Modeling the LCPDI with Refraction and Diffraction

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Summer High School Research Program 2001

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Abstract

A liquid crystal point diffraction interferometer (LCPDI) computer model was extended to calculate spreading due to diffraction as well as refraction. The software can also model cells with elliptical beads. This more general software allows the rapid investigation of LCPDIs without fabrication, which presently is neither quick, accurate, nor easy. By isolating the parameters and materials needed to make a useful device, it is hoped to develop an optimal interferometer to replace larger and more expensive commercial interferometers.

1 Introduction

Inertial confinement fusion (ICF) is a technology with the potential to power the world in the years to come. Fusion is desirable for powering electrical generators because fusion fuel is derived from hydrogen, one of the most abundant elements. Energy is liberated in fusion by joining two light atoms or nuclei into a single, heavier nucleus. The resultant nucleus is lighter than its components. The "missing mass" is converted to energy – explosively in nuclear weapons and slowly in nuclear power plants. The goal of ICF research is obtaining usable energy from nuclear fusion.

The OMEGA laser at the University of Rochester / Laboratory for Laser Energetics, the most powerful laser system of its type, researches inertial confinement fusion. OMEGA uses 60 laser beams to compress a small capsule filled with deuterium and tritium. The fuel reaches extreme density and temperature, allowing nuclear fusion to occur. Although new technologies, including smoothing by spectral dispersion (SSD) and cryogenics, have been applied to OMEGA to increase its performance, the system has yet to produce usable energy from fusion. The OMEGA laser and other ICF systems today are not advanced enough to produce more electrical energy than consumed. Even OMEGA's successor, the National Ignition Facility (NIF, currently under construction), will not "break-even" in terms of energy return. The NIF is a large step in the right direction and hopefully in the future ICF can provide the world with cheap electricity [1].

The sixty laser beams used in OMEGA are reflected by mirrors and passed through lenses many times before they reach their target. By splitting and recombining a laser beam it can be characterized by interferometry. A device called an interferometer can generate images called interferograms that are useful in finding flaws in laser beams and the optics they have encountered.

Many interferometers have been available commercially for some time. These interferometers, however, tend to be expensive, bulky, and complicated. Research is underway on a new type of interferometer, the Liquid Crystal Point Diffraction Interferometer (LCPDI) [2, 3]. The heart of an LCPDI is made from relatively inexpensive materials, fits in the palm of your hand, and has a phase-shifting ability inherent in its design. Because the liquid crystal center of an LCPDI can change refractive index with voltage, the LCPDI can easily examine a laser beam using a full range of phases. Commercial phase-shifting interferometers employ sophisticated mechanisms to the same end [4].

The LCPDI itself consists of a layer of liquid crystal sandwiched between two panes of glass. These glass windows are about 1 mm in thickness while the LC layer is less than 50 μm thick. Between the windows and in the LC are several plastic beads. These beads can be spherical, ellipsoidal, or squished to the thickness of the gap. The LC is mixed with a dye that greatly attenuates any light not passing through one of these beads.

