A Boundary Condition for Guderley's Converging Shock Problem

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The Guderley problem[1] consists of a 1-D radially converging shock wave supported to infinity. The solution consists of leveraging the group invariance of the flow equations, along with an ideal equation of state, to reduce the set of partial differential equations into a single ordinary differential equation. The result is a simplified self-similar model describing the collapse of spherical shock waves.

This model has been used for understanding the shocks in laser-driven implosions in the past.[2] In particular the model has been used to predict the neutron yield from spherical implosions.[2,3] This work sets out to make two modifications to the Guderley solution that extend its utility for understanding implosions.

The first of the modifications includes the addition of an isentropic release wave launched from some reference radius. This corresponds to the pressure no longer being supported, such as when the laser turns off in a laser-driven implosion. In this case the release wave will be launched from the ablation surface as the remaining solid density material releases into vacuum. This wave breaks the self-similarity of the solution and changes the density profile in the shocked region. The density profile of the release is

$$\rho(r) = \rho_R \left(\frac{R_{\text{out}} - r}{R_{\text{out}} - R_{\text{in}}} \right)^{\epsilon}, \tag{1}$$

with R_{out} and R_{in} referring to the inner and outer edges of the release wave, respectively. The inner edge trajectory of the release wave is known assuming it moves at the local sound speed of the material and is launched from a known location at a known time. The outer edge of the release wave is then solved for using the conservation of mass in the released region. The temperature and pressure in the released material are then solved for by using the adiabatic relationship and an ideal equation of state.

The second modification includes taking the single fluid temperature present in the Guderley solution and partitioning it between an electron fluid and an ion fluid based on their masses. Additionally, a scheme for the fluids equilibrating is introduced based on Spitzer. The temperatures are constrained to satisfy

$$T_{\text{i}} + \overline{Z} T_{\text{e}} = \left(1 + \overline{Z}\right) T_{\text{G}}, \tag{2}$$

where T_{e} and T_{i} are the electron and ion temperatures, respectively, and T_{G} is the single-fluid temperature. This leaves the single-fluid temperature unchanged, so the hydrodynamics are unperturbed. Finally, the temperatures equilibrate according to[4]

$$\frac{\mathrm{d}T_{\text{e}}}{\mathrm{d}t} = \frac{T_{\text{i}} - T_{\text{e}}}{\tau_{\text{ei}}}. \tag{3}$$

The results of the modified Guderley solution are compared to *LILAC* simulations of an 860-μm solid CH sphere driven by a 2-ns square pulse. In order to set the parameters of the Guderley solution, the initial shock pressure and the time of shock collapse were set to match the *LILAC* results. Figure 1(a) compares the trajectories from *LILAC* and the Guderley solution. The shock and release wave trajectories compare well between the two models. The shock trajectory after the release wave hits the outgoing shock is the most obvious difference between the two models. This is because the shock trajectory in the Guderley model is unchanged by the release.

Figure 1(b) compares the total x-ray yield from the shock collapse that escapes the sphere as a function of initial density of the target. As the density decreases, the release modification is less relevant because the material is much less efficient at absorbing x rays, and as the density increases, temperature equilibration is less important because the time scale for coupling is so short that the material is equilibrated instantaneously with respect to the emission time scale. Only the model that includes both modifications is able to reproduce the *LILAC* results over a broad range of the density space.

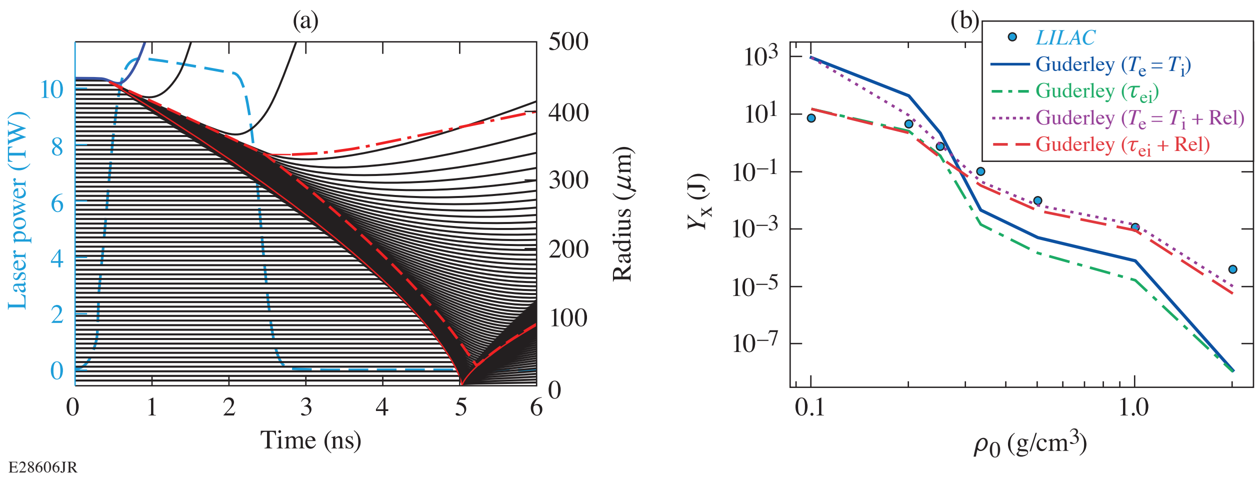

Figure 1
(a) A comparison of the wave trajectories for the modified Guderley solution (red curves) plotted over the particle trajectories from *LILAC* (black curves). The *LILAC* pulse shape is shown by blue curves. (b) A comparison of total x-ray yield in joules across different models: *LILAC* (solid circles), standard Guderley (solid blue curve), Guderley with temperature equilibration (dashed–dotted green curve), Guderley with release modification (dotted purple curve), and Guderley with release and temperature equilibration (dashed red curve). The simulations were run given the same conditions but with a scaled initial density to highlight regions where different physics played a key role.

The modified Guderley solution has proved capable of reproducing many results from *LILAC* with many fewer input parameters. This work was done with the ultimate goal of using both models to infer important physics information from spherical experiments through a Bayesian model-fitting procedure.

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