Modeling Collisional Blooming and Straggling of the Electron Beam in the Fast Ignition Scenario

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Abstract

The motion and distribution of electrons in the plasma in the fast-ignition scenario are defined by three equations that describe the spread in the direction of motion (straggling), the spread in the direction perpendicular to the motion (blooming), and the amount of energy lost as the electron travels through the plasma. A program was written that models the distribution of the electrons in the plasma and tracks their energy deposition. This model treats the beam of electrons as many parallel beams of infinitesimal width that move in a straight line. When the blooming or straggling exceeds a certain value, each beam of electrons is split into multiple beams of different weights. The model was found to require exponentially more time and memory for greater degrees of accuracy and to be sensitive to small adjustments in the splitting algorithm. The results of this model applied to a test problem were found to be very similar to the analytic predictions, with errors ranging from ~2% to 11%.

Introduction

In conventional inertial confinement fusion, ignition occurs when one of the fusion products (alpha particles) created in a central hot-spot are stopped in the high density, cold fuel, causing a propagating burn. A proposed alternative ignition method, known as "fast ignition", is to heat a part of the high density, cold fuel with a beam of relativistic electrons created by focusing a high-intensity laser into the target. The transport of these electrons is currently carried out with a straight-line model in which the electrons do not deviate from straight-line trajectories. In reality, the electrons undergo straggling (different penetration depths) and blooming (spreading of the beam) because of collisions with the background plasma.

This report describes a mathematical model and how it is applied within the straight-line algorithm. The results are then compared to the predictions of the analytic computations done by Li and Petrasso.[1]

Mathematical model

In this report we describe a method for accounting for straggling and blooming within the constraint of the existing straight-line model. As the electron beam travels through the target, the electrons collide with the plasma, depositing their energy, and changing their directions. This is described by the following expression[2], which is used in the existing model:

$$\frac{\mathrm{dE}}{\mathrm{ds}} = -\frac{2\pi r_0^2 m_0 c^2 n_i Z}{\beta^2}\left[\ln\left(\frac{(\gamma-1)\lambda_D}{2\sqrt{2\gamma}r_0}\right)^2 + 1 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2 - \left(\frac{2\gamma-1}{\gamma}\right)\ln 2 + \ln\left(\frac{1.123\beta}{\sqrt{2kT_e/m_0c^2}}\right)^2\right], \quad (1)$$

where r_0 is the classical electron radius, m_0, the electron mass, c, the speed of light, n_i, the background ion density, Z, the background average ion charge, T_e, the background electron temperature, k, the Boltzmann constant, γ, the relativistic energy divided by m_0c^2 and β, the relativistic velocity. This equation determines the amount of energy (dE) lost per distance the electron traveled (ds), but the distance traveled and distance penetrated are not the same because the electrons do not follow straight paths. Previous models have accounted for the discrepancy between the trajectory and the penetration distance by using the equation,

$\frac{dE}{dx} = \langle\cos\theta\rangle^{-1}\frac{dE}{ds}$ where dx is the penetration and $\langle\cos\theta\rangle$ is the average deviation angle from the straight line given by[1]

$$< \cos\theta >= \exp\left[-\int_{E_0}^{E} K(E')\left(\frac{dE'}{ds}\right)^{-1} dE'\right],$$

where

$$K = 4\pi n_i \left(\frac{r_0}{\gamma\beta^2}\right)^2 \left[Z^2 \ln\Lambda^{ei} + \frac{4(\gamma+1)^2}{(2^{\sqrt{(\gamma+1)/2}})^4} Z \ln\Lambda^{ei}\right]$$

and $\ln\Lambda^{ei}$ is the Coulomb logarithm.

Blooming and straggling have been added to the existing model using expressions from Ref. 1. Blooming is the deviation perpendicular to the initial direction of movement and is described by

$$\left\langle y^2 \right\rangle = \frac{2}{3}\int_{E_0}^{E} \left\langle P_1(\cos\theta)\right\rangle \left(\frac{dE'}{ds}\right)^{-1} \left(\int_{E_0}^{E'} \frac{1+2\left\langle P_2(\cos\theta)\right\rangle}{\left\langle P_1(\cos\theta)\right\rangle}\left(\frac{dE''}{ds}\right)^{-1} dE''\right) dE'. \qquad (2)$$

Straggling is the deviation parallel to the initial motion and is given by

$$\left\langle x^2 \right\rangle = \frac{2}{3}\int_{E_0}^{E} \left\langle P_1(\cos\theta)\right\rangle \left(\frac{dE'}{ds}\right)^{-1} \left(\int_{E_0}^{E'} \frac{1-\left\langle P_2(\cos\theta)\right\rangle}{\left\langle P_1(\cos\theta)\right\rangle}\left(\frac{dE''}{ds}\right)^{-1} dE''\right) dE', \qquad (3)$$

where $\left\langle P_1(\cos\theta)\right\rangle$ and $\left\langle P_2(\cos\theta)\right\rangle$ are Legendre polynomials.

