Three-Dimensional Characterization of Spherical Cryogenic Targets Using Ray-Trace Analysis of Multiple Shadowgraph Views

Introduction
In the laser-driven direct-drive approach to inertial confinement fusion (ICF), energy from many individual high-power lasers is delivered to a spherical target, causing a spherically symmetric implosion. The 60-beam, 30-kJ OMEGA Laser System is used to study direct-drive ignition (DDI), where the laser energy is deposited directly onto the target. For indirect-drive ignition (IDI), the laser energy is directed onto a metal container (a hohlraum) surrounding the target, creating x rays that deposit the energy onto the target. IDI is inherently less efficient than DDI but has less-restrictive symmetry requirements on the laser illumination.

Current designs for both DDI and IDI high-gain ICF targets require a layer of condensed hydrogen fuel that adheres to the inner surface of a spherical shell ablator. Photon energy delivered to the target ablates its outer surface, and the ablation pressure drives the fuel layer inward, compressing both it and the gaseous fuel at the target’s center. The drive pressure is varied in time such that the fuel density is compressed by a factor of as much as 4000 while remaining relatively cold. Shock waves resulting from the drive-pressure history, along with compressive work, heat the gaseous-core “hot spot” to the high temperatures needed to initiate burning the fuel.

As the fuel layer is compressed and decelerates, perturbations on the inner ice surface act as amplitude seeds for the Rayleigh–Taylor instability on the inner surface. The nonlinear growth of this deceleration-phase instability mixes the cold compressed fuel layer with the hot-spot fuel vapor, reducing fusion yield or preventing ignition. Asymmetry-induced hydrodynamics can reduce the performance of ICF targets to well below that predicted by 1-D modeling. Ignition requirements impose strong constraints on the illumination uniformity and on the sphericity of the target.

The degrading effect of an inner-ice-surface perturbation on implosion performance depends on the perturbation’s mode number, which is the ratio of the capsule’s circumference to the wavelength of the perturbation. The surface roughness is characterized in terms of a mode spectrum analogous to Fourier analysis. Since the target geometry is spherical, spherical harmonics \( Y_{\ell m}(\theta,\phi) \) form the basis functions used for the mode spectrum. Accurate surface characterization of ice layers requires reliable measurement of the layer’s surface with submicron resolution at many points distributed over the surface of a target. Hydrodynamic codes then calculate capsule implosion performance using the measured surface mode power spectrum. The benchmarking of calculated target performance with experimental results is essential for designing ignition-scale targets and specifying their allowable surface roughness with confidence.

The DDI specifications for the National Ignition Facility (NIF) require a total root-mean-square (rms) deviation of less than 1 \( \mu m \) for an ice layer with less than 0.25-\( \mu m \) rms from Fourier modes higher than \( n = 10 \). An ice-layer rms deviation of less than 1 \( \mu m \) is also required for successful IDI on the NIF. Measurement of the ice-layer radius over the entire surface with submicron resolution is required to verify success or failure at achieving the required specifications.

This article describes the optical backlit shadowgraphic 3-D characterization of cryogenic direct-drive-target ice layers at LLE using ray-trace analysis of the shadowgrams. The following sections (1) briefly describe the principles and equipment used to record a cryogenic-target shadowgram at LLE; (2) analyze the resolution of shadowgram measurements; (3) describe three-dimensional ice-layer reconstruction from multiple target views using the conventional assumption that the shadowgram bright ring can be directly related to the ice thickness based on spherically symmetric calculations; and (4) present a shadowgram analysis to which nonspherically symmetric ray tracing is added, thereby improving the 3-D ice-layer reconstruction by self-consistently calculating the effects of ice-layer asymmetries and roughness on the position of the bright ring in each view. The conclusions are presented in the final section.

Shadowgraphic Characterization of Ice Layers
Optical backlit shadowgraphy is the primary diagnostic used to measure ICF target ice-layer roughness. A shadow-
graph records the image of the light rays passing through a backlit target. The rays are reflected and refracted at the shell wall and ice-layer surfaces. Some rays are focused into characteristic rings. Ray-trace modeling of a typical cryogenic target using the PEGASUS code\textsuperscript{26} has identified the specific reflections/refractions responsible for the brightest rings (see Fig. 109.44). The most-prominent ring or “bright ring” is the result of a single internal reflection off the inner solid/vapor interface of the ice layer. The position of the bright ring in the shadowgraph is directly correlated with the position of the inner surface of the ice layer and allows the nonuniformity of the inner surface to be characterized.

