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2.B Magnetic Field Effects on Electron Heat Transport

Thermal transport in laser-produced plasmas is a topic of great importance to laser fusion. In one-dimensional situations, and where moderately steep temperature gradients are believed to occur, heat fluxes considerably smaller than those predicted by the classical theories of Spitzer[1] and Braginskii[2] have been inferred experimentally. Such results are often parametrized in terms of a "flux limiter" f,[3] with the heat flux q given as a multiple f of the "free-streaming flux" q_F, defined here as $q_F = n_e kT (kT/m)^{1/2}$, where n_e, T, and m are the electron number density, temperature, and mass, respectively, and k is Boltzmann's constant. In two-dimensional situations, the problem is more complicated because magnetic fields generated in the laser-plasma interaction process must be considered. It is the purpose of this article to discuss some of the effects of magnetic fields on thermal transport in the presence of moderately steep temperature gradients.

Many attempts have been made to understand the physical basis of reduced heat fluxes. Three different approaches may be summarized as follows:

a) Classical theories simply do not apply in the regime of interest (thermal-electron mean free path $\lambda_T \gtrsim 0.01$ times the temperature scale length L).[4] If the electron kinetic equations are solved more accurately, the correct heat flux will result. No additional physical processes are involved. This approach has recently attracted considerable attention, particularly on account of numerical solutions of the Fokker-Planck equation, reported by Bell *et al.*,[5] which indicated reduced heat fluxes corresponding to a value of f of the order of 0.1.

b) Ion-acoustic turbulence is set up in the plasma giving rise to an enhanced electron collision frequency and therefore a reduced heat flux.[6,7]

c) Magnetic fields are generated in the plasma as a result of a lack of one-dimensional symmetry;[8] the classical theory of Braginskii[2] then predicts reduced heat fluxes due to the localization of electron orbits in the magnetic fields.

The third approach is inapplicable to ideal, one-dimensional plasmas, but real plasmas are never truly one-dimensional, whether due to the finite extent of the focal spot in single-beam experiments or to the finite level of irradiation nonuniformity in a multi-beam spherical system such as OMEGA. In CO_2-laser experiments, magnetic fields have been observed to play a significant role in enhancing collisionless lateral energy transport along the target surface and away from the focal spot.[9,10] In the more collisional plasmas produced by micron- or sub-micron-wavelength lasers, the role of magnetic fields may be important in inhibiting the inward flow of energy to the ablation region as well as in affecting the lateral smoothing of irradiation nonuniformities. In the limit of small λ_T/L, the problem of heat conduction in magnetic fields was solved by Braginskii,[2] but for moderately large λ_T/L ($\gtrsim 0.01$), a more accurate kinetic treatment is required.

A full treatment of the transport problem in moderately steep temperature gradients, including both strong and weak magnetic field limits, would require, for example, a two-dimensional Fokker-Planck treatment. To date, such treatments have been computationally prohibitive. However, it was shown by Shvarts *et al.*[11] that, in one dimension, a simple local treatment, in which the anisotropic portion of the distribution function (f_1) is bounded from above by the isotropic Maxwellian distribution function (f_0), leads to results similar in many respects to those obtained by Bell *et al.*[5] from Fokker-Planck simulations. This correspondence encourages us to extend this simple local model to two and three dimensions, including magnetic fields. While some questions will remain unanswered in the absence of a full Fokker-Planck treatment, this approach may provide some insight into the respective roles of magnetic fields and kinetic effects in inhibiting thermal conduction. In particular, we shall demonstrate that a transition occurs between flux limitation and magnetic field inhibition at moderately low values of the magnetic field.

Theory

We first review the physical basis by which electron heat transport is modified by a magnetic field, with reference to a simplified model[2] illustrated schematically in Fig. 12. The heat flux across a surface element at a given point P is obtained by summing the contributions of all electrons crossing that surface element, from all angles and from either side. Each electron transports energy characteristic of the temperature of the point at which it had its last collision; thus, in the absence of magnetic fields [Fig. 12(a)], electrons whose last collision occurred on the hotter side of the surface (H) contribute more than electrons from the other side, and the net heat flux is directed antiparallel to the temperature gradient ∇T. In the presence of a magnetic field perpendicular to the temperature gradient [Fig. 12(b)], the electron orbits are circular, and a component of heat flux perpendicular to ∇T arises. This component, which will be referred to as the "transverse" heat flux, maximizes when the average heat-carrying electron traverses half a gyrational orbit between collisions. The magnetic field also inhibits the "longitudinal" heat flux component antiparallel to ∇T [Figs. 12(c) and 12(d)], because the heat flux from the hotter side is carried by electrons whose last collision may have been at a hotter point H [Fig. 12(c)] or a colder point C [Fig. 12(d)].

