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2.C Stimulated Raman Scattering in a Collisional Homogeneous Plasma

In the last two decades, the coherent nonlinear interaction of three coupled waves has received considerable attention. It occurs in the process of stimulated Raman scattering (SRS) in the plasma corona that surrounds the fuel pellet in laser-fusion experiments. In this process an incident light wave decays into a Langmuir wave and a scattered light wave. It is important to calculate the amplitudes of the daughter waves for two reasons. First, light energy that is scattered away from the pellet can no longer assist in the ablation process. Second, the breaking of the Langmuir wave generates hot electrons, which can preheat the fuel and thereby reduce the amount of compression.

The standard approach to this problem is to derive evolution equations for the amplitude of each wave, which can then be solved in a variety of ways. For instance, in cases in which the wave amplitudes depend on both space and time, one could use the Inverse Scattering Transform.[1] In temporal problems, the solutions are assumed to be homogeneous in space, and the problem reduces to that of three coupled simple harmonic oscillators whose natural frequency ω_j depends parametrically on the wavenumber k_j. Spatial problems typically involve a constant-amplitude pump wave impinging on a medium which may be finite or semi-infinite in length. One then solves a two-point boundary value problem to determine the steady-state amplitude of each wave.

Here, we consider the temporal problem for SRS in a collisional homogeneous plasma. The electromagnetic waves are collisionally damped while the plasma wave is affected by collisions and a phenomenological term representing Landau damping. This problem, with a different damping coefficient for each wave, has previously been thought intractable by analytic means.[2] In this article, we formulate a slightly different problem by introducing a driving term into the pump equation. This balances the energy lost by dissipation and permits a steady, spatially uniform oscillation of the pump wave. However, this equilibrium is unstable since the amplitude of the pump wave is above its SRS threshold value. Energy is therefore exchanged between the waves until a new and stable equilibrium condition is reached. Previously,[3] we calculated the steady-state amplitude of each wave when the system was only marginally unstable. Here, the calculation is extended to include the highly unstable regime.

The governing equations for SRS assume their simplest form when written in terms of the action amplitude of each wave. The square of the action amplitude, namely the action density, is defined to be the energy density of each wave divided by its natural frequency, which is proportional to the number density of quanta present in each wave field. The resulting simplification of the basic equations reflects the fact that physically, SRS is the decay (and recombination) of an incident pump photon into a scattered photon and a plasmon. The governing equations are

$$\left(\frac{\partial}{\partial t} + \nu_1\right) A_1 = -icA_2A_3\exp(i\delta t) + \nu_1 A_1(0), \tag{1a}$$

$$\left(\frac{\partial}{\partial t} + \nu_2\right) A_2 = -icA_1A_3^*\exp(-i\delta t), \tag{1b}$$

$$\left(\frac{\partial}{\partial t} + \nu_3\right) A_3 = -icA_1A_2^*\exp(-i\delta t), \tag{1c}$$

where

$$\delta = \omega_1 - (\omega_2 + \omega_3)$$

is the frequency mismatch, and the subscripts 1, 2, and 3 refer to the incident pump wave, the scattered electromagnetic wave, and the plasma wave, respectively. We see that the exchange of energy, made possible by the terms that are quadratic in the wave amplitudes, is modified by the effects of damping and frequency mismatch. Explicit expressions for the coupling constant c, and the damping rates ν_j, can be found in Ref. 1. The initial amplitude of the pump wave is equal to $A_1(0)$, while the daughter waves initially have noise-level amplitudes.

A standard linear analysis of Eqs. (1a)–(1c) shows that the threshold amplitude of the pump, denoted by A_{10}, is given by

$$|A_{10}|^2 = \frac{\nu_2\nu_3}{c^2}(1 + r^2), \tag{2}$$

where $r \equiv \delta/(\nu_2 + \nu_3)$ is the mismatch ratio. When the pump amplitude exceeds its threshold value, the daughter waves grow exponentially in time. The linear growth rate γ is equal to the root of

$$\gamma^2 + [(\nu_2+\nu_3) - ir(\nu_2-\nu_3)]\gamma + \nu_2\nu_3(1 + r^2) - c^2|A_1(0)|^2, \tag{3}$$

which has the largest real part. Eventually, nonlinear effects become important and modify the temporal evolution of the system.