Procedure

A program was written to model the energy deposition of a beam of electrons into a target. While the program can model a target with non-uniform density and temperature, it was tested by modeling a uniform two-dimensional slab with a density of 300 g/cm^3 and a temperature of 500 eV, for which an analytical solution is known[1]. A rectangular grid is used. The beam of electrons is modeled as many beamlets of infinitesimal width, which will be referred to as "beams" for simplicity. As each beam travels through the target, the energy deposited is calculated for each grid square it passes through. Thus, while the beams are treated as infinitesimal, the energy deposited is only accurate to the resolution of the grid. The spread of the beam is also calculated in both the blooming and straggling directions for each zone in the grid. That spread is summed over the trajectory and when it exceeds a certain level, which is a parameter of the model, the beam is split into multiple beams to represent the spread. The daughter beams have the same energy and direction, but represent different amounts of electrons. This method is illustrated in Figure 1.

The most efficient and accurate method was to split the electron beam into three daughter beams each time a split was required. Any more splitting produced too many beams, requiring too large a memory space, and having only two daughter beams required more splits, which again led to too many beams. The weights and distance between the beams were calculated so that the distribution of the beams stayed Gaussian with the correct standard deviation, obtained from Ref. 1. At each split the standard deviation of the set of beams was set so that it was halfway between the current deviation and what it would be next time the beams split. While this procedure leads to overestimation of the deviation right after the split, it averages out and does not consistently overestimate or underestimate the deviation for both straggling and blooming. The weights given to the beams were 57% of the electrons in the center daughter beam, and 21.5% for each of the off-center daughter beams, which were offset by 1.325 standard deviations. This was found to keep the distribution relatively Gaussian without requiring too many splits.

These daughter beams were then transported into the target until their spread exceeded the maximum level, in which case they were again split. They were finally stopped when their energy was negligible compared to the temperature of the target, at which point their remaining energy was deposited in the current grid space.

Results

The results of this model are compared with those predicted by the analytic model of Li and Petrasso[1] in Fig. 2. Plotted are the differences between the two models in the average penetration distance (Fig. 2a) and in the final root-mean-square deviations in the x direction (straggling, Fig. 2b) and the y direction (blooming, Fig. 2c), for increasing source energy. The average distance is modeled very closely, with errors ranging from –2% to 1% over a range of energies. The problem was modeled with various sets of parameters (such as the distance moved between splits, the distance moved during the splits, and the width of the initial beam**).** The results for straggling and blooming, shown with yellow lines in Figs. 2b and 2c, are for the best set of parameters and show small deviations from the Li and Petrasso predictions (ranging from -8% to 11%). Depending on when the beams were split, the model could either overestimate or underestimate these values at the end of the simulation. An example is shown in the dark red lines for which the errors range from –23% to –8%**.**

The difference between the predictions of two different runs of the model for the energy deposited is shown in Fig. 3 for a 10-μm beam centered at 30 μm. Figure 3a shows the difference when the blooming parameter is changed. Most of the differences are at the beam edges where the modeling of the blooming is more sensitive. The difference when the straggling parameter is changed is illustrated in Fig. 3b. In both cases the largest error is close to the source before the first split of the beamlets. In the bulk of the target the differences are about 10% or lower. These results show that the computational model provides a fairly accurate treatment of the propagation of the beam.

Future Work

The program could be improved upon in two different ways. Currently there is only one thread doing all the computations for the model, but the computations could be done in parallel. This could easily make the program many times faster. Another improvement to the program would be to reconsolidate the beams after a few splits. After about 6 splits there are 729 different beamlets; the calculation could be stopped at some stage and the beamlets in close spatial proximity could be combined. With recombining we could split more often than before, without having more beams at once. This is important because the number of beams is limited by the amount of available memory. Thus with recombining we could represent a smoother distribution with fewer beams, and not run out of memory. Using fewer beams would also speed up the model by requiring fewer computations.

Conclusions

An improved straight-line model, which includes straggling and blooming, has been developed to model the transport of relativistic electrons in the fast-ignition scheme.

