Hydrodynamic Instabilities from the Beginning to the End

Implosion at dawn

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Implosion at sunset

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Acceleration or deceleration, this is the question!

Shock transit and acceleration phase

- RM and RT growth
- RT and RM seeding:
  - outer-surface nonuniformities
  - inner-surface nonuniformities (feedout)
  - laser imprintsings

Reflected shock transit and deceleration phase

- RM and RT growth
- RT and RM seeding:
  - outer-surface nonuniformities (feedout)
  - inner-surface nonuniformities (feedthrough)

Why all this theory is so incredibly useful!
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- Keeps theorists employed.
- Improves physical understanding.
- Provides feedback to numerical simulations.
- Leads to the development of a fast, reliable, and accurate postprocessor of 1-D codes. Target design can be carried out using such a postprocessor and subsequently refined using 2-D simulations.
**Acceleration Phase**

The classical RT is just Newton’s law at work: \( F = ma! \)

\[
F = S(P_h - P_\ell) = ma = \rho_h \lambda S \ddot{\eta}
\]

\[
\frac{dP_0}{dx} = \rho_0 g = \begin{cases} 
\rho_h g & \text{heavy} \\
\rho_\ell g & \text{light} 
\end{cases}
\]

\[
P_\ell = P_0 + \left[ \frac{dP_0}{dx} \right]_\ell \ddot{\eta} = P_0 + \rho_\ell g \ddot{\eta}
\]

\[
P_h = P_0 + \left[ \frac{dP_0}{dx} \right]_h \ddot{\eta} = P_0 + \rho_h g \ddot{\eta}
\]

\[
F = ma \rightarrow S \rho_h g \ddot{\eta} = \rho_h \lambda S \ddot{\eta} \rightarrow \ddot{\eta} = k g \ddot{\eta} \rightarrow \ddot{\eta} \sim e^{\gamma t} \rightarrow \gamma = \sqrt{kg}
\]

\[
k \sim 1/\lambda
\]
The ABLATIVE RT is just Newton’s law at work again but with a restoring force: the dynamic pressure

- Newton’s law
  \[ S[P_h - (P_\ell + \rho_\ell V_b^2)] = \rho_h \lambda S \ddot{\eta} \]

- Energy balance
  \[ pV_b = q_{\text{heat}} \rightarrow \dot{V}_b = V_b \frac{\ddot{\eta}}{\lambda} \]

- Perturbed dynamic pressure
  \[ \rho_\ell V_b \dot{V}_b = \dot{m}V_b \frac{\ddot{\eta}}{\lambda} \]

- Ablation rate

- Growth rate: \( S(\rho_h g - k m V_b) \ddot{\eta} = \rho_h \lambda S \ddot{\eta} \rightarrow \ddot{\eta} \sim e^{\gamma t} \rightarrow \gamma = \sqrt{kg - k^2 \frac{\dot{m}}{\rho_h} V_b} \)
The “old”-fashioned ablative stabilization is still there but it is not very effective.

- A more accurate calculation yields additional stabilization:

\[
\gamma = \sqrt{Agk - k^2 \frac{\dot{m}}{\rho_h} V_b + 4k^2 V_a^2 - 2kV_a}
\]

“Old”-fashioned Dynamic pressure

- Atwood number: \( A = \frac{\rho_{\text{heavy}} - \rho_{\text{light}}(\lambda)}{\rho_{\text{heavy}} + \rho_{\text{light}}(\lambda)} \approx 1 \)

- The cutoff wave number depends only on the dynamic pressure:

\[
k_g = k^2 \frac{\dot{m}}{A\rho_h} V_b \rightarrow k_{\text{cutoff}} = \frac{\rho_h g}{\dot{m}V_b A}
\]

The ABLATIVE RM is NOT an instability but just damped oscillations

\[ [\rho_h g(t) - kmV_b] \ddot{\tilde{\eta}} = \rho_h \lambda \dddot{\eta} \]
\[ g(t) = U_{\text{shell}} \delta(t) \]

\[ \dot{\tilde{\eta}}(0)^+ = kU_{\text{shell}} \tilde{\eta}(0) \]
\[ \omega = \sqrt{k^2 \frac{m}{\rho_h} V_b} \]

\[ \tilde{\eta}(t) = \tilde{\eta}(0)^+ \cos(\omega t) + \frac{\dot{\tilde{\eta}}(0)^+}{\omega} \sin(\omega t) \]

- If the ablative convection of vorticity is included then the RM is damped:

\[ \tilde{\eta}(t) = \left\{ \tilde{\eta}(0) \cos(\omega t) + \frac{\dot{\tilde{\eta}}(0) + 2kV_a \tilde{\eta}(0)}{\omega} \sin(\omega t) \right\} e^{-2kV_a t} \]

Prediction of the sharp boundary model agrees with the numerical results.

