Dynamic Defocusing in Streak Tubes

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1 Abstract

Streak cameras are used at LLE to measure the time histories of the laser pulse and the optical and x-ray emissions from OMEGA targets. A streak camera operates by applying fast-risetime voltage transients to a pair of parallel plates to deflect or streak electron trajectories across a phosphor screen. At high streak speeds, the foci of the electrons move behind the screen, thereby blurring the image on the screen. A program was written in C# to model electron deflection in streak cameras. The model uses the fourth-order Runge-Kutta method to integrate the Lorentz equation of motion and trace the path of electrons through the deflection plates as the voltages change. Data from the model was collected and extrapolated to calculate adjustments needed to refocus the electrons. A common-mode focusing voltage was applied to the deflection plates to shift the focal plane back to the screen.

2 Introduction

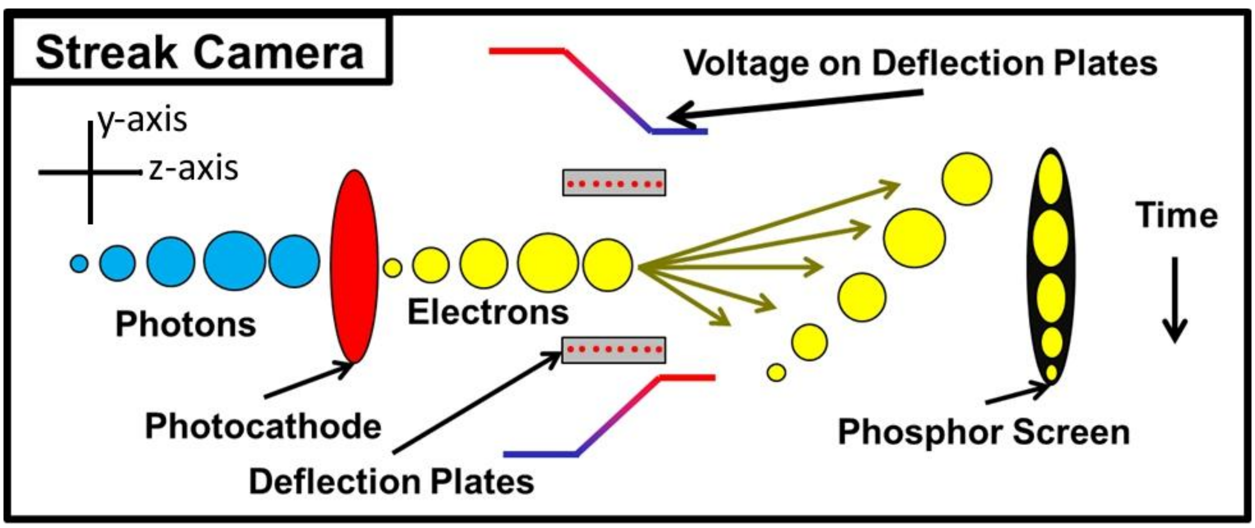

Figure 1: Simplified schematic diagram of a streak camera.
This 2-dimensional diagram illustrates electron deflection in a streak camera. The simplified schematic shows only the (y,z) plane and omits the input window and slit, the accelerating electrodes, and the focusing electrodes. Electrons produced by photons incident on the photocathode travel toward the right of the diagram and are deflected by the voltages on the deflection plates. The voltages change with respect to time according to their representations by the adjacent straight-line graphs of voltage.

The Laboratory for Laser Energetics utilizes streak cameras to measure the time histories of the laser pulse and the optical and x-ray emissions from OMEGA targets. A streak camera captures and records light pulse intensity with respect to time. As shown in Figure 1, photons entering a streak camera hit a photocathode after passing through a narrow slit. The photocathode emits electrons through the photoelectric effect, with the emitted electron current density linearly proportional to the incident photon intensity. These electrons are accelerated and focused by a set of electrodes in the streak tube.[1] The final section of the streak tube is a drift region where the electrons encounter the deflection plates. Initially the deflection plates are biased with one plate at a positive voltage and the other at a negative voltage. As the electrons pass through the deflection plates, fast-risetime voltage transients shift the voltage and reverse the polarity on the deflection plates, so that the last electron to pass through the deflection-plate field is deflected in the direction opposite to the first, with the middle electrons deflected at intermediate positions. The time at which any electron passes through the deflection field determines its angle of deflection, as shown in Figure 1. At the end of the streak tube, the deflected electrons hit a phosphor screen that emits light proportional to the number of electrons hitting a particular area.[2] To record the data, a charge-coupled device then captures a digital image of the phosphor screen from which a computer can read quantitative data. The photon intensity is transformed into a 2-dimensional image in which the temporal dimension is converted into a spatial dimension on the phosphor screen (Figure 1).

