SUBPICOSECOND ELECTRO-OPTIC SAMPLING

by

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CURRICULUM VITAE

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ABSTRACT

This dissertation documents the development of a novel, state of the art optical sampling technique for the characterization of picosecond electrical transients. The technique is based upon the interaction of subpicosecond optical pulses and the unknown electrical transient in a traveling-wave Pockels cell employed as an ultrafast intensity modulator. All facets of the design, construction, performance, and application of the system are discussed. A theoretical model for temporal performance is also introduced and applied to the experimental situations.

The Pockels, or electro-optic, sampling technique discussed here has significantly extended the regime of electrical signal analysis in the picosecond domain and is the only technique that has achieved a subpicosecond temporal resolution. The technique exploits the availability of high repetition rate, subpicosecond, mode-locked laser pulses and the ultrafast response of the Pockels effect (a special case of parametric three-wave mixing).

The system, to date, has achieved a temporal resolution of 550 femtoseconds, in excellent agreement with the results of the theoretical model. The theory indicates that resolutions of near 200 femtoseconds should be attainable. The high repetition rate of the sampling process (91 MHz)

enables the use of signal averaging techniques to enhance the sensitivity of the Pockels effect. With signal averaging, sensitivities of better than 100 μV have been observed.

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CHAPTER I

INTRODUCTION

I.A. Motivation

The availability of picosecond and subpicosecond optical pulses as generated by mode-locked lasers[1-4] has made possible the investigation of material processes in the picosecond time regime. For the most part, these studies have been of an optical nature.[75] Recently, however, an increasing amount of research is being conducted in the investigation and development of picosecond electronic materials and devices. Examples include photodetectors, photoconductive materials, and ultrafast transistors. Successful development of these new devices requires a measurement system with the ability to characterize electrical performance on a picosecond time scale.

It is well known that sampling techniques for electrical signal characterization provide superior temporal performance. In sampling systems, the temporal resolution, ideally, is determined by the duration of the sampling gate. Hence, a high resolution sampling system requires a narrow sampling gate width. For many years now, measurement techniques have been dependent on electronic sampling gates having durations of no less than 25 picoseconds. In this thesis, we propose and investigate a novel, alternative,

sampling technique that is based upon the direct use of sub-picosecond optical pulses as the sampling gates.[5-7] We can exploit the availability of such ultrashort optical pulses by employing electro-optic crystals as an interaction medium for the optical and unknown electrical signals. The electro-optic, or Pockels, effect is ideally suited for picosecond sampling because the mechanism responsible for the effect poses no temporal response limit, even into the femtosecond regime. Thus, the direct use of optical pulses enables us to achieve a similar subpicosecond temporal resolution for sampling electrical signals.

Experimental results verify the attainment of an unprecedented temporal resolution of approximately 500 femtoseconds, which represents an improvement of almost two orders of magnitude over conventional electronic sampling systems.

I.B. High speed electrical measurement schemes

This section reviews several existing techniques for the measurement of high speed electrical signals. We begin by discussing the important parameters of any electrical measurement technique. These parameters will then be used as a basis for comparing the variety of current technologies to the new technique developed in this thesis. Since we are most interested in high speed performance, the limiting tem-

poral effects of each technique will be emphasized.

I.B.1. Measurement system parameters

Most electrical measurement systems measure voltage as a function of time. Since "voltage" per se cannot be "seen" by a human observer, the measurement system can be considered to be a device that converts voltage to another type of "secondary" signal that allows the waveform to be either directly visualized, or recorded, or both. Ideally, the secondary signal should generate an exact, noise free, replica of any input waveform both in time and amplitude. The ideal case, however, is never achieved.

