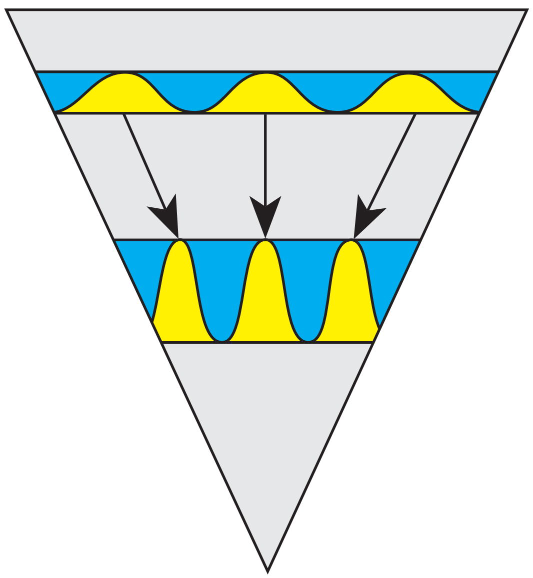
On the Bell–Plesset Effects: the Effects of Uniform Compression and Geometrical Convergence on the Classical Rayleigh–Taylor Instability

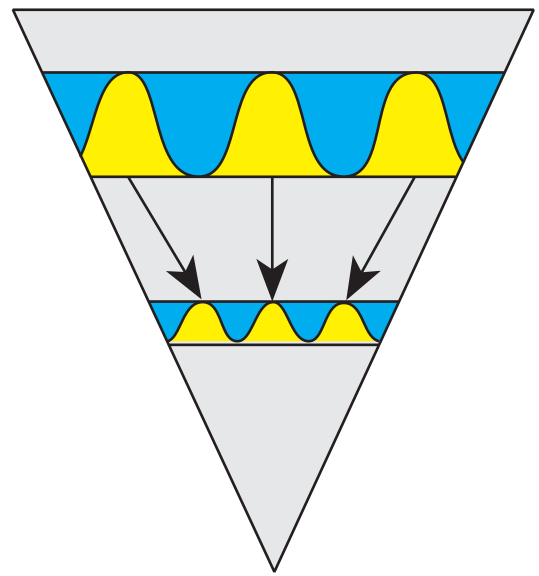
Incompressible shell

$$\frac{d}{dt}\rho = 0$$



Compressing core

$$\frac{d}{dt}\left(\rho R^{3}\right) = 0$$



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Summary

Bell–Plesset effects are formally very simple but varied in their behavior

- The classical Rayleigh–Taylor problem has been modified for arbitrary rates of compression and geometrical convergence in a way that clarifies Bell–Plesset effects on the density and radius scaling of perturbations and on stability criteria.
- Bell–Plesset effects are different for each of the pair of independent perturbation solutions.
- In the opposite limits of fast and slow Rayleigh–Taylor growth, the perturbation amplitudes scale in completely different ways.
- For fast Rayleigh–Taylor growth – the limit of interest to the deceleration phase of ICF – the scaling is simplified.
- In this limit, spatial perturbations of the surface of a uniformity compressing cylinder, or sphere, exhibit no explicit Bell–Plesset effects.

Outline

- Background
- Modified classical perturbation equation
- Illustrations
- Scaling limits
- Conclusions

Bell's introduction to convergence and compression effects based on a mass amplitude was the preferable approach

- Bell's formulation (1951) uses mass amplitudes to obtain perturbation equations for free surfaces ($\rho_1 = 0$ or $\rho_2 = 0$).
- Plesset (1954) treats only the spherical interface with no compression with an arbitrary density jump, but he uses spatial displacement amplitudes.
- Goncharov *et al.* (2000) and their Rayleigh–Taylor post-processor based on the "sharp boundary model" includes Bell–Plesset effects.