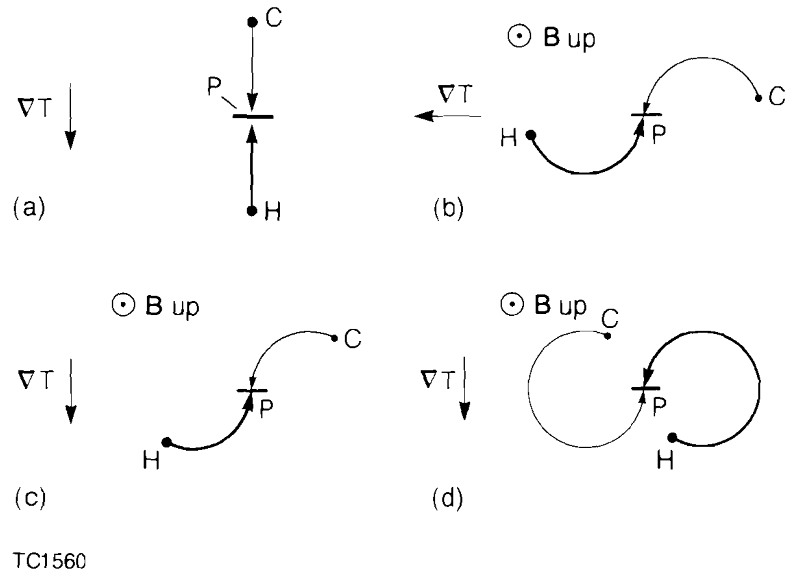

Fig. 12
Schematic showing the effects of a magnetic field (B) on the orbits of heat-carrying electrons. A net flux q, across a surface element at a point P, occurs because electrons which had their last collision at a hotter point (H) carry more energy than electrons whose last collision occurred at a colder point (C).

(a) When $B = 0$, q is directed along ∇T;

(b) When $B \neq 0$, heat can flow perpendicular to ∇T;

(c), (d) When $B \neq 0$, the heat flowing antiparallel to ∇T is reduced because of averaging over electrons whose last collisions occurred at different places on the circular orbit.

The effect of the magnetic field is usually parametrized by the Hall parameter β, defined as $\omega_c \tau_{ei}$, where ω_c is the electron gyrofrequency, and τ_{ei} is the electron-ion collision time. Here, β is the number of radians traversed around a gyro-orbit between collisions; β depends strongly on the electron velocity v, since $\tau_{ei} \propto v^3$. The longitudinal heat flux associated with electrons whose β is large is greatly reduced ($\sim\beta^{-2}$), since each point on the gyro-orbit develops an equal probability of being the location of the last collision. In typical electron distribution functions, there will be some electrons with $\beta<<1$ and others with $\beta>>1$.

Our theory starts with the Boltzmann equation for the electron distribution function f(r, v, t):

$$\frac{\partial f}{\partial t} + v \cdot \nabla f - \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\, v \times \mathbf{B}\right) \cdot \frac{\partial f}{\partial v} = C. \tag{1}$$

E and **B** are the electric and magnetic fields, (-e) the electron charge, c the speed of light, and C the collision operator. We consider a coordinate system (x, y, z) with the z axis locally aligned along the magnetic field, as indicated in Fig. 13, and we use spherical polar coordinates (v, θ, ϕ) in velocity space: $v \equiv v\mathbf{\Omega}$. We use the first two terms of a moment expansion for f:[12]

$$f(\mathbf{r}, v, t) = f_0\,(\mathbf{r}, v, t) + \mathbf{f}_1\,(\mathbf{r}, v, t) \cdot \mathbf{\Omega}, \tag{2}$$

where f_0 and $\mathbf{f}_1$ will be referred to as the isotropic and anisotropic components of f.