The accuracy with which a real system can actually measure an unknown waveform is usually described by the following parameters: temporal resolution (related to bandwidth), sensitivity, and dynamic range. A measure of the temporal response for any system is the temporal resolution which we will denote as $\mathcal{T}$. Given a perfect step function as the input signal, $\mathcal{T}$ is defined as the risetime of the secondary or output signal. Usually, the risetime is measured between the 10% and 90% points of the rising signal as shown in Figure I-1. Thus, $\mathcal{T}$ is a measure of the speed of the conversion process. The shorter $\mathcal{T}$ becomes, the more ideal the temporal conversion. The reciprocal of $\mathcal{T}$ yields an equivalent frequency, f_{max}, that corresponds to the maximum resolvable frequency and is defined as:
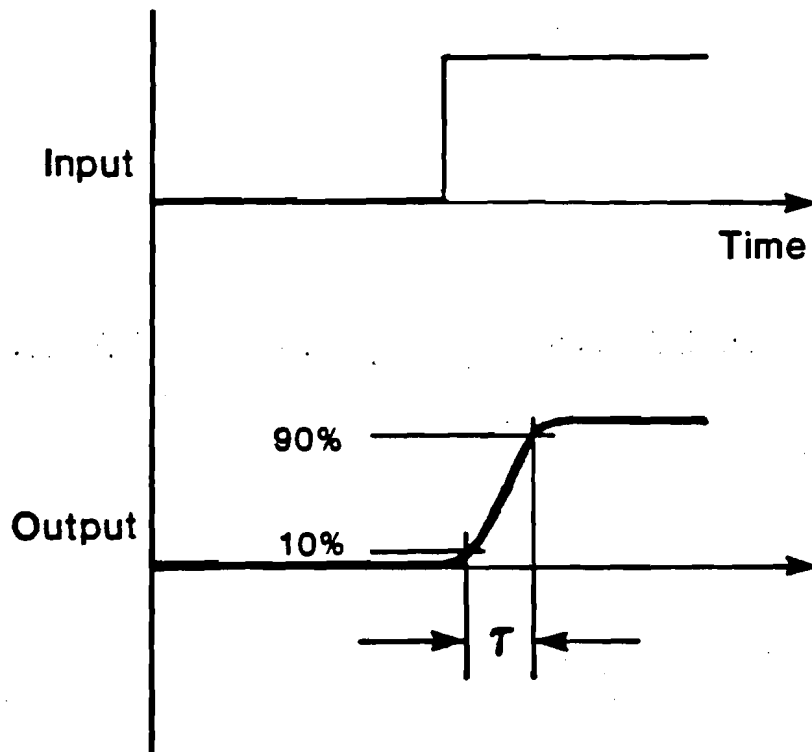
Figure I-1 Definition of risetime, T

$$f_{max} = \frac{0.35}{T},$$

<div align="right">I-1</div>

Another way of expressing the temporal performance is the bandwidth, B. The bandwidth is defined as the frequency range over which the conversion process is constant with frequency. If the low frequency range of a converter extends to DC, as is often times the case, then the bandwidth is simply f_{max}.

Minimum sensitivity is commonly defined as the applied voltage level that yields a signal to noise ratio of unity in the secondary signal. A lower sensitivity figure allows one to measure smaller input signals. The dynamic range refers to the range of voltages that can be accurately measured, which in turn defines the maximum voltage to which the system responds linearly.

In the ideal case,[76] both the minimum sensitivity and the temporal resolution, T, of a measurement system would be infinitely small. A $T=0$ implies an interaction time of zero for the conversion process. Experimentally, however, we find that to achieve a reasonable sensitivity, the conversion interaction must take a FINITE amount of time. As a result of this finite interaction time, the secondary signal is no longer a perfect replica of the input signal either in time or amplitude. These imperfections manifest themselves as either a limited high frequency capability or a reduction

in sensitivity. However, as will be seen in most cases, the temporal inaccuracy is traded off against the sensitivity in order to satisfy the specific experimental requirements. As will be seen, for many devices, $\mathcal{T}$ is exactly the interaction time between the voltage and the secondary signal. More complex devices (eg. traveling-wave oscilloscopes) can attain a $\mathcal{T}$ significantly less than the interaction time, and in doing so, increase their frequency response without sacrificing sensitivity.

I.B.2. Conventional oscilloscopes

The most common, and by far the most versatile, voltage waveform measuring device is the conventional cathode ray oscilloscope. It accepts an input voltage and converts it to a luminous plot of voltage versus time on a fluorescent screen. The secondary signal is an electron beam focussed onto the screen. The conversion is from voltage to deflection of the electron beam. The deflection is accomplished by propagating the beam of electrons through two orthogonal sets of deflection plates. The horizontal set is used to generate the temporal axis, while the vertical set has an amplified voltage replica of the input signal applied to it.