200 $\mu$m-thick DT ice, 100 TW/cm$^2$ square laser pulse, 
$\lambda = 20$ $\mu$m, $\tilde{\eta}_0 = 0.1$ $\mu$m
The RT postprocessor is based on the sharp boundary model

- \( \tilde{\eta} \) represents the surface distortion.
- The model is based on a set of coupled second-order differential equations:
  \[
  \sum_{j=1}^{N} a_j(t) \dddot{\eta}_j + b_j(t) \ddot{\eta}_j + c_j(t) \dot{\eta}_j = 0
  \]
- Physics included:
  - thermal conduction
  - ablative flow
  - compressibility
- The solution of the model requires the initial conditions: \( \eta_j(0), \dot{\eta}_j(0) \)
- All the details of sharp boundary model can be found in V. Goncharov’s PhD thesis (UR 1998).
RT seeding by rear-surface nonuniformities

Feedout
After the shock breaks out, a rarefaction wave propagates toward the ablation front while the rear surface expands; after the rarefaction wave breaks out, the ablation front accelerates.

For the analytic solution, see R. Betti et al. (PRL, 1998)

\[
g(t_{rb}) = 2.5 \, g_0
\]

\[
g_0 \equiv g(\text{steady state}) = \frac{P_a}{\rho d}
\]
Two-dimensional behavior in the presence of a rippled rear surface

(a) The shock reaches the rippled rear surface.

(b) Shock and rippled surface interact.

(c) The rippled rarefaction front is formed.

(d) The rarefaction reaches the front surface.
After the rarefaction breakout the ripple on the front surface is seeded and begins to grow

0.1 ns after rarefaction breakout
When the rippled rarefaction wave reaches the ablation front, it imprints a velocity perturbation and the ablation front develops a ripple that starts growing linearly in time.

\[ t = t_{rb} = \text{rarefaction-wave break-out time} \]

\[ \Delta t = \frac{\Delta}{C_s} \]

\[ g(t_{rb}) = \frac{5}{2} \frac{P_a}{\rho d} \]

\[ \tilde{v} = g \Delta t \]

\[ \tilde{v} = \frac{6}{5} C_s \frac{\tilde{\eta}_r(0)}{d_{\text{comp}}} \]

\[ \implies \text{This theory is valid only for } kd_{\text{comp}} < 1. \]
The predicted ablation-front-surface ripple-amplitude evolution soon after the rarefaction wave breaks out does NOT accurately reproduce the simulation results.

The theory predicts a linear growth right after the rarefaction wave breaks out.

\[ \tilde{\eta}_t = \tilde{v} \, t \]
\[ \tilde{\eta}_r(0) = 0.1 \, \mu m \]
\[ \tilde{v} \approx 0.7 \, \mu m/\text{ns} \]
\[ k \, d_{\text{comp}} = 0.033 \]
\[ P = 20 \, \text{Mbar} \]
\[ d \, (\text{CH}) = 20 \, \mu m \]
\[ d_{\text{comp}} \approx d/4 \]
In addition to the linear growth, an approximately quadratic growth occurs because of the perturbed acceleration.

Asymptotically, the acceleration of P is less than Q’s.

Later in time, the acceleration of P is greater than Q’s.

\[ \ddot{g} = g_P - g_Q = -\frac{\partial g}{\partial d} \tilde{\eta}_r(0) \]

asymptotic growth → \( \ddot{\eta}_f(t) \sim \ddot{g}t^2/2 \)
The predicted ablation-front-surface ripple-amplitude evolution is in good agreement with the simulation results when the perturbed acceleration is included.

Ablation-front-surface distortion versus time

![Graph showing ablation-front-surface distortion versus time with a quadratic growth model.]

**Sharp boundary model**

\[ \ddot{\eta}_f = \ddot{g} + \sqrt{k} g \dot{\eta}_f \]

\[ \ddot{\eta}_f(0) = 0, \quad \ddot{\eta}_f(0) = \frac{6}{5} C_s \frac{\ddot{\eta}_r(0)}{d_{\text{comp}}} \]
Both theory and simulations indicate that the transfer function for NIF is well below unity for $\ell > 30$

\[ \eta_f(t) = \eta_r(0) \left( F(kd_{\text{comp}}) \exp\left[ \int_{t_{rb}}^{t} \sqrt{k}g(t') dt' \right] \right) \]

- Transfer function from theory $F(kd_{\text{comp}}) = \frac{1}{16} \left[ \frac{1}{kd_{\text{comp}}} + \frac{2.5}{\sqrt{kd_{\text{comp}}}} \right]$
Feed-out of the rear-surface perturbation was measured for three wavelengths

- Perturbation wavelengths used were
  - 60 µm with a 0.5 µm amplitude
  - 30 µm with a 0.5 µm amplitude
  - 20 µm with a 0.5 µm amplitude

- Target foils were constructed from 20-µm-thick CH.