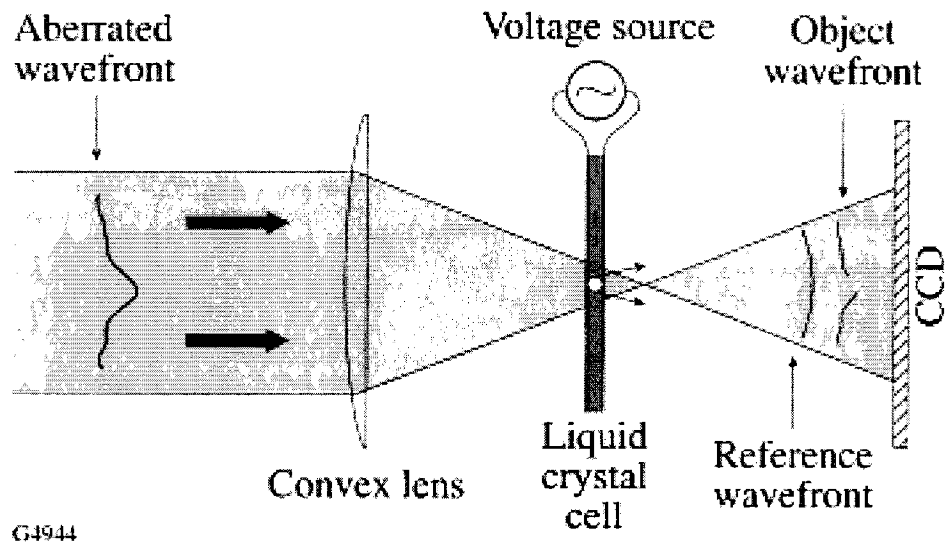


Figure 1: Schematic diagram of the LCPDI

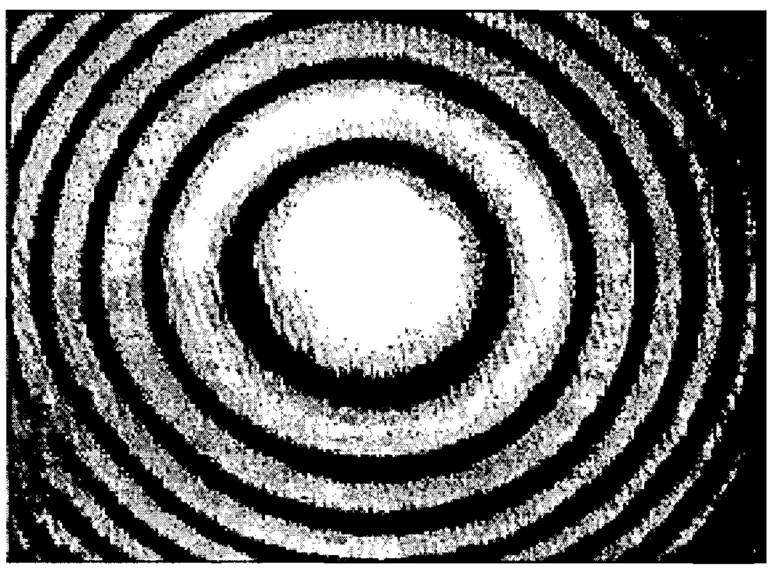


Figure 2: An interferogram, the result of the interference of the object and reference beams

To produce an interferogram (similar to Fig. 2), a laser beam is focused several microns past the center of one of these beads. A small portion of this laser energy strikes the bead and spreads out. The rest continues straight through the LCPDI cell, greatly attenuated by the dye dissolved in the LC. The small portion of spread light is assumed free of aberrations and is called the reference beam. The rest is called the object beam. The two beams, cast onto the same area of CCD camera, interfere to form a pattern of fringes called an interferogram.

Liquid crystals exhibit birefringence; their index of refraction in one direction is different from their index of refraction in the other. LCs also exhibit dielectric anisotropy; a voltage applied to the crystals causes them to rotate. By increasing the index of refraction of the LC, the object beam's optical path (number of cycles it has traveled before reaching the camera) can be adjusted and all of the phases of the object beam can be scanned.

2 Modeling the LCPDI

2.1 The Basic Model

Amy Turner, a student of the Summer High School Research Program, developed a computer model of the LCPDI [5]. This computer program tracked rays from their entry into the LCPDI to the points on which they interfered on the camera. For every ray $\vec{x_0}$ headed in the direction $\hat{c_0}$, the program calculated the point(s) it struck the spherical bead, if any:

$$\begin{aligned} \vec{x_0} + s\hat{c_0} &= \vec{x} \\ \left| \vec{f} - \vec{x} \right| &= R \end{aligned} \qquad (1)$$

where $\vec{x}$ is the intersection point, s is the distance the ray traveled before striking the sphere, $\vec{f}$ is the center of the sphere, and R is its radius. The program used Equ. (2) for calculating s:

$$s = (\hat{c_0} \cdot (\vec{f} - \vec{x_0})) \pm \sqrt{R^2 + \left| \hat{c_0} \times (\vec{f} - \vec{x_o}) \right|^2} \qquad (2)$$

It also used Snell's Law to calculate the path each ray took toward the target plane:

$$\mu_0(\hat{c_0} \times \hat{n}) = \mu_1(\hat{c_1} \times \hat{n}) \qquad (3)$$

where μ_0 and μ_1 were the indices of refraction, $\hat{n}$ was the normal at the intersection point, and $\hat{c_0}$ and $\hat{c_1}$ were the directions of the ray before and after striking the interface. The optical path of each ray was also calculated. Optical path is defined as the sum of the distance traveled in each material times the corresponding index of refraction. Each optical path calculation determines the phase of the ray when it reaches the target plane. Turner's program calculated the interferogram based on the difference in path lengths between object and reference beams.

