Angular Distribution of Amplified Spontaneous Emission -
A Comparison of Theory and Laser-Pumped Dye Amplifier Experiment

by

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Vitae

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Abstract

Amplified spontaneous emission (ASE) is stimulated emission initiated by the fluorescence in a laser amplifier in the absence of optical feedback. A measurement of the angular distribution of ASE is shown to be a practical, sensitive, and unambiguous single-shot diagnostic of gain in laser amplifier experiments. The technique is more sensitive than spectral line narrowing observations in situations in which the measurements are spatially integrated. Other direct measurement techniques, such as pulse injection or gain-length variation, are often impractical because of the lack of reproducibility of the (pulsed) experimental conditions.

A two-dimensional geometric model is presented that predicts the angular distribution of ASE from a rectangular amplifier with uniform fluorescence and gain. The radiation transport equation including gain is integrated over the amplifier to determine the total radiant intensity emitted into a given direction. Angular distribution curves are presented in terms of the amplifier length to width ratio and the gain-length product. The FWHM beam divergence is shown to be inversely proportional to the amplifier aspect ratio and to increase with decreasing gain-length product. Near threshold the beam divergence is several times larger than the inverse aspect ratio. Calculations for integrated measurements in which the gain coefficient and fluorescence rate are linear functions of a parameter (time, wavelength, space) show that an average gain coefficient may be used in the uniform gain model to good approximation at small
gain-length products. A reasonable threshold for the determination of gain from directional ASE measurements is found to occur for an average gain-length product of unity.

A transverse laser-pumped dye amplifier experiment is described which was designed to measure the ASE angular distribution from an optically freestanding amplifier with a non-uniform transverse gain profile. Measurements of the distribution are presented, and the observed dependence on the gain coefficient and the amplifier geometry compared to that predicted by the ASE theory. To simulate an optically freestanding amplifier, R6G dye was index matched to its cell, thus suppressing reflections and parasitic oscillations. The dye concentration determined the transverse gain profile. Pumping was provided by a frequency-doubled 15nsec. pulse from a Q-switched Nd$^{3+}$:Yag oscillator and Nd$^{3+}$:glass amplifier chain.

In order to simulate an inhomogeneous laser-produced plasma x-ray amplification experiment, the model is extended to the case where only a portion of the emitting region exhibits gain. The result is that the direction of greatest intensity shifts off of the amplifier axis, and the overall directionality is reduced at a given gain-length product. The directionality of ASE is shown to still provide a useful diagnostic of gain under such conditions.
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1. Introduction

1.1 Objectives

Amplified spontaneous emission (ASE) is stimulated emission initiated by the fluorescence in a laser amplifier. Optical feedback is assumed to be absent. Our principal interest is in the angular distribution of ASE from a laser amplifier and in its use as a gain diagnostic, especially for use in pulsed experiments for which the experimental conditions are not reproducible. Such an application is in the diagnosis of soft x-ray amplification in laser-produced plasmas. Reviews of such research are given by Chapline and Wood(1), and by Waynant and Elton(2).

We have modeled the directional behavior of ASE from a two-dimensional, rectangular amplifier and examined the use of the directionality as a gain diagnostic. Results from this model are compared to ASE angular distribution measurements of a transverse, laser-pumped R6G dye amplifier.

1.2 Techniques for Measuring Gain

There are several techniques for directly measuring the gain of an amplifier in an ASE experiment: 1) signal injection through the amplifier, 2) measurement of ASE as a function of amplifier length, 3) spectral line narrowing of ASE, and 4) the angular distribution of ASE.

Signal injection is the most direct and commonly used technique for the measurement of gain in an amplifier. However, a pulsed
experiment requires a synchronized signal of significantly greater spectral radiance than that from the amplifier fluorescence. In addition, suitable detectors for measuring the ratio of the input and output signals are required. These requirements are very inconvenient or impossible to meet in some experimental situations — this is especially true for laser-produced plasmas used in soft x-ray amplification experiments.

Length variation is an accepted technique for measuring the gain in ASE experiments. Allen and Peters have examined the length dependence of ASE theoretically and experimentally in CW HeNe and pulsed Ne ASE sources. Shank, et al. used length variation to measure the gain in a pulsed, laser-pumped dye amplifier. However, this technique is impractical in pulsed experiments in which the experimental parameters are not reproducible from shot to shot.

Spectral line narrowing is also a practical single shot gain diagnostic for some experiments. The emission at the linewidth center is amplified more than in the wings, thus narrowing the linewidth at high gain. Such narrowing was considered by Rigden and White who suggested using the ASE emission at 3.39 μ from a HeNe discharge as a narrow bandwidth wavelength standard. Yariv and Leite observed spectral narrowing in forward-biased GaAs junction diodes and compared the result to a model for narrowing in a homogeneously broadened medium. Their result for gL>>1 was \( \Delta \nu / \Delta \nu_H \approx (gL)^{-\frac{1}{2}} \) where \( \Delta \nu_H \) is the broadened linewidth, and \( gL \) is the gain-length product.
More recently Casperson and Yariv have presented a theory of spectral narrowing in a high gain medium including an arbitrary mixture of homogeneous and inhomogeneous broadening, saturation effects, and wavelength independent absorption\textsuperscript{(13)} They also presented experimental data for spectral narrowing of ASE in a xenon DC discharge ($\lambda=3.5\mu$m). The linewidth was observed to narrow to $\Delta\nu=14$ Mhz from an estimated Doppler width $\Delta\nu_D=100$ Mhz, comparing well to the predicted reduction factor $(gL)^\frac{1}{2}=5$.

The use of spectral line narrowing as a gain diagnostic is thus well established. However, the narrowing is a weak function of the gain-length product, and is therefore not suitable as a threshold gain diagnostic. For example, the linewidth is only narrowed to 80% of the fluorescence linewidth for $gL=1$ in an unsaturated amplifier, a 20% effect.

The directional behavior of ASE can provide a significantly more sensitive gain diagnostic than can spectral line narrowing. (Spatially resolved spectral line narrowing measurements may be more sensitive, however, in situations exhibiting regions of significant unamplified fluorescence, for example, as discussed in section 6.2). For $gL=1$ the axial ASE can exceed the fluorescence by over 70%. This can be especially important for soft x-ray amplification experiments in laser-produced plasmas. A typical amplifying geometry would be approximately cylindrical, say 50$\mu$m by 1000$\mu$m, produced by a line-focused laser (depicted in figure 1-1). This geometry is amenable to a measurement of the x-ray line emission both parallel to and perpendicular to the axis, whereas current grazing incidence grating spectrometers may not resolve the spectral line narrowing.
Figure 1-1

LASER-PRODUCED LINE PLASMA X-RAY AMPLIFICATION EXPERIMENT

TARGET

PLASMA

X-RAY FLUORESCENCE

X-RAY ASE

LASER BEAM

SPECTROMETER

ASPHERIC/CYLINDRICAL LENS
1.3 Previous Research

In this work we examine the dependence of the ASE angular distribution on amplifier parameters which include the gain-length product and the geometry. Previous authors have emphasized other aspects of ASE, most notably the length dependence and spectral line narrowing. One exception was Roess who observed ASE from a ruby laser amplifier, and used the ratio of the peak axial output and the side fluorescence to estimate the gain.\(^{14}\)

The most complete previous study of ASE was presented by Allen and Peters.\(^{3-9}\) They developed an ASE theory which considered the threshold condition for ASE; the ASE output power of an amplifier as a function of gain, length, and saturation; the ASE beam divergence and spatial coherence; and spectral line narrowing. The theory was compared to ASE power measurements of a HeNe, CW, RF excited discharge (\(\lambda=3.39\ \mu\)) and a pulsed Ne discharge (\(\lambda=0.614\ \mu\)). Their spectral line narrowing theory was not experimentally confirmed, however, and the predictions conflicted with the established theory of Casperson and Yariv.\(^{13}\) Maeda and Yariv have pointed out some deficiencies in Allen and Peters' model.\(^{15}\)

The beam divergence measurements made on the HeNe and Ne discharges are not applicable to our work. Severe wall reflections and diffraction were present in their experiment, and no calibrated measurement of the fluorescence relative to the axial ASE was made.

Shank, et al. measured the single pass gain of exciplex 4-MU and rhodamine 6G dye transversely pumped amplifiers by measuring the
ASE intensity as a function of pump power for two different dye cell lengths. \(^{(10)}\) They used a nitrogen laser (10 nanosecond pulses), line-focused at the dye cell/dye interface, to pump the dye. The output energy was measured over a 6 Å bandwidth using a monochromator and integrating photomultiplier. Measurements of the angular distribution were not reported.

Ganiel, et al. considered both ASE and signal amplification in a transversely pumped R6G dye amplifier experiment. \(^{(16)}\) They employed a dye oscillator and amplifier, each pumped by the same nitrogen laser. Total axial ASE power measurements were made as a function of pump power. Amplifier gain measurements were performed as a function of pump power and input signal strength from the dye oscillator. The average gain increased with pump power, but saturated rapidly at high ASE levels or for high input signals.

They also presented a coupled set of wavelength dependent rate equations for the population densities and the photon fluxes in both directions along the dye cell axis, thus allowing for saturation. Spontaneous emission within the solid angle subtended by the end of the medium was included as the source term. Their numerical results for the dependence of ASE and gain on the pump power and the effect of gain saturation at high signal levels compared reasonably well with the experimental results. However, they did not report on the directional dependence of the ASE.
1.4 Laser-Pumped R6G Dye Amplifier Experiment

We have experimentally examined the directional dependence of ASE from a laser-pumped R6G dye amplifier. Our objective was to demonstrate the viability and practicality of a directional ASE measurement as a single shot gain diagnostic, and to compare measurements of the angular distribution with the results of the directional ASE model.

A laser-pumped R6G dye amplifier was chosen for a number of reasons. The most important reason was that the dye solution could be index-matched to a dye cell to reduce wall reflections and thus approximate an optically freestanding amplifier. The relatively high gain attainable in laser pumped dyes allowed the amplifier size to be small (length 2.5 cm) compared to the detector distances, enabling us to measure the total radiant intensity in a given direction as assumed in the ASE model (see figure 2-1). These features, coupled with the ability to choose the dye amplifier length to width ratio (aspect ratio) and to change the transverse gain profile, enabled us to approximately model the geometry expected in a line-focus, laser-produced plasma soft x-ray amplification experiment. The analogy between the transverse gain profile in the dye amplifier and the transverse electron density profile in a laser-produced plasma is shown in figure 1-2. The gain profile in the dye amplifier depends on the pump pulse absorption depth, and thus on the dye concentration, and is shown for low and high dye concentrations.
Figure 1-2

GAIN PROFILES

ASE DYE LASER

PUMP PULSE

LOW CONCENTRATION

X-RAY PLASMA

PUMP PULSE

HIGH CONCENTRATION

TARGET
An additional advantage was the visible ASE, which permitted conventional measurement techniques to be used.

1.5 Organization of Thesis

This thesis examines the directional behavior of ASE from a laser amplifier and its use as a gain diagnostic. Chapter 2 presents a canonical ASE model, the uniform gain model, which predicts the angular distribution from a 2D, rectangular amplifier. The alteration in the directional behavior of ASE for integrated measurements is also examined, with the result that the uniform gain model is a reasonable approximation for low gain-length products. Chapter 3 discusses the laser-pumped dye experiment, in particular the experimental design and set-up. In Chapter 4 we present the experimental results for the ASE angular distribution and gain coefficient measurements. Chapter 5 discusses the experimental results and some of the limitations of the ASE model in explaining them. In Chapter 6 we discuss the application of the directional ASE model to the diagnosis of soft x-ray amplification in laser-produced plasmas. The model is extended to include a thermal fluorescence region adjacent to the amplifying region in order to determine the viability of a directional ASE measurement as a gain diagnostic under such conditions. The possibility of x-ray refraction altering the angular distribution is also discussed.
2. ASE Theory

2.1 Introduction

We now develop a canonical model of ASE from an optically free-standing laser amplifier. The objective is to describe the directional properties of ASE, develop a diagnostic for gain, and to apply the model to the laser-pumped dye experiment, and to x-ray amplification schemes in laser-produced plasmas.

2.2 Rate Equations and Radiation Transport Equation

In modeling the laser amplifier, we assume that a two level atomic system interacts with the radiation field, and that rate equations are sufficient to describe the population densities of each laser level. We consider the situations for which gain saturation is negligible - this obtains when the total stimulated emission is small compared to the total fluorescence. For a cylindrical amplifier of length $L$ and diameter $d$, this requirement is satisfied if the gain-length product, $gL$, obeys the condition

$$gL < \frac{2\ln L}{d}.$$ 

The absence of saturation assures that the radiation transport equation does not couple back to the rate equations. Thus, the population densities are explicit functions of time, position, and frequency, independent of the radiation field. The rate equations will not be solved explicitly in this work.
We describe the amplification of the radiation field in the amplifier by a steady-state, geometric (no diffraction) radiation transport equation. The transport equation is

\[ \tilde{s} \cdot \nabla I_\nu(\tilde{s}, \tilde{x}) = \left[ N_b(\tilde{x}, \nu) \sigma_{ba}(\nu) - N_a(\tilde{x}, \nu) \sigma_{ab}(\nu) \right] I_\nu(\tilde{s}, \tilde{x}) + N_b(\tilde{x}, \nu) \sigma_{ba}(\nu) \left( \frac{2h\nu^3}{c^2} \right) \] (2-1)

where \( I_\nu(\tilde{s}, \tilde{x}) \) is the specific intensity or spectral radiance at a point \( \tilde{x} \) in the medium in the direction \( \tilde{s} \) at a frequency \( \nu \). \( I_\nu(\tilde{s}, \tilde{x}) \) has units of power per unit area per steradian per unit frequency interval. Because the propagation time in the medium is assumed to be small compared to the temporal variations, the time dependence of \( I_\nu(\tilde{s}, \tilde{x}) \) may be considered explicitly. \( N_b(\tilde{x}, \nu) \) and \( N_a(\tilde{x}, \nu) \) are the population densities of the upper and lower laser levels, respectively, and in general are functions of position and frequency. \( \sigma_b(\nu) \) and \( \sigma_a(\nu) \) are the stimulated emission and absorption cross-sections and are also functions of frequency.

Two contributions to the increase in spectral radiance are considered: net stimulated emission and fluorescence. The net stimulated emission is the difference in stimulated emission, \( N_b \sigma_{ba} I_\nu \), and absorption \( N_a \sigma_{ab} I_\nu \). Fluorescence or spontaneous emission, \( N_b \sigma_{ba} \frac{2h\nu^3}{c^2} \), is often neglected in laser oscillator calculations; however, fluorescence must be included in ASE calculations because it is the sole source term. The medium is assumed to be optically thin so that resonant absorption (and thus radiation trapping) and scattering may be neglected.
2.3 2D Uniform Gain Model

2.3.1 Measurement Geometry and Radiation Transport Equation

Consider a two-dimensional section of an amplifier, rectangular in shape, of length $L$ and width $D$. The amplifier dimensions and the diameter $d$, of the detector used to measure the ASE are assumed to be small with respect to the distance $r$, separating the amplifier and detectors, as shown in figure 2-1. That is $D/L < r$ and $d/r$. The quantity we measure and wish to calculate is the radiant intensity $I(\hat{s})$ (power per steradian) in the direction $\hat{s}$, integrated over the volume of the amplifier. The direction vector $\hat{s}$ makes an angle $\Theta$ with respect to the longitudinal axis of the amplifier.

The radiation transport equation (2-1) can be written in the simplified form

$$\frac{dI(\hat{s})}{ds} = g I(\hat{s}) + K. \quad (2-2)$$

We make the approximation that the gain coefficient, $g$, and the fluorescence per unit volume per steradian, $K$, are uniform or homogeneous throughout the amplifier. Integrating equation (2-2) in the direction $\hat{s}$ along the path $l(x)$ yields

$$\tilde{I}(x) = \frac{K}{g} \left( e^{gl(x)} - 1 \right) \quad (2-3)$$

where $l(x)$ is a function of transverse coordinate $x$ as shown in figure 2-2. The radiance $\tilde{I}(x)$ must be integrated over the transverse coordinate $x$ to find the radiant intensity $I(\Theta)$ in the direction $\hat{s}$. 

Figure 2-1

MEASUREMENT GEOMETRY FOR AMPLIFIER AND DETECTOR
Figure 2-2

GEOMETICAL REGIONS
UNIFORM GAIN AMPLIFIER
\[ \tan \theta \leq \frac{D}{L} \]
2.3.2 Analysis

The rectangular geometry requires us to consider two cases: $0 \leq \tan \theta \leq D/L$ and $\tan \theta \geq D/L$ where $L/D$ is the aspect ratio of the amplifier. With $\tan \theta \leq D/L$ there are two distinct regions: region 1, a parallelogram, and region 2, two identical triangles. (see figure 2-2).

