Section 2
PROGRESS IN LASER FUSION

2.A Simplified Theory of Electron Heat Transport

The kinetic theory of electron heat flow is an important topic in the study of laser-fusion plasmas.[1-13] As the laser drives a heat front into the plasma, temperature and density profiles become so steep that characteristic scale lengths become comparable to the collisional mean free paths of the electrons that dominate thermal conduction. Under these circumstances, the classical formulation of thermal conduction,[14] based on the random walk of electrons with mean free paths small compared to typical scale lengths, breaks down. The heat flow at any given radial position becomes dependent on the surrounding temperature and density profiles, and can be significantly inhibited or enhanced compared to its classical value. Such changes in the heat flow can significantly alter the hydrodynamic evolution of the plasma. These changes can also alter the smoothing of nonuniform laser energy deposition, which occurs as the nonuniformities are transported in from the critical to the ablation surface. Finally, the kinetic theory of nonclassical transport yields a non-Maxwellian electron distribution that can alter significantly the growth rates of magnetic instabilities,[15] which may, in turn, additionally affect thermal transport.

One approach to the transport problem has been to develop numerical codes that solve the full Fokker-Planck equation [1-4] (including such features as hydrodynamic motion, energy sources from inverse bremsstrahlung or resonant absorption, and energy losses from ablation). Because of restrictions on the time step in such codes, it has not been possible to use these codes to follow the hydrodynamics over realistic time scales. Thus, other approaches, some predominantly analytic, make various simplifications in the Fokker-Planck equation;[5-13] with these models nonclassical heat fluxes are much more quickly obtained, and so may be more useful in a hydrodynamic code. One simplification, in particular, is to consider approximations appropriate to the behavior of "hot" electrons, which have velocities greater than about twice the average, or thermal, velocity.[6,7,10,12] [We define the thermal velocity as $(2T/m)^{1/2}$.] Such a model is motivated by the dominant role the hot electrons play in determining the heat transport. Here, we present some new studies and applications of such a model.

After describing our model, we consider, first of all, some of the questions of the self-consistency of our approximations. The model is then extended to three dimensions in order to study the smoothing of nonuniform energy deposition by electron transport. Finally, we develop our model heuristically to find an expression for the electron heat flux as a spatial convolution over nonlocal sources.[16]

Kinetic Model

The one-dimensional (1-D) Fokker-Planck equation for the electron distribution function $f(x, v, \mu, t)$,

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial x} - \frac{eE}{m}\left[\mu \frac{\partial f}{\partial v} + \frac{1-\mu^2}{v} \frac{\partial f}{\partial \mu}\right] = \left.\frac{\partial f}{\partial t}\right|_C , \qquad (1)$$

is the basis for our work. Here x is the inhomogeneous dimension, $\mu \equiv \hat{\underline{v}} \cdot \hat{\underline{x}}$, and

$$\left.\frac{\partial f}{\partial t}\right|_C$$

is the collision integral.[17] We analyze this equation as in the classical theory, expanding f in Legendre polynomials—here just the first two:

$$f(x, v, \mu, t) = f_o(x, v, t) + \mu f_1(x, v, t). \qquad (2)$$

In this form f_o represents the electron number and energy desnities, while f_1 represents the electron current and heat flux. Numerical work has shown[1,2] that this classical approach leads to accurate heat fluxes even for the steepest gradients encountered. We also take the steady-state limit, since the electron collision times are short compared to hydrodynamic time scales, and our source terms are independent of time. The electric field is determined by quasineutrality, which in 1-D steady state requires that the total current be zero, so that no charge accumulates at the boundaries of the plasma. Some further approximations are based on dividing the distribution into a small fraction of hot electrons and a bulk, or background, of cold electrons, which we take to obey the Spitzer-Härm solution. We then solve Eq. (1) for the nonclassical behavior of the hot electrons in the cold background. We also make the "high-Z" approximation, neglecting electron-electron collisions in comparison with electron-ion collisions, wherever possible, and further neglect electron-ion energy exchange.