A high-magnification, high-fidelity backlit optical shadowgraphy system (see Fig. 109.45) is used to diagnose the ice-layer quality. A 627-nm red-light–emitting diode (LED) provides the backlighting. A 50- to 100-\(\mu\)s pulse drives the LED to illuminate (~\(f/5\)) the target. Imaging optics (~\(f/6\)) magnify the target image on a DALSA charge-coupled-device (CCD) camera (12 bit, 1024 \times 1024) (Ref. 27) such that the camera typically images about 1.2 \(\mu\)m per pixel. The camera is triggered by the same pulse that drives the LED.

A sample shadowgram of an LLE cryogenic D\(_2\) target suspended from a beryllium “C-mount” by four threads of spider silk is shown in Fig. 109.46. The strong unbroken bright ring and mostly featureless central spot are indicative of the high quality of this ice layer. Two inner rings are also clearly visible.

**Resolution of Shadowgram Rings**

The analysis of the target image in an individual shadowgram consists of accurately determining the target center, unwrapping the image into polar coordinates, and measuring the radial positions of both the target edge and the bright ring’s peak intensity. Details of this procedure are published elsewhere.\textsuperscript{21} Here, the accuracy and resolution of these measurements are discussed.
Previously, by examining the bright-ring-measurement scatter for very smooth liquid hydrogen layers,\textsuperscript{21} the resolution of the measurements was estimated to be about 0.1 pixel (~0.12 \textmu m). The high resolution of the bright-ring measurements has been verified using precision calibration targets as described here. The calibration targets are simulated target images of photolithographed chrome on glass.\textsuperscript{28} A simulated image consists of a “perfectly” circular edge along with a bright ring (plus two fainter inner rings) with a known variation in radial position. The radial variation of the rings was calculated, using the linearized formula discussed in the next section, for an ice surface with a surface-deviation, Fourier-mode power spectrum of

\[ P_n = Cn^{-2} \]  

(1)

for Fourier modes \( n = 1 \) to 100, where \( C \) was chosen such that the spectrum meets the DDI specification. The phase of each Fourier mode was randomized. This power spectrum and the calculated bright-ring position are shown in Fig. 109.47. The precision calibration target was photolithographed with a manufacturing tolerance of 0.1 \textmu m. A shadowgram of this target taken in one of OMEGA’s Cryogenic Target Characterization Stations is shown in Fig. 109.48. This image has been analyzed using LLE’s standard shadowgram analysis routines, and the results are shown in Fig. 109.47. The measured bright-ring positions have a mean error of less than 0.1 \textmu m (within the manufacturing tolerances of the calibration target), and the total rms error of the ring measurement is about 0.01 \textmu m.

It is clear that the bright-ring position can be very accurately measured in the characterization station shadowgrams. The relationship of that ring position with the radius of the actual ice surface producing the ring is discussed in the next section.
Three-Dimensional Ice-Layer Reconstruction Using Multiple Shadowgraph Views

An important feature of LLE’s Cryogenic Target Shadowgraphy System is the use of multiple views of the target to fully characterize the ice surface. Multiple views allow a more-complete surface characterization than is possible from a single view. Even with three mutually orthogonal views, it can be shown that there is only a small chance of detecting many local ice defects.\textsuperscript{29}

In the LLE Cryogenic Target Characterization Stations, the targets are rotated to provide a large number of different views for a single camera. The maximum number of views is limited only by the step size of the rotation stepper motor, which is a few tenths of a degree. Shadowgrams are typically recorded at 15° intervals, producing a total of 48 independent views between two cameras in each characterization station. The two cameras have approximately orthogonal views: one camera views the target center from an angle of 26.56° above the equator and the second camera, located 109.96° azimuthally from the first, views the target center from 12.72° above the equator. These view angles are determined by the location of the layering sphere windows that are aligned with the OMEGA target chamber’s viewing ports, which are used to center the target at shot time. These views are not optimum for target characterization. An off-the-equator viewing angle always results in unviewable regions surrounding the rotation poles; these unviewable “polar caps” are apparent in Fig. 109.49(a).