Let us now examine the nonlinear interaction. In the absence of damping, Eqs. (1a)–(1c) yield periodic solutions which can be expressed in closed form, in terms of elliptic functions.[4] There is therefore a periodic exchange of energy between the pump wave and its daughter waves, as shown in Fig. 23.28. In this and subsequent figures, we have measured time in units of Ω^{-1}, where Ω is a naturally occurring frequency parameter of the order of ω_p. An explicit expression for Ω is given in Ref. 1. The peak energy of the backscattered wave, which we denote by E_{2p}, is related to the initial energy contained in the pump wave by the familiar Manley-Rowe relationship $E_{2p}/\omega_2 = E_1(0)/\omega_1$.

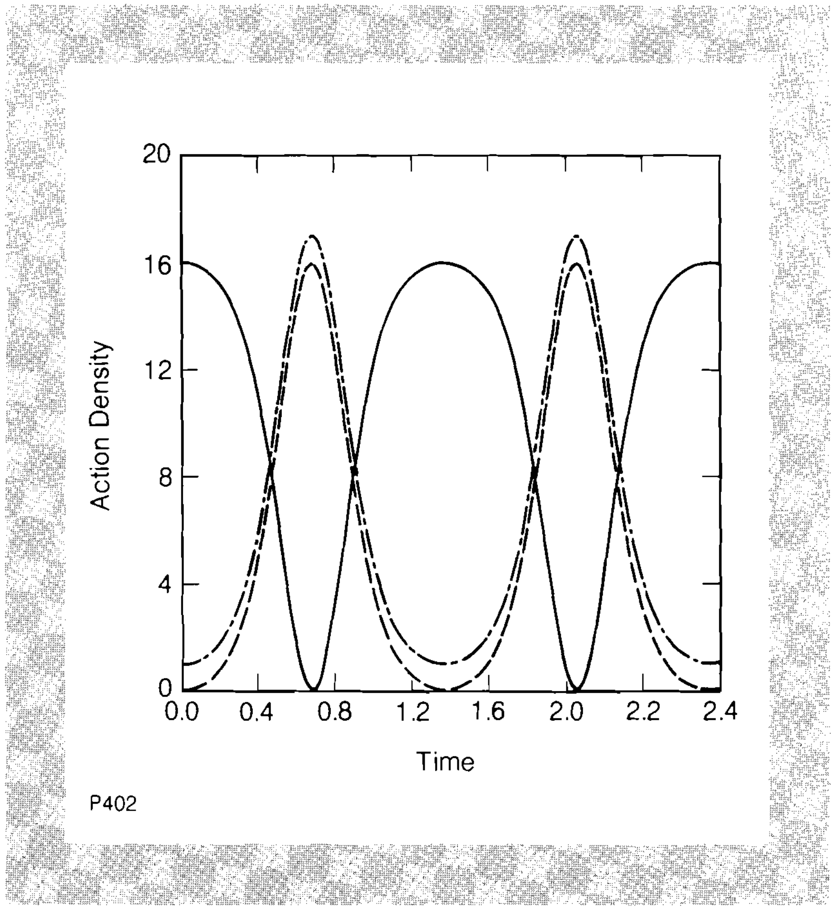

Fig. 23.28
Action density of each mode as a function of time, in units of $mnv_e^2/2\omega_p$. We have taken $v_e/c = 0.040$, $\nu_1 = \nu_2 = \nu_3 = 0$, and set the mismatch ratio equal to zero. The solid line represents mode 1, the dot-dash line represents mode 2, and the broken line represents mode 3.

A more realistic case, however, is to be found in a collisionless plasma where the two electromagnetic waves are undamped and the Langmuir wave is Landau damped. In this case, the governing equations can easily be reduced to a single equation that describes a damped nonlinear oscillator.[5] This is readily solved to determine the temporal behavior of the system. Our results are plotted in Fig. 23.29, in complete agreement with those of Fuchs and Beaudry,[5] and Hiob and Barnard.[6] However, our interpretation differs from that of Hiob and Barnard. During the first half-cycle ($t < 110\ \Omega^{-1}$), the pump wave transfers action to the daughter waves. In the terminology of quantum field theory, one pump photon decays into a scattered photon and a