- Targets with 60-µm wavelength perturbations had a front-surface amplitude = 10% of rear-surface amplitude (0.05 µm).
Calculated $\rho R$ from sharp boundary model agrees with experimentally measured optical depth.

$\lambda = 60 \ \mu m \quad kd_c = 0.6$

$\lambda = 30 \ \mu m \quad kd_c = 1.2$

$\lambda = 20 \ \mu m \quad kd_c = 1.8$

- $\rho R$ is scaled by x-ray mfp and framing camera MTF.
RT seeding by laser-intensity nonuniformities

Laser Imprintings
Hydrodynamic flow is the main imprint mechanism: velocity perturbation

- Post-shock speed depends on the ablation pressure $U_{ps} \sim \sqrt{p_a}$

\[
\tilde{v} \sim \tilde{U}_{ps} \sim \frac{1}{2} U_{ps} \frac{\tilde{p}_a}{p_a}
\]

\[
\frac{d\tilde{\eta}}{dt} = \tilde{v} \sim U_{ps} \frac{\tilde{p}_a}{p_a}
\]

\[
\eta_{vel} = \tilde{v} t \sim \frac{\tilde{p}_a}{p_a} c_s t
\]
Hydrodynamic flow is the main imprint mechanism: acceleration perturbation

- Rippled shock creates lateral mass flow.

\[
\frac{d^2\tilde{\eta}}{dt^2} = \tilde{a} \propto k \frac{\tilde{p}_a}{\rho}
\]

\[
\eta \propto \frac{\tilde{p}_a}{\rho} c_s t^2
\]

\[
\tilde{\eta} = \tilde{\eta}_{\text{vel}} + \tilde{\eta}_{\text{ac}}
\]
Thermal smoothing\(^1\) suppresses acceleration perturbations

- Laser perturbations decouple from the ablation front when \( kD_c \sim 1 \).

\[
\frac{\tilde{p}_a}{p_a} \propto \frac{I}{I} e^{-kD_c}
\]

\[
D_c \gg V_{ct}
\]

\[\text{Decoupling time } t_D \propto (kV_c)^{-1}\]

Imprint growth is reduced by thermal smoothing

- After decoupling time $t > t_D$, $\tilde{a} = 0$.

\[ \tilde{v} = \tilde{v}_D \]

\[ \tilde{v}_D \approx \frac{I c_s^2}{I/V_c} \]

\[ \eta \propto \tilde{a} t^2 \]

\[ \eta \propto \tilde{v}_D t \]

\[ \eta \propto \tilde{v}_D t \]

\[ k\text{cst} \]

\[ t_D \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ 1 \]

The most damaging modes oscillate during the shock propagation.

- Single-mode imprint ORCHID simulations

![Graph showing shock breakout and amplitude over time]

- NIF $\alpha = 3$
- Target design
- Shock breakout

Time (ns) vs. $\eta_f$ amplitude (nm)
Mode spectrum at the beginning of the acceleration phase is used as an initial condition for RT postprocessor.

- Initial conditions for RT model

\[
\begin{align*}
\bar{\eta} (\mu m) & \\
10^{-3} & 10^{-2} & 10^{-1} & 10^0 & 10^1 & 10^2 & 10^3 \\
\ell & 1 & 10 & 100 & \\
\end{align*}
\]

- RT postprocessor

\[
\begin{align*}
\sum_{j=0}^{2} & \left[ C_j \frac{d^j}{dt^j} \eta_f + D_j \frac{d^j}{dt^j} \eta_r \right] = 0 \\
\sum_{j=0}^{2} & \left[ E_j \frac{d^j}{dt^j} \eta_f + F_j \frac{d^j}{dt^j} \eta_r \right] = 0
\end{align*}
\]
The results of the model are in good agreement with multimode *ORCHID* simulations

- Multimode perturbation at the beginning of the deceleration phase.
Deceleration phase instabilities

- The simple model of a decelerating foil
- The seeds
- The growth rates
- Change of motion:
  ablation front → outer surface
  rear surface → inner surface.
The decelerating foil problem provides the basic understanding of the deceleration-phase instability.

- The shock reflected from the wall slows down the foil, which in turn compresses the gas and decelerates.
- The 1-D problem can be solved analytically leading to a clear understanding of the relevant physics issues.
The foil is slowed down both impulsively and continuously.

Evolution of pressure and density
Dec. RT seeding by outer-surface nonuniformities

FEEDTHROUGH

Ablation front/outer surface

Inner surface

Feedthrough

$\tilde{\eta}_{\text{in}} = \tilde{\eta}_{\text{out}}^{-kd}$
As expected, the foil inner surface is hydrodynamically unstable.
The deceleration-phase instability is a combination of Richtmyer–Meshkov and Rayleigh–Taylor.