With the preceding approximations, Eqs. (1) and (2) yield

$$-\frac{1}{v}\left(\frac{\partial}{\partial x} - \frac{eE}{mv}\frac{\partial}{\partial v}\right)(\lambda_o v^6)\left(\frac{\partial}{\partial x} - \frac{eE}{mv}\frac{\partial}{\partial v}\right) f_o = \frac{3}{2}\int_{-1}^{1} d\mu \left.\frac{\partial f_o}{\partial t}\right|_c^{ee} \qquad (3)$$

and

$$f_1 = -\lambda_o v^4 \left(\frac{\partial}{\partial x} - \frac{eE}{mv}\frac{\partial}{\partial v}\right) f_o. \qquad (4)$$

Here, $\lambda_o v^4$ is the mean free path for 90° scattering of an electron of velocity v by the combined angular scattering effects of ions and other electrons. The electron-electron collision term becomes

$$\left.\frac{\partial f_o}{\partial t}\right|_c^{ee} \approx \frac{1}{v^2}\frac{\partial}{\partial v}\left[f_o + \frac{T(x)}{mv}\frac{\partial f_o}{\partial v}\right], \qquad (5)$$

where T(x) is the temperature of the bulk of the electrons. The first term in Eq. (5) is a drag operator, representing loss of energy from the hot electrons to the cold background; the second term diffuses the energy of the hot electrons in the relatively cold but finite background temperature. Equation (3) thus determines f_o through the balance of spatial diffusion and energy drag and diffusion from electron-electron collisions. Equation (4) describes the resultant flux, balancing the gradients of f_o with electron scattering off ions.

We further simplify our model by neglecting the electric field in Eq. (3), an assumption we verify *á posteriori*. The electric field is also neglected in Eq. (4) for the hot electrons but must be retained for the cold electrons.

Finally, we always make some approximation to the energy diffusion term in the electron-electron collision operator. This is a reasonable procedure when the tail of the distribution is "overfull," or overpopulated, regarding the Maxwellian distribution at the local temperature and density. The physical effect of the full electron-electron collision operator is then to drag the overpopulated tail down to the Maxwellian level, which is just what the drag operator alone tends to do. In fact, Fokker-Planck code results indicate that the electron distribution is overfull throughout most of the region where the heat flux is significant.

Self-Consistency of the Model

For simplicity, our first treatment of the energy diffusion term is to neglect it. We have then solved Eq. (3), neglecting the electric field, in the presence of a hot electron source, $\delta(x)\,\delta(v-v_h)\,(n_h/v_h)$, which corresponds to a source flux of electrons at $x = 0$, of energy $(mv_h^2/2)$, and whose strength is characterized by the density n_h. We refer to this as the monokinetic solution, which is clearly of general importance.

To check our neglect of electric field effects, we consider the ratio

$$R \equiv (eE/mv)(\partial f_o/\partial v)/(\partial f_o/\partial x),$$

of the term neglected to that retained in our calculation. The electric field is computed from Eq. (4) by setting the net current caused by the hot particles and Spitzer-Härm cold particles equal to zero. It is then found that R is insensitive to the exact value taken for v_o, the velocity dividing hot electrons from cold, provided $v_h^2 \gg v_T^2$ and $(v_h/v_o)^8 \gg 1$, where v_T is the thermal velocity. R is proportional to $(n_h v_h/n v_T)$, which represents the ratio of the normalizations of the hot-electron inward current to the cold-electron return current.

A typical plot of R is shown in Fig. 21.11 for $v_h = 3v_T$. The value given for $(n_h v_h / n v_T)$ is based on a numerical model of a penetrating heat front.[18] The electric field is typically found to be of negligible importance, compared with collisional effects, as in the figure. Only at much higher velocities do we find that R exceeds one. We do not, however, expect the transport to be sensitive to the behavior of the very small number of such particles [e.g., $(n_h/n) \sim 10^{-3}$ for $(v_h/v_T) \sim 7$].