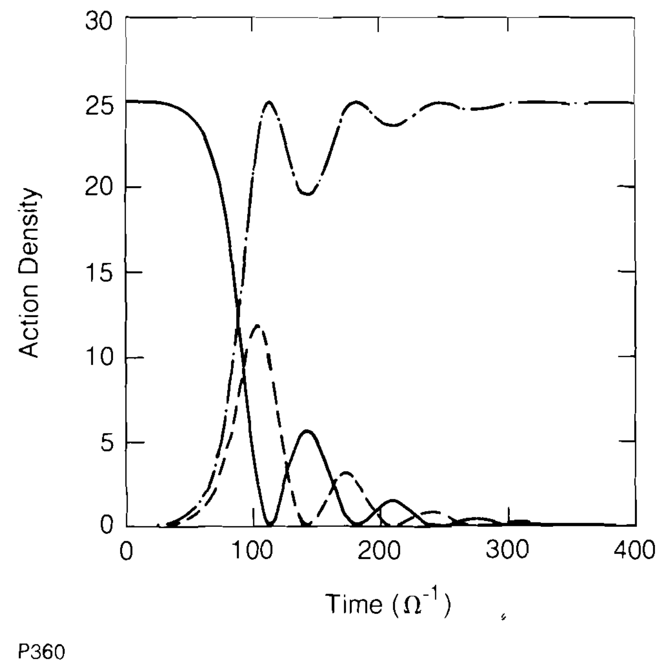

Fig. 23.29
Action density of each mode as a function of time, in units of $mnv_e^2/2\omega_p$. We have taken v_e/c = 0.040, $\nu_1 = \nu_2 = 0$, ν_3 = 0.020 Ω, and set the mismatch ratio equal to zero. The solid line represents mode 1, the dot-dash line represents mode 2, and the broken line represents mode 3.

plasmon. Because damping acts continually on the plasma wave, at time t = 110 Ω^{-1} the action density of the plasma wave is less than that of the scattered electromagnetic wave. Consequently, during the second half-cycle, when a scattered photon and a plasmon recombine to create a pump photon, we run out of plasmons before the action density of the pump can be restored to its initial value. Thus, in each complete cycle, action is irreversibly transferred to the scattered light wave. As time increases, the plasmon action density tends to zero and no recombination can take place. Hence, the system tends to the steady state $E_1 = E_3 = 0$, $E_2 = (\omega_2/\omega_1)\, E_1\,(0)$, as shown in the figure. The steady-state reflectivity, which is defined to be the ratio of the final energy density of mode 2 to the initial energy density of mode 1, is simply

$$R = \omega_2/\omega_1. \tag{4}$$

It is interesting to note that although the time taken to reach the final state depends on ν_3, the reflectivity is independent of it.

The most general case, in which all three waves are damped arbitrarily, occurs in a collisional plasma. It has been shown that a complete analytic solution to this problem cannot be obtained in terms of known functions.[2] However, Eqs. (1a)-(1c) can, of course, be integrated numerically. The result of one such integration is shown in Fig. 23.30. It can be seen that after some transient oscillatory behavior, the system tends asymptotically to a nonlinear steady state. This steady state *can* be determined analytically. After some algebra, Eqs. (1a)–(1c) yield the following expressions for the action density of each wave:

$$|A_1|^2 = |A_{10}|^2, \tag{5a}$$

$$|A_2|^2 = \frac{\nu_1}{\nu_2}|A_{10}|^2\left\{\frac{[m^2 + (m^2-1)r^2]^{1/2} - 1}{(1 + r^2)}\right\}, \tag{5b}$$

$$|A_3|^2 = \frac{\nu_2}{\nu_3}|A_2|^2, \tag{5c}$$

where m is the factor by which the initial pump amplitude exceeds its threshold value [i.e., $A_1(0) = mA_{10}$]. The interpretation of these results is straightforward.

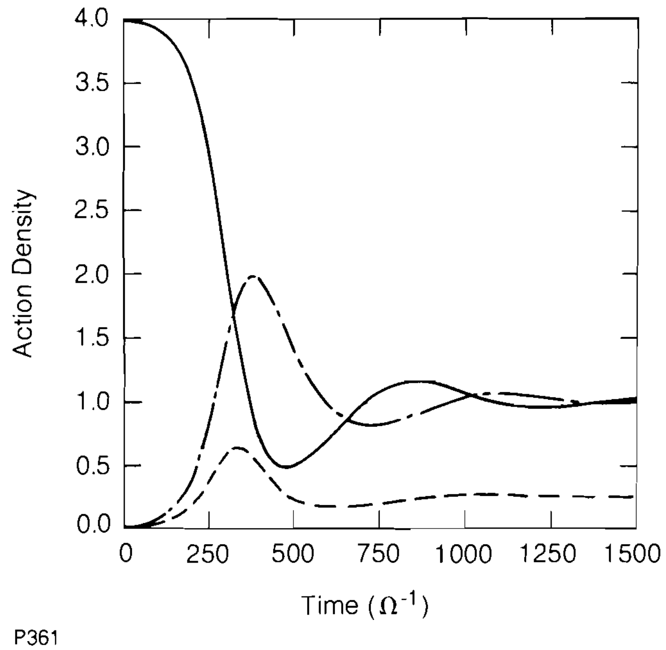

Fig. 23.30
Action density of each mode as a function of time, in units of $mnv_e^2/2\omega_p$. We have taken v_e/c = 0.040, $\nu_1 = \nu_2 = 0.005\,\Omega$, $\nu_3 = 0.020\,\Omega$, and set the mismatch ratio equal to zero. The solid line represents mode 1, the dot-dash line represents mode 2, and the broken line represents mode 3. $I/I_{th} = 4$.