The ultimate limit in temporal response for a conventional oscilloscope is dictated by its vertical deflection plate geometry.[8] A typical geometry is represented in Figure I-2. An electron traveling between the plates experiences a
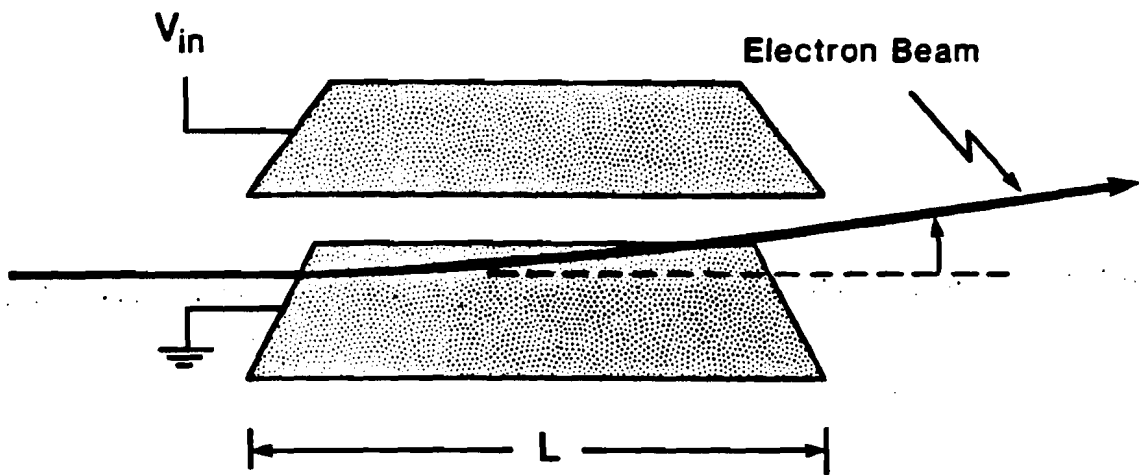
Figure I-2 Oscilloscope deflection plates

deflection proportional to both the input voltage, V_{in}, and the length of the plates, L. If the time an electron spends between the plates (transit time) is short compared to the temporal characteristics of V_{in}, then an accurate replica of the input voltage waveform will be generated. Distortion arises when V_{in} varies on a time scale less then the electron transit time, in which case an electron experiences a changing field as it passes through the plates.

Consider the application of a perfect step function to the plates. The deflection a particular electron experiences depends upon where it is with respect to the deflection plates at the onset of the step. If it is between the plates when the step function arrives, then its deflection will be proportional to the remaining distance it must travel through the plates. It can be shown that the resulting risetime, $\mathcal{T}$, is equal to the electron transit time through the plates. This effect is referred to as the transit time limitation for temporal resolution, and is an example of $\mathcal{T}$ being equal to the interaction time. The fastest oscilloscopes of this type have a $\mathcal{T}$ on the order of one nanosecond. To obtain better temporal resolution, L must be shortened and hence, sensitivity is sacrificed.

The problem of the transit time limitation is solved in most cases by some form of traveling-wave system. Here the deflection plates are either broken up into a number of seg-

ments or formed into a coiled helixical line so that the signal travels parallel to the electron beam at the same speed as the electrons. Thus, any electron is deflected by the same signal component throughout its entire interaction time. Typical maximum frequencies that can be resolved using traveling-wave oscilloscopes are on the order of a few gigahertz which corresponds to a resolution approaching 100 picoseconds. Sensitivities tend to be extremely poor, ranging from a few volts to several tens of volts per trace width as compared with about 1 mV for many conventional slower oscilloscopes. Temporal limitations arise from difficulties encountered in coupling high frequency signals into the complex deflection structures and also in generating the high frequency horizontal sweep.