Bell–Plesset effects were added to the language of the ICF community relatively recently

1. G. I. Bell, Los Alamos Scientific Laboratory, Report LA-1321 (1951).
2. M. S. Plesset, J. Appl. Phys. 25, 96–98 (1954).
3. J. B. Beck, “The Effects of Convergent Geometry on the Ablative Rayleigh–Taylor Instability in Cylindrical Implosions,” Ph.D. Thesis, Purdue University (1996).
4. W. Hsing, C. W. Barnes, J. B. Beck, N. M. Hoffman, D. Galmiche, A. Richard, J. Edwards, P. Graham, S. Rothman, and B. Thomas, Phys. Plasmas 4, 1832–1840 (1997).
5. V. N. Goncharov, P. W. McKenty, and S. Skupsky, Phys. Plasmas 7, 5118–5139 (2000).
6. P. Amendt, J. D. Colvin, J. D. Ramshaw, H. F. Robey, and O. L. Landen, Phys. Plasmas 10, 820–829 (2003).

Perturbation equations are best written in terms of a mass amplitude

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Incompressible planar approximation

$$\frac{d^2}{dt^2} A_k = \gamma_0^2 \, A_k \qquad \gamma_\pm = \pm\gamma_0$$

$$A_{k\pm} = A_{k0} \, e^{\gamma_\pm t} \qquad \gamma_0^2 = k\left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right)\ddot{R}$$

Compressible spherical solution (i.e., Bell–Plesset*)

$$\left(-\gamma_\rho - \gamma_R + \frac{d}{dt}\right)\frac{d}{dt}\left(A_\ell \rho R^2\right) = \gamma_0^2 \left(A_\ell \rho R^2\right) \qquad \gamma_R = \dot{R}/R, \gamma_\rho = \dot{\rho}_1/\rho_1 = \dot{\rho}_2/\rho_2$$

$$\gamma_0^2 = \frac{\ell(\ell+1)}{R} \frac{(\rho_2 - \rho_1)\,\ddot{R}}{\left[\ell\rho_2 + (\ell+1)\rho_1\right]} \qquad \gamma_\pm = \frac{1}{2}\left(\gamma_\rho + \gamma_R\right) \pm \sqrt{\gamma_0^2 + \frac{1}{4}\left(\gamma_\rho + \gamma_R\right)^2}$$

*G. I. Bell, Los Alamos National Laboratory, Report No. LA-1321 (1951).
M. S. Plesset, J. Appl. Phys. 25 (1), 96-98 (1954).

Perturbation equations are very similar among the three simple geometries

$\gamma_R = \dot{R}/R,\ \gamma_\rho = \dot{\rho}/\rho$	γ_0^2	$\gamma_\pm$
Planar $\left(-\gamma_\rho + \frac{d}{dt}\right)\frac{d}{dt}(A_k\rho) = \gamma_0^2 (A_k\rho)$	$k\left(\frac{\rho_2-\rho_1}{\rho_2+\rho_1}\right)\ddot{R}$	$\frac{1}{2}\gamma_\rho \pm \sqrt{\gamma_0^2 + \frac{1}{4}\gamma_\rho^2}$
Cylindrical $\left(-\gamma_\rho + \frac{d}{dt}\right)\frac{d}{dt}(A_\ell\rho R) = \gamma_0^2 (A_\ell\rho R)$	$\frac{\ell}{R}\left(\frac{\rho_2-\rho_1}{\rho_2+\rho_1}\right)\ddot{R}$	$\frac{1}{2}\gamma_\rho \pm \sqrt{\gamma_0^2 + \frac{1}{4}\gamma_\rho^2}$
Spherical $\left(-\gamma_\rho - \gamma_R + \frac{d}{dt}\right)\frac{d}{dt}\left(A_\ell\rho R^2\right) = \gamma_0^2 \left(A_\ell\rho R^2\right)$	$\frac{\ell(\ell+1)}{R}\,\frac{(\rho_2-\rho_1)\,\ddot{R}}{[\ell\rho_2+(\ell+1)\rho_1]}$	$\frac{1}{2}(\gamma_\rho+\gamma_R) \pm \sqrt{\gamma_0^2 + \frac{1}{4}(\gamma_\rho+\gamma_R)^2}$