In this model, f_0 will be treated as a known function (e.g., a Maxwellian at the local temperature). The anisotropic component $\mathbf{f}_1$ is then obtained in terms of f_0 by substituting Eq. (2) into Eq. (1) and integrating Eq. (1) over $\mathbf{\Omega}\, d\mathbf{\Omega}$. We find:

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\nabla f_0 - \frac{e}{m}\mathbf{E}\,\frac{\partial f_0}{\partial v} - \omega_c\, \hat{z} \times \mathbf{f}_1 = \mathbf{C}_1, \tag{3}$$

where $\omega_c = eB/mc$ is the electron gyrofrequency, and $\hat{z}$ is a unit vector in the z direction. We assume that only electron-ion Coulomb scattering contributes to the collision integral $\mathbf{C}_1$, and we use[13] $\mathbf{C}_1 = -\mathbf{f}_1/\tau_{ei}(v)$, where the velocity-dependent collision time $\tau_{ei}(v)$ is proportional to v^3. We also drop the time derivative in Eq. (3), thereby assuming a quasi-static state. Then, solving Eq. (3) for $\mathbf{f}_1$, we obtain

$$\mathbf{f}_1(v) = -\frac{\tau_{ei}}{(1+\beta^2)}\left[1+\beta^2\,\hat{z}\hat{z}\,\cdot + \beta\,\hat{z}\,\times\right]\left(v\nabla f_0 - \frac{e}{m}\mathbf{E}\,\frac{\partial f_0}{\partial v}\right), \tag{4}$$

where $\beta(v) = \omega_c\tau_{ei}(v) = \beta_0\,(v/v_0)^3$, and $v_0 = (kT/m)^{1/2}$.

So far, we have followed Braginskii's treatment. Now we make use of the prescription of Shvarts *et al.*[11] We assume that the i component of Eq. (4) is satisfied for $v \le v_i^*$, and that

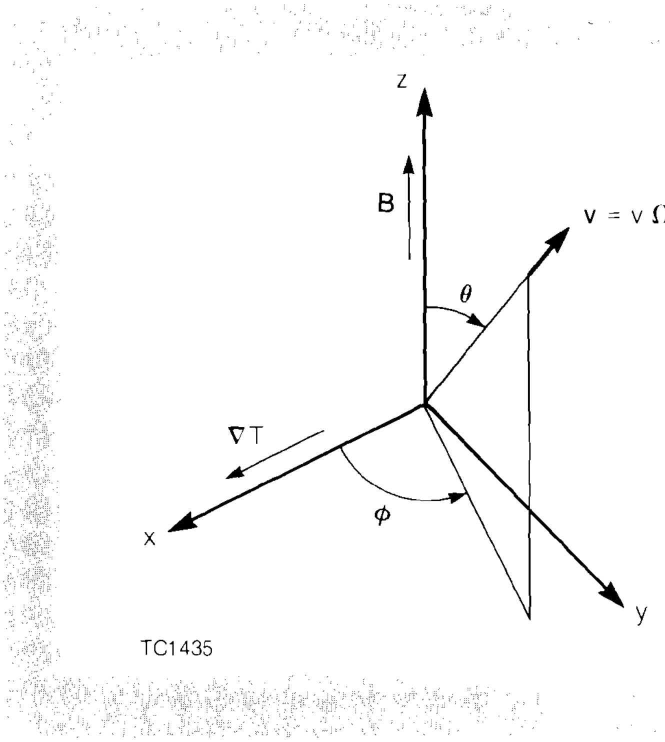

Fig. 13
Coordinate system. The z axis is taken along the magnetic field, and spherical coordinates about this axis (v, θ, φ) are used to describe velocity space. The x axis is chosen to lie along the temperature gradient. The heat flux has two components, "longitudinal" along the x axis, and "transverse" along the y axis.

$$f_{1i}(v) = \delta_i f_0(v) \tag{5}$$

for $v \geq v_i^*$, where the parameters δ_i are assigned values of order unity to ensure, at least, that the electron distribution function in Eq. (2) will never be negative. The cutoff velocities v_i^* are to be determined by iteration, and $f_{1i}(v)$ must be continuous at $v = v_i^*$. In the illustrative calculations below, we use $\delta_i = 0.67$ for each i. The detailed results will, of course, depend somewhat on the δ_i, but in view of the generally close agreement between this approach[11] and Fokker-Planck calculations[5] in one dimension, we may reasonably expect our results to yield some useful insight.