In this model, after the electrons have accumulated a certain amount of straggling or blooming during their trajectory, they are split into three electrons. The direction and energy of each electron is chosen so as to provide the straggling and blooming predicted by analytic predictions such as those of Li and Petrasso. This model agrees with the analytic model within ~10% for the energy deposited, for a wide range of initial electron energies.

References.

1. C. K. Li and R. D. Petrasso, Phys. Rev. E **73**, 016402 (2006).

2. C. K. Li and R. D. Petrasso, Phys. Rev. E 70, 067401 (2004).

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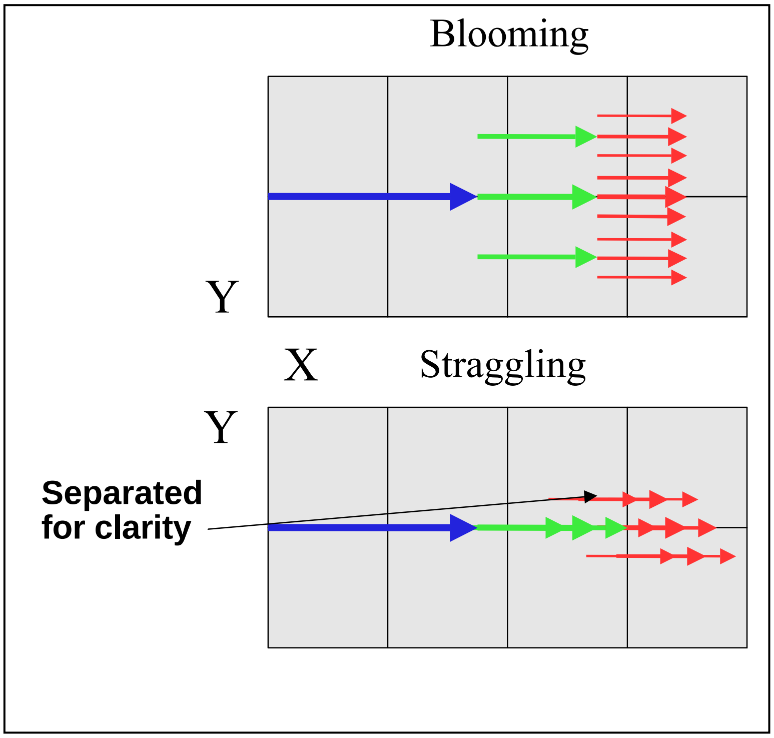


Figure 1. Diagram showing how an electron beamlet is split for blooming or straggling.