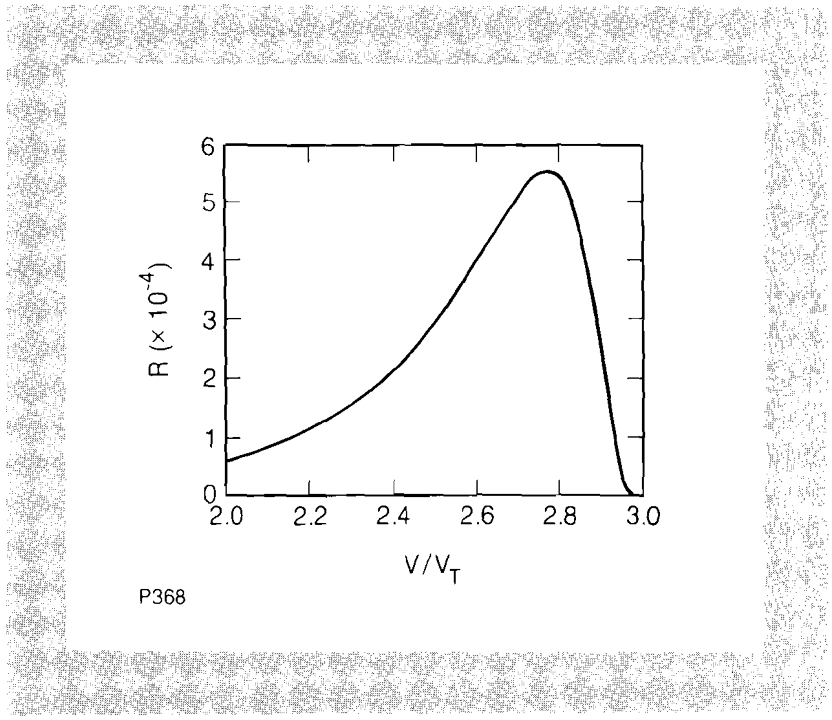

Fig. 21.11
Self-consistency of neglect of electric field in computation of hot-electron distribution. Ratio R of electric field term neglected to terms retained in Eqs. (3) and (4) is plotted. Case shown is for monokinetic solution with hot-electron source velocity $v_h = 3\ v_T$, $Z = 4$, and $(n_h v_h / n v_T) = 3.3 \times 10^{-3}$.

A second issue of self-consistency is the value of (f_1/f_o). If $(f_1/f_o) > 1$, f can become negative, while for $(f_1/f_o) > 3$, the distribution would imply that more than all of the particles are moving in one direction.[2,3] For the monokinetic solution, (f_1/f_o) is found to be proportional to $xv^4/(v_h^8 - v^8)$. Thus, (f_1/f_o) becomes large at large x and v, where there are the fewest particles, since particles are dragged to lower energies as x increases. [There are no particles at $v = v_h$, where (f_1/f_o) is singular.] Numerical computation confirms this behavior.

Smoothing from Electron Transport

To study smoothing, we generalize Eqs. (2)–(4) to three dimensions. Neglecting the electric fields, $(\partial^2/\partial x^2)$ is replaced by ∇^2 in Eq. (3), while $(\partial/\partial x)$ is replaced by $\hat{v} \cdot \nabla$ in Eq. (4), with μf_1 replaced by f_1 in Eq. (2). In the equation for f_o we neglect magnetic fields, which can arise in three dimensions, as well as the electric field and the energy diffusion term. We also take the density to be constant, which is plausible for the foot, or preheat, region of a heat front. As a model for nonuniform energy deposition, we consider a source $S^{(1)}$ localized at $x = 0$, with plane wave behavior in the transverse directions, and a Maxwellian distribution in v, $S^{(1)} = A\delta(x)\, e^{ik_y y} e^{ik_z z} \exp(-v^2/v_h^2)$ with

$$A = n^{(1)} (\pi^{1/2} v_h)^{-3}.$$

In practice, $S^{(1)}$ is a small perturbation on the zero order source and yields a corresponding perturbation in f_o, $f_o^{(1)}$. Both f_o and $f_o^{(1)}$ are easily reduced to quadrature.