The current and heat-flux vectors are obtained by solving Eqs. (4) and (5) iteratively for the components of $\mathbf{f}_1$ and by performing the appropriate integrals over velocity space. By way of comparison, full Fokker-Planck calculations evaluate $f_0(v)$ by solving Eq. (1). Results may be obtained for a given electric field (in which case the current and heat flux are calculated), or for a given current (in which case the electric field and heat flux are calculated). Details are given in Ref. 14.

Illustrative Results

The magnetic-field-induced modifications to the heat flux are presented in terms of the Hall parameter β_0 for thermal electrons, given by

$$\beta_0 = \omega_c \tau_{ei}(v_0) = 5.10\times10^{21}\, BT^{3/2}/[n_e\,(Z+1)], \tag{6}$$

where n_e is measured in cm^{-3}, T in keV, and B in MG. (If the electron-ion momentum-transfer collision time τ_e given by Braginskii[2] had been used in place of $\tau_{ei}(v_o)$, β_o would be a factor of 3.8 higher.) The effect of electron-electron collisions is approximated here by using (Z+1) in place of Z, and the Coulomb logarithm is taken to be 10. For a typical plasma of Z=4 and T=1 keV, we obtain the useful relationship

$$\beta_o \approx (10^{21}/n_e)B, \tag{7}$$

or, at the critical density for 1-μm Nd:glass laser radiation, $\beta_o \simeq B$, where B is in units of MG.

Magnetic fields of the order of a megagauss have been observed in the coronas of laser-fusion targets through Faraday rotation,[15] while fields of the order of 0.1 MG are harder to detect and are often considered unimportant. However, even values of β_o as low as 0.1 are sufficient to modify the heat flux significantly, because the β corresponding to the electrons which carry the bulk of the heat is at least an order of magnitude higher.

We will restrict ourselves to the geometry of Fig. 13, with **B** aligned in the z direction and ∇T in the x direction. The current, heat flux, and electric field will all lie in the x-y plane, their longitudinal and transverse components being parallel to the x and y axes, respectively. In all cases the heat flux **q** will be expressed relative to the free-streaming flux q_F ($=n_e m v_o^3$); the ratio q/q_F may be thought of as the effective flux limiter.

In order to illustrate the main features of this model, we will consider the case where the ratio of the thermal-electron mean free path λ_T (defined as $v_T \tau_{ei}$, where $v_T = \sqrt{3}\, v_o$) to the temperature scale length L_x is relatively high (0.1), and where the current **J** is zero. (The model applies equally to the case of non-zero current, for which results are given in Ref. 14.) The normalized heat fluxes q_x/q_F and q_y/q_F are given in Fig. 14 as functions of β_o. Here, as elsewhere, the solid lines denote q_x/q_F and the dashed lines q_y/q_F, for bounded $\mathbf{f}_1$. For the purposes of comparison, the thin lines denote the same quantities for unbounded f_1 (Braginskii's results[2]). The Braginskii result for $\beta_o = 0$ (off-scale) is $q_x/q_F = 0.57$. We note that for $\beta_o \gtrsim 0.2$, there is little difference between the bounded and unbounded results for either component. Therefore, even for a relatively small magnetic field, there is no need to invoke a flux limiter.

The asymptotic behavior of **q** at large β_o is suggested from the form of Eq. (4): $q_x \sim \beta_o^{-2}$ and $q_y \sim \beta_o^{-1}$. From Fig. 14, it is evident that there is a strong reduction of q_x/q_F (to 0.04), even for $\beta_o = 0.2$. At higher values of β_o the transverse component exceeds the longitudinal component. The transverse heat flux has a peak at very low β_o (0.03 in the Braginskii case, 0.1 in the bounded case); it should maximize when the β of heat-carrying electrons [$\sim \beta_o(v/v_o)^3$] is of the order of π, on the basis of the simple physical picture discussed earlier [see Fig. 12(b)]. Indeed, taking $\beta_o = 0.1$ and $v/v_o \simeq 3.2$ (see Fig. 15), we find $\beta \simeq 3$.