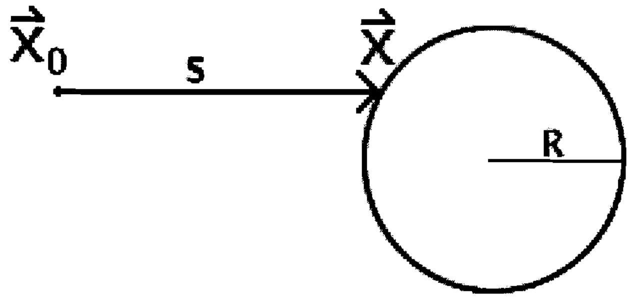


Figure 3: The ray $\vec{x_0}$ with direction $\hat{c_0}$ intersecting a circle

Rohit Rao, another Summer High School student, extended Turner's model to include the intensity of each ray in the calculation [6]. With Rao's extension each ray was tracked with a complex electric field. At each point in the target plane, the electric fields were summed with Eqn. (4) and the intensity was calculated with Eqn. (5):

$$E_{total} = E_{10}e^{-i\phi_1} + E_{20}e^{-i\phi_2} \tag{4}$$

$$I = \frac{1}{2\mu_0 c}\left|E_{total}\right|^2 \tag{5}$$

where E_{10} and E_{20} were the complex electric fields of interfering rays, ϕ_1 and ϕ_2 were their phases (calculated from optical path length), and I was the final intensity used to plot the interferogram. Turner's program with Rao's extension created graphs comparable to experimental images.

2.2 Modeling the ellipsoid

Turner and Rao's program modeled the LCPDI assuming the central bead was a sphere. Often times when an LCPDI is constructed, the central bead is "squished" and the resulting bead is shaped more like an ellipsoid. The program has been extended to account for ellipsoidal beads. The extension works similarly to the spherical code – for every ray $\vec{x_0}$ headed in the direction $\hat{c_0}$, the program calculates the point(s) it strikes the ellipsoid, if any:

$$\begin{aligned} \vec{x_0} + s\hat{c_0} &= \vec{x} \\ a^2b^2 &= a^2|\vec{x}|^2 - (\vec{x} \cdot \vec{v})^2 \\ \vec{v} &= \langle 0, \sqrt{a^2+b^2}, 0 \rangle \end{aligned} \tag{6}$$

where $\vec{x}$ is the intersection point, s is the distance the ray travels before striking the ellipsoid, a is one-half the bead's major axis, and b is one-half the bead's minor axis (usually equal to the cell gap). For simplicity's sake the ellipsoid's center is assumed to be $\langle 0,0,0 \rangle$. $\vec{v}$ is the vector from $\langle 0,0,0 \rangle$ to a focus. The program uses Equ. (7) for calculating s:

$$s = -\frac{a^2 c_{01} x_{01} + b^2 c_{02} x_{02} \pm ab\sqrt{c_{01}^2(a^2 - x_{02}^2) + c_{02}^2(b^2 - x_{01}^2) + 2x_{01}x_{02}c_{01}c_{02}}}{a^2 c_{01}^2 + b^2 c_{02}^2} \tag{7}$$

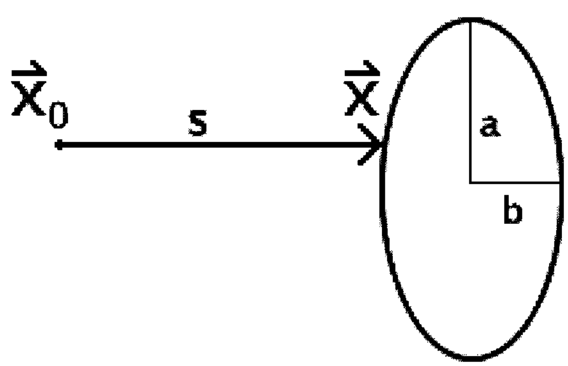


Figure 4: The ray $\vec{x_0}$ with direction $\hat{c_0}$ intersecting an ellipse

2.3 Modeling diffraction

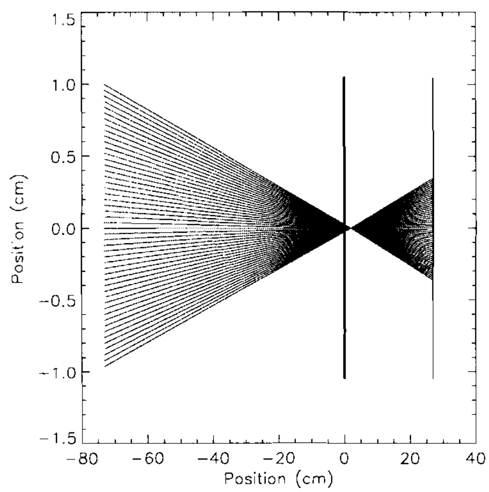
Turner and Rao's program, even amended to include the ellipsoidal code, modeled only the effects of refraction and optical density. Diffraction, however, is a significant influence on the LCPDI and now can be modeled with the program. The intensity of each ray as it strikes the target (camera) plane is a function of every point (x, y) on the plane of the LCPDI's window. The complex electric field at a given point P on the target plane can be calculated with the Fresnel-Kirchoff diffraction integral [7, p. 380]:

$$E(P) \;=\; \frac{1}{4\pi} ik \iint_D \frac{e^{-iks}}{s} E(x, y) \left[\hat{c_p} \cdot \hat{n} + \hat{c_o} \cdot \hat{n}\right] dx\, dy \qquad (8)$$

where $k = \frac{2\pi}{\lambda}$, D is the plane of the LCPDI's window, s is the distance traveled by the ray from the window to the target, $E(x, y)$ is the complex electric field at some point (x, y) on the window plane D, $\hat{c_p}$ is the direction from (x, y) to P, $\hat{c_o}$ is the direction of the ray at (x, y), $\hat{n}$ is the normal of the window, and i is the imaginary unit.