In region 1 the amplification length $l_1(x) = L / \cos \theta$ is independent of position. The radiant intensity is found by integrating equation (2-3) over the transverse coordinate $x$

$$ I_1(\theta) = \int_0^D \frac{K g}{\cos \theta} \left( e^{gL / \cos \theta} - 1 \right) dx $$

$$ I_1(\theta) = (D \cos \theta - L \sin \theta) \frac{K g}{\cos \theta} \left( e^{gL / \cos \theta} - 1 \right) . $$

In region 2 the amplification length $l_2(x) = \frac{X}{\cos \theta \sin \theta}$ is a function of position. Integrating the radiance over the transverse coordinate $x$, we obtain the radiant intensity $I_2(\theta)$

$$ I_2(\theta) = 2 \int_0^{L \sin \theta} \frac{K g}{\cos \theta \sin \theta} \left( e^{gx / \cos \theta \sin \theta} - 1 \right) dx $$

$$ I_2(\theta) = \frac{2K}{g} \left[ \frac{\cos \theta \sin \theta}{g} \left( e^{gL / \cos \theta} - 1 \right) - L \sin \theta \right] . $$
The total radiant intensity is found by summing (2-5) and (2-6)

\[ I(\theta) = I_1(\theta) + I_2(\theta) \quad \text{for } \tan\theta \leq D/L \]  

\[ I(\theta) = \frac{K}{g} \left[ (e^{gL/cos\theta} - 1) \left( \frac{2}{g} \sin\theta \cos\theta + D \cos\theta - L \sin\theta \right) - 2L \sin\theta \right]. \]

The symmetry between \( D \) and \( L \), and between \( \cos\theta \) and \( \sin\theta \), allows us to write the solution for \( \tan\theta > D/L \) by replacing one quantity of each pair by the other in (2-7).

\[ I(\theta) = \frac{K}{g} \left[ (e^{gD/sin\theta} - 1) \left( \frac{2}{g} \cos\theta \sin\theta + L \sin\theta - D \cos\theta \right) - 2D \cos\theta \right] \]

2.4 Results of Uniform Gain Model

2.4.1 ASE Length Dependence

The length dependence of the ASE along the amplifier axis (\( \theta=0 \)) is

\[ I(\theta=0,L) = \frac{KD}{g} (e^{gL} - 1) \]

and is shown in figure 2-3 for \( g=KD=1 \). The dependence is linear for \( gL<<1 \), and exponential for \( gL>>1 \). Note that there is no abrupt threshold length for the onset of stimulated emission as for a laser cavity. The dependence on the gain coefficient, \( g \), is identical to that for length if the fluorescence, \( K \), is proportional to \( g \) when \( L \) is fixed. This dependence on \( g \) becomes sub-exponential if the fluorescence, \( K \), is also fixed.
Figure 2-3

ASE vs LENGTH

![Graph showing ASE vs LENGTH with logarithmic scales for both axes, illustrating exponential and linear-no-gain gain relationships.](image)
A value for the gain coefficient can be obtained from measurements of the axial ASE using two different amplifier lengths. Using (2-9) to find $I(\Theta=0, L)=I_L$ and $I(\Theta=0, L/2)=I_{L/2}$, and then combining we obtain the relationship for the gain coefficient

$$g = \frac{2}{L} \ln \left( \frac{I_L}{I_{L/2}} - 1 \right).$$  \hspace{1cm} (2-10)

This result was previously obtained and applied to the measurement of gain in a nitrogen laser pumped dye amplifier by Shank, et al.\(^2\)

The technique requires that the gain and fluorescence be repeatable over two shots if the experiment is pulsed.

2.4.2 Use of Directional ASE as a Gain Diagnostic

A single shot gain diagnostic can be obtained by simultaneously measuring the ASE on axis, $I_A=I(\Theta=0)$, and the fluorescence perpendicular to the axis, $I_F=I(\Theta=\pi/2)$, and examining the ratio. Consider an amplifier for which the transverse gain-length product is $gD<<1$, in which case $I_F$ is essentially fluorescence. We then obtain the ratio

$$\frac{I_A}{I_F} = \frac{e^{gL} - 1}{gL}.$$  \hspace{1cm} (2-11)

A similar result was obtained by Roess for ruby gain measurement.\(^3\)

In figure 2-4 we have plotted $gL$ as a function of $I_A/I_F$ on a log scale, allowing a direct estimate of $gL$ from a simultaneous measurement of $I_A$ and $I_F$.

The threshold ratio $I_A/I_F=2$, marked in figure 2-4, corresponds to $gL=1.26$ - approximately unity. This ratio was chosen to guarantee
Figure 2-4

GAIN-LENGTH PRODUCT vs. ASE TO FLUORESCENCE RATIO
UNIFORM GAIN MODEL

\[ gL \]

\[ \frac{I_A}{I_F} \]
an unambiguous and definitive demonstration of gain in pulsed and non-reproducible laser amplifier experiments, e.g. x-ray amplification in laser-produced plasmas. This threshold criteria is somewhat arbitrary, but is practical in application to such experiments. It should also be noted that at threshold, the stimulated emission and fluorescence on axis are equal.

A slight correction to the estimated gain coefficient must be made if the amplification in the transverse direction is significant, e.g., single pass amplification of 5% or more. This correction is plotted for L/D ratios of 10 and 20 in figure 2-5. In general, it is only important at high gain or small amplifier aspect ratios, as can be seen from these examples.

It is important to realize that the emission from the amplifier is also directional if there is absorption instead of gain, i.e., if $g$ is negative. Then the radiant intensity is smaller in the axial direction than in the transverse direction. To distinguish this condition the orientation of the amplifier must be determined independently or from angular distribution measurements, e.g., beam divergence measurements.

2.4.3 Angular Distribution

The angular distribution of ASE from the amplifier is shown in figure 2-6 for a range of $gL$ where the fluorescence, $K$, is proportional to $gL$. The relative ASE radiant intensity is plotted on a log scale versus the azimuthal angle $\Theta$ for L/D=20. Only the positive $\Theta$
Figure 2-5

ASE RATIO \( I(\theta = 0)/I(\theta = \pi/2) \) vs. \( gL \)

TRANSVERSE AMPLIFICATION

\( L/D = 10, 20, \infty \)
Figure 2-6

ASE ANGULAR DISTRIBUTION
FOR DIFFERENT $gL$

RELATIVE ASE RADIANT INTENSITY $I(\theta)$

AZIMUTHAL ANGLE $\theta$

$L/D = 20$

$gL = 5$
$gL = 4$
$gL = 3$
$gL = 2$
$gL = 1.254$
$gL = 0.8$
axis is shown because the distribution is symmetric about the axis.
The threshold ratio, \( I(\Theta=0)/I(\Theta=\pi/2) = 2 \), occurs for \( gL = 1.254 \).

On axis the intensity increases exponentially with \( gL \), as discussed previously. At large angles, say \( 5D/L < \Theta < \pi/2 \), the emission is principally fluorescence and the increase is linear with \( gL \) (because we have assumed \( K = gL \)). A fluorescence measurement need not be made perpendicular to the axis, but only at a sufficiently large angle such that the amplification is negligible. The rapid decrease in intensity as \( \Theta \) increases is due simply to the decrease in the amplification length.

We may characterize the directionality or collimation of the ASE from the amplifier by a FWHM (full-width at half-maximum) beam divergence, which is plotted in units of \( D/L \), versus \( gL \) in figure 2-7. Units of \( D/L \) are appropriate because the angular distribution scales linearly with respect to \( D/L \) for \( \Theta \ll 1 \) if \( D/L \ll 1 \). We find that the beam divergence is proportional to \( D/L \) under these conditions and that it decreases to an asymptotic limit of approximately \( D/L \) as \( g \) increases. An expression for \( \Theta_{\text{FWHM}} \), valid for \( gL > 5 \), is

\[
\Theta_{\text{FWHM}} \approx \frac{D}{L} \left(1 - \frac{2}{gL}\right)^{-1}.
\]  

(2-12)

As \( g \) is decreased the beam divergence increases - for \( gL \approx 3.5 \), \( \Theta_{\text{FWHM}} \approx D/L \) - and becomes undefined at threshold. An estimate of the amplifier aspect ratio may thus be obtained from a measurement of the beam divergence well above threshold. Near threshold a more complete measurement of the angular distribution (including the fluorescence) is required to determine the aspect ratio.
Figure 2-7

ASE BEAM DIVERGENCE vs gL

BEAM DIVERGENCE - FWHM (UNITS D/L)

gL: GAIN-LENGTH PRODUCT
2.4.4 Summary

We have presented the results for the directional dependence of ASE from a rectangular amplifier. We may use these results to estimate the gain-length product and approximate geometry of a laser amplifier given measurements of the ASE angular distribution. A reasonable threshold gain-length product of unity is found for a practical measurement ratio of roughly two for the ASE in the axial and transverse directions.

2.5 Directional Behavior of ASE with Non-uniform Gain and Fluorescence

The directional behavior of ASE emission from a laser amplifier can change from that predicted by the uniform gain model under a number of conditions, and thus change the gain estimates based on that model. For example, experimental conditions may require that a measurement be integrated over a temporal pulse shape or spectral line profile; the gain and fluorescence distributions may be spatially non-uniform; or unamplified fluorescence (or noise) may degrade the directionality of the ASE and thus reduce its viability as a gain diagnostic. We will consider spectral or temporal integrations in section 2.6, multiple integrations in section 2.7, linear transverse gain and fluorescence profiles in section 2.8, and additional fluorescence in section 6.2.
2.6 Spectral or Temporal Integration

An integration over a temporal pulse shape may be necessary if, for example, the detectors used measure energy or do not have sufficient time resolution. Film is an energy detector and is the most commonly used detector in x-ray amplification experiments in laser-produced plasmas; film was also used to record the angular distribution in the laser-pumped dye experiment. For the laser plasma experiments, the time response of either PD or PMT would be insufficient to resolve the subnanosecond x-ray pulse.

A second integration is obtained when the spectral line profile is not resolved. In the laser plasma experiments spectral line measurements will be made at soft x-ray wavelengths (~100Å) using grazing incident x-ray spectrometers which cannot, in general, resolve the x-ray line profile. In the dye amplifier experiment measurements were integrated over the spectral profile.

Different models may be employed to describe the temporal or spectral line shape. Specifically, the spectral line shape may be Lorentzian, such as in a pressure broadened or homogeneous medium; it may be Gaussian, as for an inhomogeneously, e.g. Doppler, broadened medium; or it may have an approximately triangular spectral profile as for R6G dye in the laser-pumped dye experiment. The temporal pulse shape is experiment dependent; in the dye experiment the fluorescence is approximately Gaussian in time.

In the analytic modeling to follow, we will approximate the gain and fluorescence profiles by a linear function. This is a good
approximation for the spectral and spatial profiles in the dye amplifier experiment, but is only a rough approximation for the temporal profile.

2.6.1 Analysis

We begin with the ASE angular distribution derived using the uniform gain model. From (2-7) the radiant intensity for \(\tan \Theta \leq D/L\) is

\[
I(\Theta) = \frac{K}{g} \left[ (e^{gL/\cos \Theta} - 1) \left( \frac{2}{g} \sin \Theta \cos \Theta + D \cos \Theta - L \sin \Theta \right) + 2L \sin \Theta \right].
\]

We allow the gain coefficient, \(g\), and fluorescence, \(K\), to be functions of time (or frequency). Specifically, we choose linear (triangular) gain and fluorescence profiles:

\[
g(t) = \begin{cases} g_0 t & 0 < t \leq 1 \\ 0 & t > 1 \end{cases}, \quad K(t) = \begin{cases} K_0 t & 0 < t \leq 1 \\ 0 & t > 1 \end{cases}
\]

where \(g_0\) and \(K_0\) are the peak gain and fluorescence. Note that \(K(t)\) is taken to be proportional to \(g(t)\); this is approximately true in the laser-pumped dye experiment.

We rewrite equation (2-7) for \(I(\Theta, t)\), changing to a dimensionless variable \(x_0 = \frac{g_0 L}{\cos \Theta}\),

\[
I(\Theta, t) = \frac{K_0 L^2}{x_0 \cos \Theta} \left[ (e^{x_0 t} - 1) \left( \frac{2 \sin \Theta}{x_0 t} + \frac{D}{L} \cos \Theta \sin \Theta \right) - 2 \sin \Theta \right]. \quad (2-13)
\]

To obtain the energy \(E(\Theta)\) we integrate \(I(\Theta, t)\) over \(t\)

\[
E(\Theta) = \int_0^1 I(\Theta, t) dt. \quad (2-14)
\]
Substituting for $I(\Theta,t)$ from (2-13) into (2-14) we obtain

$$E(\Theta) = \frac{K_oL^2}{x_o\cos\Theta} \int_0^1 \left[ (x_o t)^{-1} (\frac{2\sin\Theta}{x_o t} \frac{D}{L} \cos\Theta - \sin\Theta) - 2\sin\Theta \right] dt \quad (2-15)$$

To evaluate this expression analytically, we expand the exponential in a series

$$e^{x_o t} - 1 = \sum_{n=1}^{\infty} \frac{(x_o t)^n}{n!}, \quad (2-16)$$

and substituting into (2-15)

$$E(\Theta) = \frac{K_oL^2}{x_o\cos\Theta} \int_0^1 \left[ 2\sin\Theta \sum_{n=1}^{\infty} \frac{(x_o t)^{n-1}}{n!} + \left( \frac{D}{L} \cos\Theta - \sin\Theta \right) \sum_{n=1}^{\infty} \frac{(x_o t)^n}{n!} - 2\sin\Theta \right] dt.$$

Combining terms and changing summation indices

$$E(\Theta) = \frac{K_oL^2}{x_o\cos\Theta} \int_0^1 \left[ 2\sin\Theta \sum_{n=2}^{\infty} \frac{(x_o t)^{n-1}}{n!} + \left( \frac{D}{L} \cos\Theta - \sin\Theta \right) \sum_{n=2}^{\infty} \frac{(x_o t)^n}{n!} \right] dt.$$

Integrating

$$E(\Theta) = \frac{K_oL^2}{x_o\cos\Theta} \left[ 2\sin\Theta \sum_{n=2}^{\infty} \frac{x_o^{n-1}}{nn!} + \left( \frac{D}{L} \cos\Theta - \sin\Theta \right) \sum_{n=2}^{\infty} \frac{x_o^n}{n!} \right]. \quad (2-17)$$

Substituting for $x_o = \frac{g_o L}{\cos\Theta}$ we obtain for $\tan\Theta \leq D/L$

$$E(\Theta) = \frac{K_o L}{g_o} \left[ 2\sin\Theta \sum_{n=2}^{\infty} \frac{(g_o L/\cos\Theta)^{n-1}}{nn!} + \left( \frac{D}{L} \cos\Theta - \sin\Theta \right) \sum_{n=2}^{\infty} \frac{(g_o L/\cos\Theta)^n}{n!} \right]. \quad (2-18)$$

By symmetry the result for $\tan\Theta > D/L$ is

$$E(\Theta) = \frac{K_o D}{g_o} \left[ 2\cos\Theta \sum_{n=2}^{\infty} \frac{(g_o D/\sin\Theta)^{n-1}}{nn!} + \left( \frac{D}{L} \sin\Theta - \cos\Theta \right) \sum_{n=2}^{\infty} \frac{(g_o D/\sin\Theta)^n}{n!} \right]. \quad (2-19)$$
2.6.2 Angular Distribution

The energy angular distribution of ASE from the amplifier, \(L/D=20\), is shown in figure 2-8 for a range of peak gain-length product \(g_0L\); the peak fluorescence, \(K_0\), is proportional to \(g_0L\) with \(L\) fixed. These curves are very similar to those obtained using the uniform gain model, however there are important differences.

The major difference is that the energy increases more slowly than the peak intensity as \(g_0L\) is increased. The decreased directionality in the energy is due simply to the lower amplification with respect to the peak over most of the pulse. This is shown graphically in figure 2-9(for \(N=1\)) where the ASE to fluorescence ratio is plotted versus the peak gain-length product \(g_0L\) for both energy and peak intensity. A larger peak gain coefficient is required for the energy to achieve the same directionality as the peak intensity, thus increasing the detection threshold.