Figure 15 shows, plotted as functions of v/v_o, the x and y components of f_1/f_o (upper graphs) and $(v/v_o)^5\, f_1$ (lower graphs)

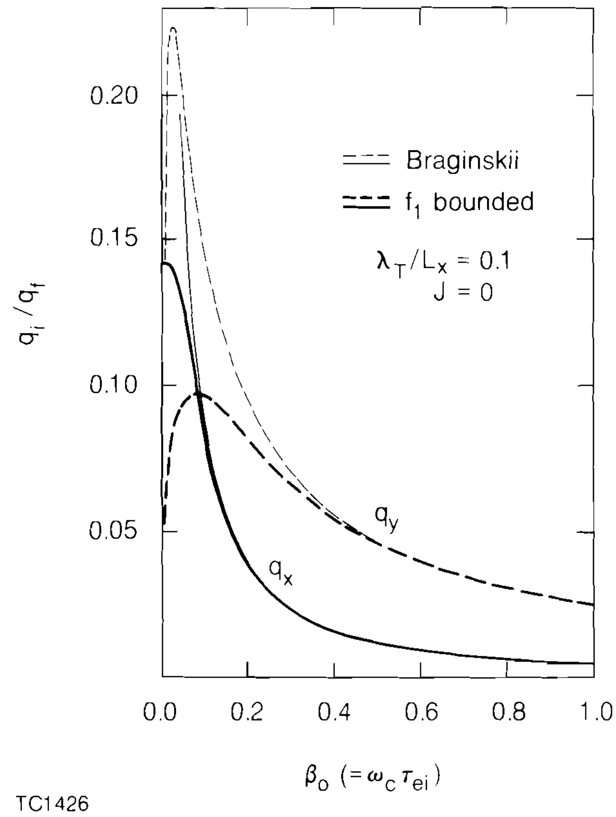

Fig. 14
Dependence of heat flux on β_0 $(=\omega_c \tau_{ei})$ for $\lambda_T/L_x = 0.1$ and zero current ($J = 0$). Solid curves, q_x; dashed curves, q_y. Light curves, Braginskii theory. Only for low values of β_0 is it necessary to modify the Braginskii theory.

corresponding to the fluxes plotted in Fig. 14, for various values of β_0. For β_0 =0, a limit on f_{1x}/f_0 is needed to avoid large values of this ratio. No limit is needed for the higher values of β_0 shown, since the magnetic field limits f_{1x}/f_0 in the Braginskii theory, and f_{1x} is well-behaved throughout the whole velocity range. This maximum, and the minimum corresponding to the low-velocity return current, both decrease in amplitude as β_0 increases.

In our model it is always necessary to limit f_{1y}/f_0 at some velocity. For low values of β_0 (e.g., β_0 = 0.2), a strong transverse flow of high-velocity electrons is partially limited. As β_0 increases (e.g., β_0 = 0.6), the cutoff point moves to higher velocities, and the electrons which carry the bulk of the energy flow are unaffected.

The areas under the lower graphs [of $(v/v_0)^5\, f_{1i}(v)$] are proportional to the heat fluxes in the respective directions. In each case, the maximum occurs at $v/v_0 \sim 3$. The integrated transverse flux is clearly greater than the longitudinal flux, and both decrease with increasing β_0.

In Fig. 16, the heat fluxes q_x/q_F and q_y/q_F are shown as functions of λ_T/L_x for various values of β_0. The classical Braginskii[2] or Spitzer-Härm[1] result for $\beta_0 = 0$ is also plotted. The Braginskii curve would of