I.B.3. Sampling oscilloscopes

A different approach is possible when the signal to be observed is repetitive. This approach involves an amplitude sampling technique using a short gating pulse of about 25 ps duration. The basic waveforms are illustrated in Figure I-3. Samples of the input signal are taken at ever increasing times, T, at each recurrence of the signal. Thus, the conversion process is not continuous, as in the oscilloscope, but rather is an accumulation of discrete portions, or samples. These samples are amplified, lengthened in time, and subsequently displayed on a conventional oscillo-
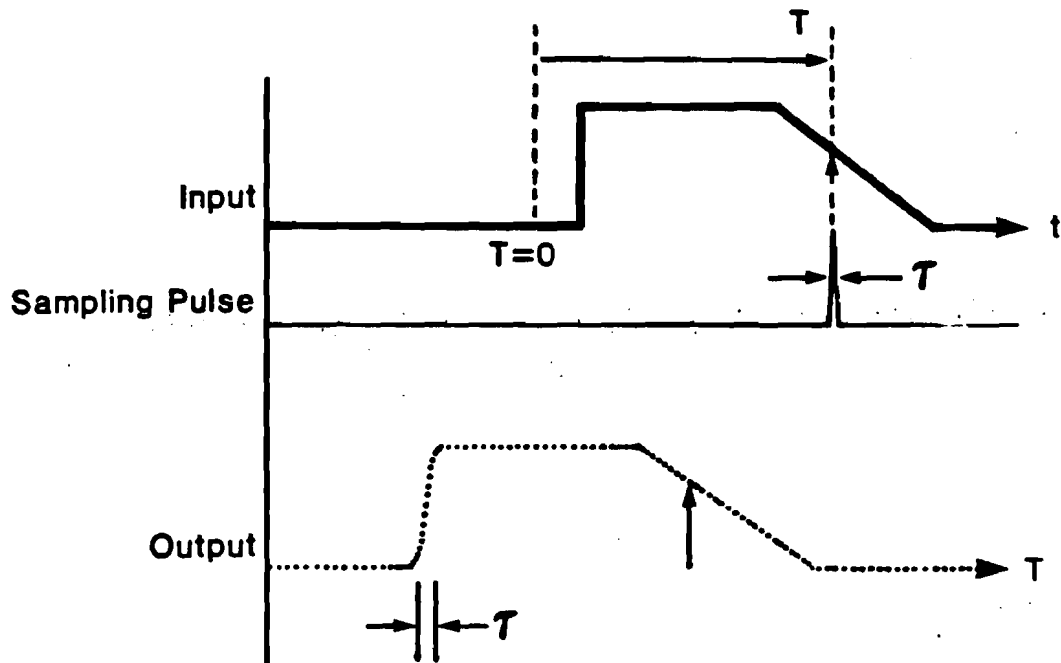
Figure I-3   Sampling waveforms

scope as a function of T. The sampling process allows the oscilloscope to operate at a sweep speed much slower than would be required to observe the signal directly.

The temporal resolution of the sampling technique is, however, still dictated by the interaction time. Here, the input signal interacts with the sampling gate pulse to generate the proportional sample voltage. The temporal resolution, τ, is therefore determined solely by the sampling pulse width, and manifests itself as a risetime in exactly the same way as before. Figure I-3 shows the resultant risetime of τ given a step function input.

The best sampling oscilloscopes developed to date have specified temporal resolutions of 25 ps,[9] but due to temporal instabilities that randomly vary the position of the sampling pulse, called jitter, the effective resolution is reduced to approximately 100 ps. Sensitivity is on the order of a few millivolts with a linearity range limited to about 10 V. In addition, spurious signals of a few millivolts in amplitude and several nanoseconds in duration can be emitted from the input of the sampling gate[10] possibly interfering with the input signal. The sampling oscilloscope has been used for many years not only because of its availability, but also because of the lack of a better alternative.

I.B.4. Superconducting sampling

Due to efforts to increase the speed of computer circuits, much work has been done in the field of superconductive switches. Some switches, known as Josephson devices (JD), combine high speed operation with low-power dissipation, but being superconductive, require a cryogenic environment. Recently, Faris[11] has demonstrated that current pulses in the picosecond regime can be generated and measured using JD's as sampling gates. Tuckerman,[12] subsequently has employed similar JD's as sampling gates to develop a sampling system that has achieved a temporal resolution of under 10 ps with a sensitivity of near $10 \mu V$. The major obstacle, of course, is the cryogenic environment which presents great difficulties in being able to couple high speed signals into or out of the sampling device. A 10 GHz transmission line to room temperature has been implemented, but otherwise the sampler is limited to analyzing high speed signals that are generated within the low temperature environment.

I.B.5. Indirect optical (photoconductive) sampling

An attractive alternative to conventional sampling gates are photoconductive switches. These devices are based on semi-insulating materials that become conducting when they absorb light. The photoconductivity process can be exceedingly fast, and if the activating light is in the form

of picosecond pulses, then many photoconductors can be employed as sampling gates on the same time scale. The photoconductive switch is essentially a direct replacement for the conventional electronic sampling gate, operated in an entirely analogous manner.