3 Dynamic Defocusing at High Streak Speeds

The rate of change of the voltage with time, $\frac{dV}{dt}$, on the deflection plates determines the rate of the electron beam deflection, i.e. the streak speed. This streak speed can be altered by adjusting the risetime and magnitude of the voltage transients applied to the deflection plates. Higher streak speeds allow for greater time resolution, as the temporal profile of the pulse is mapped to larger distances on the phosphor screen. However, experimental evidence from streak cameras at high streak speeds have shown decreased temporal resolution due to beam defocusing. At slow streak speeds, the electron beam focuses at the phosphor screen. At high streak speeds, the focal plane of the beam begins to move behind the phosphor screen. Therefore the image of the pulse on the phosphor screen becomes blurred and is comparably wider. The degree of defocusing increases as $\frac{dV}{dt}$ is increased, since the electrons experience a greater change in voltage while passing through the deflection region.

4 Deflection model

4.1 Program Outline

The program *RungeKutta* was created in the C# programming language in order to further analyze and simulate dynamic defocusing in streak cameras. It is named after the algorithm central to the calculation of the electron trajectories in the program. The program models the drift region of the streak tube comprising the deflection plates and the phosphor screen, but excludes the accelerating and focusing electrodes. The electrons enter the deflection region through a small aperture. The model numerically solves the electron force equation to calculate the electron path, tracking the electrons in a rectangular coordinate system. The force and acceleration are calculated from a series of line charges representing the deflection plates, allowing the trajectories to be integrated through the Runge-Kutta method to track electron position. The program reads input data necessary for simulating dynamic deflection, including voltage, streak speed, deflection plate location, and initial electron positions and velocities from the accelerating and focusing electrodes. The data from the calculations is output to two ASCII files: one that contains detailed numerical data from the model, and another that contains data points that can be displayed in graphical form by a program named *Eoplot*.

4.2 Existing Code

Previously, a program for static electron deflection had been written by Dr. Paul Jaanimagi. This program models the deflection region on a numerical grid, solving the Laplace equation for the voltage at each grid point from which the electric fields and electron trajectories can be calculated.

To analyze dynamic defocusing, a program that included deflection plates with dynamic voltages and variable start times for the electrons was written. The previous code was modified to accommodate the new features required to model dynamic electron deflection.

4.3 Deflection Plate Modeling

The deflection plates in the program were modeled through a series of line charges arranged along the x-direction perpendicular to the plane of Figure 1, passing through the red dots shown in Figure 1. The electrons were given coordinates on a rectangular coordinate system. The electrons (travelling in the Y-Z plane) in the program were subjected to forces depending on their distances from the line charges on the deflection plates.

According to Coulomb's law, the force $\boldsymbol{F}$ acting upon two electrically charged particles is,

$$\boldsymbol{F} = k_e \frac{q_1 q_2}{r^2} \tag{1}$$

where k_e is the proportionality constant, defined as $k_e \approx 8.98755 \times 10^9 \, N \cdot m^2/C^2$, q_1 and q_2 are the point charges, and r is the distance between the two points.

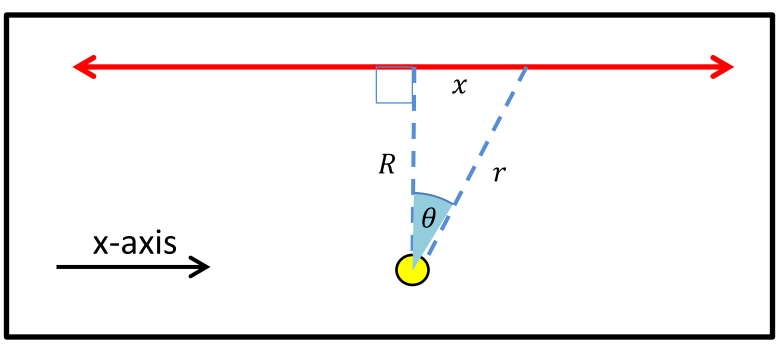

Figure 2: Electron distance from a line charge.
This diagram illustrates the variables involved in Eq. 2. The yellow circle represents the electron, the red line represents the line charge, and the blue dashed lines represent distance.