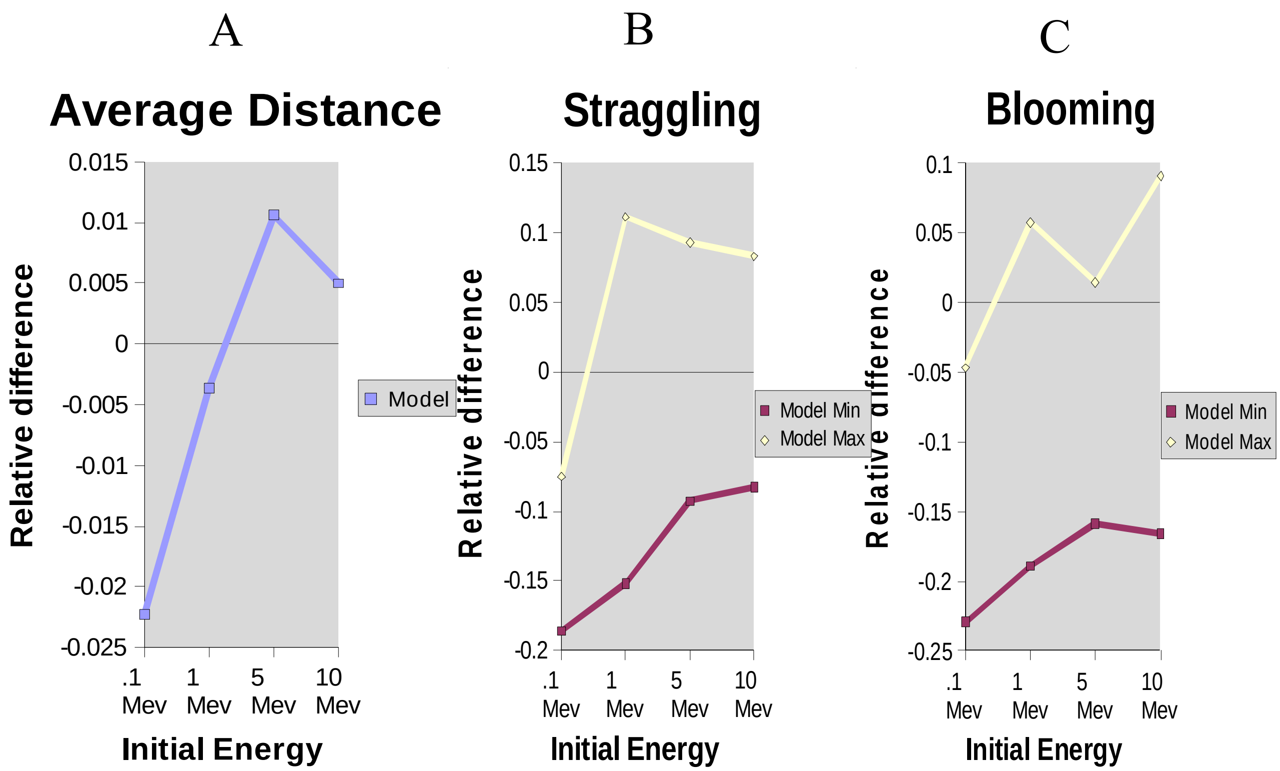
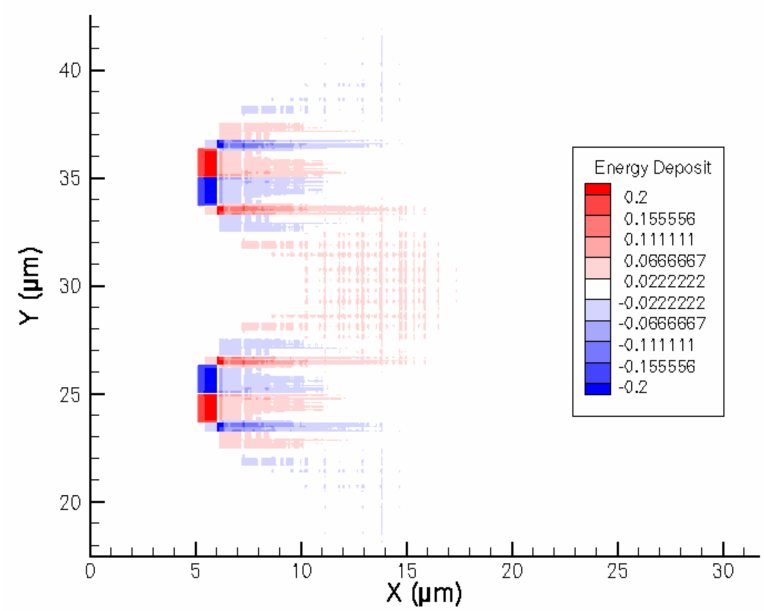


Figure 2. Differences between Li and Petrasso's predictions and those of the model for various source energies: a) average penetration; b) straggling; c) blooming. The yellow lines are for the best set of parameters; the dark red line is for another set.



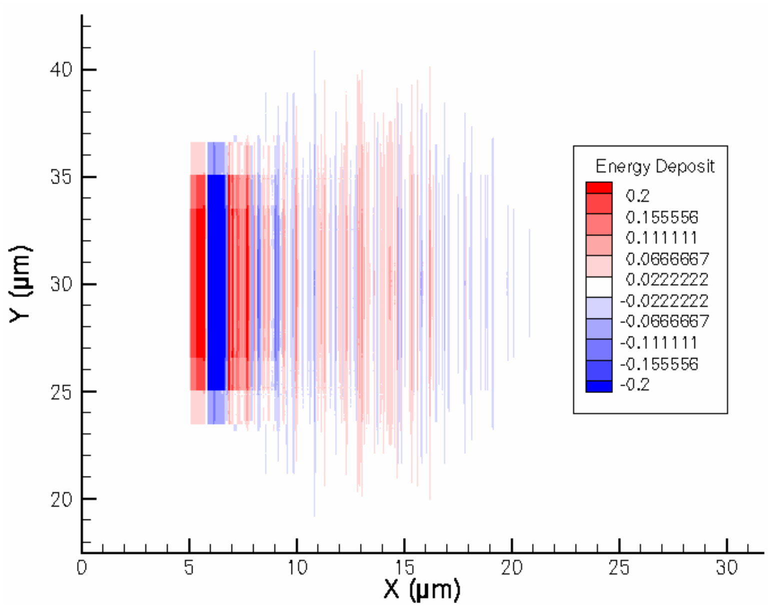


Figure 3. Fractional differences in energy deposition of the model for a 10-µm beam centered at 30 µm, between two different trials. In a) the two runs compared had different blooming parameters, and in b) they had different straggling parameters.