Figure I-4 depicts a typical sampling arrangement employing a photoconductive gate.[13] A train of picosecond optical pulses is generated by a mode-locked laser and divided in two. The beam splitter yields two perfectly synchronized pulse trains. An unknown waveform is optically triggered by one of the trains and propagated along a strip transmission line that has a photoconductive material positioned somewhere along its length. As the signal passes the photoconductor, an optical pulse from the second train illuminates the gate and allows a sample of the unknown signal to be extracted along a second transmission line. Successive samples of the signal are accumulated and averaged. The sampling process occurs at the repetition rate dictated by the pulse train, which is typically around 100 MHz. A proportional replica of the unknown signal is generated by varying the relative delay between the two pulse trains, while mapping the sampled values as a function of the delay.

The most significant aspect of this sampling scheme that differentiates it from previous methods is that it is optically controlled. The use of optical pulses coupled
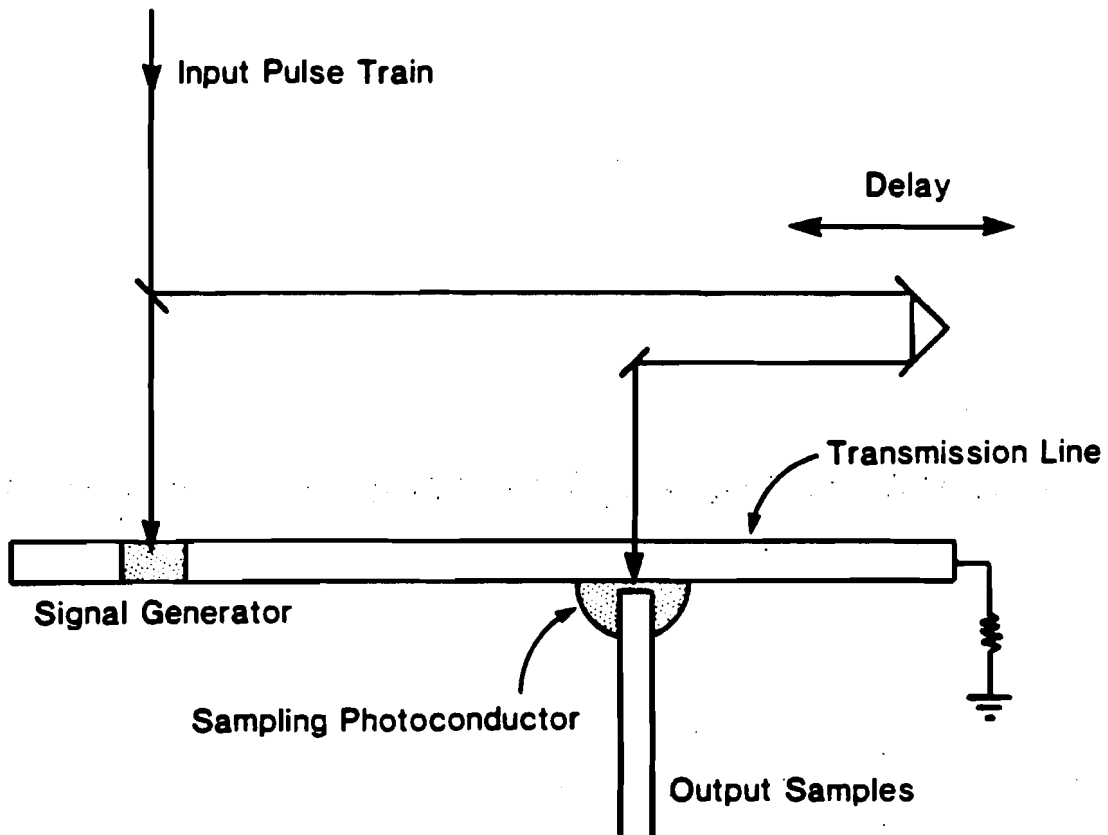
Figure I-4 Indirect optical (photoconductive) sampling
(see Reference 13)

with the photoconductive effect leads to several distinct advantages over conventional electronic sampling schemes.

The temporal resolution, $\mathcal{T}$, for photoconductive sampling is precisely the duration of the photo-induced conductivity, which is determined by the length of the optical pulse and the photoconductive material characteristics. It has been shown (see Appendix B) that the risetime of the gating pulse follows the integral of the optical excitation pulse, while the fall time is generally dictated by carrier recombination kinetics of the photoconductor. Many materials have been investigated,[14-16] but the most recent work by Auston[17] indicates that gate widths, and hence temporal resolutions, of less than three picoseconds have been obtained by using radiation damaged silicon as the photoconductor. We refer to this gating technique as "indirect" optical sampling, because the sampling gate, although optically triggered, still depends on electronic material parameters to determine its ultimate duration.