2.6.3 Average Gain Coefficient Using Uniform Gain Model

Another way of interpreting the results is to compare them to those of the uniform gain model. The uniform gain model is the logical choice for estimating the gain in an experiment for which the gain may be a function of time and the exact behavior is unknown. In this case, we can define an average gain coefficient, \(\bar{g}\), for use in the uniform gain model. If the functional form is known, we may be able to relate the average and peak gain coefficients.
Figure 2-8

ASE ANGULAR DISTRIBUTION FOR DIFFERENT PEAK $g_o L$

ONE INTEGRATION
$E(\theta) = I(\theta, N = 1)$
$L/D = 20$

RELATIVE INTEGRATED ASE RADIANT INTENSITY $E(\theta)$

AZIMUTHAL ANGLE $\theta$

$g_o L$

6

5

4

3

2.5

2

1.5

1

0° 5° 10° 15° 20°
Figure 2-9

ASE TO FLUORESCENCE RATIO vs. PEAK $g_0L$

$N = 0, 1, 2, 3$ INTEGRATIONS

RATIO ASE TO FLUORESCENCE $\frac{I(0, g_0, N)}{I(\pi/2, g_0, N)}$

PEAK GAIN-LENGTH $g_0L$
To determine $\bar{g}$ in the uniform gain model, we require that $E(\Theta)$ have the same functional form as $I(\Theta)$ in the limit $\bar{g} \to 0$. We may do this by equating the ASE to fluorescence ratios in the two models. We require that
\[
\lim_{g_0 \to 0} \frac{E(0, g_0)}{E(\pi/2, g_0)} = \frac{I(0, \bar{g})}{I(\pi/2, \bar{g})}.
\] (2-20)

Using equations (2-7), (2-8), and (2-16) we obtain the following equations for average intensity on axis
\[
I(0, \bar{g}) = \frac{K_0 D}{\bar{g}} \sum_{n=1}^{\infty} \frac{(gL)^n}{n!}
\] (2-21)
and for the fluorescence
\[
I(\pi/2, \bar{g}) \approx K_0 DL, \bar{g}D \ll 1.
\] (2-22)

Similarly from equations (2-18) and (2-19) we obtain equations for the energy on axis
\[
E(0, g_0) = \frac{K_0 D}{g_0} \sum_{n=2}^{\infty} \frac{(g_0 L)^{n-1}}{n!}
\] (2-23)
and for the fluorescence
\[
E(\pi/2, g_0) \approx K_0 DL/2, g_0 D \ll 1.
\] (2-24)

We may determine $\bar{g}$ in terms of $g_0$ by equating the ratios and expanding the series
\[
\frac{2}{g_0 L} \sum_{n=2}^{\infty} \frac{(g_0 L)^{n-1}}{n!} = \frac{1}{\bar{g}L} \sum_{n=1}^{\infty} \frac{(\bar{g}L)^n}{n!}.
\] (2-25)
The average gain-length product may be expanded in terms of the peak product \( g_o L \) and substituted into (2-25) to obtain

\[
\bar{g}L = \sum_{m=1}^{\infty} a_m (g_o L)^m.
\]  

(2-26)

The first few terms of the expansion of (2-26) are

\[
gL = 0.667 (g_o L) + 0.0185 (g_o L)^2 + 0.00041 (g_o L)^3 + \ldots.
\]

In the limit as \( g_o \to 0 \) we find that \( \bar{g} = 2/3g_o \).

This result indicates that the use of an average gain in the uniform gain model is a good approximation to the integrated measurement at low gain. This can be seen more readily by examining the energy ASE to fluorescence ratio \( E(0, g_o) / E(\pi/2, g_o) \) plotted versus \( \bar{g}L \) with \( \bar{g} = 2/3g_o \) in figure 2-10. We see that the fit to the homogeneous model is good for \( \bar{g}L < 2 \), but that the energy ratio increases more rapidly than predicted by the uniform gain model at higher gains when using the average gain of \( 2/3g_o \). This is equivalent to the average gain increasing more rapidly than the peak gain, as can be seen from the first few terms of the expansion of \( \bar{g}L \) in terms of \( g_o L \). The angular distribution thus fits the uniform gain model using an average gain coefficient for small gain, but becomes more directional than predicted as the gain is increased.

The consequence of using the average gain approximation in an integrated measurement is that the peak gain coefficient, \( g_o \), will be overestimated by a directional ASE measurement at large gains if the peak gain is assumed to be \( g_o \approx 3/2\bar{g} \). The peak gain will be
Figure 2-10

ASE TO FLUORESCENCE RATIO vs. AVERAGE $\bar{g}_L$

$\bar{g}_L = 2/3 \ g_0 L$

$N = 0, 1, 2, 3$ INTEGRATIONS
underestimated, however, if the average gain measured is assumed to be the peak gain. This places boundaries on the estimated value of \(g_o\) when using the average gain approximation.

2.6.4 Gain Coefficient by Length Variation

An average gain coefficient can also be determined by using the expression derived from the uniform gain model (2-10) and the values of ASE energy on axis at length \(L\) and \(\frac{L}{2}\); this technique was used in the dye amplifier experiment. We will define this average gain coefficient as

\[
\bar{g} = \frac{2}{L} \ln \left[ \frac{E(0, g_o, L)}{E(0, g_o, L/2)} - 1 \right].
\]  (2-27)

This expression for \(\bar{g}\) may be evaluated using equation (2-18).

The ratio of ASE energy to fluorescence

\[
\frac{E(0, g_o, L)}{E(\pi/2, g_o, L)}
\]

is plotted versus \(\bar{g} L\) in figure 2-11. Notice that the uniform gain model \((N=0)\), using \(\bar{g} L\) as determined from the length variation measurement, predicts a larger energy ratio than would actually be observed in the experiment \((N=1)\). This is equivalent to saying that the average gain as defined by length variation will be underestimated by the uniform gain model and a measurement of the ASE to fluorescence ratio.
Figure 2-11

ASE TO FLUORESCENCE RATIO vs. AVERAGE $\bar{g}L$
(deduced by length variation)

$N = 0, 1, 2, 3$ INTEGRATIONS
2.7 Multiple Integrations Over Gain Profile

In some experiments it may be necessary to integrate the ASE over more than one variable, e.g. time, frequency, position, polarization, etc. Because both the gain and fluorescence may be functions of any or all of these variables, we must consider the change in the directional behavior of ASE when making integrated measurements. We will then ascertain how these changes alter the use of the directional behavior as a gain diagnostic.

2.7.1 Analysis

As before, we will assume the gain coefficient, \( g \), to be proportional to the fluorescence, \( K \); this requires the ground state laser level population to be negligible. Both gain and fluorescence are approximated by linear functions in \( N \) variables \( t_i; i=1, N \)

\[
g = g(t_1, t_2, ..., t_N) = \begin{cases} g_0 t_1 t_2 ... t_N, & 0 < t_i < 1 \\ 0, & t > 1 \end{cases} \tag{2-28}
\]

\[
K = K(t_1, t_2, ..., t_N) = \begin{cases} K_0 t_1 t_2 ... t_N, & 0 < t_i < 1 \\ 0, & t > 1 \end{cases}
\]

The radiant intensity, \( I(\Theta, t_1...t_N) \), is obtained in a similar manner to equation (2-13). Using the dimensionless variable \( x_0 = g_0 L / \cos \Theta \), we obtain for \( \tan \Theta \leq D/L \)

\[
I(\Theta, t_1...t_N) = \frac{K_0 L^2}{x_0 \cos \Theta} \left[ 2 \sin \Theta \sum_{n=2}^{\infty} \frac{(x_0 t_1 ... t_N)^{n-1}}{n!} \right]
\]

\[
+ \left( \frac{D}{L} \cos \Theta - \sin \Theta \right) \sum_{n=2}^{\infty} \frac{(x_0 t_1 ... t_N)^{n-1}}{(n-1)!} \right].
\]
The radiant intensity integrated over all the variables is

\[ I(\Theta, N) = \int_0^1 \cdots \int_0^1 I(\Theta, t_1, \ldots, t_N) dt_1 \cdots dt_N. \]  

(Equation 2-30)

Evaluating (2-30) we obtain

\[ I(\Theta, N) = \frac{K_{oL}}{g_o} \left[ 2 \sin\Theta \sum_{n=2}^{\infty} \frac{x_o^{n-1}}{nN!} + \left( \frac{D}{L} \cos\Theta - \sin\Theta \right) \sum_{n=2}^{\infty} \frac{x_o^{n-1}}{nN-1!} \right] \]  

(2-31)

for \( \tan\Theta \leq D/L \).

By symmetry for \( \tan\Theta > D/L \), using \( x_o = \frac{g_o D}{\sin\Theta} \), we obtain

\[ I(\Theta, N) = \frac{K_{oD}}{g_o} \left[ 2 \cos\Theta \sum_{n=2}^{\infty} \frac{x_o^{n-1}}{nN!} + \left( \frac{L}{D} \sin\Theta - \cos\Theta \right) \sum_{n=2}^{\infty} \frac{x_o^{n-1}}{nN-1!} \right]. \]  

(2-32)

2.7.2 Directional Behavior

The integrated intensity ratio of axial ASE to fluorescence, \( I(0, N)/I(\pi/2, N) \), is plotted for \( N=0,1,2,3 \) as a function of the peak gain-length product, \( g_oL \), in figure 2-9. The uniform gain model is represented by \( N=0 \), while \( N=3 \) represents integrations over three gain and fluorescence parameters, e.g. time, frequency, and space. Each integration at a given \( g_o \) reduces the ASE ratio and the overall directionality. One consequence is that the minimum detectable peak gain coefficient, \( g_o \), increases with each integration process. For example, using a threshold ratio of two, the minimum \( g_o \) increases from roughly 1.3 to 3.5 with three integrations. A second consequence is that the uniform gain model will underestimate the peak gain \( g_o \) more severely with each integrated measurement.
2.7.3 Average Gain Coefficient Using Uniform Gain Model

We may also interpret these results by comparing them to the uniform gain model results when using an average gain coefficient, \( \bar{g} \). The functional form of the integrated intensity is required to match the uniform gain model in the limit \( g_o \to 0 \). This condition is satisfied if the intensity ratios are equal

\[
\lim_{g_o \to 0} \frac{I(\theta=0,g_o,N)}{I(\theta=\pi/2,g_o,N)} = \frac{I(\theta=0,\bar{g})}{I(\theta=\pi/2,\bar{g})}.
\] (2-33)

Substituting the integrated intensities from (2-31) and (2-32) and assuming \( g_o \ll 1 \) we obtain the ratio in terms of a series expansion

\[
\frac{I(\theta=0,g_o,N)}{I(\theta=\pi/2,g_o,N)} = \frac{2^N}{g_o^L} \sum_{n=2}^{\infty} \frac{(g_o L)^{n-1}}{n^{N-1} n!}.
\] (2-34)

Using equations (2-21), (2-22), and (2-34) and equating series

\[
\frac{2^N}{g_o^L} \sum_{n=2}^{\infty} \frac{(g_o L)^{n-1}}{n^{N-1} n!} = \frac{1}{\bar{g} L} \sum_{n=1}^{\infty} \frac{(\bar{g} L)^n}{n!}.
\] (2-35)

In the limit \( g_o \to 0 \) we find that \( \bar{g} \approx (2/3)^N g_o \).

\[
(2-36)
\]

Each integration over a linearly varying gain and fluorescence profile thus reduces the average gain, \( \bar{g} \), to a value two-thirds that of the peak gain for gains near threshold.

The axial ASE to fluorescence ratio may be plotted versus \( \bar{g} L \) instead of \( g_o L \). This family of curves is shown in figure 2-10 for \( N=0,1,2,3 \). For \( \bar{g} L < 1 \) the deviation from the uniform gain model is less than \( \sim 10\% \); however, for \( \bar{g} L > 1 \) the ASE to fluorescence ratio
increases more rapidly with increasing $\bar{g}_L$ or $K$ ($\bar{g}_L \approx g_o \approx K$) than for the uniform gain model. The deviation also increases with the number of integrations, $N$.

2.7.4 Angular Distribution

The angular distribution for $L/D=20$ is shown for $N=0,1,2,3$ in figures (2-6), (2-8), (2-12), and (2-13) respectively. The integrated intensity is plotted on a relative logarithmic scale versus azimuthal angle $\Theta$ for a range of $g_o L$ with the fluorescence $K=\bar{g}_o L$. These distribution curves are very similar for a given axial ASE to fluorescence ratio and different $N$. The important difference between each set of curves is that this ratio increases more rapidly with $g_o L$ for $N+1$ than for $N$ at a given ASE ratio. This difference becomes more pronounced for $\bar{g}_L>1$ than for $\bar{g}_L<1$ where the curves are almost identical for different $N$.

The difference in the family of curves for different $N$ is of prime importance in fitting experimental data over a wide range of $g_o L$ in which several integrations have been made, e.g. in the laser-pumped dye experiment.

2.7.5 Gain Coefficient by Length Variation

This difference also appears in comparing the average gain coefficient determined from length variation and the uniform gain model to the axial ASE to fluorescence ratio. We may define an average gain coefficient in the same manner as (2-27)
ASE ANGULAR DISTRIBUTION
FOR DIFFERENT PEAK $g_0L$
TWO INTEGRATIONS
$I(\theta, N = 2) \quad L/D = 20$

RELATIVE INTEGRATED ASE RADIANT INTENSITY $I(\theta, N = 2)$

AZIMUTHAL ANGLE $\theta$
Figure 2-13

ASE ANGULAR DISTRIBUTION
FOR DIFFERENT PEAK $g_0L$
THREE INTEGRATIONS
$I(\theta, N = 3), L/D = 20$

RELATIVE INTEGRATED ASE RADIANT INTENSITY $I(\theta, N = 3)$

AZIMUTHAL ANGLE $\theta$
In figure 2-11 we have plotted the ASE to fluorescence ratio $I(0,N,g_0,L)$ versus $gL$ rather than $g_oL$ for $N=0,1,2,3$. The agreement between the integrated curves, $N=1,2,3$, and the uniform gain model, $N=0$, is quite good; however, the integrated ratio becomes progressively smaller than predicted by the uniform gain model as either $gL$ or $N$ is increased. This difference is important principally in the comparison of data in the dye laser experiment.

2.8 Linear Transverse Gain and Fluorescence Profile

2.8.1 Introduction

The gain coefficient and fluorescence are not constant in some laser amplifiers. In particular, a linear transverse gain and fluorescence profile is a good approximation in the transversely laser pumped R6G dye amplifier experiment. At low pump power such that bleaching isn't significant, absorption of the pump pulse in the dye will produce an exponentially decaying pump along the axis of the pump pulse. The gain is approximately proportional to the fluorescence in R6G dye due to the wavelength offset of the absorption and emission bands. Thus the transverse gain and fluorescence profiles are both exponentially decaying functions and can be approximated by a linear function.

Laser produced plasma x-ray amplification experiments are also expected to have non-uniform transverse gain and fluorescence

\[ g = \frac{2}{L} \ln \left[ \frac{I(0,N,g_0,L)}{I(0,N,g_0,L/2) - 1} \right]. \quad (2-37) \]
profiles, although not necessarily linear. A laser-produced plasma is created by focusing a pulse from a high peak power laser, e.g., 50GW Nd^3+:glass laser system into a 100\mu diameter area on a solid target material. The plasma electrons are accelerated by the optical electric field and collisionally transfer their energy to the ions in the plasma. Energy deposition occurs predominantly at the point of critical electron density (10^{21}\text{cm}^{-3} for 1.06\mu light) where the plasma frequency is resonant with the optical field. The electron density is typically an exponentially decaying function of displacement from the target material.

A population inversion between two electronic states of an ion may be produced by electron collisional excitation or recombination, for example. The population inversion (and thus gain) might be expected to follow the electron density over some distance from the target where the density and temperature are suitable. Thus the gain is likely to be an exponentially decaying function of displacement normal to and away from the target.

A line-focus laser produced plasma would be used in an x-ray amplifier experiment, and gain (if present) would have a non-uniform profile transverse to the longitudinal plasma axis. The linear transverse gain and fluorescence profile approximation allows an estimate of what effect the non-uniform gain will have on the directional behavior of ASE and its use as a gain diagnostic.
The similarity between the expected transverse gain profile in the dye amplifier experiment and the electron density profile in a laser-produced plasma is shown in figure 1-2. In the dye amplifier experiment the gain profile is controlled by the dye concentration.

2.8.2 Linear Transverse Gain Model

We now consider the linear transverse gain model for ASE from a laser amplifier. Conforming to the canonical model, the amplifier is assumed to be rectangular with length $L$ along the $y$-axis and width $D$ along the $x$-axis. The direction vector $\mathbf{s}$ is at an angle $\theta$ with respect to the $y$ axis, as shown in figure 2-14. The gain coefficient, $g(x)$, and the fluorescence rate, $K(x)$, are linear functions of the transverse coordinate $x$. In general, we let the profiles be represented by $f(x)$:

$$g(x) = g_m f(x)$$
$$K(x) = K_m f(x)$$

where $g_m$ is the median gain coefficient and $K_m$ is the median fluorescence rate. Specifically, $f(x)$ is a straight line of slope $m$ such that

$$\frac{1}{D} \int_{0}^{D} f(x) \, dx = 1.$$ 

(2-39)

A suitable function is

$$f(x) = m(x - D/2) + 1$$

$$0 \leq x \leq D; \quad |m| \leq 2/D.$$ 

(2-40)
Figure 2-14

COORDINATE SYSTEM FOR AMPLIFIER WITH LINEAR TRANSVERSE GAIN PROFILE
Figure 2-15 shows this profile for \( m=0 \), the uniform gain case; and for \( m=\pm 2/D \), the most extreme linear profiles.

In Appendix 1 we prove a theorem which shows that the emission from the above amplifier is symmetric about the longitudinal axis because the gain coefficient is taken to be proportional to the fluorescence. Appendix 2 presents the analysis for the integration of the radiation transport equation (2-2).

2.8.3 Results

The angular distribution is shown in figure 2-16 for a range of peak gain-length products, \( g_0L \), where \( g_0 = 2g_m \). We have set \( L/D = 15 \), \( m = 2/D \) (gain is zero at one side), and \( K_m \propto g_m \). The radiant intensity \( I(\Theta) \) is plotted on a relative log scale versus the azimuthal angle \( \Theta \).