The force acting upon an electron due to a line charge can be obtained by integrating this equation along the line. Although the deflection plates in streak tubes do not extend to infinity along the X-axis of the rectangular coordinate plane in the model, it is possible to model deflection plates as line charges because the points on the line that are far from the center provide a negligible force on the electron, and thus can be ignored. The electric field for a line charge is radial from symmetry. The field at a distance R from the line (shown in Figure 2) can be derived through an integral of the point charge equation,

$$\boldsymbol{E_R} = 2\int_0^\infty k_e \frac{dq}{r^2}\cos\theta \tag{2.1}$$

where dq is an infinitesimal differential charge on the line, r is the distance from the point charge to the differential charge, and θ is the angle between R and r. Substituting into Eq. 2.1 λdx for dq, with λ as the charge density and dx as the differential distance on the line charge, $({}^{R}\!/_{r})$ for $\cos\theta$, and $\sqrt{x^2 + R^2}$ for r, with x as the distance on the line charge, results in,

$$\boldsymbol{E_R} = 2k_e\lambda R\int_0^\infty \frac{dx}{(x^2+R^2)^{3/2}} \tag{2.2}$$

and solving Eq. 2.2,

$$\boldsymbol{E_R} = \frac{2k_e\lambda}{R} \tag{2.3}$$

With this electric field, the Lorentz force equation is used to calculate the force acting on the electron based on its position relative to the field created by the point representing a line charge. The Lorentz equation is the following,

$$\boldsymbol{F} = q[\boldsymbol{E} + (\boldsymbol{v}\times\boldsymbol{B})] \tag{3}$$

where q is the particle charge, $\boldsymbol{E}$ is the electric field, and $(\boldsymbol{v}\times\boldsymbol{B})$ is the cross product between the instantaneous velocity $\boldsymbol{v}$ and the magnetic field $\boldsymbol{B}$ vectors. Because there are no static magnetic fields in the streak tube, $\boldsymbol{B}$ is set to zero. Substituting $\boldsymbol{E_R}$ from Eq. 2.3,

$$\boldsymbol{F} = \frac{2k_e \lambda q}{R} \tag{4.1}$$

where q is the charge of an electron. From Eq. 4.1, the force acting on an electron isn't determined by the inverse-square law, as Eq. 1 would show. Instead, the force is proportional to the inverse of the distance.

Because of the extreme speeds that the streak tube electrons move, ($\boldsymbol{v} \approx 0.2c$), where c is the speed of light, relativistic effects must be taken into account when calculating the movement of the electrons. As the electron's velocity approaches the speed of light, its kinetic energy is transferred into mass, decreasing the acceleration from the force caused by the deflection plates. The Newtonian formula for kinetic energy KE,

$$KE = \frac{1}{2} m_O \boldsymbol{v}^2 \tag{4.2}$$

where m_o is the electron rest mass, does not accurately account for the altered electron mass from relativistic effects. Therefore, a relativistic force equation is necessary to calculate electron acceleration. This can be derived from the derivative with respect to time of relativistic momentum $\boldsymbol{p}$ given by,

$$\boldsymbol{p} = \frac{m_O \boldsymbol{v}}{\sqrt{1 - (\boldsymbol{v}^2/c^2)}} \tag{4.3.1}$$

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F} \tag{4.3.2}$$

Combining Eq. 4.3 with Eq. 3, an equation for instantaneous electron acceleration can be derived, by which,

$$\frac{m_O}{\sqrt{1 - (\boldsymbol{v}^2/c^2)}} \frac{d\boldsymbol{v}}{dt} = q[\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B}) - \boldsymbol{v}\,(\boldsymbol{v} \cdot \boldsymbol{E})/c^2] \tag{4.4}$$

with $\boldsymbol{B} = 0$ in this work.

4.4 Electron Path Calculations

The electrons are each assigned an initial position, trajectory angle, and energy. These variables are gathered by the program from the input file, and are inserted into the equations of the program.