With $f_o^{(1)}$ we study the radial smoothing of the energy density,

$$\epsilon^{(1)} = n^{(1)}T^{(1)} = \int d^3v \frac{mv^2}{2} f_o^{(1)} , \tag{6}$$

and of the energy deposition rate $\dot{Q}^{(1)}$ which is determined by the drag operator in the equation for f_o,

$$\dot{Q}^{(1)} = \frac{2\pi m}{\lambda_o (z+1)} \int_o^\infty dv\, v^2 \frac{\partial}{\partial v} f_o^{(1)} , \tag{7}$$

where z is the ion charge. (We have extended the integrals to v = 0, since their behavior is dominated by the high-energy contributions.) An analytic estimate[10] of $\epsilon^{(1)}$ indicates that $\epsilon^{(1)} \propto H(\lambda(x) - x)\exp(-k_\perp x)$, where $\lambda(x)$ is the velocity-averaged, energy-loss mean free path and H is the Heaviside step function. These two factors represent the intrinsic smoothing of the transverse variation and the loss of energy to the cold background. When $k_\perp \lambda$ is less than one, the smoothing factor $e^{-k_\perp x}$ has little effect, while for $k_\perp \lambda$ greater than one it is dominant. This behavior is approximately verified by the numerical evaluations of $\epsilon^{(1)}$ and $|\dot{Q}^{(1)}|$ shown in Fig. 21.12. The specific case considered is the same as the one in Ref. 7, i.e., the material is gold and the hot-electron temperature is 10 keV. The dimensionless unit of length in Fig. 21.12 corresponds to 0.041 μm at solid density (19.5 g/cm^3). From the figures we see the $e^{-k_\perp x}$ behavior of $\epsilon^{(1)}$ emerge as $k_\perp$ increases. The ratio of slopes for the straight line figures approximately corresponds to $e^{-k_\perp x}$ behavior. Furthermore, as x increases the curves asymptotically approach this behavior, since $\lambda(x)$ increases with x as the distribution $f_o^{(1)}$ becomes weighted toward higher energies. The figures of $|\dot{Q}^{(1)}|$ display the same qualitative behavior as the $\epsilon^{(1)}$ figures, amplified by the fact that they result from higher velocity moments of the distribution.

Formulation of Delocalized Heat Flow

Returning to Eq. (3), we develop a heuristic model for nonlocal thermal conduction in steep temperature gradients. For this purpose the energy diffusion term must be retained in some form to account for the effect of a given temperature profile T(x). We include it by taking

$$f_o = f_M = [n(\ell)/\pi^{3/2} v_T^3(\ell)]\exp[-v^2/v_T^2(\ell)]$$

in the diffusion term. This formulation guarantees that in the limit of high collisionality the Spitzer-Härm limit, $f_o = f_M$, is recovered. We also neglect all explicit electron energy sources and sinks. We neglect the electric field in Eq. (3), but retain it in Eq. (4), so that we can extend the resulting distribution to all velocities. The resulting expression for the heat flux q is easily obtained from Eqs. (3)–(5). This heat flux is in the simple form of a convolution of sources propagated from all points along the heat front.

The appropriate physical properties of the nonlocal heat-flux formula are easily verified. In the limit of high collisionality (or, equivalently, weak

