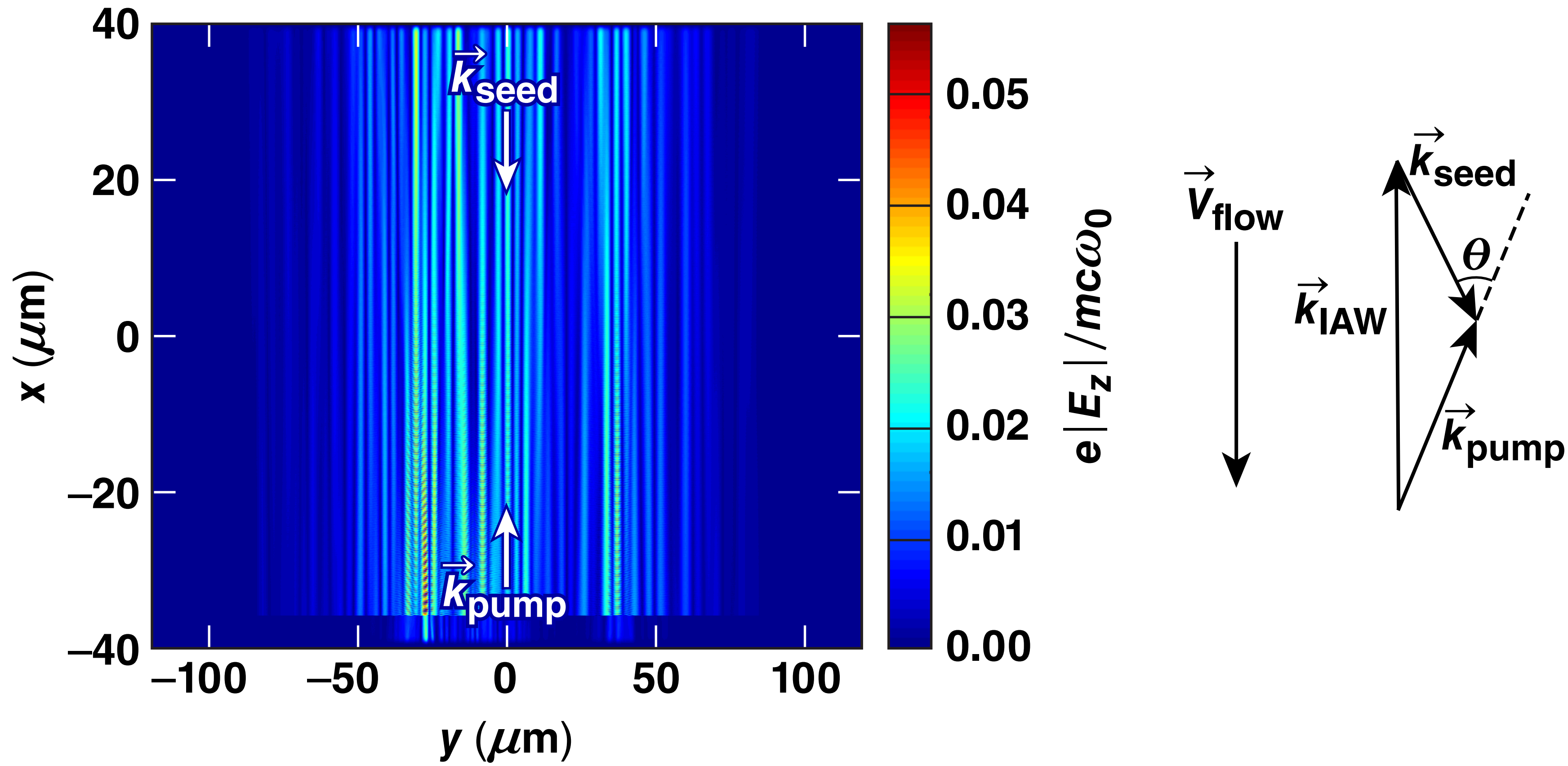

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