The ratio of on axis ASE to fluorescence is identical to the energy ratio calculated for a spatially uniform gain profile where the integration is over a triangular temporal pulse shape. (section 2.6.2) In each case the gain is uniform when propagating along the amplifier axis; therefore a transverse spatial or temporal integration give the same result. We may then define an average gain coefficient

\[
\bar{g} = \frac{2}{3}g_0 = \frac{4}{3}g_m, \tag{2-41}
\]

as was done for the temporal integration (2-26), such that the uniform gain model predicts similar results at low gain.
Figure 2-15

LINEAR PROFILE FUNCTION $f(x)$

- $m = 2/D$
- $m = 0$
- $m = -2/D$

Axes:
- $x$-axis: $D/2$ to $D$
- $y$-axis: 0 to 2
Figure 2-16

ASE ANGULAR DISTRIBUTION
FOR DIFFERENT PEAK \( g_o L \)

LINEAR GAIN AND
FLUORESCENCE PROFILE
\( m = 2/D, L/D = 15, L/D = 20 \)

RELATIVE ASE RADIANT INTENSITY \( I(\theta) \)

AZIMUTHAL ANGLE \( \theta \)
The angular distributions are not identical at intermediate angles, however, because the gain is spatially non-uniform along the direction of propagation. The distribution is narrower for this spatially integrated profile because the effective amplifier width, the region for which the gain and fluorescence is significant, is less than the full width D. We may estimate the effective or average width \( \bar{D} \) by comparing the distribution to that for the uniform gain model at large angles, \( \tan \theta >> D/L \). For the uniform gain model from (2-8)

\[
I(\theta) \approx \frac{KL}{g} \sin \theta \left( e^{\bar{D}/\sin \theta} - 1 \right)
\]  

(2-42)

and for the linear gain model from (A-19)

\[
I(\theta) \approx \frac{K_m L}{g_m} \sin \theta \left( e^{D/\sin \theta} - 1 \right).
\]

(2-43)

Using \( g \approx 4/3g_m \) and \( K\bar{D} = K_mD \) to equalize the fluorescence, we obtain for the effective width

\[
\bar{D} \approx 3/4D.
\]

(2-44)

Using this effective width, \( L/\bar{D} = 20 \) for \( L/D = 15 \) in figure 2-16.

The fit of the uniform gain model to the linear gain case is good near threshold. However, the spatially integrated distribution is more cusped at small angles, \( \theta < D/L \), than for uniform gain - this results in a slightly smaller beam divergence above threshold than predicted by the uniform gain model.
For the low gain regime in which we are predominantly interested, the uniform gain model satisfactorily predicts the angular distribution from an amplifier with linear transverse gain coefficient and fluorescence profiles provided an average gain coefficient, \( \bar{g} \), and width \( \sigma \) are used.
3. Laser-Pumped R6G Dye Amplifier Experiment

3.1 Purpose of Experiment

The laser-pumped dye amplifier experiment was designed to simulate an optically freestanding ASE source or amplifier with a non-uniform transverse gain profile. Our objective was to measure the ASE angular distribution and to compare the observed dependence of the distribution on the gain coefficient and the amplifier geometry to that predicted by the ASE theory. This would demonstrate the viability and practicality of an angular distribution measurement as a gain diagnostic.

3.2 Experimental Set-Up

The experimental set-up for the dye amplifier ASE experiment is shown schematically in figure 3-1. R6G dye in an index matched quartz dye cell was optically pumped by a 15 nsec. pulse from a frequency doubled, Nd\(^{3+}\):Yag oscillator and glass laser amplifier system. The angular distribution of ASE from the dye was measured at discrete angles using silicon photodiodes, and continuously about the longitudinal axis using Tri-X film. The pumped length of the dye cell was varied using an aperture, thus permitting a gain measurement independent of the angular distribution measurements.

Scale drawings of two experimental table configurations are shown in figures 3-2 and 3-3. The entire table was covered by a flat black light-tight box to block both amplifier flashlamp light and room light. Two 45\(^{0}\) dielectric mirrors with \(R_{\text{max}}\) at 532nm were
ASE LASER PUMPED DYE EXPERIMENT

Figure 3-1

Nd\textsuperscript{3+}: Nd\textsuperscript{3+: YAG}
SINGLE MODE Q-SWITCHED OSCILLATOR

Nd\textsuperscript{3+}: GLASS
AMPLIFIER

APERTURE

20 nsec

15 nsec

FREQUENCY DOUBLING CRYSTAL

CDA

PD

FLUORESCENCE

FILM

AXIAL ASE

DYE CELL

CRYSTAL

CYLINDRICAL NEGATIVE LENS

APERTURE

APERTURING AMPLIFIER

Nd\textsuperscript{3+}: GLASS

PD

PD

ASE LASER PUMPED DYE EXPERIMENT
SET UP DYE AMPLIFIER EXPERIMENT

- Beam Tube
- PD4
- Orange Glass Filter
- Alignment HeNe
- Cell
- Aperture
- Orange Glass Filter
- Negative Lens
- Cylindrical Lens
- 532 nm Dielectric Mirrors
- Light-Tight Box
Figure 3-3

SET UP DYE AMPLIFIER EXPERIMENT

- BEAM TUBE
- PD4 FILTER
- BEAM TUBE
- PD3
- I I I ALIGNMENT HeNe
- PD1 CELL FILTERS
- RF APERTURE
- C3 NEGATIVE LENS
- CYLINDRICAL LENS
- MIRRORS LIGHT-TIGHT BOX
- ENERGY PD E
- HEIDELBERG WEDGE
- 532 nm DIELECTRIC MIRRORS
- 10 cm
used to align the pump beam on the experimental table and into the dye cell. These mirrors also filtered out the unconverted 1.06\(\mu\)m beam.

The pump beam was vertically focused by a positive cylindrical lens (creating a horizontal line focus) and then expanded by a negative spherical lens as shown in figure 3-4. This lens configuration produced a diverging beam with an elliptical cross-section; at the dye cell the horizontal magnification was eight while the vertical magnification was approximately 1.6. The central portion passed through a rectangular aperture to control the distribution of pump light to the dye. Half of the pump beam (away from the detectors) was blocked by the aperture for one-half length amplifier shots.

Pump beam alignment was facilitated by using a coincident beam from a HeNe laser to center the beam through the dye cell and to ensure proper beam focusing. The final alignment was obtained by photographing the beam transmitted through the dye cell on Polaroid 410 film using only the frequency doubled oscillator pulse. This also provided a qualitative estimate of the pump beam uniformity. A quantitative estimate based on 1.06\(\mu\)m beam profile measurements (figure 3-7A) and the horizontal magnification is shown in figure 3-7B. For a typical pump beam profile, approximately 35% non-uniformity obtained.

A Herschel wedge was inserted in the pump beam to reflect approximately 4% of the beam to each photodiode, PDE and PDP. PDE measured the pump beam energy and was calibrated using a Gentec
FOCUSING GEOMETRY

SIDE VIEW

DYE CELL

NEGATIVE LENS

CYLINDRICAL LENS

$F_1 = 17$cm

$F_2 = 2.5$cm

$F_2$

$11.5$cm

$18$cm

$F_3/h_1 = 1.6$

$h_3/h_1 = 8$

Figure 3-4
calorimeter. PDP monitored the pump beam temporal profile. A fraction of this energy (due to beam expansion and obscuration of the beam) pumped the dye.

The R6G fluorescence power was temporally resolved by PD4. PD4 was located perpendicular to the dye cell axis and 28° above the horizontal plane. This orientation, combined with a beam tube and a 560nm cut-off, long-pass orange glass filter, prevented significant pick up of the green pump beam. PD4 was sufficiently distant from the cell (29cm.) for the fluorescence to be isotropic over the collection angle.

The ASE along the dye cell axis was time resolved by PD1 as shown in figure 3-2. A small beamsplitter (1cm.sq.) directed 25% of the axial ASE to PD1. PD alignment was obtained using a HeNe beam centered along the dye cell bore. Pump light was excluded by the use of a beam tube and an orange glass filter.

The temporally integrated angular distribution was measured simultaneously using 4"x5" panachromatic Tri-X film located behind the beamsplitter in a standard film holder. A black cardboard light box connected the film holder and dye cell, preventing stray light from fogging the film.

A second configuration is shown in figure 3-3 in which the ASE was measured along the dye cell axis by PD1 and in the horizontal plane by PD2 and PD3. Orange glass filters: 560nm cut-off, long-pass, were used on the PD's.
3.3 ASE Diagnostics: Measurement and Calibration

3.3.1 Photodiodes

The ASE photodiodes PD1-PD4 each consisted of an HP5082-4207 photodiode and 55mm focal length glass lens mounted in a brass housing. The lens served to magnify the 1mm diameter collection area of the photodiode, thus increasing the output voltage. It also limited the acceptance angle of the detector. Black photographic tape was used to line the PD housing to minimize reflections. Each PD was calibrated relative to PD1 using an expanded and chopped HeNe beam. No absolute calibration was made.

The ASE photodiode signals were recorded using Tektronix 7844 dual beam scopes at 5nsec/cm. and photographed on Polaroid 410 film. These scope traces were digitized on an HP9830 desk top computer to obtain the relative peak power and energy. The relative accuracy in reading the trace was approximately ±5% or ±.5mv, whichever's greater. For the integrated energy, the accuracy was typically ±7% due to bias on the baseline.

Similarly the laser (1.064μ) and pump beam (532nm) power and energy diagnostics consisted of Hewlett-Packard HP5082-4220 or HP 5082-4207 photodiodes mounted in brass housings with opal glass diffusers to spatially integrate the beam.

All PD's were reverse biased by a high speed capacitor, charged by a 90V battery through a 1MΩ resistor. The load resistor was 50Ω for impedance matching to the RG-8 foam cable. Power measurements
were made using a Tektronix 519 for the oscillator and 7844 for the pump beam. System rise time was 2-3nsec. For energy measurement the output was taken across an RC integrator with R=100KΩ and C=0.1μf.

3.3.2 Angular Distribution from Film

The temporally and spectrally integrated angular distribution was recorded on Kodak Tri-X Professional Pan 4"x5" sheet film using the configuration shown in figure 3-2. The film was normal to the longitudinal dye cell axis at an effective mean distance from the dye cell of 20cm. On axis the radiant intensity was averaged over an internal angle $\Delta \Theta \approx 0.007$; this increased to $\Delta \Theta \approx 0.02$ at the edge of the field of view: $\Theta_{FOV} \approx 0.13$ or 7.5°. This corresponds to an external angle (due to refraction at the cell/air interface) $\Theta_{FOV} \approx 11^0$.

Film from a series of shots were developed simultaneously with D76 at 75°F for 7 minutes using 4 gallon tanks with nitrogen burst agitation. Very uniform development was obtained using this technique, enabling one calibration curve to be used for each series.

The film was calibrated by the use of an ND 0.3 step wedge placed along the edge of the film on every other shot - the irradiance on the step wedge was fairly uniform. In addition, the on axis energy was obtained from PD1 readings on each shot. A D-log E calibration curve was generated from densitometer traces of the step-wedge exposures and from the on axis density versus PD1 energy curves.
The angular distribution was obtained from a densitometer scan across the film. This trace was digitized on an HP9830 desk top computer and converted to an intensity versus angle plot using the digitized D-log E curve. Refraction at the glass cell/air interface was corrected to obtain the angle internal to the cell. Amplitude corrections were made for the $\cos^3 \Theta$ decrease in exposure due to film angle and increased distance from the source at an external angle $\Theta$, and for the beamsplitter transmission on axis. The random error in reducing the densitometer data was estimated to be approximately $\pm 5\%$ of the intensity. The development uniformity, based on the densitometer traces of uniformly exposed film, corresponded to approximately 5% variation at the film center and 10% near the edges.

3.4 Laser System Description

The laser system is shown approximately to scale in figure 3-5. It consisted of a pulsed Nd$^{3+}$:Yag oscillator, two Nd$^{3+}$:glass amplifiers, apodizing aperture, frequency doubling CDA crystal, dielectric mirrors, and optical diagnostics.

The Nd$^{3+}$:Yag oscillator was an Apollo unit using a helical flashlamp. We modified it for passive Q-switching using BDN dye by incorporating a dye cell into the laser cavity and by replacing the mirror mounts with more stable Aerotech mounts. The cavity consisted of a flat dielectric mirror with $R_{\text{max}}$ at $1.06\mu$ and a resonant reflector for longitudinal mode control. An adjustable iris aperture was used to select the TEM$^{00}_{\infty}$ spatial mode. The output polarization was horizontal.
Figure 3-5

LASER SYSTEM

532nm MIRRORS

CELL

EXPERIMENTAL TABLE

CDA CRYSTAL

532nm MIRROR

PD3

PD4

APERTURE

AMP 3

2m

AMP 2

BS

PD1

PD2

YAG OSC

1 METER

SCALE

1.06μ DIELECTRIC MIRRORS

DIELECTRIC MIRRORS
The oscillator was operated on a single longitudinal mode in order to produce a pulse with no temporal modulation. (Any modulation would be doubled in the CDA crystal and cause severe gain and fluorescence modulation in the dye amplifier. Another experiment using the laser system required a temporally coherent pulse\(^{(1)}\). Mode selection was achieved by use of a two period resonant reflector, a short (≈30cm) optical cavity for large longitudinal mode spacing, and passive Q-switching. This allowed a sufficient buildup time of the optical intensity for one longitudinal mode to dominate.

The resonant reflector is a Hercher/Watts design,\(^{(2-3)}\) consisting of two 0.125" fused silica optical flats separated by a 1.00" long spacer. The reflection coefficient is an oscillatory function of frequency with calculated periods of \(\Delta \sigma_1 = 1.086 \text{cm}^{-1}\) and \(\Delta \sigma_2 = 0.167 \text{cm}^{-1}\); \(R_{\max} = 0.4\). A small difference in the reflection coefficients of two adjacent modes causes one mode to become dominant.

The oscillator pulse was then split by a 1.06μ dielectric beamsplitter. Half of the pulse went to photodiodes PD1 and PD2 which monitored the temporal pulse shape and energy respectively. The typical oscillator energy was four millijoules (as measured using a Gentec calorimeter) in a 20nsec, approximately Gaussian pulse.

The other half of the pulse was amplified by two \(\text{Nd}^{+3}:\text{glass}\) amplifiers. Each amplifier utilized a two flashlamp head with modified elliptical cavity. The glass rod in Amp 2 was 12mm. diameter, the rod in Amp 3 was 19mm diameter, both 46cm long. The
single pass, small signal gain in each amplifier was adjusted by controlling the flashlamp pump voltage—the maximum small signal gain measured was approximately 80.

An apodized aperture was used between the amplifiers to reduce the beam diameter and divergence and to produce a smooth spatial beam profile (i.e. eliminate Fresnel rings produced by a simple circular aperture). The apertures were photographed on Kodak High Resolution plates; the normalized radial transmission function was $T(r) = \frac{1}{(r/r_0)^n}$, where $r_0 = 0.28 \text{cm}$ and $n = 8$. Each aperture was index matched in water to reduce reflections and prevent beam deviation due to plate wedge.

The 1.06$\mu$m spatial beam profile was measured at the same distance as the crystal using the set-up shown in figure 3-6. Spatially separated multiple reflections from the mirror-beamsplitter stack were recorded on Kodak High Speed Infrared film. Densitometer traces of the beam profile for each reflection and the measured reflection values were used to calibrate the film and determine the radial intensity distribution. The aperture profile used typically produced the radial beam profile shown in figure 3-7A. A hot spot is present in the center. Deviations in the aperture diameter of different batches altered the central spot intensity—an increase in aperture diameter decreased the central intensity.

The amplified pulse was then frequency doubled by an angle tuned CDA crystal, 13mmx13mmx30mm, housed in an index matching cell. The temporal and spatial intensity distributions at 532nm
Figure 3-7

A. NORMALIZED 1.06\mu m BEAM PROFILE

B. NORMALIZED 532 nm BEAM PROFILE ALONG CELL AXIS
are squared with respect to the 1.06\(\mu\) pump because the conversion efficiency is proportional to the 1.06\(\mu\) pump power. This reduced the pulse width to 15 nsec. and narrowed the spatial beam profile. The 532 nm output was vertically polarized.

Laser diagnostic photodiodes PD3 and PD4 are shown in figure 3-5. PD3 monitored the 1.06\(\mu\) input energy to Amp 3 by viewing the reflection off the rod input face. PD4 measured the 1.06\(\mu\) energy transmitted through the CDA crystal which is proportional to the energy output of Amp 3 for small frequency doubling conversion efficiencies. This was used in conjunction with PDE to monitor the conversion efficiency.

