Non-LTE Speed of Sound, Irreversibility, and Thermodynamic Consistency

Aluminum at one-tenth solid density

\[ \gamma_1 = \frac{5}{3} \]

\[ \gamma_1 = \frac{4}{3} \]

\[ T \text{ (eV)} \]

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Summary

Non-LTE modification of the speed of sound in a plasma follows the modification of the ionization

- The speed of sound in radiating plasma is obtained from a self-consistent collisional-radiative (CR) model based on the non-equilibrium thermodynamics of irreversible radiation emission.

- The speed of sound and the adiabatic indices are modified through CR effects on the equation of state.

- Time-dependent ionization kinetics causes dispersion and damping of the sound waves.

- The model is formulated in terms of a single ionization transition, but it accepts equivalent ionization parameters from larger models.
Outline

- Thermodynamics, nonequilibrium and irreversibility
- Modification of adiabatic compressibility
- Results: adiabatic index and the speed of sound
- Time-dependent kinetics, dispersion, and damping
Non-LTE EOS effects appear as modified adiabatic indices and modified sound speed in many applications

- R. M. More et al.* have considered the response of multilevel atomic systems to be small departures from radiative LTE.

- Shock compression is sensitive to $\delta \gamma$, e.g., maximum $\rho_2/\rho_1 = (\gamma + 1)/(\gamma - 1)$.

- Astrophysical self-gravitating objects (or layers) collapse when ionization drives $\gamma$ below 4/3.

- Sensitive ionization transitions occur below the “CRE density” (e.g., 1/10 solid for Al, near solid for Ti) in expanding plasmas in lab astrophysics, z-pinches, x-ray lasers, interpulse ablation coronas, etc.

- Irreversible relaxation is due to finite relaxation times (even in LTE) damp and disperse sound waves and other transient, particularly near $\omega \tau_s \sim 1$.

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The thermodynamics of an ideal plasma is modified to include entropy production due to irreversible radiation

Thermodynamics: \( dE = dQ - pdV \), \( TdS = dE + pdV - AdN^+ \)

Ideal gas: \( pV = NkT \), \( E = \frac{3}{2}NkT + \int_0^{N^+} \chi(n)dn \)

Model I: 2-species ionization

\[
\begin{align*}
\frac{dN^+}{dt} &= K_f N^0 - K_r N^+ = 0 \\
A &= kT \ln \left( 1 + \frac{R_r}{n_eC_r} \right)
\end{align*}
\]

\( e^- + I^2 \rightarrow 2e^- + I^{z+1} : K_f = n_eC_f \)

\[
\begin{align*}
2e^- + I^{z+1} &\rightarrow e^- + I^z \\
e^- + I^{z+1} &\rightarrow hv + I^z \quad : K_r = n_e2C_r + n_eR_r
\end{align*}
\]

Model II: phenomenological

\( A = BkT \)
The adiabatic compression index and the ionization law are constrained by thermodynamic consistency

1st law: \( \gamma_1 = \frac{5/2 + \eta_T (5/2 + \chi/kT) + \eta_V (\chi/kT)}{3/2 + \eta_T (3/2 + \chi/kT)} \); \( p \propto \rho \gamma_1 \)

\[ \eta_T = \frac{T}{N^+} \left( \frac{\partial N^+}{\partial T} \right)_V, \quad \eta_V = \frac{V}{N^+} \left( \frac{\partial N^+}{\partial V} \right)_T \]

Entropy as function of state: \( \frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T} \) constrains \( \eta_T \) and \( \eta_V \)

Model I: 2-species ionization \( \eta_T/\eta_V = (3/2 + \chi/kT)(1 + f) - f/2, \quad f = R_r/(n_e C_r) \)

Model II: phenomenological \( \eta_T/\eta_V = 3/2 + \chi/kT \)

Speed of sound: \( c_S^2 = \gamma_1 \frac{p}{\rho} \)

* Cf. J. P. Cox and R. T. Giuli (1968)
Non-LTE modification of the plasma compressibility follows the modified ionization temperature dependence.

Aluminum at one-tenth solid density

Average ionization

Adiabatic index $\gamma_1$

$\gamma_1 = 5/3$

$\gamma_1 = 4/3$
Non-LTE modification of the plasma speed of sound reflects the corresponding modification of the compressibility

Aluminum at one-tenth solid density

Adiabatic index $\gamma_1$

\[ T \text{ (eV)} \]

\[ \gamma_1 = 5/3 \]

\[ \gamma_1 = 4/3 \]

Speed of sound

\[ C_s \text{ (\mu m/ns)} \]

\[ T \text{ (eV)} \]

\[ \gamma_1 = 5/3 \]

\[ \gamma_1 = 4/3 \]
Non-LTE modification of the plasma compressibility follows the modified ionization temperature dependence.

Carbon at one-tenth critical density \((z \equiv 5)\)

**Average ionization**

- LTE
- CRE

**Adiabatic index \(\gamma_1\)**

- \(\gamma_1 = 5/3\)
- \(\gamma_1 = 4/3\)
Non-LTE modification of the plasma speed of sound reflects the corresponding modification of the compressibility.

Carbon at one-tenth critical density ($z = 5$)

**Adiabatic index $\gamma_1$**

- $\gamma_1 = 5/3$
- $\gamma_1 = 4/3$

**Speed of sound**

- $\gamma_1 = 5/3$
- $\gamma_1 = 4/3$
The finite plasma equilibration time introduces sound wave dispersion and damping

\[ \frac{dN^+}{dt} = n_e C_f N^0 - \left(n_e C_r + n_e R_r\right) N^+ \approx 0 \]

Apply method of R. Haase (1969) in LTE

Adiabatic relaxation time:

\[ \tau_S^{-1} = \left[ \frac{\partial}{\partial N^+} \left( \frac{dN^+}{dt} \right) \right]_{s,v} = n_e C_f \left[ \frac{1 + 4z}{2z} + \frac{(3/2 + \chi/kT)^2}{3(1 + z)} \right] \]

\[ n_e C_f = 1.75 \, \text{ps}^{-1} \left( \frac{kT}{500 \, \text{eV}} \right)^{-3/2} \left( \frac{n_e}{n_c} \right) \exp \left( -\chi/kT \right) \]

Sound wave:

\[ N^+ - N_0^+ \propto \exp \left[ i(\omega t - kx) \right] \]

\[ c_S^2 = \frac{c_A^2 + i\omega \tau_S c_N^2}{1 + i\omega \tau_S} \]

\[ \omega \tau_S \ll 1, \quad c_S = c_A = \sqrt{\frac{\gamma_1 p}{\rho}} \]

\[ \frac{c_S k}{\omega} \approx 1 - \frac{i\omega \tau_S}{2} \left( \frac{5}{3\gamma_1} - 1 \right) \]

\[ \omega \tau_S \gg 1, \quad c_S = c_N = \sqrt{\frac{5p}{3\rho}} \]

\[ \frac{c_S k}{\omega} \approx 1 - \frac{i}{2\omega \tau_S} \left( 1 - \frac{3\gamma_1}{5} \right) \]

* Cf. Zel’dovich and Raizer II (1967); A. H. Nelson and M. G. Haines (1969)
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