Electron Acceleration by a Laser Pulse in a Plasma

The motion of a charged particle in an electromagnetic field is a well-known paradigm of physics.[1,2] Suppose that the field is associated with a laser pulse of finite extent propagating in a vacuum. As the pulse overtakes the particle, the particle gains energy and momentum at the expense of the pulse. At the peak of an intense pulse, the particle has considerable (longitudinal) momentum in the propagation direction of the pulse. This fact suggests that intense pulses[3] might be used to accelerate particles. Unfortunately, the energy associated with the transverse particle motion is wasted, and it is difficult to extract the particle from the middle of the pulse. One cannot simply wait until the pulse has overtaken the particle completely because the pulse leaves the particle at rest, albeit displaced a finite distance in the direction of the pulse. If the pulse is of finite transverse extent, the particle can be expelled from the pulse.[4] However, this transverse expulsion is difficult to control.

In this article, we describe the motion of a charged particle in the field of an idealized laser pulse propagating through a medium and show that the particle can be accelerated efficiently and extracted easily. We then discuss briefly some issues relevant to electron acceleration in a plasma. For a detailed review of laser-driven electron acceleration schemes, we refer the reader to the paper by Sprangle, Esarey, and Krall.[5]

The motion of a particle, of charge q and mass m, in an electromagnetic field is governed by the equation[6]

$$d_\tau\left(u_\mu + a_\mu\right) = u^\nu \partial_\mu a_\nu \,, \tag{1}$$

where u^μ is the four-velocity of the particle, τ is the proper time of the particle multiplied by c, and a^μ is the four-potential of the field multiplied by q/mc^2. For a circularly polarized field

$$a^\mu = \left(0, 0, a\cos\phi, a\sin\phi\right)/\sqrt{2}\,. \tag{2}$$

We assume that the phase ϕ and amplitude a are functions of t and x alone. In this case, it is well known that

$$d_\tau\left(\mathbf{u}_\perp + \mathbf{a}_\perp\right) = 0\,, \tag{3}$$

which reflects the conservation of transverse canonical momentum. For future reference, notice that Eq. (3) does not imply that $\mathbf{u}_\perp$ attains all values of $\mathbf{a}_\perp$; it implies that the values of $\mathbf{a}_\perp$ to which the particle has access are reflected in the corresponding values of $\mathbf{u}_\perp$. Using Eq. (3), one can rewrite the first two components of Eq. (1) as

$$d_\tau \gamma = \frac{1}{2}\partial_t u_\perp^2, \quad d_\tau u_\parallel = -\frac{1}{2}\partial_x u_\perp^2\,. \tag{4}$$

The right sides of Eqs. (4) are the t- and x-components of the ponderomotive four-force.

We assume that $\phi = t - sx$, where s is the inverse phase speed of the pulse, and $a = a(\psi)$, where $\psi = t - rx$ and the inverse pulse speed $r > 1$. Although these assumptions are based on the known characteristics of a low-intensity pulse, which may differ from those of a high-intensity pulse, the only requirements of the following analysis are that the pulse propagates without distorting significantly and that its propagation speed is less than c. The propagation characteristics of high-intensity pulses have been studied by Decker and Mori.[7]

Since $u_\perp^2$ is independent of ϕ, it follows from Eqs. (4) that

$$d_\tau\left(r\gamma - u_\parallel\right) = 0\,. \tag{5}$$

For a particle that is at rest initially, $u_\parallel = r(\gamma - 1)$. Since $r > 1$, longitudinal momentum is produced more efficiently in a medium than in a vacuum. By combining this result with the definition of the Lorentz factor γ, one can show that

$$\left(r^2 - 1\right)u_\parallel^2 - 2ru_\parallel + r^2 u_\perp^2 = 0\,. \tag{6}$$

It follows immediately that

$$u_\parallel = \frac{r \pm r\left[1 - \left(r^2 - 1\right)u_\perp^2\right]^{1/2}}{r^2 - 1} \tag{7}$$

and

$$\gamma = \frac{r^2 \pm \left[1 - \left(r^2 - 1\right)u_\perp^2\right]^{1/2}}{r^2 - 1}. \tag{8}$$

Equation (6) describes an ellipse that intersects the origin and has semi-major and semi-minor axes of length $r/\left(r^2-1\right)$ and $1/\left(r^2-1\right)^{1/2}$, respectively, as shown in Fig. 67.15. The minus (–) signs in Eqs. (7) and (8) correspond to the left half of the ellipse, whereas the plus (+) signs correspond to the right half. It follows from Eqs. (7) and (8) that $v_\parallel = 1/r$ at the intermediate point B. This information allows the particle motion to be determined qualitatively.