3.5 **Dye Cell Design**

The dye cell construction is shown in figure 3-8. There are two components: the quartz dye cell and the glass matching cell. The R6G dye solution was index matched to the quartz cell \(n_D=1.458\) to reduce reflections and suppress parasitic oscillations. Specifically, the objective was to reduce the grazing incidence reflections within the cylindrical bore of the dye cell (such reflections could alter the ASE angular distribution), and to prevent significant feedback from the dye cell windows. The dye cell was externally index matched by liquid contained in the matching cell in order to prevent parasitics due to ring mode reflections off the outside surfaces of the dye cell. The walls of the matching cell were not parallel to reduce feedback from reflections off the sides, the top
DYE CELL DESIGN

DESIGN CONSIDERATIONS:

- **INDEX MATCHING & BAFFLES REDUCE REFLECTIONS & SUPPRESS PARASITIC FEEDBACK**
- **DYE CELL ASPECT RATIO COMPARABLE TO LASER PRODUCED X-RAY PLASMAS**
- **TRANSVERSE GAIN PROFILE ADJUSTABLE**
and the bottom. Baffles made from black rubber prevented feedback along the axis of the dye cell and prevented ring modes due to total internal reflections off the corners of the matching cell.

The quartz dye cell was fabricated by boring a 2mm hole with a diamond drill through the center of a polished quartz block. A piece of 1/16" music wire and 25µ, 15µ, and 9µ grit grinding power were used to grind the walls smooth. The hole was then polished by running four strands of waxed cord dampened with polishing compound back and forth through the hole. Polishing was continued until negligible scattering could be seen off the walls with the cell in air.

A quartz window was optically contacted to one end of the cell and sealed around the outside edge with RTV to prevent the index matching liquid from breaking the optical contact. The other window was temporarily sealed after filling the cell with dye solution by applying RTV around the edge of the window.

3.6 Index Matching Liquids

The index matching liquid had to be a suitable solvent for R6G dye, in addition to matching the index of the cell. The most common solvents for R6G: ethanol ($N_D=1.36$) and water ($N_D=1.34$) have indices that are too low, lower than any glass. Higher index solvents: ethylene glycol ($N_D=1.432$) and benzyl alcohol ($N_D=1.538$) have been successfully used with R6G; these solvents were mixed in a volumetric ratio of 31:10 to match the index of quartz ($N_D=1.458$) for the current experiment.
This solvent is not ideal, however. The ethylene glycol and benzyl alcohol solution exhibits greater wavelength dispersion (due to benzyl alcohol's high dispersion) than quartz. The index of quartz\(^{(5)}\) and the solution index as computed from component data\(^{(6)}\) are shown versus \(\lambda\) in figure 3-9. The difference in dispersion is \(\Delta n = 0.0005\) over the wavelength range, 560nm to 600nm, in which the dye predominantly emits. The index of the dye solution is necessarily too high at short wavelengths and too low at long wavelengths. We will discuss the effect of this condition in section 5.4.1.

Evaporation of the more volatile benzyl alcohol caused a reduction of index with time and the formation of index gradients and striae in the solvent. This difficulty was controlled by the regular replacement of the solvent and the use of a cover on the matching cell.

The solvent's index is also more temperature dependent than quartz; \(-\frac{dn}{dT}\) is 0.0003\(^{\circ}\)C\(^{-1}\) for the solvent as compared to 10\(^{-5}\)\(^{\circ}\)C\(^{-1}\) for quartz. This necessitated conducting each experimental run while the ambient temperature was constant. Possible bulk heating of the dye by the pump pulse and the subsequent lowering of the index is unlikely because low pump energies (10 millijoules maximum) and slow repetition rates (\(\approx\) one shot every five minutes) were used.

Other potential solvents included glycerin \((N_D=1.475)\) mixed with either ethylene glycol or ethanol, and benzyl alcohol mixed with ethanol or methanol. The mixtures using alcohol separated into layers after sitting for a few hours; striae developed due to
INDEX OF REFRACTION vs. $\lambda$ (20°C)

1.462

31:10 ETHYLENE GLYCOL & BENZYL ALCOHOL

QUARTZ

1.460  1.460  1.458  1.456

$\lambda$(nm)

530  550  570  590  610  630
evaporation of the alcohol. The mixtures using glycerin were too viscous and retained striae. An attempt to mix the components ultrasonically caused the glycerin to break down.

3.7 Measurement of the Index of Refraction

The index of refraction of the ethylene glycol and benzyl alcohol mixture was measured using a HeNe (λ=632.8nm) laser and a Hilger Chance Refractometer as shown in figure 3-10. The refractometer consists of two right angle prisms; the specimen is placed in the V-block. The specimen index \( n \) is computed from the angular deviation \( \Theta \) of a collimated beam normally incident on one prism and the prism index \( n_0 \)

\[
n^2 = n_0^2 + \sin \Theta \sqrt{n_0^2 - \sin^2 \Theta}.
\]

The prism index was calibrated with respect to a block of quartz \((N_{632.8} = 1.4570)\) using the relationship

\[
n_0^2 = N^2 + \frac{1}{2} \sin^2 \Theta_N - \sin \Theta_N \sqrt{N^2 - 3/4 \sin^2 \Theta_N}.
\]

we found \( \Theta_N = -0.133 \), \( n_0 = 1.525 \).

The precision in the measurement of the dye solution index with respect to that of quartz may be estimated from

\[
\Delta n \approx \frac{1}{2} \Delta \Theta, \quad \Theta \ll 1.
\]

The precision to which we could measure the angle was \( \Delta \Theta \approx 0.0004 \), corresponding to \( \Delta n \approx 0.0002 \) at 632.8nm.
For an exact index match to quartz at 580nm, the index of the solution at 632.8nm must be set lower than quartz by $\Delta n = 0.0005$. In practice, the solution index was adjusted at the ambient lab temperature (typically $17^\circ C$) by allowing a very slight amount of benzyl alcohol to evaporate from the solution, lowering the index to the target value. The dye cell was filled with a syringe and the quartz window sealed in place with RTV. A visual inspection of the dye cell was made to insure a good index match and the absence of air bubbles. The veracity of the index match was determined by illuminating the cell (thus causing the dye to fluoresce, predominantly in the yellow-green) and looking for either grazing incidence reflections or total external reflections along the cylindrical bore of the dye cell. This technique produced a good index match in the range 570nm to 590nm.

3.8 R6G Dye

R6G is an organic dye with large-bandwidth, homogeneously-broadened absorption and emission spectra; emission occurs at longer wavelengths than absorption. This behavior can be explained by considering a typical energy level diagram (figure 3-11). See also: Schäfer.\(^{(9)}\)

A typical dye molecule has two distinct sets of electronic states: the singlet and triplet states. Each state has discrete vibrational sublevels, which are further split by many closely spaced rotational levels. Collisions and rapid ($\tau \sim 10^{-12}$ sec)
Figure 3-11

ENERGY LEVEL DIAGRAM FOR ORGANIC DYES

S₂

S₁

EXCITATION

Emission

S₀

K⁻¹

Erot

T₁

T₂

ABSORPTION

τs₁

τT₁
changes in the solution state broaden the levels to a near continuum. At room temperature the lowest vibration and rotation levels in the $S_0$ singlet state are populated.

Optical pumping is achieved by absorption from the ground state to upper levels of the $S_1$ singlet state. The lowest vibration-rotation levels of $S_1$ are populated by extremely rapid non-radiative decay of the upper levels. A radiative transition to the higher levels of the ground state can then occur; this shifts the emission spectrum, relative to the absorption spectrum, to longer wavelengths. A population inversion is created between the lowest $S_1$ levels and the upper $S_0$ levels; ground state absorption and other losses may cause net absorption, however.

Nonradiative intersystem crossing from the $S_1$ singlet state to the $T_1$ triplet state competes with fluorescence. The radiative lifetime is $\tau_{S_1}$ and the crossing rate is $K_{ST}$. Assuming only fluorescence and intersystem crossing depopulate the $S_1$ level, the quantum yield for fluorescence is: $\phi = (1 + \tau_{S_1} K_{ST})^{-1}$. For R6G: $\tau_{S_1} \approx 5 \times 10^{-9}$ sec, $K_{ST} \approx 5 \times 10^{-8}$ sec, and $\phi \approx 0.9$ - these values are typical for good laser dyes.

The triplet state lifetime ($\tau_{T_1}$) is typically very long ($\tau_{T_1} \approx 10^{-3}$ sec) because the triplet-singlet radiative transition to the ground state is spin forbidden. For pump pulses longer than $K_{ST}^{-1}$, the inversion can be depleted by intersystem crossing - the population is trapped in the triplet state. Singlet state depletion and triplet-triplet absorption can severely reduce the fluorescence
and net gain. To alleviate this problem, very short pump pulses $(t_p \approx 15 \text{ nsec})$ were used in the dye amplifier experiment.

The absorption and emission spectra obtained by Snavely for $10^{-4}$ molar R6G dissolved in ethanol are given in (10). We measured the absorption spectrum for $1.2 \times 10^{-4}$ molar R6G in ethylene glycol using a Cary 14 spectrophotometer; this is shown in figure 3-12. The peak absorption cross-section is $\sigma_{O1} = 3.4 \times 10^{-16} \text{ cm}^2$ for $\lambda = 535 \text{ nm}$. Similar results were obtained with benzyl alcohol as the solvent. These absorption spectra are very similar to the R6G in ethanol singlet state absorption spectra. We therefore expect the measured emission spectrum of R6G in ethanol to be satisfactory for estimating the behavior of ASE from the dye amplifier.

The radiation transport equation in the dye may be written, adopting the notation of Ganiel, et al. (11)

$$\frac{d}{ds} I(\tilde{s}, t, \lambda) = \left[ N_1(t) \sigma_e(\lambda) - N_0(t) \sigma_{O1}(\lambda) - N_T(t) \sigma_T(\lambda) \right] I(\tilde{s}, t, \lambda) + \tau_{s1}^{-1} E(\lambda) N_1(t)$$

(3-4)

where $I(\tilde{s}, t, \lambda)$ is the spectral radiant intensity. $N_1(t)$, $N_0(t)$, and $N_T(t)$ are the population densities of the upper and lower singlet and triplet states, respectively. Note that the total population density

$$N = N_1 + N_0 + N_T.$$  

(3-5)
Figure 3-12

ABSORPTION SPECTRA R6G IN ETHYLENE GLYCOL
1.2 x 10^{-4} molar

OPTICAL DENSITY (1 mm cell)

\( \lambda \) (nm)

\( \sigma_{01} \times 10^{16} \text{ cm}^2 \)
Similarly, \( \sigma_e(\lambda) \), \( \sigma_{01}(\lambda) \), and \( \sigma_T(\lambda) \) are the stimulated emission cross-section, ground state absorption cross-section, and triplet state absorption cross-section, respectively. \( \mathcal{E}(\lambda) \) is the fluorescence spectrum, normalized so that \( \int \mathcal{E}(\lambda) d\lambda = \phi \), the quantum yield for fluorescence. \( \mathcal{E}(\lambda) \) is related to \( \sigma_e(\lambda) \) by

\[
\sigma_e(\lambda) = \frac{\lambda^4 \mathcal{E}(\lambda)}{8\pi c n^2 \tau_S}\tag{3-6}
\]

where \( n \) is the refractive index of the dye solution.

The fluorescence term \( \tau_S^{-1} \int_{S_1} E(\lambda) N_1(t) \) (corresponding to \( K \) in equation 2-2) is a linear function of the excited state population density \( N_1 \). The wavelength dependence \( E(\lambda) \) was taken from Snively. \( E(\lambda) \) is very approximately triangular in shape, with the peak occurring at about 550nm, and may be approximated by a linear function in \( \lambda \) for \( \lambda > 550 \text{nm} \).

In contrast, the gain coefficient

\[
g(t,\lambda) = N_1(t) \sigma_e(\lambda) - N_0(t) \sigma_{01}(\lambda) - N_T(t) \sigma_T(\lambda) \tag{3-7}
\]

depends on \( N_1(t) \) and \( \lambda \) in a more complicated way. Because the pulse duration is relatively short compared to the triplet state cross-relaxation time, the triplet state population \( N_T \) should be negligible during most of the pulse, and cause significant absorption only near the end of the pulse. This could account for the slight lag (\( \approx 1 \text{ nsec} \)) of the fluorescence peak observed with respect to the ASE peak.

The dominant loss mechanism, however, is ground state absorption at short wavelengths - this increases as \( N_0 \) is increased and/or \( N_1 \)
is decreased, e.g. at low pump levels. Figure 3-13 shows the peak gain coefficient plotted versus \( \lambda \), calculated for \( N=1.1 \times 10^{16} \text{cm}^{-3} \) and several values of \( N_1 \). \( N_T \) has been neglected and the values of \( E(\lambda) \) are taken from Snavely while the values for \( \sigma_{01}(\lambda) \) with \( n=1.46 \) are taken from figure 3-12. The curves scale in \( g, N_1, \) and \( N \). At longer wavelengths (\( \lambda \geq 570\text{nm} \) for the curves shown) \( g \) is approximately proportional to \( N_1 \). At shorter wavelengths absorption rapidly exceeds the gain.

A 560nm cut-off orange glass filter was used to filter out both the pump beam at 532nm and the short wavelength fluorescence. The transmission was less than 0.01 below 540nm, 0.12 at 550nm, 0.59 at 560nm, 0.83 at 570nm, and 0.90 for \( \lambda > 580\text{nm} \). However, absorption was still measured at low pump levels in the dye experiment because the absorption shifts to longer wavelengths at low pump levels, thus occurring within the transmission band of the filter.
Figure 3-13
CALCULATED GAIN COEFFICIENT vs. $\lambda$
R6G DYE — $N = 1.11 \times 10^{16}$
4. Experimental Results

4.1 Introduction

In this chapter we present results from the R6G dye amplifier experiment. Two basic types of data were obtained: 1) temporally resolved fluorescence and ASE data from photodiodes at discrete angles; and 2) the temporally integrated angular distribution within ±8° of the dye cell axis, recorded on Tri-X film. Both were spectrally integrated using a 560nm cut-off, long-pass filter. These data were obtained for two different dye concentrations (and thus for different transverse gain profiles) over a wide range of laser pump power.

The principal objective of the experiment was to measure the ASE angular distribution from the amplifier and to compare these measurements to the ASE model presented in Chapter 2, specifically considering the gain-length product gL and the amplifier aspect ratio L/D. Energy angular distribution data, as recorded on film, are presented for the two series of shots. These ASE profiles are fit to theoretical curves and the resulting values of gL and L/D are compared to independent measurements. The energy gain-length product was independently determined from the temporally integrated photodiode measurements of the axial ASE for two different dye cell lengths.

The photodiode data are presented for both the temporally resolved peak power and for the integrated pulse energy. The peak power data for each shot provides a value for the peak gain coeffi-
cient which is better defined than for energy since the ASE and fluorescence are not averaged over time. In both cases, the measured axial ASE to fluorescence ratio is examined as function of the measured $\bar{g}_L$ and compared to the theoretical dependence in order to test the validity of the measurement as a gain diagnostic. The measured $\bar{g}_L$ is also examined as a function of the fluorescence in order to verify the assumption that the gain coefficient is proportional to the fluorescence.

4.2 Experimental Run: Shots 527-542 and 566-589 – 1.85x10^{-5} Molar R6G

In this series of measurements the R6G dye concentration was 1.85x10^{-5} molar (1.1x10^{16} cm^{-3}), which corresponds to an optical density $OD=0.34$ across the 2mm amplifier diameter. At low pump intensity this corresponds to a 50% drop in the pump intensity across the cell, while at high pump intensity, bleaching of the dye provided a fairly uniform pump intensity across the cell.

4.2.1 Gain Determination from Peak Power Data

To determine the gain independently of the angular distribution measurements, we used the axial ASE data from PD1 for the two amplifier lengths in equation (4-1) or (2-10) where $I_L \equiv I(\theta=0,L)$

$$\bar{g}_L = 2\ln \left(\frac{I_L}{I_{L/2}} - 1\right). \quad (4-1)$$

The value of $I_{L/2}$ corresponding to $I_L$ was obtained using the fluorescence value from PD4 as a reference. The fluorescence for length
L/2 was assumed to be half that at length L for a given gain coefficient and fluorescence rate. (Pump power and energy measurements were not a reliable reference because only the central portion of the pump beam illuminated the dye.) Values of $I_L$ and $I_{L/2}$ intermediate to the data points were obtained by curve fitting and interpolation.

We may estimate the random error $\Delta gL$ in the gain-length product value so obtained by differentiating (4-1) to obtain

$$\Delta gL \equiv \frac{2}{e^{gL/L/2}} \Delta \left( \frac{I_L}{I_{L/2}} \right),$$

(4-2)

where $\Delta (I_L/I_{L/2})$ is the error in the ASE ratio. For a given error $\Delta (I_L/I_{L/2})$, the error $\Delta gL$ is largest at low gain. The random error in either $I_L$ or $I_{L/2}$ was estimated to be $\pm 5\%$, thus we may take $\Delta (I_L/I_{L/2}) \approx 0.07$, the root mean square.