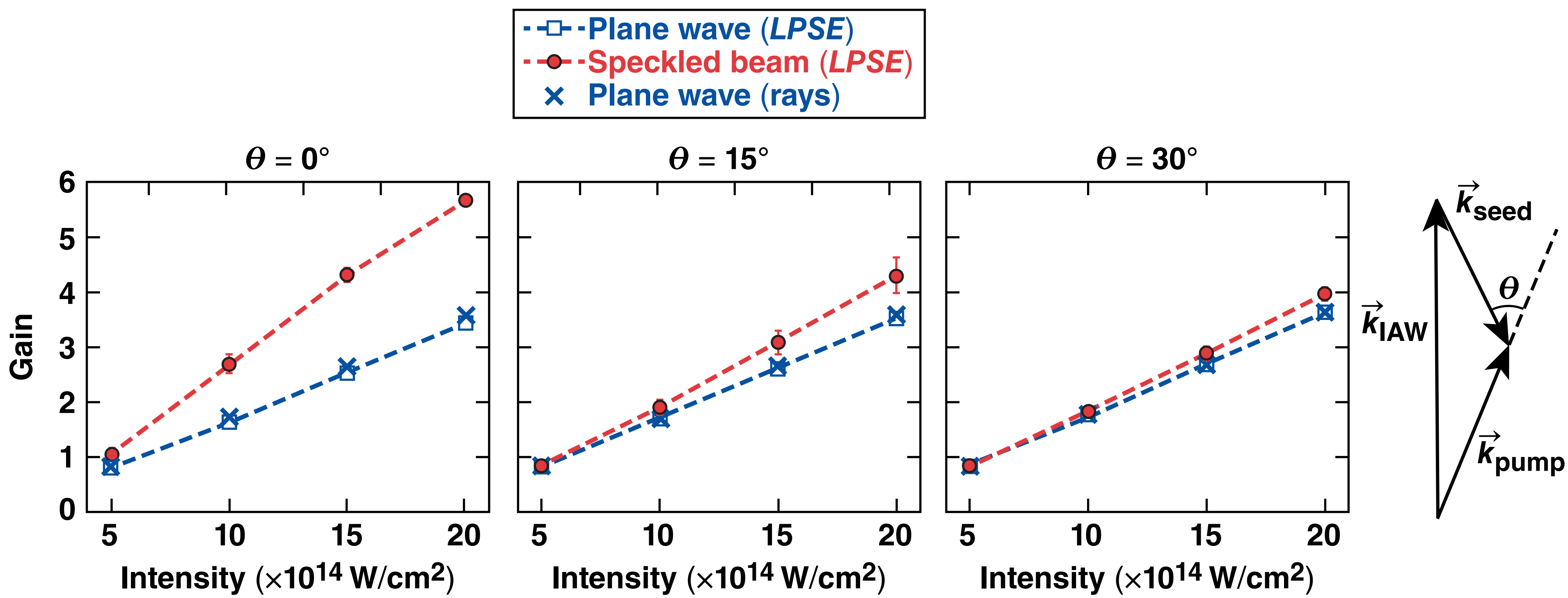

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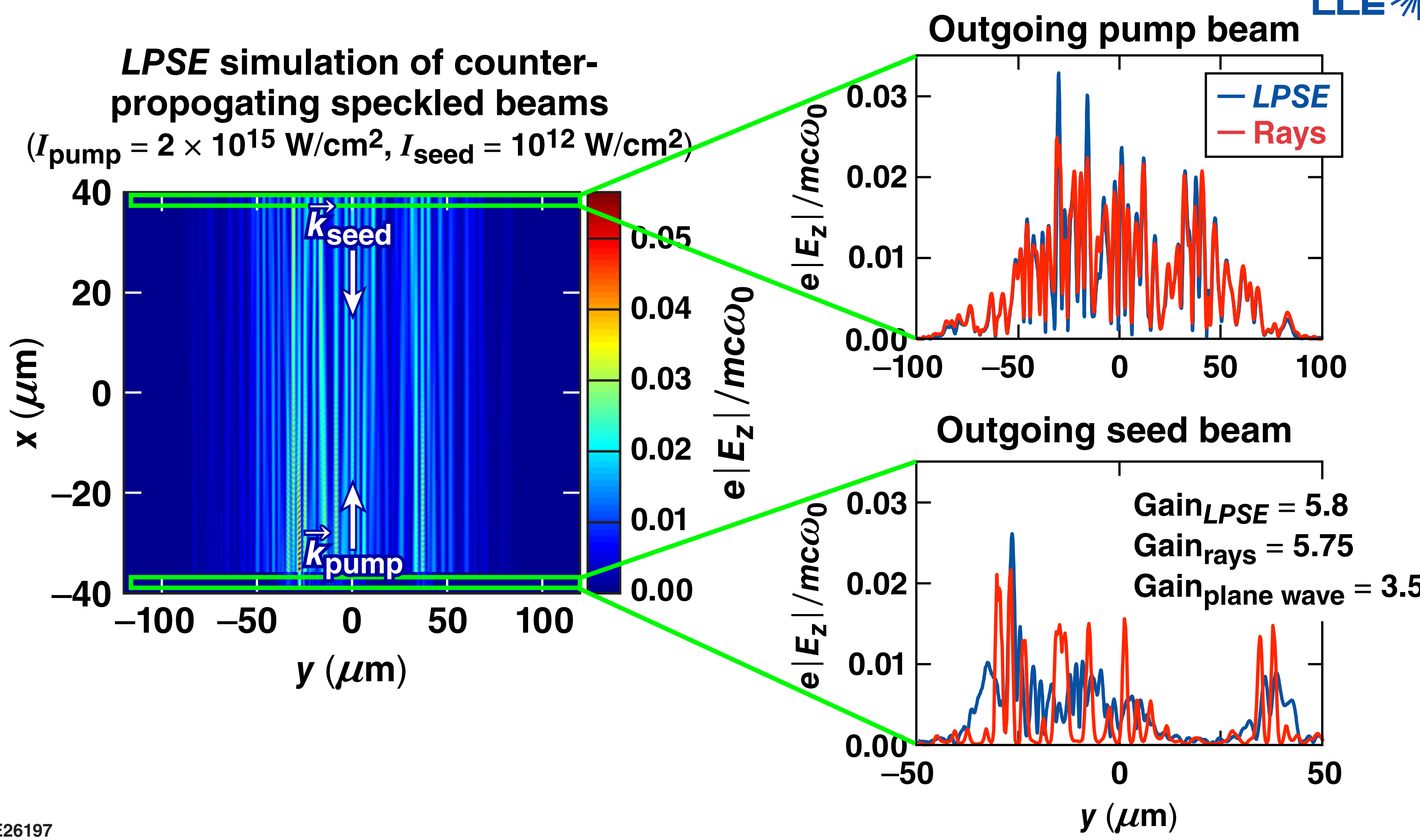