From Eq. (1a), we see that the beating of the two daughter waves gives rise to an electric field at the pump frequency. When δ is equal to zero, the self-generated field is out of phase with the pump field by exactly π radians, so its effect is to decrease the net field at the driving frequency. A nonlinear steady state cannot occur unless the pump amplitude is equal to its threshold value. This determines the strength of the self-generated field, which in turn determines the amplitude of each daughter wave. When δ is nonzero, in addition to being depleted, the pump wave is subject to a nonlinear phase shift that further inhibits the transfer of energy to the daughter waves. In Fig. 23.31 we have plotted the action density of mode 2 as a function of m, for several values of the mismatch ratio.

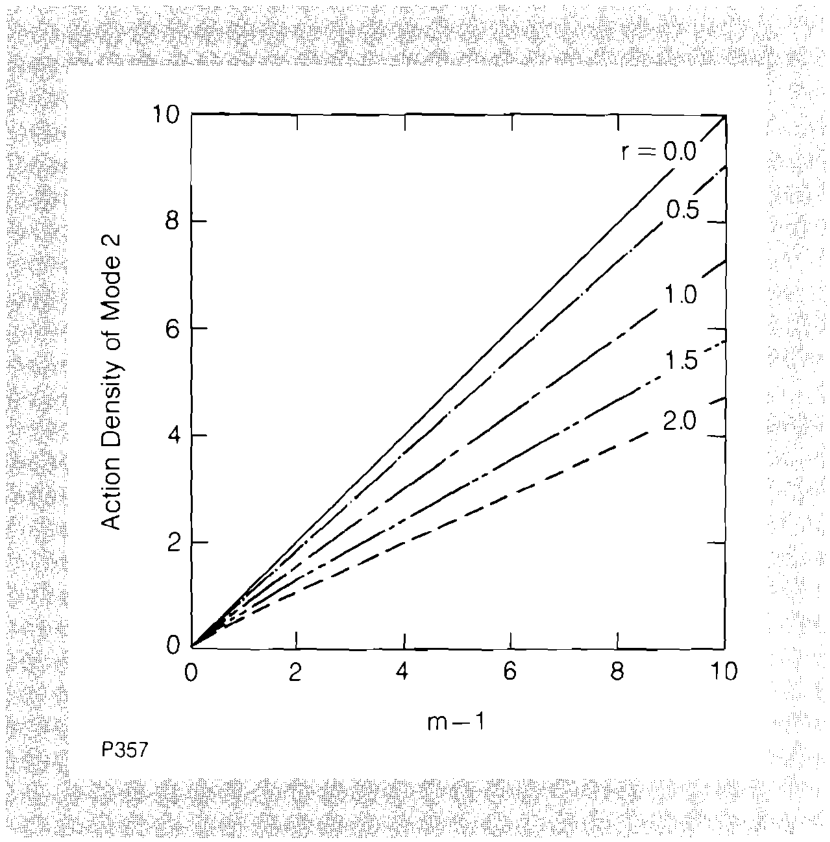

Fig. 23.31
Steady-state action density of mode 2 as a function of m, in multiples of $(\nu_1/\nu_2)\ |A_{10}|^2$. Results are shown for several values of the mismatch ratio r.

The relationship between the final action densities of modes 2 and 3 is also easily explained. In this three-wave interaction, plasmons and scattered photons are created in pairs. In steady state, the loss of each due to damping must therefore occur at the same rate. Hence $2\nu_2|A_2|^2 = 2\nu_3|A_3|^2$, which is just Eq. (5c).

Rewriting Eq. (5b) in terms of energy densities, we obtain the following expression for the reflection coefficient. This is again defined to be the ratio of the final energy density of the scattered electromagnetic wave to the initial energy density of the pump wave.

$$R = \left(\frac{\omega_2 \nu_1}{\omega_1 \nu_2}\right) \left\{\frac{[m^2 + (m^2 - 1)r^2]^{1/2} - 1}{m^2\,(1 + r^2)}\right\}. \tag{6}$$

This is shown in Fig. 23.32. Notice how the reflection coefficient reaches a maximum, as a function of increasing initial pump intensity, and decreases thereafter. This type of behavior has recently been observed in long-plasma-scale-length experiments performed by Herbst *et al.*[7] A quantitative comparison of our theoretical predictions with experimental results is now under way.