The instantaneous electric field acting on an electron from one line charge is calculated by inserting the distance between the electron and the line charge and the instantaneous charge of the line charge into Eq. 2.3 to give $\boldsymbol{E_R}$, which has Y and Z components. The process is repeated with the other line charges, which may have different distance and charge values, after which the field components are added and substituted into Eq. 4.4 to give the total instantaneous acceleration.

The program functions by repeating the calculations for the change in position and velocity over a time step based on the instantaneous acceleration. The code utilizes the fourth order Runge-Kutta approximation method to solve the differential equations,

$$X" = f(t, X, X') \tag{5.1}$$

$$X_{n+1} = X_n + h\left[X'_n + \frac{1}{6}(k_1 + k_2 + k_3)\right] + O(h^5) \tag{5.2}$$

$$X'_{n+1} = X'_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{5.3}$$

$$k_1 = hf(t_n, X_n, X'_n) \tag{5.4}$$

$$k_2 = hf(t_n + \frac{h}{2}, X_n + \frac{h}{2}X'_n + \frac{h}{8}k_1, X'_n + \frac{k_1}{2}) \tag{5.5}$$

$$k_3 = hf(t_n + \frac{h}{2}, X_n + \frac{h}{2}X'_n + \frac{h}{8}k_1, X'_n + \frac{k_2}{2}) \tag{5.6}$$

$$k_4 = hf(t_n + h, X_n + hX'_n + \frac{h}{2}k_3, X'_n + k_3) \tag{5.7}$$

where $X"$ is the combined acceleration of the electron along the Y and Z axis, X' is the combined velocity of the electron, X is the position of the electron, h is the step size, n is the step number, and O is the error. The function $X" = f(t, X, X')$ used to calculate the k values is the acceleration equation from Eq. 4.4. The program repeats these steps for each electron, recalculating the acceleration, velocity, and change in position at each time step until they arrive at the phosphor screen.

4.5 Static Deflection Voltages

The initial conditions for the electron beam are set such that it converges on axis at the phosphor screen when no voltage is applied across the deflection plates (the middle group of trajectories in Figure 3). When the voltages on the deflection plates are set to a constant value, the electrons follow a static deflection trajectory, in which all the electrons are deflected in the same direction. For slow streak speeds, multiple code runs with static deflection at different voltages can be overlaid in order to model ideal dynamic deflection.

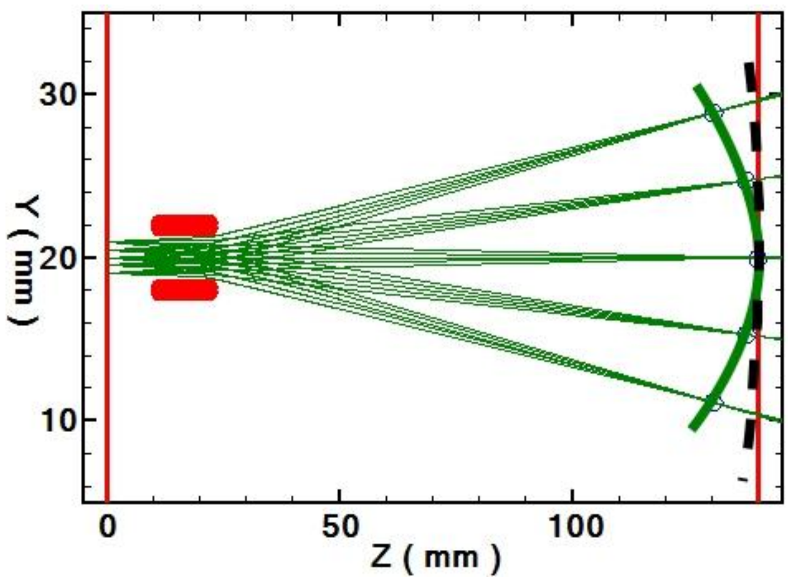

Figure 3: Static deflection focal plane.
The superposition of 5 code runs with different static voltages shows the parabolic focal plane. Graphical output of model data of a static deflection was displayed with *Eoplot*. The thick red lines represent deflection plates, the thin green lines are the electron trajectories, the thick green curve is the parabolic focal plane, and the dashed black line is a circular focal plane.