The standard method of shadowgram analysis assumes that the ice surface position along a great circle perpendicular to the shadowgraph view can be uniquely determined from the observed bright-ring position for that view\textsuperscript{17,22–25} by characterizing the ice-to-bright-ring relationship using a ray-trace study of spherically symmetric targets with varying ice thickness. At LLE, ray-trace modeling of a typical cryogenic target using the \textit{PEGASUS} code\textsuperscript{26} has identified how the shadowgram ring positions vary with the target parameters such as the shell outer radius and thickness, the D\textsubscript{2}-ice thickness, the shell index of refraction, and the D\textsubscript{2} index of refraction. The \textit{PEGASUS} code is two dimensional and assumes spherical symmetry in the target. A linearized formula derived from this modeling is used to determine the inner-ice-surface radius from the position of the bright ring for given target parameters.

A 3-D representation of the ice layer can be constructed from the ice-surface positions determined from the multiple shadowgram views. Figure 109.49 shows a target’s inner ice surface reconstructed from a target rotation of 24 separate views. The surface is dominated by low-mode-number asymmetries, but very different Fourier modes are observed for any given great circle. For this data set, the ice-surface 1-D rms roughness of the individual great-circle observations varies from 2.6 μm to 5.3 μm with an average value of 3.5 μm. This particular data set was selected for the following reasons:

- The outer surface is very smooth and symmetric and should have little effect on the bright ring.
- The optical distortion from collection optics was well minimized for these images.
- The bright ring is smooth and has few breaks.

![Figure 109.49](image)

Three-dimensional representation of a cryogenic inner ice surface (μm) displayed (a) on a spherical surface, (b) using the Aitoff projection with contour lines, and (c) using the Aitoff projection with surface elevation. These displays are constructed by interpolating all the individual data from the many great-circle observations to an evenly spaced (θ,φ) surface grid. The dotted lines in (a) show the location of the actual great circles observed in the individual shadowgraphs. The “polar caps” crossed by none of the great-circle observations are clearly visible.
As mentioned earlier, computer modeling of a spherical implosion, including instability growth, requires an ice-roughness spectrum described in terms of spherical-harmonics-basis functions $Y_{\ell m}(\theta, \phi)$ on the ice surface:

$$R(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} A_{\ell m} Y_{\ell m}(\theta, \phi) \quad (\mu m).$$

This description gives a Legendre-mode power spectrum and total surface variance of

$$P_{\ell} = \frac{1}{4\pi} \sum_{m=-\ell}^{\ell} |A_{\ell m}|^2 \quad (\mu m^2),$$

$$\sigma^2_{\text{rms}} = \sum_{\ell=1}^{\infty} P_{\ell} \quad (\mu m^2),$$

respectively. The $P_{\ell}$ spectrum represents an average over all azimuthal modes.

The $P_{\ell}$ spectrum for high mode numbers can be inferred from the Fourier power spectra of the many great circles observed. If one assumes that the surface perturbations are randomly distributed, the great-circle 1-D Fourier-mode power spectrum, averaged over many great circles, can be mapped to an equivalent Legendre-mode power spectrum. The assumption of randomly distributed perturbations limits the applicability of the mapping to higher mode numbers. At LLE the ice-surface positions are directly fit to spherical harmonics to determine the lower mode numbers (up to some $\ell_{\text{max}}$).

The results of a direct $Y_{\ell m}(\theta, \phi)$ fit are shown in Figs. 109.50 and 109.51 for a fit up to $\ell_{\text{max}} = 10$. The surface reconstruction in Fig. 109.51 based on the low-mode-number fit is a good match to the data shown in Fig. 109.49. The Legendre power spectrum $P_{\ell}$ corresponding to this fit along with the higher mode numbers determined by the mapping method is displayed in Fig. 109.50. Target reconstructions using the standard analysis have successfully detected low-mode asymmetries in the ice layer, allowing the identification and correction of the sources of the layering sphere temperature isotherm asymmetries that cause them.

The maximum mode number fit, $\ell_{\text{max}}$, is limited by the largest space between sampled points on the surface. For typical LLE targets, the largest gap in the surface data occurs at the unviewable polar cap of the target. The maximum mode number that can be reliably fit is also reduced by the many smaller surface gaps between great-circle measurements, noise in the data, uneven surface weighting (sections crossed by several great circles are more heavily weighted), and the fact that the data do not agree at “cross-over” points (see the next section). The sum of these effects typically limits the direct surface fit to mode numbers up to about $\ell_{\text{max}} = 10$. The exact limit varies with each data set. The results of fitting too high an $\ell_{\text{max}}$ are shown in Figs. 109.52 and 109.53, where the fit has been extended to $\ell_{\text{max}} = 16$. The power spectrum in Fig. 109.52 is rising as mode numbers approach
\( \ell_{\text{max}} \), a trend not observed in the Fourier analysis of the bright rings. The combination of a too high \( \ell_{\text{max}} \) along with the spaces between data circles and data mismatch at the great-circle crossover points produces a fit with more structure than seen in the individual bright rings. This increased structure and “crinkling” are evident in Fig. 109.53.