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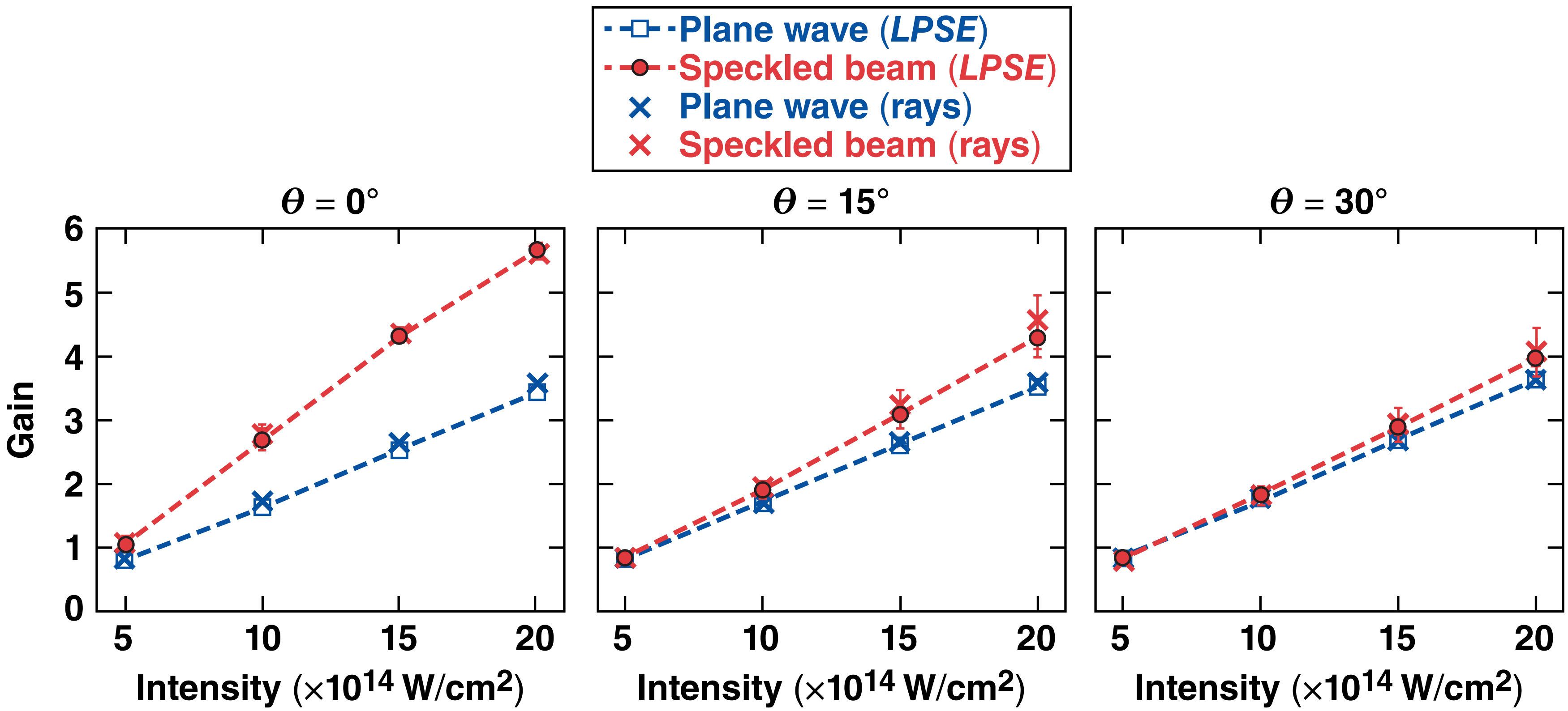