If the deflection angle for a certain deflection plate voltage remained constant regardless of electron height, the result would show a circular focal plane (the dashed black line in Figure 3), with its center at the deflection plates. However, the focal plane takes a parabolic shape tangent to the screen on axis, as seen by the thick green curve in Figure 3. The parabolic focal plane can be attributed to the fact that the impulse varies with electron height. Electrons passing between the deflection plates that are closer to the positive plate accelerate in the direction of electron propagation when approaching the deflection region, thus spending less time in the deflection region and resulting in a reduced deflection angle. On the other hand, electrons passing closer to the negative plate decelerate when approaching the deflection region, spending more time between the plates and increasing the deflection angle of the beam.

4.6 Dynamic Deflection Voltages

For high streak speeds, a realistic model of the electron trajectories of streak cameras must include dynamic voltages since the deflection plates change polarity as the electron beam passes through. This allows for the sweep pattern of electron trajectories to be simulated in one run of the program, and also allows electron behavior and path in dynamic deflections to be analyzed.

In the dynamic model, the values of the charge on the deflection plates shift over time, with one going from a positive to negative potential and the other from negative to positive. The electron beam now consists of a sequence of time-delayed pulses synchronized to the maximal $\frac{dV}{dt}$ change at the deflection plates.

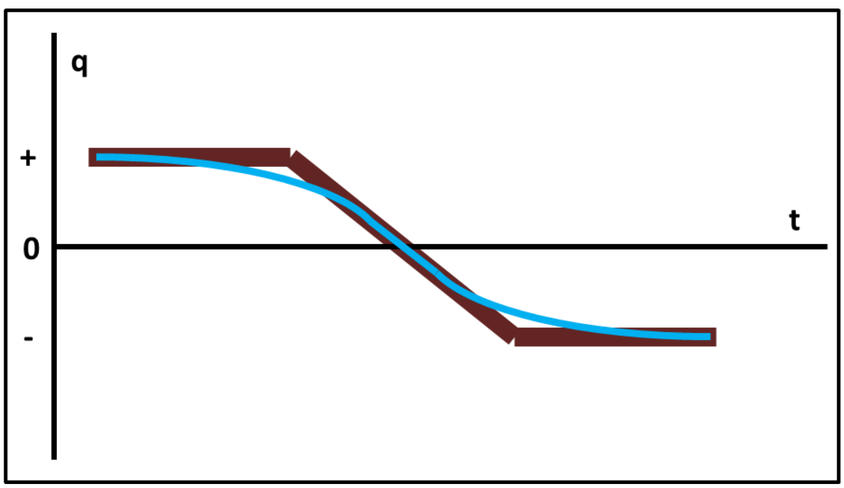

Figure 4: Sigmoid function and linear charge curve.
Curves model change in charge over time. The black represents a linear charge curve, and the blue, a charge curve modeled after a sigmoid function.

Ideally, the charge curves for streak cameras would be linear, with the potentials on each plate shifting at a constant rate. In reality, the charge curve is not so perfect, as it is limited by the capabilities of the equipment. To model the charge curve in *RungeKutta*, a sigmoid function was used (Figure 4):

$$q(t) = a + \frac{(b-a)}{1 + \mathrm{e}^{\frac{-(t-t_0)}{\lambda}}} \tag{6}$$

where $q(t)$ is the charge of a point on the deflection plate at time t, a is the initial charge of the deflection plate, b is the final charge, t_0 is the time at which the curve reaches its midpoint, and λ is a value proportionate to the risetime.

Since electromagnetic waves can only travel at the speed of light, according to Maxwell's equations, it was necessary to include retarded potentials in the model. Therefore, because of the time it takes for the information to reach the electron, the electron experiences the force due to a previous charge on the plates. This delay of information is proportional to the distance of the electron from the deflection plates, as shown in the equation for retarded time,

$$t_r = t - \frac{r}{c} \tag{7}$$

where r is the distance between the electron and the line charge. This equation changes the charge that the electron encounters from each point on a deflection plate to the charge at the deflection plate point from an earlier distance-determined time.