**Ray-Trace Analysis**

Despite the success of the standard analysis, it is well known\(^{18-20,24}\) that the assumption that the ice-surface position along a great circle perpendicular to the shadowgraph view can be directly correlated to the observed bright-ring position for that view is valid only for perfectly spherical symmetry. Kozioziemski et al.\(^{18}\) showed that a shift in the ice layer along the viewing axis will alter the bright-ring position and “significantly shift the apparent ice-layer thickness.” This effect can be easily seen in Fig. 109.54, which shows the ray path of the bright ring for a target layer shifted along the viewing axis. For imperfect ice layers, Koch et al.\(^{19,20}\) note that “correlation depends on the height and curvature of the imperfection.” To illustrate this, the bright-ring radii predicted by 3-D ray tracing of a simulated ice surface constructed from spherical harmonics for opposing views are shown in Fig. 109.55. The predicted bright rings show general similarities in the low-mode structures but differ greatly in the higher-mode detail. This explains why the ice surfaces determined by the standard method do not agree at the cross-over points of the views.

If the shadowgrams were viewed along the equator and exactly opposing views could be measured, the average position of two bright rings would show a good correlation to the ice-surface position along the great circle perpendicular to the views.\(^{32}\) In this case the standard method can accurately be applied to the averaged bright ring. If one has nonequatorial views, exactly opposing views cannot be recorded. A study of two above-the-equator views in the Cryogenic Target Characterization Stations for OMEGA rotated 180° about the polar axis shows asymmetries affect the bright ring. When viewed from the right, the bright ring appears lower than when viewed from the left. The standard analysis would determine a quite-different ice thickness for each view. For an unshifted layer, the rays on both sides would be at the same height and the bright ring would appear the same for both views.
axis showed that only the lowest modes can be determined with any accuracy by averaging two bright rings. To self-consistently and accurately determine the 3-D ice surface from shadowgram bright-ring measurements requires the modeling of the effect that the ice-surface asymmetries and defects have on the bright-ring position and including this modeling into the shadowgram analysis. Kozioziemski et al. accomplished this in a limited fashion by using interferometry to measure the P1 ice-layer mode along the viewing axis, then correcting the bright-ring position for the effect of the P1 based on a ray-trace study. The ray-trace shadowgram analysis at LLE uses 3-D ray tracing to simultaneously fit the bright-ring measurements for all views (typically 48 different views) to a multimode (up to $\ell = 18$) spherical-harmonic representation of the ice layer.

Producing a simulated full shadowgram for a nonspherically symmetric ice layer can take days of CPU time due to the large number of ray-trace calculations required. For this fitting analysis, where many iterations of varying a large number of spherical-harmonic components is required, an alternative was found based on the observation that for spherically symmetric targets, the peak intensity of the bright ring is centered on rays whose paths on both sides of the target are along the viewing axis. This is a poor approximation for asymmetric layers such as a melted layer that is very offset from the view angle, but it is a good approximation for the quasi-symmetrical case of a typical well-layered OMEGA cryogenic target.

With the above assumption, one need only follow one ray for each measured bright-ring position used in the fitting. The rays are launched backward from the measured bright-ring positions along their viewing angles and followed through the target and out the other side where the divergences of the rays' final paths from the view angles are recorded. Nonlinear fitting iterations are employed to adjust the spherical-harmonic description of the ice surface, minimizing the divergence of all the rays from the viewing angles. Typical total fitting times are of the order of several hours to a day, depending on the number of measurements (typically 180 points from each of 48 views) and the number of spherical-harmonic components fit, which varies as $(\ell_{\text{max}} + 1)^2$. The nonlinear fitting routine constrains the maximum peak-to-valley variation of the ice surface to be similar to the maximum variation in the bright-ring position, preventing large peak-to-valley structures from occurring on the surface between the data rings or in the polar cap.

An example of the results from the ray-trace analysis with $\ell_{\text{max}} = 16$ is shown in Figs. 109.56–109.58. The ice-surface Legendre-mode power spectrum is well behaved up to this $\ell_{\text{max}}$ (Fig. 109.56), and the surface reconstructs show less anomalous structure (Fig. 109.57) than the standard method.