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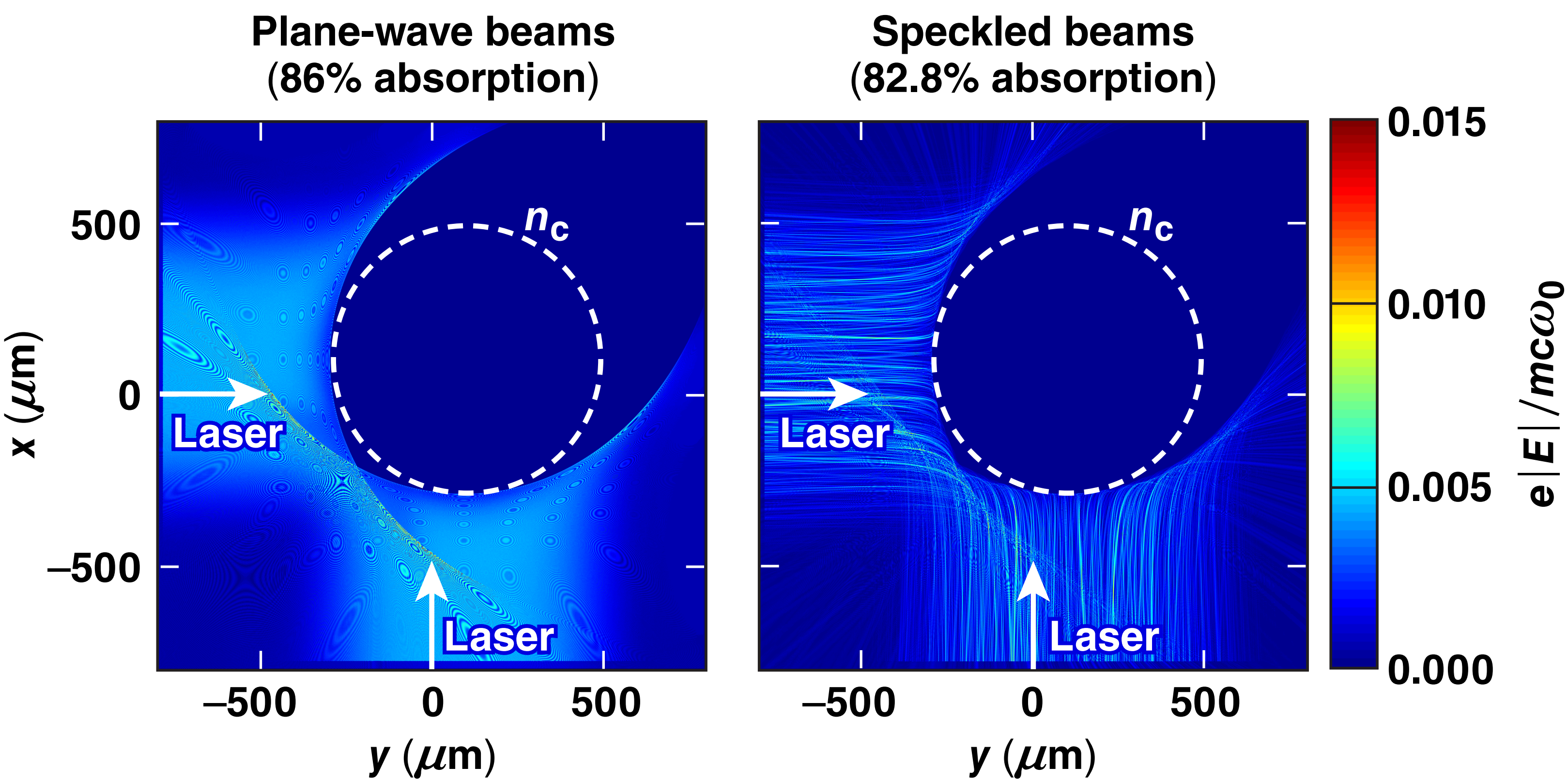