5 Results and Approaches

5.1 Defocusing in the Model

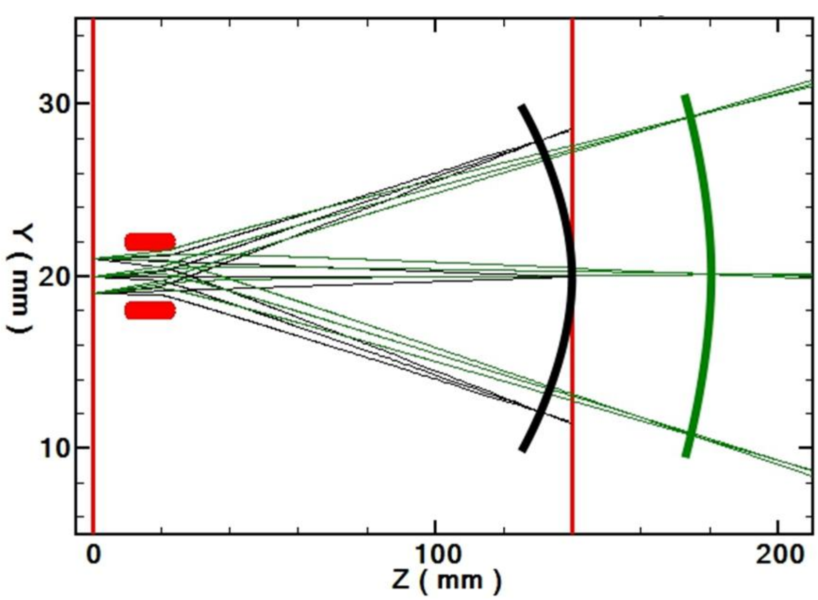

Figure 5: Overlay of static and dynamic deflection
A plot of the electron paths and focal plane from a static deflection is shown in black. The electron paths and focal plane from a dynamic deflection are in green. The shifted and defocused dynamic focal plane is due to a high streak speed.

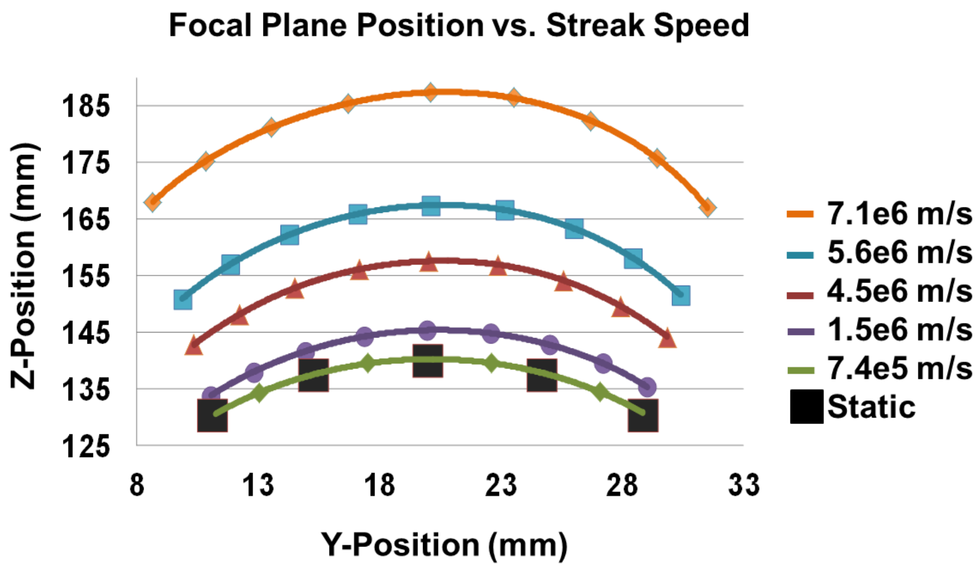

Figure 6: Focal plane position vs. streak speed from the dynamic deflection model. Graphs of the focal plane position at various streak speeds show the trend that focal plane displacement increases with streak speed.

Dynamic deflection code runs from *RungeKutta* demonstrate the same defocusing effects that streak cameras exhibit. At high sweep speeds, the simulated focal plane moves behind the phosphor screen, defocusing the image (Figure 5). Data from the model in Figure 6 show the dynamic defocusing increasing with sweep speed, following the same trend as the experimental data from streak cameras. At slower streak speeds, the focal plane is identical to that of a static deflection. It gradually moves further behind the phosphor screen as the streak speed increases. Data from dynamic deflection code runs indicate that the distance $D(s)$ the focal plane moves at each streak speed s can be fit to an equation of the form,

$$D(s) = As^B \tag{8}$$

where A and B are constants.