Due to the integral relationship between the optical pulse and the gating pulse, there is essentially no jitter between the two signals. If the unknown signal can also be generated in a jitter-free manner with respect to its optical triggering pulse, then the timing between the signal and the sampling gate also becomes free of jitter. The precise synchronism between signals enables the temporal resolution

to be truly limited by only the gate width and not by fluc-tuating relative waveform positions, as was the case in electronic sampling.

Several systems utilizing photoconductors as sampling gates have been experimentally investigated and implemented.[10,13,16,18] Auston and Smith[16,19,20] have demon-strated the versatility of their system by characterizing the electrical behaviour of high-speed semi-conductors, pho-todetectors and field-effect transistors. The voltage sensitivity of this method is excellent due to the inherent-ly low-noise nature of photoconductive devices. Sensitivities as low as 10 μV can be attained with the use of signal averaging techniques.[13]

I.B.6. Summary

We have found that conventional oscilloscopes, although extremely versatile, have temporal resolutions limited to the regime of greater than several hundred picoseconds. Sampling oscilloscopes, employing electronic sampling gates can attain effective temporal resolutions near 25 ps but are limited by jitter and noise in the sampling circuitry. Superconducting electronics is able to sample with better than 10 ps resolution, but only in the severely restricted cryogenic environment. The best temporal resolution achieved before the work described in this dissertation, employs picosecond optical techniques to control photocon-

ductive sampling gates (indirect optical sampling). Although the precision of optical control and triggering is exploited, the temporal resolution of 3 ps that has been achieved is still limited by material parameters. Table I-1 summarizes the key parameters for the variety of measurement systems and techniques we have considered in this review. The table also includes the achievements of this thesis work on electro-optic sampling for comparative purposes.

I.C. Overview of dissertation

Chapter II begins by introducing the concept of direct electro-optic sampling. This is followed by a discussion of the electro-optic effect. We consider its suitability for sampling electrical signals and the method of implementation. Several electro-optic materials and types of high speed modulators are compared resulting in the decision to use lithium tantalate in a traveling-wave Pockels cell geometry. Section II.C. considers the electrical and optical characteristics of the traveling-wave geometry. A simple theory for predicting the temporal resolution is derived and applied to several modulators. The theory indicates that a resolution of a few hundred femtoseconds is achievable. The chapter concludes with an introduction to the various sources of noise and considers how they affect the overall sensitivity of the electro-optic technique.

Technique	Temporal Res. (Bandwidth)	Sensitivity	Maximum Voltage	Comments
Conventional Oscilloscope	1 ns (350 MHz)	10 μV	10 V	Readily available
Trav. Wave Oscilloscope	100-300 ps (1-3 GHz)	1-10 V	100 V	Insensitive
Sampling Oscilloscope	100 ps (3 GHz)	1 mV	10 V	Jitter
Supercond. Sampling	10 ps (30 GHz)	10 μV	100mV	Cryogenic
Ind. Optical Sampling	3 ps (100 GHz)	10 μV	>1 kV	Need ps laser; No Jitter Material limit
Electro-optic Sampling	500 fs ($\sim$1 THz)	<50 μV	>1 kV	Need sub-ps laser No Jitter; Optical Limit

TABLE I-1 Comparison of high speed measurement techniques

Chapter III considers the experimental system and also presents a variety of results. The construction of the key elements is detailed together with a description of the laser and the overall system configuration. Calibration procedures for both time and sensitivity are presented, followed by an outline of the measurement procedure. Section III.C. contains results demonstrating the temporal resolution and sensitivity as well as the effects of dispersion for several modulators. A few demonstrative applications of picosecond electronics are also included. The chapter concludes with an experimental investigation of noise sources and an evaluation of the effectiveness of signal to noise enhancement techniques.

Chapter IV is a summary of the results and conclusions of the thesis work.

Appendix A discusses the method of auto-correlation by which the laser pulsewidth is determined. Appendix B elaborates on the electrical signal generated by a photoconductive detector.

CHAPTER II

THEORY OF ELECTRO-OPTIC SAMPLING

In this chapter we present the theoretical basis for choosing the electro-optic effect as the mechanism for sampling picosecond electrical transients. A brief introduction will serve to delineate the differences between the technique we have developed and previous ones. The second section describes the origin and implementation of the electro-optic effect in birefringent media. Several materials and types of modulators are considered. This discussion is followed by an analysis of the operation of the traveling-wave modulator geometry. The modulator's electrical and optical properties are established and a theoretical model is derived to predict its ultimate temporal resolution. We also discuss the major sources of noise, their effect on system sensitivity, and how the application of signal recovery techniques enhances the overall signal to noise ratio. Finally, we present an illustrative discussion of the transient measurement process.