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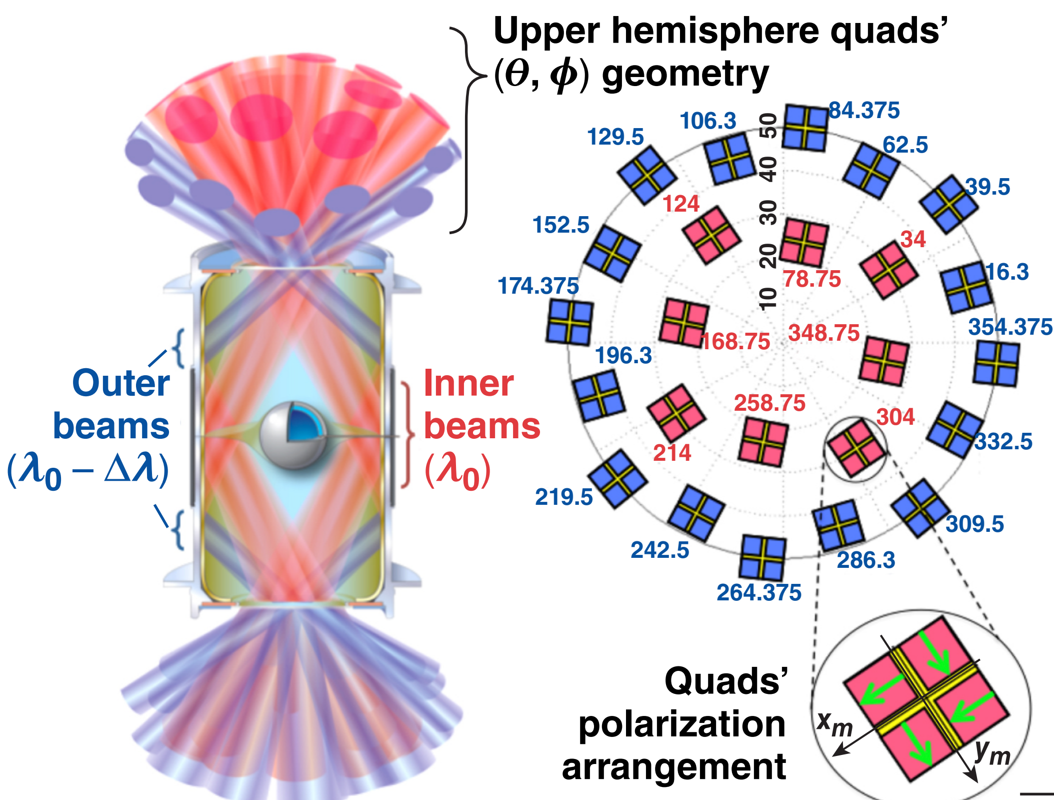

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