![Figure 109.55]

Bright-ring radii predicted by 3-D ray tracing of an asymmetric ice surface for opposing views show very different structures. The standard method of shadowgram analysis assumes that both views will be identical and will depend solely on the ice radius at the great circle perpendicular to the view. In fact, the bright-ring radii also depend on the angle of the ice surface relative to each view, and the bright ring may not be centered on rays that strike the ice surface at the great circle.

![Figure 109.56]

Legendre-mode power spectrum $P_\ell$ of the ice surface. The solid dots correspond to the low-mode-number (up to $\ell_{\text{max}} = 16$) ray-trace fit while the dashed line redispalyes the standard method fit (up to $\ell_{\text{max}} = 16$). The ray-trace analysis is well behaved up to higher mode numbers than the standard method. The circles are the result from mapping the average Fourier-mode spectrum of the many great circles to determine the higher Legendre modes.
Figure 109.58 shows a measured bright ring taken from one of the 48 different views of a $\text{D}_2$-ice layer in an OMEGA cryogenic target. The ice surface determined using the standard method with $\ell_{\text{max}} = 16$ (Fig. 109.53) would produce the bright ring shown by the dotted line according to ray-trace calculations using that surface. The standard deviation between the measured bright rings for the 48 different views and their standard method predictions is 1.5 $\mu$m.

The ice surface determined by the ray-trace analysis (Fig. 109.57) gives the bright-ring prediction shown by the solid gray line in Fig. 109.58. This surface produces a much better match to the observed bright ring. The standard deviation between the measured bright rings for the 48 different views and their ray-trace analysis predictions is 0.8 $\mu$m, a reduction of 45% from the standard method.

**Summary and Discussion**

It has been shown that the bright-ring position can be measured very precisely, but accurately correlating the bright-ring position to an ice-surface position is difficult. The standard method of applying spherically symmetric bright-ring calculations is inaccurate for asymmetric ice layers. Incorporating asymmetric ray tracing directly into the bright-ring analysis allows a self-consistent fitting of the bright rings from multiple views to an ice surface. Ray-tracing analysis reduced the error between the measured bright rings (for 48 different views) and those predicted for the fitted ice surface by 45% in comparison with the ice surface determined by the standard analysis.

It may be possible to further improve the performance of the ray-trace shadowgram analysis by

- modeling the bright-ring position, directly taking into account the uncollimated illumination of the actual shadowgraphy instead of assuming that the ring is centered on rays parallel to the viewing angles,
- fitting the optical differences between the views (magnification, focal position, etc.),
- adding some localized (e.g., spherical wavelet) defects to the ice-surface fitting to account for bright-ring features too localized to be fit by spherical harmonics and a reasonable $\ell_{\text{max}}$, and
- including the effects of outer-surface perturbations on the bright ring that are believed to be responsible for some sharp features in the bright ring.

It is important to note that as the ice-layer quality improves and becomes more symmetric, the accuracy of the standard method improves. Initial studies of DT cryogenic targets for OMEGA$^{34}$ indicate that beta-layered DT targets are very smooth and symmetric and good candidates for accurate standard analyses. These very symmetric layers may still benefit from ray-trace analysis by isolating the effects of outer-surface perturbations on the bright ring that can be even larger than the actual ice-surface effects.

LLE is building a cryogenic fill-tube target station that will allow validation of this ray-trace modeling and shadowgram
Three-Dimensional Characterization of Spherical Cryogenic Targets Using Ray-Trace Analysis

analysis. The station will possess target rotation capabilities and equatorial views for both shadowgraphy and x-ray phase-contrast layer diagnostics, allowing a direct comparison of ray-trace shadowgraphic analysis with (1) the standard analysis; (2) the standard analysis using averaged bright rings from opposing views; and (3) x-ray-phase-contrast direct measurements of the ice surface.

ACKNOWLEDGMENT
This work was supported by the U.S. Department of Energy Office of Inertial Confinement Fusion under Cooperative Agreement No. DE-FC52-92SF19460, the University of Rochester, and the New York State Energy Research and Development Authority. The support of DOE does not constitute an endorsement by DOE of the views expressed in this article.

REFERENCES
27. DALSA, Waterloo, Ontario, Canada, N2V 2E9 (http://www.dalsa.com).