5.2 Common Mode Voltage

A common mode voltage (CMV) applied to both deflection plates adjusts the focus of the electron beam by altering the deflection path of the electrons. A negative CMV focuses the electron beam (Figure 7). Using a negative CMV, it is possible to compensate for the dynamic defocusing created by the high sweep speeds and reposition the focal plane back at the phosphor screen.

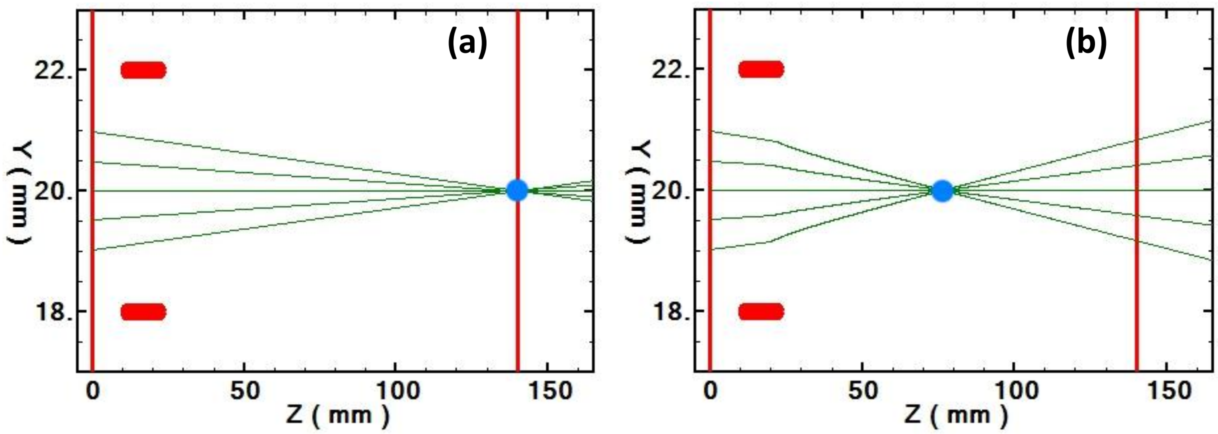

Figure 7: Effect of common mode voltage.
Graphs of (a) static shot with zero CMV and (b) static shot with a negative CMV demonstrate the focusing effects of a negative CMV.

5.3 Correction of Focal Plane

There are two main methods to achieve a negative CMV with the streak camera deflection plates. One method involves a time-shift of the voltage curve of one deflection plate, so that the voltage begins to decrease on the plate with the positive potential before the voltage on the other plate changes. The second method involves the addition of the CMV to the voltages on both plates.

The time-shift method creates a CMV that varies over time. Use of purely the time-shift method to compensate for extreme degrees of defocusing at high sweep speeds distorted the focal plane shape due to the inconsistency of the CMV. Drastic time shifts inverted the shape of the focal plane. Though this method wasn't further researched in this project, it may not be practical, due to the fact that the detailed risetimes of the voltage transients are very difficult to modify.