An important feature of Eq. (6) is its sensitivity to the threshold amplitude of the pump. Thus, any mechanism that alters the threshold can change the steady-state reflectivity. One such mechanism is the breaking of the electron plasma wave, which gives rise to a hot-electron tail on the electron distribution function. This augments the cold Landau damping by an amount

$$\nu_{Lh} = \left(\frac{\pi}{8}\right)^{1/2} \frac{\omega_p}{(k_3\lambda_D)^2} \frac{n_h}{n_c} \left(\frac{T_c}{T_h}\right)^{3/2} \exp\left[\frac{-1}{2(k_3\lambda_D)^2}\left(\frac{T_c}{T_h}\right) - \frac{3}{2}\left(\frac{T_c}{T_h}\right)\right]. \quad (7)$$

In some cases, ν_{Lh} can be of comparable magnitude to max (ν_{ei}, ν_{Lc}). One must then work with a self-consistent threshold that is itself a function of the incident laser intensity. Offenberger *et al.*[8] have shown that this can significantly affect the reflectivity. In general, the reflectivity will be reduced.

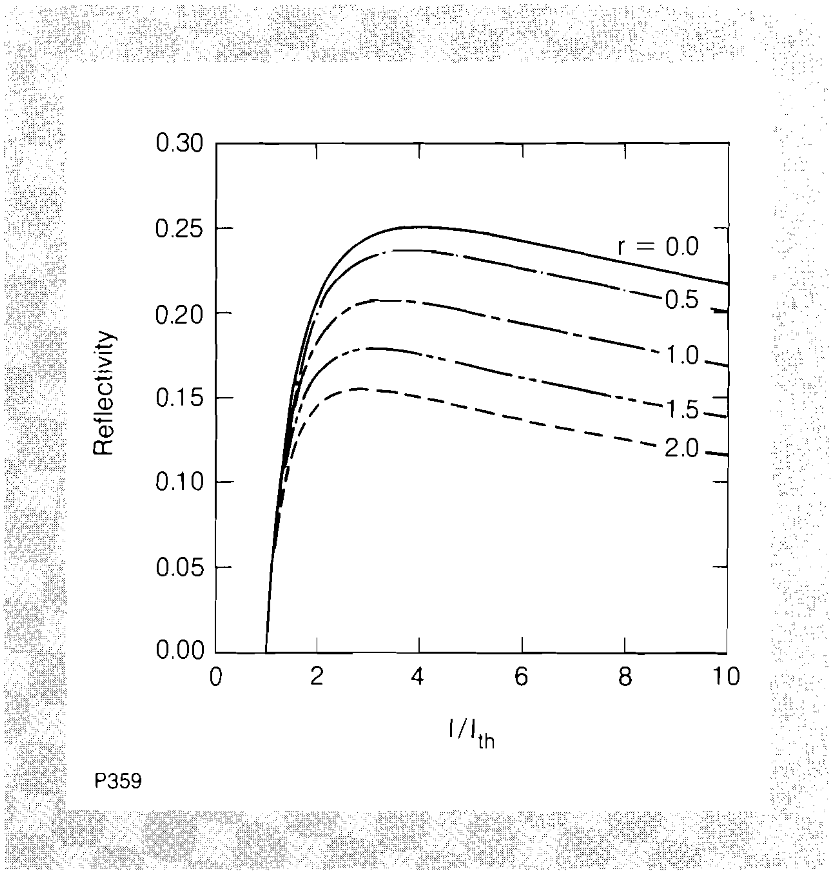

Fig. 23.32
Steady-state reflectivity of the plasma as a function of incident laser intensity, in multiples of $(\omega_2\nu_1/\omega_1\nu_2)$. Results are shown for several values of the mismatch ratio r.

In summary, SRS was considered in a homogeneous plasma, with all three waves damped. The nonlinear saturation of the instability was examined for incident pump amplitudes well in excess of threshold. A nonlinear steady state cannot occur until the pump field is reduced to its threshold amplitude. This determines the magnitude of the self-generated field at the pump frequency, which in turn determines the saturated amplitude of each daughter wave. The resultant reflection coefficient differs from a previous calculation[5] in that it reaches a maximum, as a function of increasing initial pump intensity, and decreases thereafter. This type of behavior has recently been observed in experiments conducted with long-scale-length plasmas.

ACKNOWLEDGMENT

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