We have plotted the peak power ASE signal from PD1 versus the fluorescence signal from PD4 on a log-log graph for two dye cell pump lengths, $L=2.5\text{cm}$ and $L/2=1.25\text{cm}$, in figures 4-1 and 4-2. The experimental configuration used for shots 527-542 is shown in figure 3-3. ASE versus fluorescence PD data from a second experimental run, shots 566-589 are also plotted for lengths L and L/2. The experimental set-up differed from that for shots 527-542 by allowing the angular distribution to be recorded on film (figure 3-2). The PD data have been scaled to facilitate a comparison with the first data set.

The measured average values of $\bar{g}L$ are plotted versus the peak fluorescence signal in figure 4-3. A straight line is fitted to
Figure 4-1

ASE POWER $I(\theta = 0)$ vs. FLUORESCENCE POWER $I(\theta = \pi/2)$

DYE CELL LENGTH = 2.5 cm

○ SHOTS 527-534
● SHOTS 566-577
Figure 4-2

ASE POWER $I(\theta = 0)$ vs. FLUORESCENCE
POWER $I(\theta = \pi/2)$

DYE CELL LENGTH = 1.25 cm

$ASE I(\theta = 0)$

$\text{FLUORESCENCE } I(\theta = \pi/2)$

○ SHOTS 535-542
● SHOTS 578-589
Figure 4-3

AVERAGE GAIN-LENGTH PRODUCT vs. FLUORESCENCE POWER

SHOTS 527-534 and 566-577

\[ \tilde{g}_L \]

\[ \text{FLUORESCENCE POWER (relative)} \]
Figure 4-4

ASE POWER RATIO $I(\theta = 0)/I(\theta = \pi/2)$ vs. $\bar{g}L$

- SHOTS 527-534
- SHOTS 566-577

UNIFORM GAIN MODEL

$N = 2$  
$L/D = 16$
these points. We find that $g_L$ is approximately a linear function of the fluorescence, although the effects of absorption are apparent at low fluorescence levels. Deviations in $g_L$ of 0.2 from the straight line are present at high gain, slightly more than the estimated error.

In figure 4-4 we have plotted the ratio of the peak ASE to fluorescence, $I(\Theta=0)/I(\Theta=\pi/2)$, versus the measured gain-length product, $g_L$, and compared it to two theoretical curves. One theoretical curve is for two integrations (spectral and spatial), taken from figure 2-11, with the additional assumption that there is amplification in the transverse direction and that the effective aspect ratio is $L/D_{\parallel}=16$ (see section 2.8). The second curve is from the uniform gain model with $L/D_{\parallel}\rightarrow\infty$, figure 2-5, the curve assumed in the absence of other information. The data is consistent with either curve at low gain, due to the error bars, while for $g_L>2$, the data fits the integrated measurement curve better.

4.2.2 Discrete Peak Power Angular Distribution - Shots 527-534

The off axis ASE was also measured at $-6^\circ$ and $-12^\circ$ with respect to the longitudinal axis for shots 527-534 using the set-up shown in figure 3-3. Figure 4-5 shows the peak power at each angle along with a theoretical curve. Each curve was fitted on the basis of the shape of the ASE profile not the fluorescence magnitude, which was previously considered. The resulting value of $g_L$ was determined primarily by the ASE to fluorescence ratio, while the ASE values at
Figure 4-5

ASE POWER ANGULAR DISTRIBUTION
SHOTS 527-534

PEAK ASE POWER

SHOT
528
527
529
530
532
533
534

θ (degrees)
Table 4-1

Measured $\bar{q}_L$ for Shots 527-534

<table>
<thead>
<tr>
<th>Shot</th>
<th>Measured $\bar{q}_L$</th>
<th>Theory $g_L$</th>
<th>L/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>527</td>
<td>2.33</td>
<td>2.4</td>
<td>16</td>
</tr>
<tr>
<td>528</td>
<td>2.60</td>
<td>2.6</td>
<td>16</td>
</tr>
<tr>
<td>529</td>
<td>1.56</td>
<td>1.6</td>
<td>20</td>
</tr>
<tr>
<td>530</td>
<td>0.96</td>
<td>1.1</td>
<td>20</td>
</tr>
<tr>
<td>531</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>532</td>
<td>0.26</td>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>533</td>
<td>0.06</td>
<td>0.1</td>
<td>20</td>
</tr>
<tr>
<td>534</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>
intermediate angles determined the beam divergence and thus the inferred amplifier aspect ratio.

Table 4-1 lists the measured $\tilde{g}_L$ and the $g_L$ and $L/D$ obtained from the ASE profile on each shot. The measured values of $\tilde{g}_L$ and the theoretical fit to the ASE profile compare very well for $g_L>1$ (shots 527-530). An amplifier aspect ratio of $L/D=16$ was inferred from the data for the two high gain shots, while $L/D=20$ fit the intermediate gain shots better.

This pump power dependence of the aspect ratio is not predicted by the ASE model assuming that the gain coefficient is proportional to the fluorescence. However, bleaching of the dye at high pump intensity probably increased the width of the amplifying region to the full dye cell width. At low pump intensities the pump pulse decayed exponentially with displacement transverse to the dye cell axis, which resulted in ground state absorption limiting the width of the amplifying region, thus increasing the aspect ratio.

For $g_L<1$ the curves based on the axial ASE to fluorescence ratios fit the intermediate data poorly. These intermediate points were lower than the fluorescence by up to 10%. Consequently, a higher gain coefficient and aspect ratio may be inferred from this enhanced peaking along the axis when disregarding the fluorescence. Possible explanations for this peaking will be discussed in 5.4.

4.2.3 Energy Data - Shots 566-577

The ASE and fluorescence energies were obtained by integrating the PD power traces from each shot. Figure 4-6 shows the axial ASE
Figure 4-6

ASE ENERGY $E(\theta = 0)$ vs. FLUORESCENCE ENERGY $E(\theta = \pi/2)$

SHOTS 566-589

- SHOTS 566-577  $L = 2.5$ cm
- SHOTS 578-589  $L = 1.25$ cm
Figure 4-8

ASE ENERGY RATIO $E(\theta = 0)/E(\theta = \pi/2)$ vs. $\bar{g}L$

SHOTS 566-577
energy, \( E(\Theta=0) \), plotted versus the fluorescence energy \( E(\Theta=\pi/2) \), for the two dye cell pump lengths. The gain coefficient for each full length shot was computed in the same way as in the peak power measurement. An estimate of the error \( \Delta gL \) was made using \( \Delta(E_L/E_{L/2})=0.1 \).

Figure 4-7 shows \( gL \) plotted versus the fluorescence energy on each shot. A linear curve only approximately fits the data. Absorption is apparent for low fluorescence values, and \( gL \) is increasing somewhat faster than linearly with respect to the fluorescence. Figure 4-8 shows the ASE to fluorescence energy ratio \( E(\Theta=0)/E(\Theta=\pi/2) \) plotted as a function of \( gL \). The theoretical curve for three integrations with amplification in the transverse direction for an aspect ratio of \( L/D^2=16 \) fits the data reasonably well.

4.2.4 Angular Distribution - Shots 566-577

Figure 4-9 shows the ASE profile in the horizontal plane obtained from Tri-X film on each shot as discussed in section 3.3.2. These curves are spatially, spectrally, and temporally integrated. A semi-log scale is used to display the results, compressing the intensity scale, and the positive \( \Theta \) direction is taken toward the pumped side of the dye cell.

The family of curves shows the expected trend of increased directionality as the fluorescence and gain increase. We compare these limited ASE profiles to the uniform gain model to obtain another estimate of \( gL \). In fitting to the data the shape of the ASE profile was considered, rather than the fluorescence or ASE magnitude. The theoretical curves so obtained are shown in figure 4-10.
Figure 4-9

ASE ANGULAR DISTRIBUTION
SHOTS 566-577

RELATIVE ASE RADIANT INTENSITY

θ (degrees)

SHOT
576
577
574
575
573
572
571
570
569
567
566
Figure 4-10

THEORETICAL ANGULAR DISTRIBUTION CURVE FIT

SHOTS 566-577

RELATIVE ASE RADIANT INTENSITY

\[ \theta \text{ (degrees)} \]
Table 4-2

Measured \( \bar{g}_L \) for Shots 566-577

<table>
<thead>
<tr>
<th>Shot</th>
<th>( \bar{g}_L ) Power PD</th>
<th>( \bar{g}_L ) Energy PD</th>
<th>( g_L ) Theory</th>
<th>( L/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>566</td>
<td>-0.14</td>
<td>-0.03</td>
<td>0.2</td>
<td>40</td>
</tr>
<tr>
<td>567</td>
<td>0.13</td>
<td>0.02</td>
<td>0.3</td>
<td>40</td>
</tr>
<tr>
<td>568</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>569</td>
<td>0.36</td>
<td>0.11</td>
<td>0.5</td>
<td>40</td>
</tr>
<tr>
<td>570</td>
<td>0.49</td>
<td>0.15</td>
<td>0.4</td>
<td>40</td>
</tr>
<tr>
<td>571</td>
<td>0.79</td>
<td>0.37</td>
<td>0.8</td>
<td>25</td>
</tr>
<tr>
<td>572</td>
<td>0.94</td>
<td>0.45</td>
<td>0.9</td>
<td>25</td>
</tr>
<tr>
<td>573</td>
<td>2.04</td>
<td>1.52</td>
<td>1.6</td>
<td>20</td>
</tr>
<tr>
<td>574</td>
<td>2.55</td>
<td>1.86</td>
<td>2.0</td>
<td>20</td>
</tr>
<tr>
<td>575</td>
<td>2.15</td>
<td>1.72</td>
<td>1.8</td>
<td>20</td>
</tr>
<tr>
<td>576</td>
<td>3.16</td>
<td>2.55</td>
<td>2.6</td>
<td>16</td>
</tr>
<tr>
<td>577</td>
<td>2.51</td>
<td>2.01</td>
<td>2.0</td>
<td>20</td>
</tr>
</tbody>
</table>
Table 4-2 lists the gain coefficients for each shot obtained from the PD power, PD energy, and ASE profile measurements. The curve fit value of L/D is also listed. At high gain, shots 573-577, the agreement between the PD energy and ASE profile gain coefficients is very good. However, for low average gain shots, 566-572 (gL<1), gL deduced from the beam profile is considerably higher than obtained from PD energy measurements, typically by 0.3 to 0.4. Conversely, the beam profile is peaked by up to 20% more than expected from the energy gain measurements. This peaking was also observed for the power angular distribution measurements, and is discussed further in section 5.4.

The aspect ratio deduced from the high gain shot was approximately L/D≈16. This is the same as the average aspect ratio of the cylindrical dye cell. For the intermediate gain shots a better fit was obtained for L/D≈20 to 25. At low gain L/D≈40 was inferred. This increase in aspect ratio at low gain and fluorescence may have been due to a narrower gain region caused by absorption. At low pump powers, ground state absorption may have exceeded the gain after the pump pulse penetrated some depth into the dye cell.

4.3 Experimental Run: Shots 638-669 - 3.7x10⁻⁵ Molar R6G Dye

In this series of data the R6G dye concentration was 3.7x10⁻⁵ molar (2.3x10¹⁶ cm⁻³), corresponding to an optical density OD≈0.68 across the 2mm amplifier. The high absorption provided for an exponentially decaying pump, transverse to the dye cell axis.
4.3.1 Gain Determination from Peak Power Data

The peak ASE signal from PD1 is plotted versus the fluorescence signal from PD4 on a log-log graph for two dye cell pump lengths, L=2.5cm and L/2 = 1.25cm, in figure 4-11. The experimental configuration used is shown in figure 3-2. Film was used to record the angular distribution about the axis simultaneously with the PD measurements.

The gain was determined using the axial ASE data from PD1 for the two amplifier lengths, as discussed in section 4.2.1. Figure 4-12 shows the measured average values of $\tilde{g}L$ versus the measured fluorescence signal with error bars estimated as in section 4.2.1. The gain coefficient is approximately a linear function of the fluorescence. However, extrapolation indicates that absorption is important at low fluorescence, and that the gain is lower at a given fluorescence value than for the $1.85\times10^{-5}$ molar R6G dye.

Figure 4-13 shows the ratio of the peak ASE, $I(\Theta=0)$, to fluorescence, $I(\Theta=\pi/2)$, plotted versus the measured gain-length product for each shot. Also shown are the theoretical curve for the uniform gain model (figure 2-5) and the curve for two integrations (spectral and spatial), taken from figure 2-11, with the additional assumption of amplification in the transverse direction for an aspect ratio $L/D=20$. The data best fits the curve for integrated measurements, although at low gain the measured ratio $I_A/I_F$ is lower than predicted. This is likely due to the presence of absorption over part of the spectrum at low pump levels as discussed in 3.8. Absorption is not included in the model.
Figure 4-11

ASE POWER $I(\theta = 0)$ vs. FLUORESCENCE POWER $I(\theta = \pi/2)$

SHOTS 638-669

- SHOTS 638-655 $L = 2.5$
- SHOTS 656-669 $L = 1.25$ cm

FLUORESCENCE $I(\theta = \pi/2)$ vs. ASE $I(\theta = 0)$
Figure 4-12

AVERAGE GAIN-LENGTH PRODUCT VS. FLUORESCENCE POWER

SHOTS 638-655

FLUORESCENCE POWER (relative)

\( \bar{g}_L \)
Figure 4-13

ASE POWER RATIO $I(\theta = 0)/I(\theta = \pi/2)$ vs. $\bar{g}L$

SHOTS 638-655

ASE POWER RATIO $I(\theta = 0)/I(\theta = \pi/2)$

$N = 2$
$L/D = 20$
4.3.2 Energy Data - Shots 638-669

The ASE energy, $E(\Theta=0)$, is plotted versus the fluorescence energy, $E(\Theta=\pi/2)$, for the two pump lengths in figure 4-14. An average $ar{g}L$ was determined as before, and is plotted versus the fluorescence energy in figure 4-15. The data approximately fit a straight line, with absorption inferred at low fluorescence values as before. At a given fluorescence value, the gain is lower than for the $1.85 \times 10^{-5}$ molar dye concentration. This was probably caused by the larger ground state population in the $3.7 \times 10^{-5}$ molar dye solution.

Figure 4-16 shows the ASE to fluorescence energy ratio, $E(\Theta=0)/E(\Theta=\pi/2)$, plotted versus $\bar{g}L$ for both the data and two theoretical curves. The data is reasonably consistent with the curve for three integrations (spectral, spatial, temporal) at high gain. However, at low gain the measured ratio was considerably lower than the theoretical curve. This deviation is probably due to absorption at low pump levels near the beginning and end of the pump pulse. The integrated axial ASE signal would thus be lower than the model predicts.

4.3.3 Angular Distribution - Shots 640-655

The angular distribution in the horizontal plane for each shot is shown in figure 4-17; the positive $\Theta$ axis is toward the pumped side of the dye cell. A semi-log scale is used, compressing the intensity range. These curves are spatially, spectrally, and tempor-
Figure 4-14

ASE ENERGY $E(\theta = 0)$ vs. FLUORESCENCE ENERGY $E(\theta = \pi/2)$

SHOTS 638-669

- SHOTS 638-655  $L = 2.5 \text{ cm}$
- SHOTS 656-669  $L = 1.25 \text{ cm}$
Figure 4-15
AVERAGE GAIN-LENGTH PRODUCT vs. FLUORESCENCE ENERGY
SHOTS 638-655
Figure 4-16

ASE ENERGY RATIO $E(\theta = 0)/E(\theta = \pi/2)$ vs. $\bar{g}L$

SHOTS 638-655
Figure 4-17

ASE ANGULAR DISTRIBUTION
SHOTS 640-655

RELATIVE ASE RADIANT INTENSITY

\[ \theta \text{ (degrees)} \]
THEORETICAL ANGULAR DISTRIBUTION CURVE FIT
SHOTS 640-655

RELATIVE ASE RADIANT INTENSITY

SHOT
655
653
652
651, 644
645
647, 648, 649
643
641
640

θ (degrees)
Table 4-3

Measured $\bar{g}_L$ for Shots 640-655

<table>
<thead>
<tr>
<th>Shot #</th>
<th>$\bar{g}_L$ Power PD</th>
<th>$\bar{g}_L$ Energy PD</th>
<th>Theory $g_L$</th>
<th>L/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>640</td>
<td>0.18</td>
<td>0.06</td>
<td>0.3</td>
<td>40</td>
</tr>
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<td>641</td>
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<td>-</td>
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<td>0.9</td>
<td>25</td>
</tr>
<tr>
<td>644</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
<td>25</td>
</tr>
<tr>
<td>645</td>
<td>1.90</td>
<td>1.29</td>
<td>1.4</td>
<td>25</td>
</tr>
<tr>
<td>646</td>
<td>1.02</td>
<td>0.69</td>
<td>0.9</td>
<td>25</td>
</tr>
<tr>
<td>647</td>
<td>1.45</td>
<td>0.97</td>
<td>1.2</td>
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</tr>
<tr>
<td>648</td>
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<td>1.08</td>
<td>1.2</td>
<td>25</td>
</tr>
<tr>
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<td>0.97</td>
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<td>25</td>
</tr>
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<td>2.3</td>
<td>20</td>
</tr>
<tr>
<td>655</td>
<td>3.67</td>
<td>2.73</td>
<td>2.9</td>
<td>20</td>
</tr>
</tbody>
</table>
ally integrated and were obtained from Tri-X film as described in section 3.3.2.