Initially, the pulse overtakes the particle. As it does so, $u_\perp^2$ increases and the representative point $\left(u_\parallel, \left|u_\perp\right|\right)$ moves from A, which corresponds to the leading edge of the pulse, toward B. If $u_\perp^2 < 1/\left(r^2-1\right)$ at the peak of the pulse, the representative point does not reach B. Since $v_\parallel < 1/r$, the pulse overtakes the particle completely and the representative point moves back toward A, which now corresponds to the trailing edge of the pulse. Eventually, the particle is at rest. This scenario is illustrated in Fig. 67.16(a). However, if $u_\perp^2 = 1/\left(r^2-1\right)$ before the particle reaches the peak of the pulse, the particle is repelled by the pulse, because the *x*-component of the ponderomotive four-force is positive, and the representative point continues toward C, which corresponds to the leading edge of the pulse. Eventually, $u_\perp = 0$, $u_\parallel = 2r/\left(r^2-1\right)$, and $\gamma = \left(r^2+1\right)/\left(r^2-1\right)$. This scenario is illustrated in Fig. 67.16(b). The Palmer theorem[8] does not apply to this scenario because the ponderomotive terms in Eqs. (4) are nonlinear and the pulse is propagating through a medium.

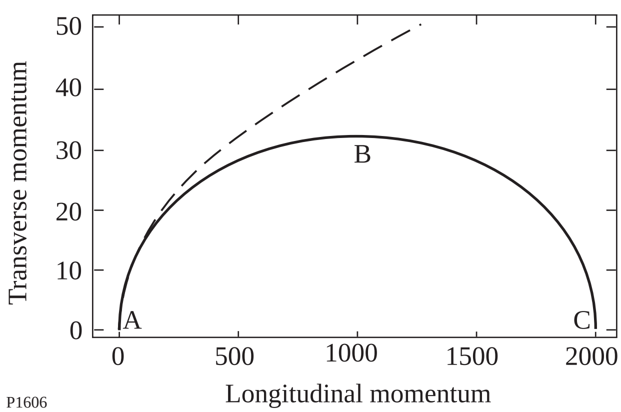

Figure 67.15
Diagram of the relation between longitudinal and transverse momentum implied by Eq. (6). The broken line corresponds to $r = 1$ (vacuum) and the solid line corresponds to $r = 1.0005$ (medium).

During this interaction, the phase of the pulse evolves according to $d_\tau\phi = \gamma\left(1 - sv_\parallel\right)$. Initially, $d_\tau\phi = 1$ and a positively charged particle rotates in an anticlockwise direction.[9] As the particle's speed increases, its rate of rotation decreases. For media in which $r < s$, the direction in which the particle rotates can change if the pulse is sufficiently intense.

For reasons that will become clear shortly, consider the acceleration of a particle that is traveling in the direction of the pulse with initial momentum u_A and initial energy γ_A. The analysis of particle acceleration is simplest in the pulse frame, in which the four-potential is time independent and, hence, the particle energy is constant; energy is exchanged between the longitudinal and transverse degrees of freedom. Because the four-potential is transverse, it has the same amplitude in both the laboratory and pulse frames. Let $\gamma_P = r/\left(r^2-1\right)^{1/2}$ be the Lorentz factor associated with the pulse and $u_P = \left(\gamma_P^2 - 1\right)^{1/2}$. Initially, the particle is moving to the left in the pulse frame. The particle will be repelled if, at some point in the pulse, the energy associated with its transverse motion equals its initial energy. In this case the particle regains its initial energy as it moves to the right. It follows from these observations and Eq. (2) that, in the laboratory frame,

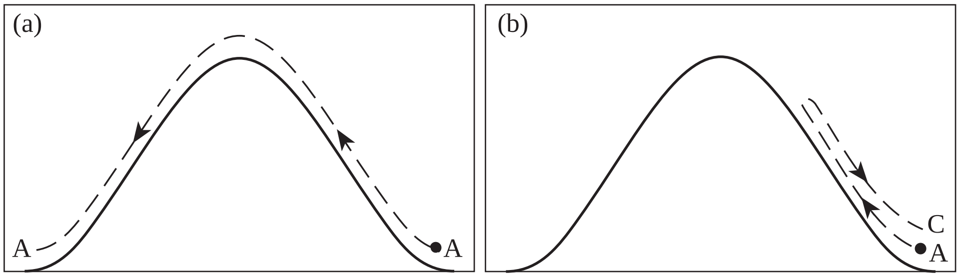

Figure 67.16
Illustration of the particle motion for a low-intensity pulse (a) and a high-intensity pulse (b).