II.A. Introduction to direct electro-optic sampling

To date, the use of optical pulses in the characterization of picosecond electrical signals has been of an indirect nature. Although the jitter-free property of the

optical pulse train has been exploited, the sampling gate itself has always been an electronic material property (photoconductivity) induced by the optical pulse. The goal of this work is to utilize picosecond and sub-picosecond optical pulses directly as the sampling elements. This approach implies that the optical pulse itself is the carrier of information about the amplitude of the electrical signal obtained at the instant of sampling.

We have chosen the linear electro-optic effect (Pockels effect) as the means of impressing electrical information onto the optical pulse. The Pockels effect alters the state of polarization of the optical beam in such a way that, viewed through a linear polarizer, the intensity of the transmitted beam changes linearly with the amplitude of the electrical signal. As will be seen, the electro-optic effect does not limit the speed of response. Temporal resolution is determined entirely by optical pulse properties and simple geometrical considerations. The electro-optic crystal is essentially passive and presents itself only as an interaction medium for the optical and electrical signals. For this reason, there is no jitter introduced between the optical and electrical signals during the sampling process. The absence of jitter allows the optical pulsewidth to be the final criterion of temporal resolution.

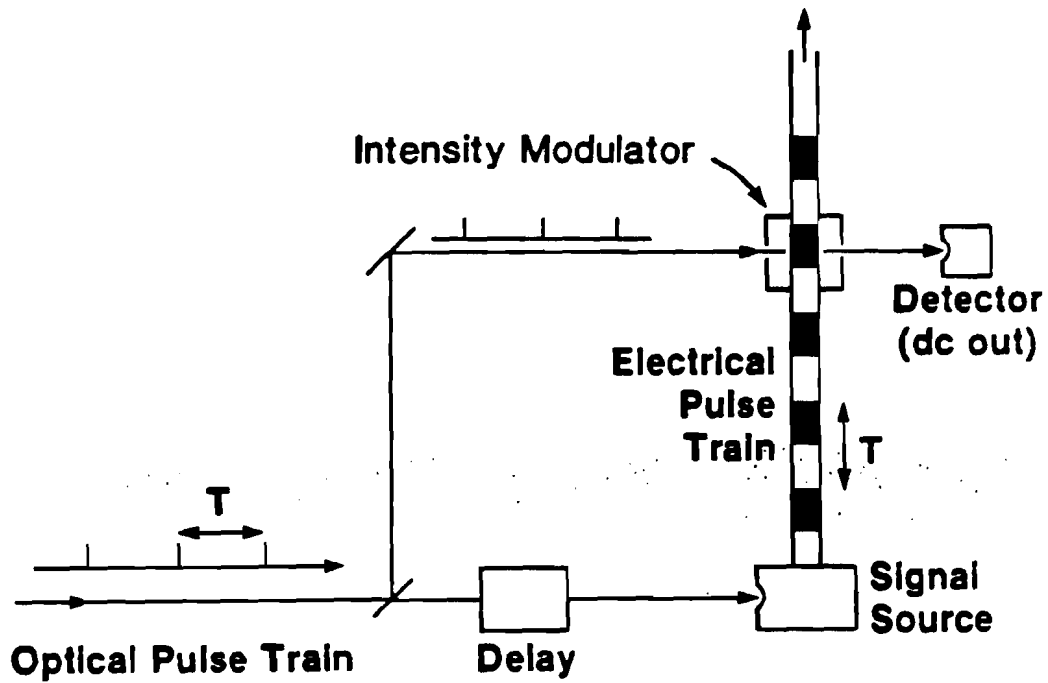Figure II-1 depicts the conceptual elements in a direct

Figure II-1 Direct optical sampling

optical sampling system. A high repetition rate (1-100 MHz) train of short optical pulses is split, with one of the resultant beams triggering the electrical signal source while the other beam goes to the electro-optic modulator. The signal source generates a jitter free train of electrical pulses to be measured at the same rate as its incident optical pulse train. These two trains meet in perfect synchronism at the modulator. Because of this synchronization, each optical pulse interacts with precisely the same corresponding point of each electrical pulse as it passes through the electro-optic medium. In this way, the intensity of each optical pulse is altered by the exact same amount, in proportion with the amplitude of the electrical signal seen at that point in time. Thus, a slow detector measuring the average intensity of the output beam will yield a DC signal proportional to the amplitude of the sampled electrical signal. By slowly sweeping the optical delay, the detector signal will follow the amplitude of the unknown electrical signal.