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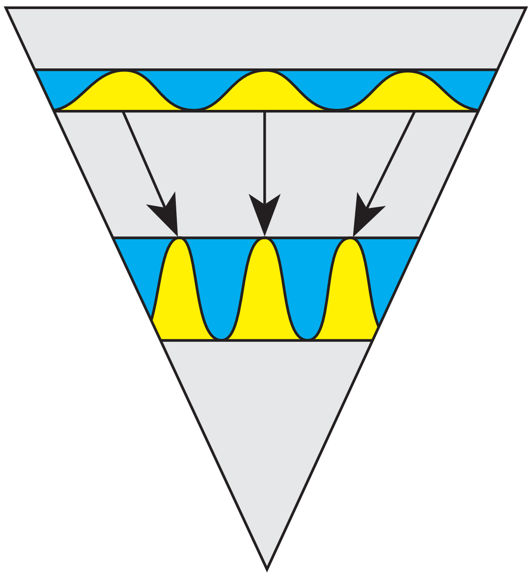
The solution of constant mass amplitude occurs only as one solution in the "accelerationless" limit

Incompressible shell



$$\frac{d\rho}{dt} = 0 \xrightarrow{?} A \propto R^{-2}$$

$$A = \left(2A_0 + \dot{A}_0/\gamma_R\right)\left(R_0/R\right) - \left(A_0 + \dot{A}_0/\gamma_R\right)\left(R_0/R\right)^2$$

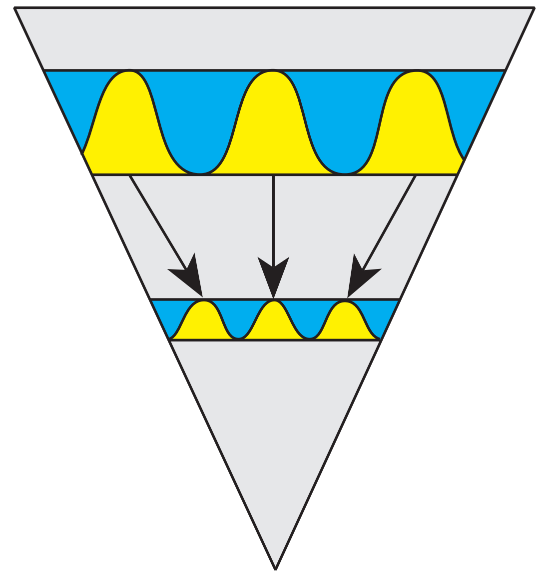
$$\frac{d}{dt}\left(A\rho R^2\right) = 0$$