the repelling threshold

$$a_B^2 = \left[\left(\gamma_P \gamma_A - u_P u_A\right)^2 - 1\right] \tag{9}$$

and the corresponding gain in particle energy

$$\delta\gamma = 2\left(u_P^2 \gamma_A - \gamma_P u_P u_A\right). \tag{10}$$

The energy gain and repelling threshold are plotted as functions of the initial energy in Fig. 67.17, for the case in which $\gamma_P = 30$. Energy is measured in units of the particle rest mass. If the particle is at rest initially, $a_B^2 = 2\left(\gamma_P^2 - 1\right) = \delta\gamma$; the energy gain is large, but so is the required pulse intensity. Since the repelling threshold decreases more rapidly than the energy gain as the initial energy is increased, the intensity requirement can be minimized by injecting preaccelerated particles. For example, if the injection energy is 7, the repelling threshold is 8.3 and the energy gain is 120. If the injection energy is 15, the repelling threshold is 1.1 and the energy gain is 45.

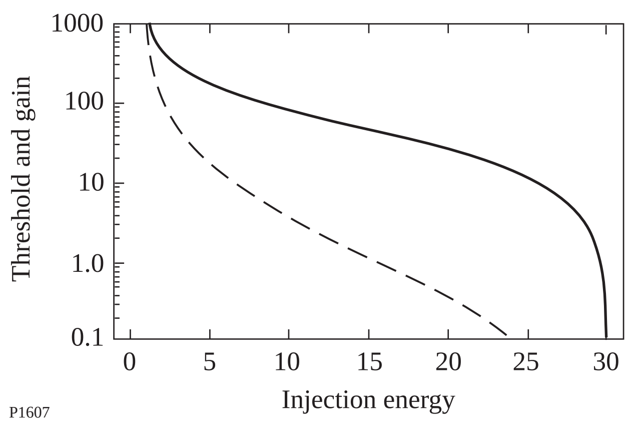

Figure 67.17
Threshold intensity a_B^2 required to repel the particle (broken line) and the corresponding gain in particle energy (solid line) plotted as functions of the particle injection energy for the case in which $\gamma_P = 30$. Energy is measured in units of the particle rest mass.

With a_B^2 fixed, the injection energy required to ensure that the particle is repelled by the pulse and the corresponding energy gain are functions of γ_P. By inverting Eq. (9), one finds that

$$\gamma_A = \gamma_P \mu_B - \left[\left(\gamma_P^2 - 1\right)\left(\mu_B^2 - 1\right)\right]^{1/2}, \tag{11}$$

where $\mu_B = \left(1 + a_B^2/2\right)^{1/2}$ is a measure of the pulse intensity. When $\gamma_P > \mu_B \gg 1$, Eq. (11) reduces to

$$\gamma_A \approx \frac{1}{2}\left(\frac{\gamma_P}{\mu_B} + \frac{\mu_B}{\gamma_P}\right) + \frac{1}{8}\left(\frac{\gamma_P}{\mu_B^3} + \frac{\mu_B}{\gamma_P^3} - \frac{2}{\gamma_P \mu_B}\right). \tag{12}$$

When $\gamma_P \sim \mu_B$, the first and second terms of Eq. (11) are adequate, and, when $\gamma_P \gg \mu_B$, the first and third terms are adequate. In the latter limit Eq. (10) reduces to

$$\delta\gamma \approx 2\gamma_P \mu_B - \gamma_P/\mu_B. \tag{13}$$

The injection energy and energy gain are plotted as functions of γ_P in Fig. 67.18 for the case in which $a_B^2 = 10$. The approximate formulas (12) and (13) are accurate and show clearly how the injection energy and the energy gain scale with γ_P and μ_B.