Direct optical sampling has been employed before. In two cases, Auston[21] and LeFur[22] used the technique on a single shot basis to observe kilovolt electrical transients in a Pockels cell with a resolution of approximately 25 ps. More recently, a high repetition rate system was employed by Alferness et al.[23] to characterize the speed of electro-optic waveguide modulators with an effective tempo-

ral resolution of approximately 60 ps.

This work investigates the limits of a direct electro-optic sampling system as a measurement tool for the characterization of unknown electrical transients. Optical and electrical considerations of the Pockels cell are discussed in detail as well as the effects of electronic signal processing. We will demonstrate that sub-picosecond temporal resolution and voltage sensitivities of less than one hundred microvolts are possible.

II.B. The electro-optic effect

In this section we treat various aspects of the electro-optic effect and its application. We first present a discussion of the origin of the electro-optic mechanism and the particular properties which apply in our situation. This discussion is followed by a description of how the electro-optic effect is implemented in an intensity modulator, addressing the questions of linearity and sensitivity. The next section compares some commonly used electro-optic materials and reaches the conclusion that lithium tantalate is the optimal medium in this application. Finally we discuss the bandwidth aspects of various types of high speed lithium tantalate modulators in current use.

II.B.1. The electro-optic mechanism

The electro-optic, or Pockels, effect is a change in the optical dielectric properties of a medium (usually a crystal) in response to an applied electric field. The effect is commonly realized as a change in the index of refraction. In order to more completely understand the limits of this mechanism we must investigate the behaviour of the electronic charges within the medium.

The application of an external electric field to a medium displaces both the ions in the lattice and the electron orbits from their unperturbed positions and orientations. The displacements create electric dipoles whose macroscopic manifestation is the electric polarization, P. In any particular material, P is a function of the applied electric field E. The polarization can be represented by the following power series:[24]

$$\bar{P}_i(\omega_\ell) = \chi_{ij}^{(1)} \bar{E}_j(\omega_\ell) + \chi_{ijk}^{(2)} \bar{E}_j(\omega_m) \bar{E}_k(\omega_n) + \cdots \qquad \text{II-1}$$

where $\chi(1)$ and $\chi(2)$ are tensors that relate the vectors P and E. i, j, and k are the cartesian indices that run from 1 to 3 and l, m, and n represent different frequency components. Each term is summed over all repeated indices according to the Einstein sum convention. We include only those terms that are of interest here. χ is a tensor, because in many materials, especially crystals, E and its

induced P are not necessarily colinear.

The $\chi(1)$ term is dominant and is an extremely good approximation for P with small applied electric fields. It is this term that applies to all linear optics and yields the common index of refraction, n, and optical dielectric constant, $\mathcal{E}$, as follows:

$$n = \sqrt{\mathcal{E}} = \left(1 + 4\pi \chi^{(1)}\right)^{1/2} . \qquad \qquad \textit{II-2}$$

The $\chi(2)$ term (containing 27 elements) gives rise to optical mixing, second harmonic generation and the Pockels effect. It exists only for crystals lacking an inversion symmetry. Otherwise, all components of the tensor vanish. In the general case, P can be written:

$$\bar{P}_i \binom{\omega_m + \omega_n}{\omega_m - \omega_n} = \chi_{ijk}^{(2)} \, \bar{E}_j \, (\omega_m) \, \bar{E}_k \, (\omega_n) \qquad \textit{II-3}$$

which relates fields of three different frequencies. This equation describes the phenomenon of optical three-wave mixing. If $\omega_m = \omega_n$ then P has a frequency of $2\omega_n$ and second harmonic generation arises. If one of the fields is at DC, then $\chi(2)$ gives rise to the Pockels effect, where the input and output frequencies are the same, and the effective $\chi(2)$ becomes a function of the DC field, ie.

$$\bar{P}_i (\omega_\ell) = \left(\chi_{ijk}^{(2)} \, E_k \, (DC) \right) \bar{E}_j \, (\omega_\ell) . \qquad \textit{II-4}$$