Linear growth of a 10-μm-wavelength perturbation
Reverse Feedout

Dec. RT seeding by outer-surface nonuniformities
Reverse feedout is also a RT seed during the deceleration phase
The outer surface perturbations seed the deceleration RT

- The feedout is back!
Deceleration phase RT instability

The Growth Rates
Not much is known about the RT growth rates during the deceleration phase

- The only mention of deceleration-phase RT growth rates is in Lindl’s book:

\[
\gamma_{cl} = \left[ \frac{\text{kg}}{1 + k\text{L}} \right]^{1/2}
\]

- Lindl’s estimate for spherical implosions is \( L \sim 0.2 \) \( R_{\text{hot spot}} \).
- For NIF, \( R_{\text{hot spot}} \sim 50 \) to 100 \( \mu \text{m} \) \( \rightarrow \) \( L \sim 10 \) to 20 \( \mu \text{m} \).
- Such a large \( L \) has a strong stabilizing effect.

The heat flux leaving the hot-spot is deposited on the shell surface causing mass ablation from the shell into the hot spot. The hot-spot mass increases in time.
The ablation velocity is determined from the energy balance.

- Hot-spot temperature profile: \( T_{hs} = T_0 \left( 1 - \frac{r^2}{R_{hs}^2} \right)^{2/5} \)
- Use the EOS: \( p_b V_b = 2 \rho_b V_b T_b / M_i = 2 \dot{m} T_b / M_i \)
- Ablation velocity: \( V_a = \frac{\dot{m}}{\rho_{shell}} = 0.2 \frac{M_i \kappa_{Spitz}(T_0)}{\rho_{shell} R_{hot \, spot}} \)
The density-gradient scale length is small

- Balance of heat flux to the shell and internal energy flux leaving the shell

\[ pV_a \approx -\kappa(T_{sh}) \frac{dT_{sh}}{dr} \approx \kappa(T_{sh}) T_{sh} \frac{1}{\rho_{sh}} \frac{d\rho_{sh}}{dr} \]

- The density-gradient scale length is found using the formula for the ablation velocity:

\[ L_m = \left[ \frac{1}{\rho} \frac{d\rho}{dr} \right]_{\text{min}}^{-1} \approx 1.6 \frac{M_i \kappa(T_{sh})}{\rho_{sh} V_a} = 8R_{\text{hot-spot}} \left( \frac{T_{\text{shell}}}{T_0} \right)^{5/2} \]

- For NIF: \( L_m \sim 1 \text{ \mu m} \)
Planar Model

Planar simulations reproduce the behavior of ICF capsule implosions

- Hot-spot temperature and radius and peak shell density have the same order of magnitude in planar and spherical cases.

**Initial distribution**

- **Density** (g/cm³)
  - Wall
  - Shell

**Stagnation**

- **Density** (g/cm³)
- **Temperature** (keV)

**X (μm)**

**X (μm)**
Ablation velocity is significant during the deceleration phase.
A significant reduction in RT growth rates is due to ablation

\[ \gamma = \frac{\sqrt{kg}}{1+KL_m} \]
Theory and 1-D *LILAC* simulations yield the same value of the ablation velocity

- From theory using $T_{hs} = 11.5 \text{ keV}$, $\rho_{sh} = 325 \text{ gr/cm}^3$, $R_{hs} = 65 \text{ }\mu\text{m} \rightarrow V_a = 25 \text{ }\mu\text{m/ns}$
- From simulations: $V_b = 100 \text{ }\mu\text{m/ns}$, $\rho_b = 60 \text{ gr/cm}^3$, $V_a = \rho_b$; $V_b/\rho_{sh} = 20 \text{ }\mu\text{m/ns}$
Theoretical NIF linear growth factors are significantly reduced by mass ablation

- NIF deceleration phase: \( g = 10^4 \, \mu \text{m/ns}^2 \), \( V_a = 20 \, \mu \text{m/ns} \), \( R_{hs} = 70 \, \mu \text{m} \), \( L_m = 1 \, \mu \text{m} \).
- Duration of deceleration phase \( \sim 100 \, \text{ps} \).

**NIF cutoff:** \( \ell \approx 190 \)
Conclusions

- Two stages of the deceleration-phase instability are observed: deceleration by a series of shocks and continuous deceleration.

- Mass ablation through the inner surface and finite density-gradient scale length are the most important stabilizing effects.

- A significant reduction in the RT instability growth rate is due to the mass ablation.

- The inner-surface density-gradient scale length is lower than its standard estimate (~0.2 hot-spot radius).

- The cutoff wave number of the deceleration-phase RT for NIF is approximately $\ell_{\text{cutoff}} = 150$ to 200.