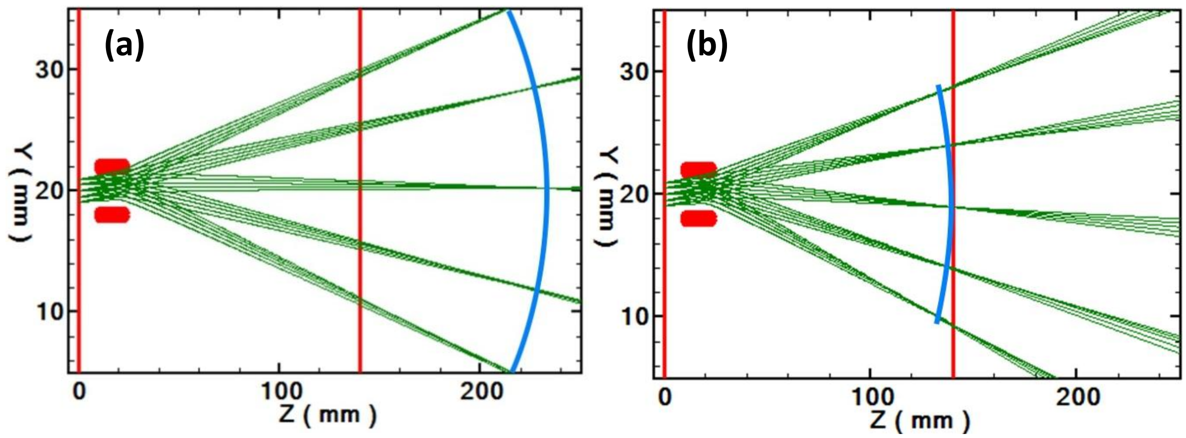

Figure 8: Correction for dynamic defocusing with a negative CMV.
Same as Figure 7, except applied to a dynamic shot with a high streak speed. The defocusing in (a) is corrected in (b) by a negative CMV.

The addition of a CMV to both plates was effective in preserving the shape of the focal plane as well as repositioning it back at the phosphor screen (Figure 8). Focal plane data with static deflection and a negative CMV can be fit to an equation in the form,

$$D(V) = Le^{KV} - L \tag{9}$$

where $D(V)$ is the distance the focal plane moves at negative CMV value V, L is the distance between the deflection plates and the phosphor screen, and K is a constant.

Using the equations,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \tag{10.1}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \tag{10.2}$$

with Eq. 10.1 being the double thin lens equation and Eq. 10.2 the triple thin lens equation, with f as the resultant focus of three lenses with foci f_1, f_2, and f_3, it is possible to calculate an equation so that a streak speed value corresponds to a correcting negative CMV value.

With the foci values of the model substituted into Eq. 10.2, the equation becomes,

$$\frac{1}{L} = \frac{1}{L} + \frac{1}{f_d} + \frac{1}{f_{cm}} \tag{11.1}$$

$$-\frac{1}{f_d} = \frac{1}{f_{cm}} \tag{11.2}$$

with L substituted for f, the intended resultant focus, and for f_1 as the focus of the focusing electrode, f_d as the focus of the defocusing lens, and f_{cm} as the focus of the negative CMV focusing lens. The foci f_d and f_{cm} in Eq. 11.2 can be found by using the double lens equation,

$$\frac{1}{f_{L+d}} = \frac{1}{L} + \frac{1}{f_d} \tag{11.3}$$

$$\frac{1}{f_{L+cm}} = \frac{1}{L} + \frac{1}{f_{cm}} \tag{11.4}$$

where f_{L+d} represents the resultant focus of the focusing electrode lens and defocusing lens, and f_{L+cm} the resultant focus of the focusing electrode lens and the negative CMV focusing lens. Substituting Eq. 11.3 and Eq. 11.4 into Eq. 11.2,

$$\frac{1}{f_{L+cm}} = \frac{2}{L} - \frac{1}{f_{L+d}} \tag{11.5}$$

and substituting Eq. 8 and Eq. 9 into Eq. 11.5 after adding L to both,

$$\frac{1}{Le^{KV}} = \frac{2}{L} - \frac{1}{As^B + L} \tag{11.6}$$

results in an equation with CMV and streak speed as variables.

6 Conclusion

A model for dynamic deflection in streak cameras, *RungeKutta*, has been written in C#. The model demonstrates the same defocusing effects that streak cameras exhibit. The effects of streak speed on dynamic defocusing in the model have been analyzed. Data from the model show the trend of the degree of dynamic defocusing increasing with streak speed. The data on dynamic defocusing from the model can be fit into an equation and extrapolated to predict the effects of dynamic defocusing under various streak speeds.

A negative common mode voltage (CMV) was applied through two methods to correct the defocusing. The time-shifted voltage curve method proved ineffective in the model due to distortion of the focal plane. The model has shown that a constant negative CMV addition to the voltages of both deflection plates successfully corrected the focal plane. With a negative CMV, it is possible to run deflection shots with high streak speeds without sacrificing resolution.

Future work on dynamic defocusing in streak cameras may further improve the accuracy of *RungeKutta* or explore additional methods such as time-shifted voltage curves, modifying the focusing electrode, or a combination of methods to improve the efficiency of the correction.

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[2] *Guide to Streak Cameras.* Hamamatsu Photonics K.K.: Hamamatsu, Japan. September, 1999. Web. <www.hamamatsu.com>