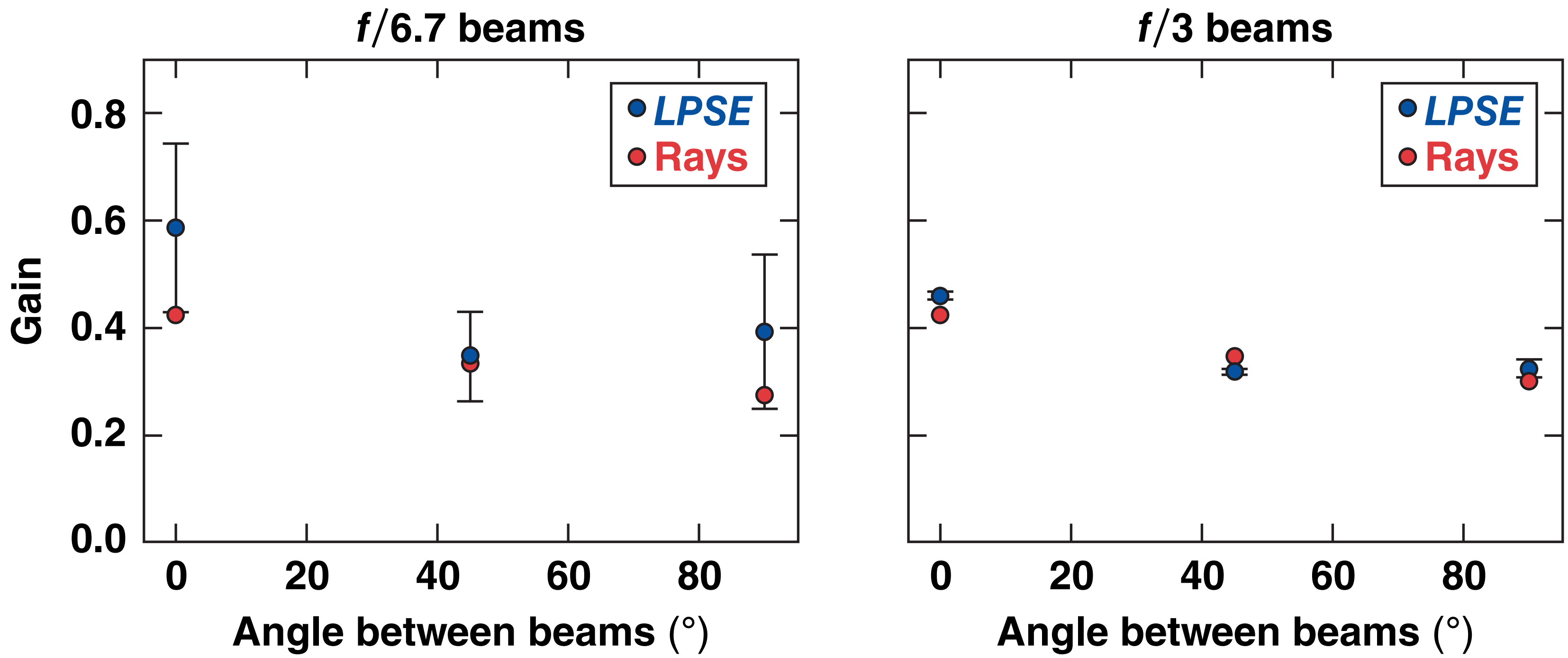

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