Subject to the constraints described between Eqs. (4) and (5), the preceding analysis is directly applicable to the accel-

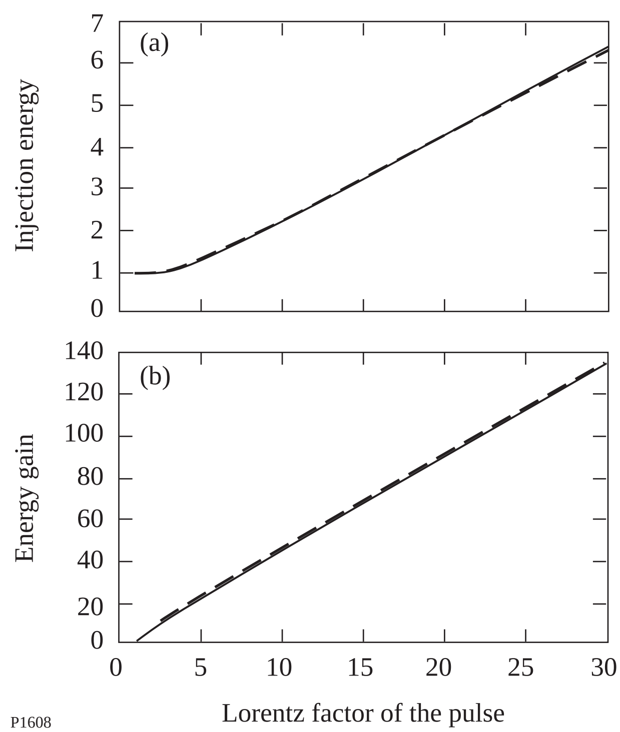

Figure 67.18
Injection energy required to ensure that the particle is repelled (a) and the corresponding gain in particle energy (b) plotted as functions of the Lorentz factor associated with the pulse for the case in which $a_B^2 = 10$. Energy is measured in units of the particle rest mass. The broken lines represent the approximate results [Eqs. (12) and (13)], whereas the solid lines represent the exact results [Eqs. (10) and (11)].

eration of a test electron in a plasma. However, one must also consider the motion of the background electrons under the influence of the pulse.

In the absence of collective effects, the background electrons behave like independent test particles. Provided that $a_B^2 < 2\left(\gamma_P^2 - 1\right)$, the background electrons pass freely through the pulse: although there is a temporary exchange of energy between the pulse and the background electrons (which is why the pulse speed is less than c), none of the pulse energy is lost to the plasma. It is for this reason that the test electron must be preaccelerated.

In the presence of collective effects, the compression of the electron fluid by the leading edge of the pulse produces a charge nonuniformity that, in turn, produces a longitudinal electric field. This longitudinal field modifies the motion of the electron fluid and the conclusions of the preceding paragraph. The electron-fluid momentum equation can be written in the form[10]

$$\partial_t\left(u_\parallel - a_\parallel\right) = -\partial_x \gamma, \tag{14}$$

from which follows a conservation equation similar to Eq. (5). By using this conservation equation, one can show that the longitudinal potential evolves according to

$$\frac{d^2 a_\parallel}{dt^2} = \frac{1}{r^2-1}\left\{r - \frac{r-a_\parallel}{\left[1+\left(r-a_\parallel\right)^2 - r^2 - \left(r^2-1\right)u_\perp^2\right]^{1/2}}\right\}. \tag{15}$$

In the low-intensity regime, Eq. (15) reduces to the equation for a driven linear oscillator, the behavior of which is well known: A short pulse provides an impulse to the plasma, which continues to oscillate after the pulse has passed. This oscillation, which is referred to as the wake of the pulse, can be removed by a second impulse that counteracts the first. In contrast, a long pulse produces an adiabatic plasma response. A pulse of infinite duration leaves no wake, whereas a pulse of finite duration leaves a wake, the amplitude of which depends on the ratio of the plasma period and the duration of the pulse. In the high-intensity regime the intrinsic plasma oscillation is nonlinear.[11] However, the plasma response is similar to that described above, as shown in Figs. 67.19 and 67.20. In Fig. 67.19(b) the first pulse loses energy to the second pulse at the same rate as the pulse loses energy to the wake in Fig. 67.19(a). Thus, a short pulse, which contains only a small amount of energy initially, is depleted quickly, whereas a long pulse, which contains a large amount of energy initially, is depleted slowly.