2.4 Implementation

The program itself is written in PV-WAVE and is executed with a Unix-style shell script and Makefile. The program operates in two dimensions – on the yz plane with the direction of propagation being z. As input it accepts several arguments including the number of rays to be used for raytracing, resolutions of the window and target planes, wavelength of the laser being used, geometry (lengths and distances), refractive indices, and optical density of the liquid crystal. The program uses these parameters to create an array of light rays, each with an initial position, direction, and complex electric field. Typically these rays form a spherical phase front without aberrations focused slightly past the LCPDI cell, but this can be modified. The rays propagate through air, the first glass window, the layer of liquid crystal, possibly the ellipsoidal bead in the center, and the second glass window, refracting and becoming attenuated as they go. At the interface between the second glass window and air, the rays are interpolated into an array of directions and complex electric fields. An array is also created on the target (camera) plane. The electric field of each point on the target plane is calculated discretely with the Fresnel-Kirchoff diffraction integral with the region of integration being the xy plane – the program assumes radial symmetry about $y = 0$ to extend the window arrays into two dimensions. From the array of complex electric fields at the target plane, a graph of the interference pattern and several auxiliary graphs are produced. The interference graph simulates a lineout through the center of an actual interferogram.



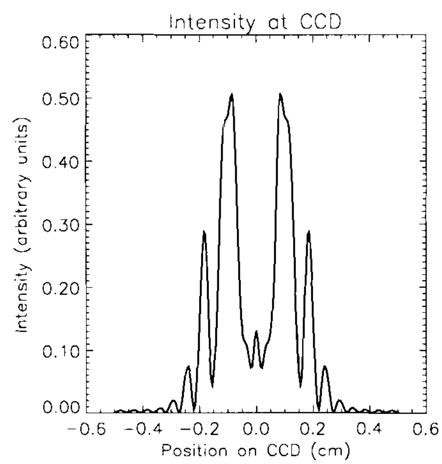


Figure 5: A schematic diagram and theoretical lineout, both output of the program

3 Results

The ability of the model to accept many parameters allows a variety of experimental conditions to be examined. This allows investigation of the properties of the LCPDI without fabrication, saving both time and materials. The model allows rapid investigation of the effect of each variable on the interferogram. Changing a variable in an experimental device can be difficult (focal length or laser wavelength) or impossible (cell gap or width of bead) and can often affect more than one variable – e.g. a change in the voltage over the liquid crystal may affect the dyes' absorbance. The program can simulate devices not currently feasible to produce. The effect of darker dyes, different wavelengths, and more precise components can be studied before the devices are constructed. An interesting experimental phenomenon related to voltage increase and contrast can be explored with the help of the model. In a typical cell, when the voltage over the LC is zero, the interferogram shows good contrast – the difference between the brightest fringe and the darkest fringe is large. When the voltage is increased the LC's refractive index changes and effects a change in the object beam's phase. Sufficient voltage shifts the object beam's phase by π. This phase shift causes a loss of contrast. When more voltage is applied and the object beam's phase shift approaches 2π, the contrast is restored. This phenomenon and others like it can be studied intensively out of the laboratory with this model.

4 Conclusions

An LCPDI computer model was extended to calculate spreading due to diffraction as well as refraction. The software can also model cells with elliptical beads. Through its flexibility, the LCPDI can be studied intensively outside the laboratory. The LCPDI is promising interferometric technology and the software model can be an aid to its development.

5 References

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6 Acknowledgments

One million thanks go to my project advisor and summer program coordinator, Dr. R. S. Craxton. Without his mathematical and technical explanations, help with debugging, and infinite patience, none of this study would have been possible. I thank B. Klehn, N. Turner, M. Guardalben, and K. Marshall for their experimental work and for familiarizing me with their laboratory. Another thanks goes to the IAL regulars, including M. Gao, M. Fiedler, and K. Monajati for their help and companionship. Special thanks go to L. Stein, for contributing his math and technical expertise, and R. Mukamel, for his coding wisdom and eagerness to share it. I lastly acknowledge all the scientists at the LLE, teaching us in more ways than one. You made this summer both educational and enlightening – thank you.