We may obtain an estimate of the spread in the data by considering the three groupings of shots with overlapping profiles; 647, 648, and 649; 644 and 651; and 653 and 654. These variations are approximately 10%, and are consistent with the estimated error in obtaining and reducing the data.

The family of curves shows the expected trend of increased directionality as the fluorescence and gain increase. We may compare the limited angular ASE profile obtained on each shot to the theory to obtain another estimate of the average gain coefficient. Using the uniform gain model and an appropriate aspect ratio, the theoretical curves were fitted to the data to obtain gL. These curves are shown in figure 4-18. Note that the fluorescence values obtained were only approximately proportional to gL.

The values of gL obtained from the PD power, PD energy, and ASE profile measurements are collected in Table 4-3. The energy gain coefficient values are fairly consistent with those from the beam profile, allowing for experimental error. However, the gain coefficient from the beam profile is typically larger by approximately 0.2. Conversely, the beam profile is approximately 0-15% more directional than that deduced from the energy gain coefficient. A similar result was obtained for the 1.85x10^{-5} molar RG solution.
4.4 Polarization Measurement

The emission from the dye was partially polarized. A peak power, spectrally integrated measurement with PD's and polarizers and a 6.2x10^{-5} molar dye solution indicated that the vertical polarization was approximately 60% stronger than the horizontal component. This is consistent with the vertically polarized, transverse pump used in the experiment. The dye transition moments in absorption and emission are parallel, thus the emission polarization should be principally vertical. Rotational relaxation-diffusion of the molecules before emission tends to depolarize the emission. (See Schäfer\(^1\)) This relaxation time was apparently comparable to the fluorescence lifetime due to the viscous solvent used: ethylene glycol and benzyl alcohol. Consequently both the emission and gain were polarization dependent; the measurements were integrated over polarization in addition to wavelength, space, and time.
5. Discussion of Experimental Results and Theory

5.1 Integrated Measurements

The ASE measurements discussed in the previous sections were integrated over wavelength, transverse spatial profile, time, and polarization. Average gain coefficients were determined which fit even the uniform gain model approximately. This is understandable, considering that we found that the uniform gain model with an average gain coefficient reasonably approximated the integrated theory at low gain (section 2.5). In fact, it is experimentally difficult to distinguish between the two models at low gain based only on the functional form of the data.

The ASE measurements were also integrated over a circular cylinder, rather than the two-dimensional rectangle assumed in the model. Consequently, the cylinder must be characterized by a mean width, D, which may be approximately related to the diameter d by

\[ D = \alpha \, d. \]

A reasonable estimate for \( \alpha \) is \( \alpha \approx 0.8 \). This value is slightly more than the weighting based on equal perimeters of a circle and square. For the dye cell with \( L/d = 12.5 \), this implies \( L/D \approx 16 \). A similar aspect ratio was measured at high gain at the lower dye concentration (section 4.2).
5.2 Absorption

Absorption was not included in the ASE models discussed in Chapter 2. Its presence can alter the angular distribution in measurements integrated over both gain and absorption. In particular it may account for the ASE to fluorescence ratio being lower than predicted with no absorption at low gain in shots 638-655, and in part for the larger effective aspect ratios observed at low gain in all the runs.

At low fluorescence levels absorption may be inferred from the gain coefficient versus fluorescence measurements shown in figures 4-3, 4-7, 4-12, and 4-15. The absorption occurred at a lower fluorescence level for the lower dye concentration. Absorption may also be inferred from the temporally resolved ASE and fluorescence traces. The ASE peak was observed to occur slightly earlier (≈1 nsec) than the fluorescence peak and then to fall off faster. A similar hysteresis was observed by Babenko, et al. in polymethine dye solutions and was attributed to triplet-state absorption. \(^1\)

Ground state absorption in R6G at short wavelengths (\(\lambda < 570\text{nm}\)) is significant at low pump power and fluorescence levels, as discussed in section 3.8. This absorption, not considered in the directional ASE model, preferentially reduces the axial ASE with respect to the transverse fluorescence, due to the longer path length.

The ASE measurements were integrated over a range of absorption and gain values, however, with more weight given to the high gain
contributions in the determination of the average gain. This resulted in a lower ASE to fluorescence ratio for a given average gain coefficient than predicted by the integrated model with no absorption. The effect was most dominant at low gain in shots 638-655 (figures 4-13 and 4-16) because the dye concentration, and thus ground state absorption, was higher. Similarly, the ratio was lower for the energy measurements than for peak power because the energy measurement integrated over the lower fluorescence and higher absorption portions of the pulse.

5.3 Beam Divergence and Amplifier Aspect Ratio

The beam divergence and amplifier aspect ratio correspond quite well for both dye concentrations at high gain. For the low dye concentration this was $L/D = 16$ (figure 4-10), and for the high concentration $L/D = 20$ (figure 4-18). A smaller beam divergence, and thus larger aspect ratio, for the high concentration data is consistent with the smaller transverse absorption depth in the dye at the pump wavelength. From the linear gain profile model (section 2.8), the effective width $\overline{D} = 3/4 D$ (2-44) where $D$ is the total width. This is consistent with the above ratios.

At low gain and fluorescence values in figures 4-9 and 4-17, the aspect ratio corresponding to each beam profile is larger than predicted by the uniform gain model, in which the geometry does not change. This decrease in beam divergence or increase in aspect ratio may have been due in part to absorption and in part to an
index mismatch between the dye and quartz cell. A discussion of
the second possibility is deferred to section 5.4.3.

The effective width of the amplifying region depends on the
depth to which the pump pulse produces gain. At low pump levels,
ground state absorption and the decay of the pump pulse into the
dye limit the gain to a region smaller than the full cell width.
At intermediate pump levels, absorption is reduced and the width
increases, and at high pump levels bleaching of the dye may permit
relatively uniform pumping over the full width. The observed
behavior of the beam divergence in figures 4-9 and 4-17 is consistent
with this explanation.

5.4 Angular Shift and Peaking of ASE

Two deviations from theory were observed in the angular distribu-
tion for both dye concentrations. For low or negligible gain shots
(see figures 4-5, 4-9, and 4-17), the ASE peaked by up to 20% more
than predicted from the PD gain measurements. This behavior may be
interpreted in terms of a larger gain and aspect ratio than otherwise
measured, as discussed in section 5.3. Also, the ASE peak was
shifted off axis by approximately $-0.5^0$, most noticeably in shots
640-655.

These observations are most satisfactorily explained by refrac-
tions and reflections due to an index mismatch between the cell and
the dye, and possibly due to index gradients in the dye. The
following sections examine these effects.
5.4.1 Grazing Incidence Reflection and Refraction Due to Index Mismatch

There was a small index mismatch between the quartz dye cell and the dye, as discussed in section 3.6. Both wavelength dispersion and temperature induced index differences were present. Three effects due to this index mismatch have occurred at the dye cell wall: reflection, refraction, and scattering. These effects may partially account for the observed peaking on axis when the PD measured gain was negligible.

Consider the case for which \( \frac{\Delta n}{n} > 0 \) with \( \Delta n = n_2 - n_1 \) where \( n_2 \) is the index of quartz and \( n_1 \) is the index of the dye. Referring to figure 5-1, we observe that refraction occurs at the dye cell wall - the external angle \( \Theta_2 \) is larger than the internal angle \( \Theta_1 \). Assuming \( \Theta_1 << 1, \Theta_2 << 1, |\Delta n/n| << 1 \), then

\[
\Theta_2^2 = \Theta_1^2 + \frac{2\Delta n}{n}.
\]

Consequently, the critical angle \( \delta = \left( \frac{2|\Delta n|}{n} \right)^{\frac{1}{2}} \) is excluded from the emission through the dye cell wall, thus decreasing the emission at small angles with respect to either axial or transverse directions.

Fresnel reflections also occur at the dye cell wall. Assuming grazing incidence and small index differences, the intensity reflection coefficient may be approximated by

\[
R \approx \left\{ \left[ \left( \frac{2\Delta n}{n\Theta_1^2} + 1 \right)^{\frac{1}{2}} - 1 \right]^2 \right\}^2.
\]
Figure 5-1

REFRACTION AT DYE CELL WALL

$\theta_1$, $\theta_2$

$n_1$, $n_2$

QUARTZ, DYE

$n_2 > n_1$, $n_2 < n_1$
When this expression is evaluated, we find that $R < 7\%$ for $0 \leq \frac{\Delta n}{n\theta_1^2} < 1$; but that total internal reflection occurs for $\frac{\Delta n}{n\theta_1^2} < -\frac{1}{2}$.

Refraction for $\frac{\Delta n}{n} < 0$ results in an external angle, $\theta_2$, smaller than the internal angle, $\theta_1$. Combined with total internal reflection, this tends to increase the far field emission observed at small angles.

We may obtain an estimate of the effect of refraction by assuming that the maximum or minimum intensity, depending on the sign of $\Delta n$, occurs at the critical angle. Referring to figure 5-2, for $\frac{\Delta n}{n} > 0$ the intensity is reduced since emission from the shaded portion of the amplifier is excluded. If

$$\theta_2 = \left(\frac{2\Delta n}{n}\right)^{1/2} \frac{D}{L} \quad \text{then} \quad \frac{\Delta I}{I_0} \approx \frac{1}{2} \frac{L}{D} \left(\frac{2\Delta n}{n}\right)^{1/2}.$$  (5-3)

Similarly for $\frac{\Delta n}{n} < 0$, the intensity is increased by the refraction at one interface and total internal reflection at the other.

The dispersion in the dye (figure 3-9) with respect to quartz would be expected to cause enhancement for short $\lambda$ ($\Delta n < 0$) and depletion for long $\lambda$ ($\Delta n > 0$). If the intensities in both wavelength bands were comparable, then the effect should approximately cancel. In the extreme case of $\frac{\Delta n}{n} \approx 0.0007$, due in part to dispersion and in part to index changes with room temperature, we estimate

$$\frac{\Delta I}{I_0} \approx 0.3$$  (5-4)
Figure 5-2
REFRACTION AND REFLECTION IN CELL

$\theta_2$

$\Delta n/n > 0$

INTENSITY REDUCED

$\Delta n/n < 0$

INTENSITY INCREASED
for a rectangular amplifier with L/D=16. The observed peaking on axis (15-20%) under negligible gain conditions is consistent in magnitude with this estimate.

5.4.2 Scattering

The observed peaking on axis might also have been due in part to scattering at the dye cell wall interface at small angles. This could reduce the intensity over a wider angular range than could the reflection-refraction effect. However, approximately 20-30% scattering would be required to produce the observed peaking on axis. Although no quantitative measurement was made to determine the scattering, the scattering off of a polished surface should be far lower than this.

5.4.3 Refraction Due to Transverse Index Gradient

The small (≈0.5°) angular offset of the ASE peak in the high dye concentration shots was most likely caused by grazing incidence reflections off the side of the dye cell toward the pump beam - the pump and, therefore, emission are greatest on that side.

However, a second possibility was reported by Loiko, et al.: refraction due to a transverse index gradient in the dye solution. For R6G in ethanol they measured a deviation of 11mrad at 2x10^{-3} mole/l and 29mrad at 4x10^{-2} mole/l. Of the three mechanisms they suggested, an exponentially decreasing heating of the dye and subsequent creation of a transverse index gradient was unlikely in our laser-
pumped dye experiment because of the low (< 10 millijoule) pump energy and low repetition rate (one shot every five minutes). Also this effect should be pump power dependent, which was not observed in our experiment.

Other suggested causes were shock waves due to the pump pulse or an index gradient due to an increasing ground state population in the transverse direction. These do not account for the deviation either since no pump power dependence was observed.

5.5 Non-Uniform Pump Along Length of Dye Cell

Based on 1.06μ beam measurements discussed in 3.4, the estimated non-uniformity of the pump beam along the length of the dye cell was up to 35%. The maximum pump power was at the center of the dye cell, and so, presumably, were the gain and fluorescence.

This variation in gain and fluorescence along the dye cell axis y should not have substantially altered the axial ASE with respect to the uniform gain case. The gain coefficient may be integrated over length to give an average gain-length product

$$\bar{g}_L = \int_0^L g(y) \, dy$$

This average gain-length product (and similarly averaged fluorescence) give identical results to a uniform gain coefficient for the axial ASE and transverse fluorescence if the assumption that the gain coefficient is proportional to the fluorescence holds. In the dye experiment this was approximately true for $\bar{g}_L > 1$. 
For low fluorescence values, absorption was significant, thus removing the above proportionality. In this case, the non-uniform pump may have affected the directionality.
6. Application of the ASE Angular Distribution to Laser-Produced Plasma X-Ray Amplification Experiments

6.1 Introduction

We have analytically and experimentally considered the directionality behavior of ASE sources for which the gain is proportional to the fluorescence. However, certain ASE sources may exhibit gain which is not identically distributed with the fluorescence, including, in particular, certain x-ray amplification schemes in laser-produced plasmas. For such cases we may employ a two region model in which one region exhibits both gain and fluorescence while the adjacent region produces only fluorescence. We allow for this latter fluorescence to be considerably more intense than in the gain region. Taking these added factors into account we are interested to know how the directional properties are altered and whether these properties still provide a viable gain diagnostic.

The particular ASE source we ultimately wish to model is a laser-produced, line-focus plasma used in x-ray amplification experiments. A typical irradiation geometry was shown in figure 1-1 where a pulse from a high peak power laser, e.g. a 50-500GW Nd\(^{3+}\)-glass laser system, is focused into a line, say 50\(\mu\)m by 1000\(\mu\)m, on a solid target material. The plasma electrons are accelerated by the optical electric field and collisionally transfer their energy to the ions in the plasma. Energy deposition occurs predominantly at the point of critical electron density \((10^{21}\text{cm}^{-3})\) for
1.06\,\mu m light), the point at which the plasma frequency is resonant with the optical field. The electron density profile is typically an exponentially decaying function of displacement from the target material, while the electron temperature profile may be slowly varying over a fairly wide region.

A population inversion between two ion electronic states might be produced by electron collisional excitation or recombination, for example. The fluorescence from a transition between these states might be expected to follow the electron density profile; however, gain (if present) cannot be assumed to do so. Gain may be quenched, for example, in high density regions where the fluorescence may be significant.

A pertinent example is the experimental demonstration of a population inversion dependent on recombination in a laser-produced plasma reported by Bhagavatula and Yaakobi.\(^{(1)}\) A population inversion was measured between the n=4,5 and n=3 levels of Al\(^{+11}\) at an electron density \(N_e=10^{20}\,\text{cm}^{-3}\) by spatially resolving the x-ray intensities of the resonance line series. A stepped target was employed, consisting of an aluminum slab with a magnesium plate in front of it (figure 6-1) - this rapidly cooled the expanding Al plasma enhancing the relative rate of three-body recombination which preferentially populates high-lying excited states.

The time averaged electron density and population inversion were measured as a function of displacement from the target, comparing well with their computed values. While the electron density was an exponentially decaying function of displacement from the target,
Figure 6-1

CONFIGURATION OF LASER PLASMA POPULATION INVERSION EXPERIMENT OF BHAGAVATULA AND YAAKOBI
the population inversion ratio increased with displacement, exceeding one in the vicinity of the magnesium plate. The gain and fluorescence regions in the plasma were thus displaced with respect to each other; the dominant fluorescence occurring closer than the amplifying region to the solid target.

6.2 Two-Region ASE Model

6.2.1 Assumptions

We will model this situation by considering two adjacent rectangular regions each of length L and width D, as shown in figure 6-2. The amplifying region exhibits uniform gain and fluorescence - the gain coefficient is g and the fluorescence rate per unit volume per unit solid angle is $K_g$. The other region produces thermal fluorescence only with a fluorescence rate $K_F$. In this geometry the presence of the thermal region will be said to give rise to side-loading. The ratio of the fluorescence produced in the two regions will be varied to simulate different degrees of side-loading.

The angular distribution of the ASE radiant intensity may be calculated by integrating the radiation transport equation (2-2) over both regions for a specific direction vector $\hat{s}$ which makes an angle $\Theta$ with the longitudinal axis shown in figure 6-2. We assume the same measurement geometry as in figure 2-1 where the amplifier-to-detector distance is much larger than either the amplifier or detector dimensions. The analysis is presented in Appendix A.3.
Figure 6-2

TWO REGION AMPLIFIER GEOMETRY
6.2.2 Results

The relative ASE radiant intensity, \( I(\theta) \), is plotted in a series of graphs (see figures 6-3, 6-4, 6-5) on a log scale as a function of azimuthal angle \( \theta \) (units D/L, D/L<<1). Note that positive \( \theta \) is taken toward the side of the axis containing the amplifying region. Each graph is for a different gain-length product, gL, and contains a series of curves for different fluorescence ratios \( k_F/k_G \). For these graphs the fluorescence in the amplifying region is held constant as gL is changed.