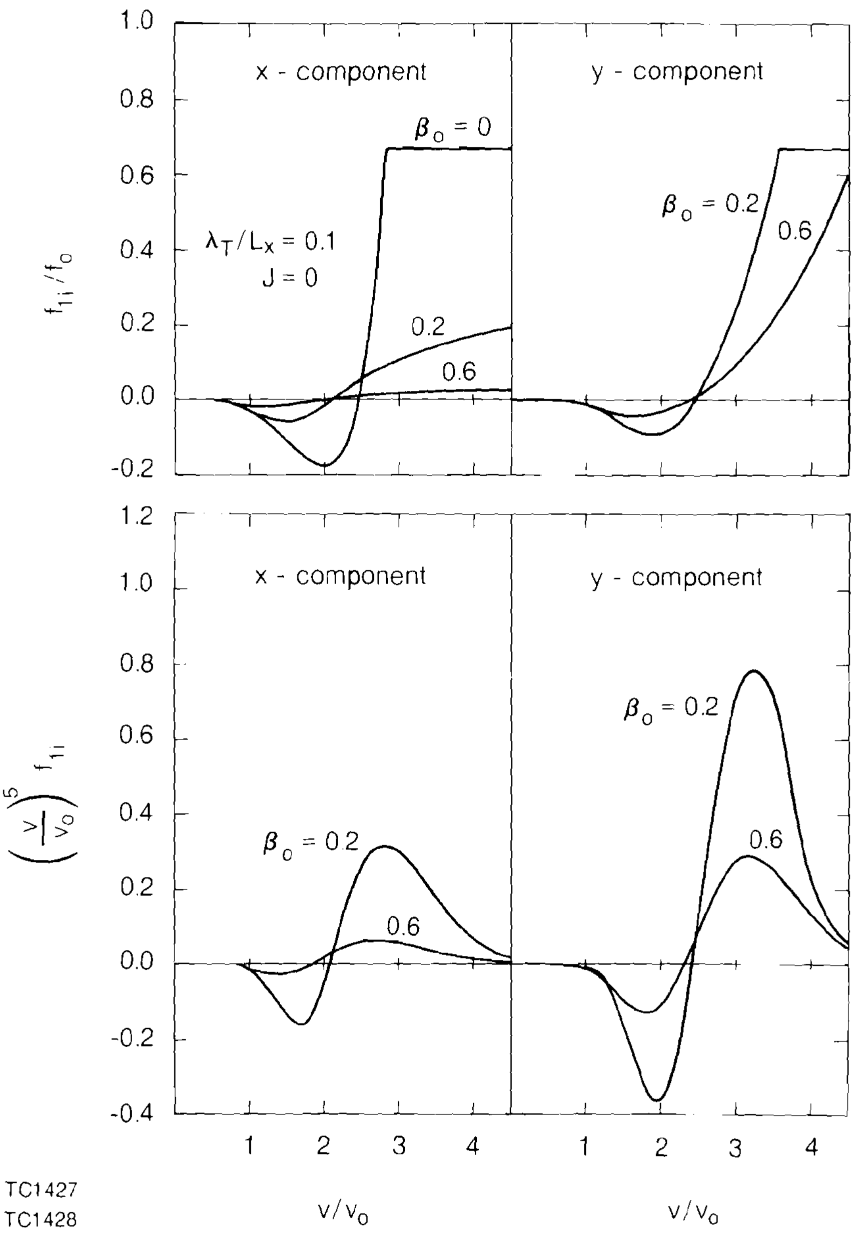

Fig. 15
Distribution functions corresponding to the fluxes plotted in Fig. 14. The upper plots, of the perturbed distribution function f_i/f_o, show that for β_o as small as 0.2, only the transverse (y) component need be bounded. The lower plots, of the normalized heat flux $(v/v_o)^5 f_i$, show that in most cases the dominant contributions to the heat flux come from electrons with velocities $\sim 3v_o$.

course be a straight line on a linear/linear plot. There is a region in the figure ($\lambda_T/L_x \sim 0.1$, $0.1 \lesssim \beta_o \lesssim 0.3$) where both components of the heat flux are of the order of a few percent (0.03-0.1) of the free-streaming value. This value of λ_T/L_x is typical of what may occur in laser-produced plasmas at moderately high intensities, and it is arguable that the inhibition commonly observed can be explained by very modest values of magnetic field. It must, however, be noted that the effective flux limiter implied by Fig. 16 is a strong function of both λ_T/L_x and β_o, and might lead to a greater diversity of experimentally inferred flux limiters than has been observed to date.