$$\gamma_0 = 0$$

$$R = R_0 e^{\gamma_R t}$$

$$\gamma_R \equiv \dot{R}/R$$

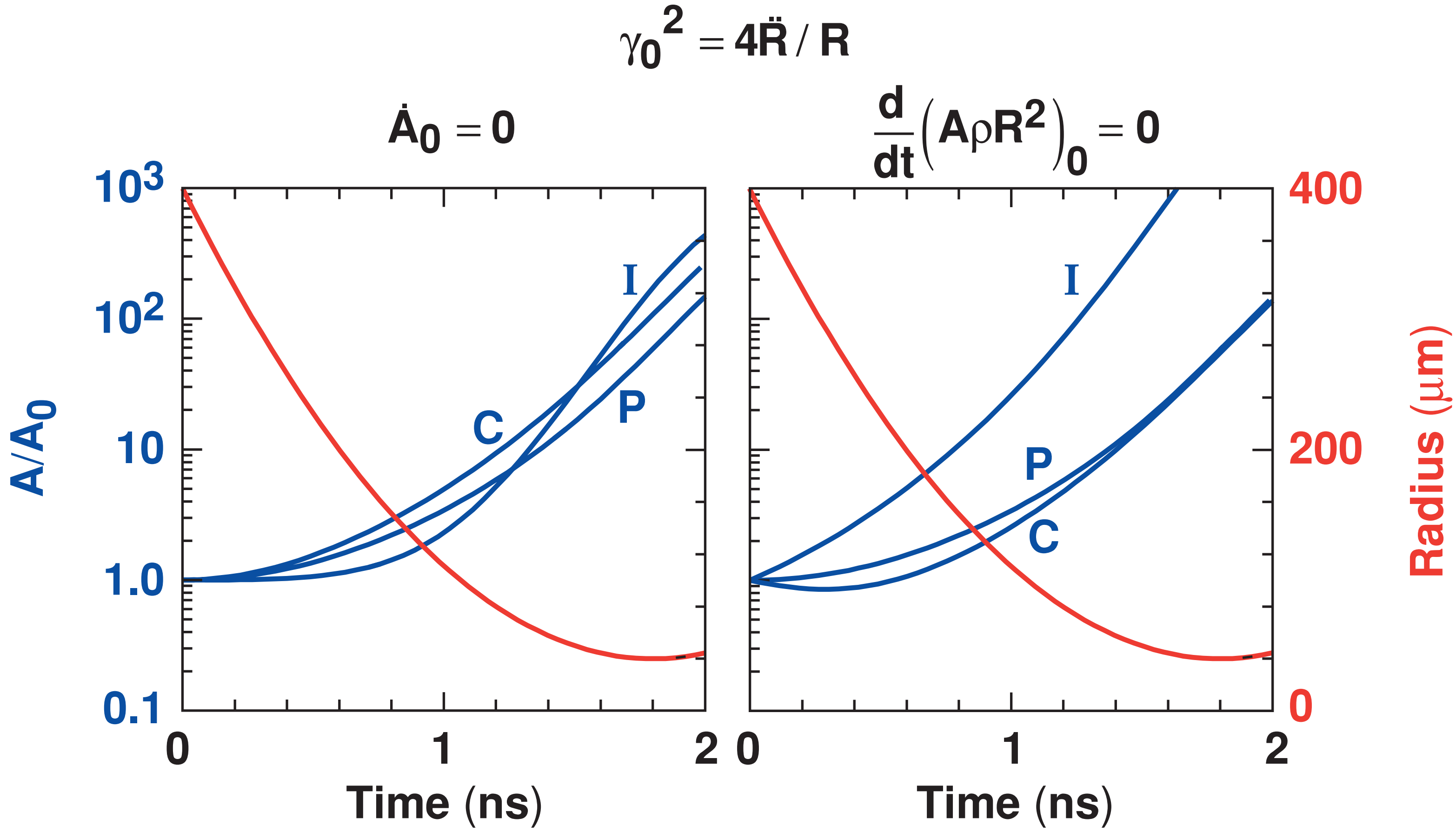
Compressing core



$$\frac{d}{dt}\left(\rho R^3\right) = 0 \xrightarrow{?} A \propto R$$

$$A = \frac{1}{2}\left(A_0 - \dot{A}_0/\gamma_R\right)\left(R_0/R\right) + \frac{1}{2}\left(A_0 + \dot{A}_0/\gamma_R\right)\left(R/R_0\right)$$

Compression and convergence complicate the time dependence of perturbations of decelerating interfaces



I = incompressible shell
C = compressible sphere
P = incompressible planar

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Bell-Plesset effects appear as density-radius scaling factors in the limits of fast and slow Rayleigh–Taylor growth

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I. The "accelerationless" limit of small γ_0 $\left(\gamma_0 \ll \gamma_\rho,\ \ \gamma_0 \ll \gamma_R\right)$, the "coasting" phase:

$$A_\pm = \left[1,\ \rho^{-1}\right], \qquad A_\pm = \left[R^{-1},\ (\rho R)^{-1}\right], \qquad \text{and } A_\pm = \left[R^{-1},\ \left(\rho R^2\right)^{-1}\right]$$

- The second solutions are indistinguishable from unperturbed flow.

II. The large ℓ or WKB limit, $\left(\gamma_0 \gg \gamma_\rho,\ \ \gamma_0 \gg \gamma_R\right)$, the "deceleration" phase:

$$A_\pm = \left[\rho^{-1/2}, \qquad \rho^{-1/2} R^{-1}, \qquad \rho^{-1/2} R^{-3/2}\right] \gamma_0^{-1/2} \exp\left(\pm \int^t \gamma_0 \, dt'\right)$$

- For uniform compression of a constant cylindrical or spherical mass,

$$A_{M\pm} = \gamma_0^{-1/2} \exp\left(\pm \int^t \gamma_0 \, dt'\right),$$ showing no explicit Bell–Plesset effects.

- Bell–Plesset effects are not an instability and do not affect stability criteria.

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