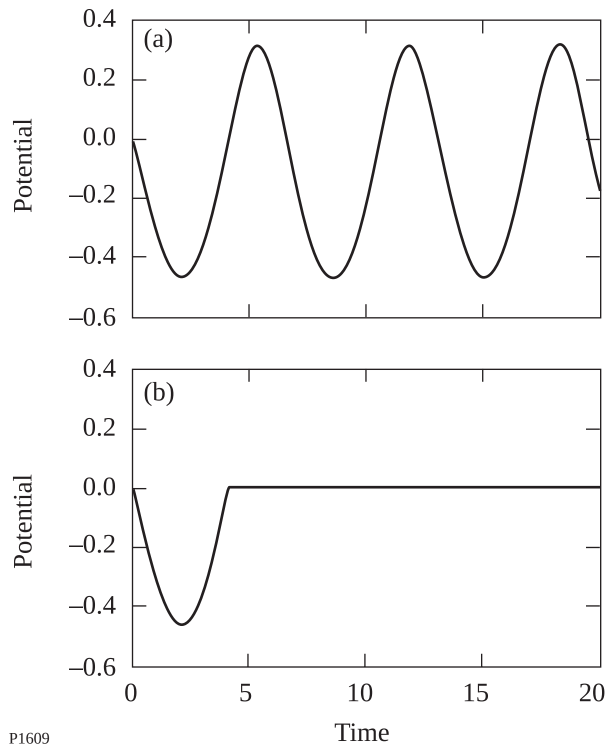

Figure 67.19
Longitudinal vector potential (wake) generated by a pulse of intensity $a^2 = 10\sin^2\left(t/t_P\right)H(t)H\left(t_P - t\right) + 10\sin^2\left[\left(t-t_D\right)/t_P\right]H\left(t_D\right)H\left(t_D + t_P - t\right)$ for the case in which γ_P = 30 and t_P = 0.1. Time is measured in units of the inverse of the electron plasma frequency: (a) $t_D = \infty$; (b) t_D = 3.99.

It is clear from Figs. 67.19 and 67.20 that a longitudinal field is generated within the pulse. The density nonuniformity associated with this longitudinal field modifies the transverse current and, hence, the pulse itself. Two-dimensional effects are also important. The pulse must be wide enough for the test electron to complete its transverse rotations without being deflected by the transverse ponderomotive force. This force also tends to expel background electrons from the neighborhood of the pulse.[5,12] If this expulsion is slow, the reduction in group speed of the pulse will be produced by the plasma through which most of the pulse will propagate. If the expulsion is rapid, most of the pulse will propagate in a plasma channel, or waveguide, and the reduction in group speed of the pulse will be produced by the waveguide. All of these effects must be included in a detailed study of this acceleration scheme.

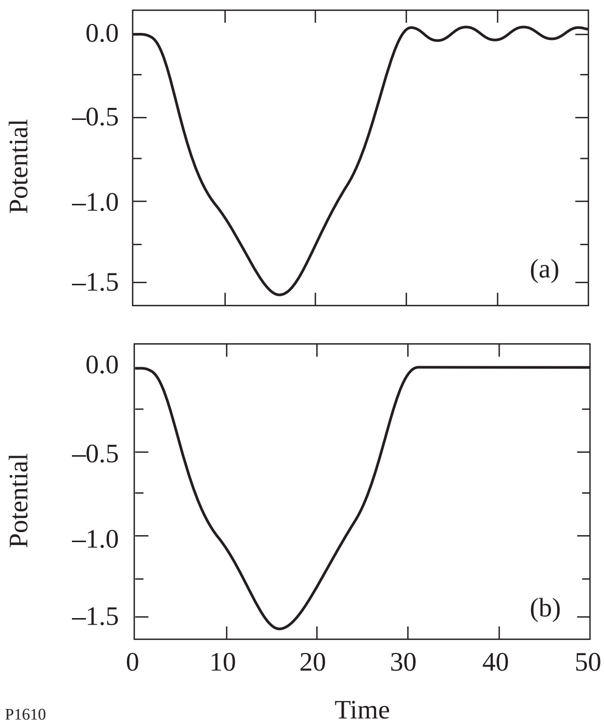

Figure 67.20
Longitudinal vector potential (wake) generated by a pulse of intensity $a^2 = 10\sin^2(t/t_P)H(t)H(t_P - t)$ for the case in which γ_P = 30. Time is measured in units of the inverse of the electron plasma frequency: (a) t_P = 10.0; (b) t_P = 10.3.

In summary, the motion of an electron in the electromagnetic field associated with a circularly polarized laser pulse in a plasma was studied. It appears possible to increase significantly the energy of a preaccelerated electron. While wake generation renders this acceleration scheme less than ideal, its simplicity is noteworthy. The wake fields produced by short laser pulses have been observed recently.[13,14] Future experiments will study the interaction of a preaccelerated bunch of electrons with such wake fields. One would only need to change the timing of the electron bunch in these experiments to test the scientific feasibility of this scheme.

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