Figure 16 includes values of λ_T/L_x up to 1.0, but the regime of validity of this theory probably does not extend beyond $\lambda_T/L_x = 0.1$,[11] at least in the magnetic-field-free case. Beyond this limit, the heat flux is dominated by nonlocal contributions from electrons whose mean free path exceeds the temperature-gradient scale length L_x. Conversely, for smaller λ_T/L_x or for larger β_0, the nonlocal contribution decreases.

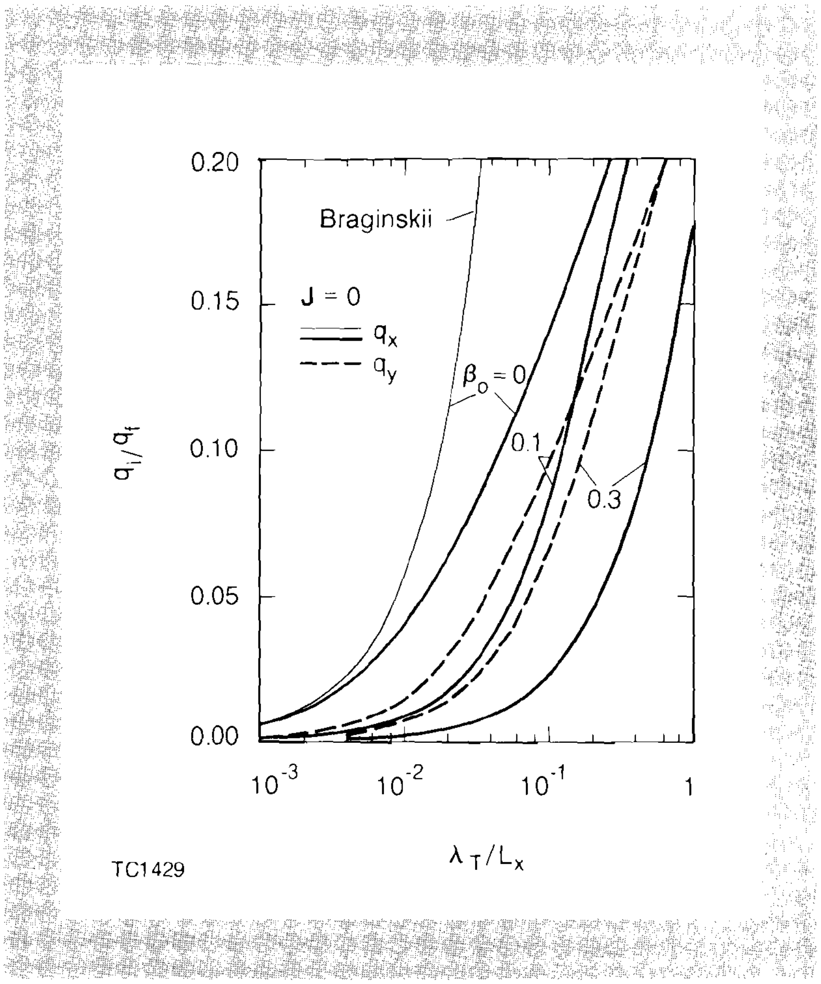

Fig. 16
Dependence of heat flux on λ_T/L_x for $J = 0$ and $\delta_x = \delta_y = 0.67$, for various β_0. Solid curves, q_x; dashed curves, q_y. Light curve: Braginskii (or Spitzer-Härm) theory for $\beta_0 = 0$. In the range $10^{-2} < \lambda_T/L_x \leq 10^{-1}$, typical of many experiments, modest magnetic fields give rise to an effective flux-limiter (q/q_F) in the range 0.01-0.1.

Summary

We have investigated the relationship between flux limiting and magnetic-field-induced transport inhibition, using a simple model which describes the transition between these two regimes in a physically reasonable way and yields some useful insight. While it would be unwise to advocate classical magnetic field inhibition as the primary explanation for the small flux limiter inferred from experiments, it is clear that these magnetic field effects deserve more careful consideration. For parameters corresponding to typical Nd:glass irradiation experiments, strong inhibition occurs for fields as small as 100 kG, an order of magnitude smaller than the megagauss fields which have been observed. At shorter wavelengths such as a third of a micron, magnetic field effects are probably less (since the collision time at the critical density scales as the square of the laser wavelength), but could still be significant.