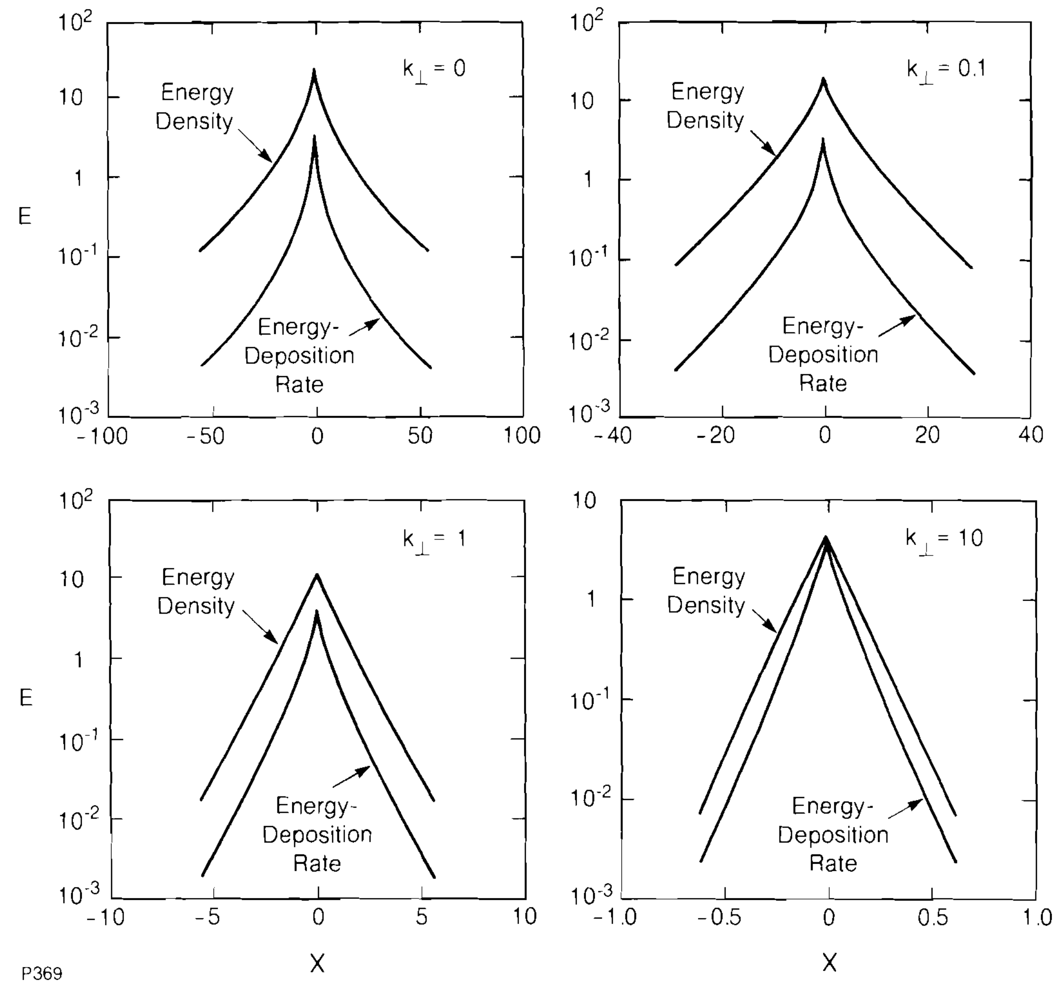

Fig. 21.12
Radial smoothing of nonuniform energy deposition. The perturbed hot-electron energy density $E^{(1)}$ and the perturbed hot-electron energy deposition rate $\dot{Q}^{(1)}$ are plotted for a δ-function source at $x = 0$. The unit of length is the geometric mean of the mean free paths and $k_\perp$ is the magnitude of the transverse k-vector of the perturbation. The $k_\perp = 0$ case is identical to the zero-order heat transport case. At $k_\perp = 10$, the wavelength of the perturbation completely controls the heat penetration.

gradients in n and T), the classical result is recovered. In the limit of an infinitely steep temperature gradient, a "flux-limited" heat flux is computed,

$$q \rightarrow \frac{1.4}{(Z+1)^{1/2}} q_{FS} \,,$$

where $q_{FS} \equiv nT(T/m)^{1/2}$ is the free-streaming heat flux. This result is always less than q_{FS} and can be reduced further by including the effect of electron-electron collisions in Eq. (4), which leads to an additional factor of $(Z+0.24)(Z+4.2)^{-1}$; for example, $q \rightarrow (0.3)q_{FS}$ for $z = 5$.

Summary

We have developed a simplified electron thermal transport theory and have found this model to be self-consistent in some important respects. A simple generalization of the model to study smoothing indicates that transverse nonuniformities in energy density and heat flux decay as $\exp(-k_\perp x)H(\lambda(x) - x)$. Different smoothing behavior may depend on the inclusion of magnetic field effects on the hot particles,[19] or effects of the electric field on the intermediate velocity particles, which have not been included here.

A heuristic generalization of our model gives the heat flux as a nonlocal convolution of given density and temperature profiles. Unlike a similar published result,[11] our formula is independent of any arbitrary or phenomenological parameters.

Continuing work on our model[20] includes comparison of our heat flux results to those of a full Fokker-Planck code. We are also increasing the efficiency of numerical evaluation of our formula for the heat flux; this will allow computation of temperature and density profiles from the time-dependent hydrodynamic equations.

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