First, consider the curve for \( k_F/k_G=0 \) and gL=1.26 in figure 6-3. This is the threshold angular distribution for a rectangular amplifier with uniform gain and fluorescence discussed in section 2.4.3. The distribution is symmetric in \( \theta \). However, as \( k_F/k_G \) is increased, the distribution becomes asymmetric (due to the amplification of the fluorescence from the thermal region for positive \( \theta \) but not for negative \( \theta \)). A consequence is that the maximum in ASE is shifted off the longitudinal axis toward the amplifying region by an angle ranging from D/L to 2D/L. The angular distribution approaches a constant shape in the limit \( k_F/k_G \rightarrow \infty \) - this is already seen for \( k_F/k_G=30 \). Qualitatively the same behavior is displayed at larger gL, for example see figure 6-4, gL=2.27, and 6-5, gL=4; however the angular offset decreases with increasing gL.

The maximum or peak ASE to fluorescence ratio is plotted versus gL in figure 6-6 for two cases: \( k_F/k_G=0 \) and \( k_F/k_G \rightarrow \infty \). This ratio is a measure of the directionality. For \( k_F/k_G=0 \) the result
Figure 6-3

ASE ANGULAR DISTRIBUTION WITH SIDE-LOADING

\[ gL = 1.26 \]

ASE RADIANT INTENSITY (RELATIVE)

AZIMUTHAL ANGLE \( \theta \) (UNITS D/L)
Figure 6-4

ASE ANGULAR DISTRIBUTION WITH SIDE-LOADING

g_L = 2.27

ASE RADIENT INTENSITY (RELATIVE)

AZIMUTHAL ANGLE \theta (UNITS D/L)

K_F/K_G

0
1
2
3
4
5
10
20
50
100
200
500
Figure 6-5

ASE ANGULAR DISTRIBUTION WITH SIDE-LOADING

$g_L = 4$

- ASE RADIANT INTENSITY (RELATIVE)
- AZIMUTHAL ANGLE $\theta$ (UNITS D/L)

$\theta_{HWHM}$
Figure 6-6

RATIO: PEAK ASE/FLUORESCENCE vs gL FOR UNIFORM GAIN AND SIDE LOADING
is that of the uniform gain model discussed in section 2.4.2. With the presence of fluorescence from the thermal region, the ASE to fluorescence ratio is considerably reduced at a given $g_L$ as compared to that for the uniform gain case. This difference is significant if the gain is to be deduced from a measurement of the ASE to fluorescence ratio. For example, to achieve a threshold ratio of two, a $g_L \approx 2.27$ (figure 6-4) is required instead of $g_L \approx 1.26$; almost a factor of two increase in $g_L$ over the uniform gain case is necessary to obtain the same directionality in ASE. Thus, we find that $g_L$ will always be underestimated when applying the uniform gain model to a situation which actually exhibits side-loading; the technique is viable, however.

A different situation exists for negative $\Theta$. In this case the unamplified fluorescence from the thermal region rapidly swamps the ASE from the amplifying region as $K_F/K_G \to \infty$. This is true for all $g_L$, although the directionality is greater at larger $g_L$ as can be seen in figures 6-4 and 6-5. For example, with a fluorescence ratio $K_F/K_G = 10$ threshold is achieved for $g_L \approx 4$ (figure 6-5). This conclusion also holds if the ASE measurements are made out of the plane containing the amplifier and adjacent fluorescence region.

The presence of the fluorescence region also increases the beam divergence over that for uniform gain. Specifically the half width half maximum beam divergence is larger in the positive $\Theta$ than negative $\Theta$ direction, a manifestation of the beam asymmetry. This can be readily seen in figure 6-5.
6.2.3 Limitations

The two-region model discussed so far has potential limitations in application to probable laser-produced plasma ASE sources. We have not explicitly treated thermal fluorescence regions of various relative widths - the model may easily be extended to these cases. The results in the extreme case of a large and dominant thermal fluorescence region is that the directionality due to gain disappears. In this case the directional ASE behavior is not a viable gain diagnostic.

Another possible difficulty with this model is the assumption that the thermal fluorescence region is optically thin, e.g. resonant scattering and absorption were neglected. An optically thick medium would behave as a surface emitter and contribute principally transverse to the amplifier axis. The overall directionality could be seriously decreased from that predicted using the optically thin model. This problem may warrant further investigation if significant amplification is discovered in high density plasmas, for example.

6.3 Refraction Due to an Index Gradient

An index gradient transverse to an amplifier axis can cause refraction which results in the peak ASE shifting off axis. Although the refractive index in a laser-produced plasma is nominally unity at x-ray wavelengths, a sufficient index gradient may be present in high aspect ratio (L/D$\approx$100), high density plasmas ($N_e \approx 10^{21}$ cm$^{-3}$)
to cause significant refraction at soft x-ray wavelengths ($\lambda > 100A$). The possibility of refraction must be considered in diagnosing an x-ray ASE experiment in a laser-produced plasma (refraction was suggested by Sillvast, et al., as a possible effect in laser plasma gain experiments$^2$).

We may estimate the refractive index in a plasma due to free electrons from

$$n^2 = 1 - \left( \frac{\omega_p}{\omega} \right)^2 \quad (6-1)$$

where $\omega$ is the incident frequency and $\omega_p$ the plasma frequency

$$\omega_p^2 = \frac{4\pi Ne^2}{m} \quad (3) \quad (6-2)$$

The expression for $n$ may be approximated for $\omega >> \omega_p$ by

$$n \approx 1 - \delta , \quad (6-3)$$

where $\delta \equiv \frac{2\pi Ne^2}{m\omega^2}$

is the deficit in index with respect to unity. Substituting for $e=4.803 \times 10^{-10}$ esu, $c=2.998 \times 10^{10}$ cm/sec and $m=9.109 \times 10^{-28}$ g we obtain for $\delta$

$$\delta \approx 4.5 \times 10^{-30} Ne \lambda^2 \quad \lambda \quad [\frac{\text{Å}}{}]. \quad (6-4)$$

Approximate values for $\delta$ are given in table 6-1 for a range of values. We find that $\delta \approx 10^{-4}$ for $Ne=10^{21}$cm$^{-3}$ and $\lambda=130A$, possible conditions in current plasma experiments.
The refraction angle may be estimated by the use of the ray vector equation (4)

\[
\frac{d}{ds} \left( n \frac{d\vec{r}}{ds} \right) = \nabla n \tag{6-5}
\]

or \((\hat{s} \cdot \nabla n) \hat{s} + n \frac{d\vec{s}}{ds} = \nabla n\)

Note that \(\hat{s} = \frac{d\vec{r}}{ds}\) is the unit vector in the direction of propagation and \(\vec{r}\) is the position vector as shown in figure 6-7. If we take \(\nabla n = \frac{\Delta n}{\Delta y} \hat{j}\) then equation (6-5) reduces to

\[
\frac{\Delta n}{\Delta y} \cos \theta = n \frac{d\phi}{ds}. \tag{6-6}
\]

If we assume that the index gradient and propagation direction are perpendicular and that \(\theta \ll 1\), then the angular deviation is

\[
\Delta \theta \approx \frac{\Delta n}{n} \frac{s}{\Delta y}. \tag{6-7}
\]

We have listed \(\Delta \theta\) in table 6-2 for several values of \(\frac{\Delta n}{n}\) and \(\frac{\Delta y}{s} \approx D/L\) where \(s\) is taken to be the amplifier length, \(L\), and \(\Delta y\) is set equal to the amplifier width \(D\). For \(\frac{\Delta n}{n} \approx 10^{-4}\) we find that the refraction angle \(\Delta \theta \approx 10^{-2}\) is comparable to \(D/L \approx 10^{-2}\). Refraction begins to substantially alter the angular distribution at this point - for smaller \(\frac{\Delta n}{n}\) or larger \(D/L\) refraction is not significant.

In a line-focused, laser produced plasma the electron density gradient has components normal to and pointing toward the slab target, as well as tangential to the slab target, pointing toward the center of the line-focus plasma. The index gradient is in the direction opposite to the density gradient, thus x-ray radiation
REFRACTION OF RAY BY INDEX GRADIENT

Figure 6-7
will refract away from the plasma and slab target - the plasma is expected to behave as a lens and wedge because the gradient decreases with displacement from the plasma and target. This is depicted in figure 6-8.

Index gradients in laser-produced plasmas at soft x-ray wavelengths may cause significant refraction. This refraction may alter the angular distribution and limit the effective amplification length in ASE experiments. Further analysis of this effect may be warranted if high density ($>10^{21} \text{cm}^{-3}$) and high aspect ratio plasmas or X-UV wavelengths ($\lambda>100\text{A}$) are investigated.
Figure 6-8

REFRACTION IN LASER-PRODUCED PLASMA

PLASMA

TARGET
Table 6-1

Refractive Index Deficit in Plasma

<table>
<thead>
<tr>
<th>$N_e , (cm^{-3})$</th>
<th>$\lambda , (\text{A})$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$</td>
<td>10</td>
<td>$4.5 \times 10^{-7}$</td>
</tr>
<tr>
<td>$10^{21}$</td>
<td>100</td>
<td>$4.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$10^{21}$</td>
<td>130</td>
<td>$7.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>$10^{21}$</td>
<td>600</td>
<td>$1.6 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 6-2

Refracti9on Angle with Transverse Index Gradient

<table>
<thead>
<tr>
<th>$\Delta n/n$</th>
<th>$D/L$</th>
<th>$\Delta \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>$10^{-1}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>$10^{-2}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$10^{-1}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$3 \times 10^{-2}$</td>
<td>$3 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
7. Conclusion

We have presented an ASE model that predicts the angular distribution of ASE from a two-dimensional rectangular amplifier in terms of the gain-length product and the amplifier aspect ratio. The ASE beam divergence decreases with increasing gain-length product or increasing amplifier aspect ratio. Approximately a factor of two ratio between the axial ASE and fluorescence obtains for a gain-length product of unity, a practical measurement threshold.

Measurements of the ASE angular distribution from a transverse laser-pumped R6G dye amplifier were presented and analyzed in terms of the model. The model was shown to reasonably describe the directional behavior of the emission and to give an average value of the gain-length product which corresponded to that obtained by length variation. Absorption over a portion of the integrated ASE pulse reduced the directionality of ASE at low gain, however, and thus reduced the gain estimate based on the directionality.

We have shown that the directional behavior of ASE can be a sensitive and practical diagnostic of gain. However, measurements made in only two directions, e.g. parallel and perpendicular to an assumed amplifier axis, are insufficient to differentiate between gain and absorption. Additional data regarding the amplifier geometry must be obtained, e.g. from a more complete angular distribution measurement. For example, in complex systems, such as laser-produced plasmas, the angular distribution of x-ray ASE, if present, may be affected by non-uniform gain, absorption, and fluorescence, or by
refraction due to index gradients at high plasma densities. Under such conditions, provision must be made to measure the emission off the assumed axis in order to obtain the peak of the ASE angular distribution.
References


3-5 Dynasil Fused Silica Catalog 702.


A.1 Theorem on Symmetry of ASE Angular Distribution

We consider a theorem on ASE symmetry, which is applicable to a wide range of sources. The theorem is: "For an ASE system in which the gain and fluorescence are arbitrary functions of position, the ASE angular distribution will be symmetric about any geometric plane of symmetry if and only if the gain coefficient is proportional to the fluorescence." The transverse laser pumped dye amplifier is a pertinent example; both the gain and fluorescence approximate exponentially decaying functions of the coordinate in the direction of the pump beam. Although the gain and fluorescence are quite inhomogeneous, the approximate proportionality results in a symmetric ASE distribution about the plane perpendicular to the pump beam.

We use the radiation transport equation (2-2) with the alteration that the gain coefficient \( g(s) \) and fluorescence \( K(s) \) are functions of position in the \( \vec{s} \) direction. To prove symmetry, we need to show that the specific intensity in the \( \vec{s} \) direction, \( \tilde{I}(\vec{s}) \), is identical to that in the \( -\vec{s} \) direction, \( \tilde{I}(-\vec{s}) \), after traversing the medium. We change notation from direction and position vector \( \vec{s} \) to position coordinate \( x \) and write the transport equation for each direction:

\[
\frac{dI_+(x)}{dx} = g(x) I_+(x) + K(x) \quad \text{positive x direction} \quad (A-1)
\]

and

\[
-\frac{dI_-(x)}{dx} = g(x) I_-(x) + K(x) \quad \text{negative x direction.} \quad (A-2)
\]
Equation (A-1) is integrated from x=0 to x=a to determine $I_+(a)$, while (A-2) is integrated to find $I_- (0)$. Assuming no signal injection, the general solutions to the transport equations are:

$$I_+(a) = e^\int_0^a g(x) dx \int_0^a e^{-\int_x^a g(x') dx'} dx \quad (A-3)$$

and

$$I_- (0) = e^\int_0^a g(x) dx \int_0^a e^{-\int_x^a g(x') dx'} dx \quad (A-4)$$

Symmetry requires that $I_+(a)=I_- (0)$; setting equations (A-3) and (A-4) equal we obtain:

$$\int_0^a K(x) \begin{bmatrix} -e^\int_x^a g(x') dx' & -e^\int_0^a g(x') dx' \\ -e^{-\int_x^a g(x') dx'} & -e^{-\int_0^a g(x') dx'} \end{bmatrix} dx = 0 \quad (A-5)$$

For the symmetry to be independent of the specific geometry, equality (A-5) must be independent of the value $x=a$. Let $K(x)=\alpha(x)g(x)$ - assume some functional form between the fluorescence and gain, thus

$$\int_0^a g(x) \alpha(x) \begin{bmatrix} -e^\int_x^a g(x') dx' & -e^\int_0^a g(x') dx' \\ -e^{-\int_x^a g(x') dx'} & -e^{-\int_0^a g(x') dx'} \end{bmatrix} dx = F(a) = 0 \quad (A-6)$$

To establish functions $\alpha(x)$ such that $F(a)=0$ and is independent of $a$, take the derivative of (A-6) with respect to $a$:

$$\frac{dF(a)}{da} = \frac{d}{da} \left\{ e^\int_0^a g(x') dx' \int_0^a g(x) \alpha(x) e^\int_0^a g(x') dx' dx - \int_0^a g(x) \alpha(x) e^{-\int_0^a g(x') dx'} dx \right\}$$

Using relationship (A-6) and setting $\frac{dF(a)}{da} = 0$ we obtain

$$\frac{dF(a)}{da} = -g(a) \int_0^a g(x) \alpha(x) e^\int_x^a g(x') dx' dx$$

$$-g(a) \alpha(a) \left( e^\int_0^a g(x') dx' - 1 \right) = 0 \quad (A-7)$$
Similarly
\[
\frac{d^2 F(a)}{da^2} = -g'(a) \int_0^a g(x) \alpha(x) e^{-\int_0^x g(x')dx'} \, dx
\]
\[
+ \left[ g(a) \alpha'(a) + \alpha(a) g'(a) \right] \left[ 1 - e^{-\int_0^a g(x')dx'} \right] = 0 .
\]

Using equation (A-6) to simplify equation (A-8)
\[
\frac{d^2 F(a)}{da^2} = g(a) \alpha'(a) \left[ 1 - e^{-\int_0^a g(x')dx'} \right] = 0 .
\]

Therefore \( \alpha'(a) \equiv 0 \) for all \( a \), and \( \alpha \) is a constant, \( \alpha(a) \equiv \alpha \).

The form of equation (A-3) for \( I_+(a) \) simplifies with \( K(x) = \alpha g(x) \)
\[
I_+(a) = e^{\int_0^a g(x)dx} \alpha \int_0^a g(x)e^{-\int_0^x g(x')dx'} \, dx
\]
\[
I_+(a) = \alpha \left[ e^{\int_0^a g(x)dx} - 1 \right]
\]
\[
\alpha = \frac{K(x)}{g(x)}
\]

Thus, we have shown that a necessary and sufficient condition for the ASE distribution to be symmetric about some geometric plane of symmetry is that the gain and fluorescence be proportional when both are otherwise arbitrary functions of position.

A.2 Integration of Radiation Transport Equation in Linear Transverse Gain Model

A.2.1 Analysis for \( \tan \Theta \geq D/L \)

We begin with the analysis for \( \tan \Theta \geq D/L \). The three geometrical regions are shown in figure A-1; the amplification length \( l(x, \Theta) \) in
Figure A-1

GEOMETRICAL REGIONS

\[ \tan \theta \geq \frac{D}{L} \]
the direction \( \hat{s} \), at an angle \( \Theta \), is shown at the transverse coordinate \( x \). Using the results from the symmetry theorem (A-10), the intensity integrated along the length \( l(x,\Theta) \) is

\[
\tilde{I}(x,\Theta) \equiv \tilde{I}(x) = \frac{K_m}{q_m} g_m l(x,\Theta) f(x) \, dl' \tag{A-11}
\]

We change variables and substitute for \( f(x) \) in (A-11), where

\[
l'(x,\Theta) = x / \sin \Theta, \quad dl'(x) = \frac{dx}{\sin \Theta}.
\]

We then obtain

\[
\tilde{I}(x) = \frac{K_m}{q_m} \left[ e^{\frac{g_m (\frac{1}{2} mx^2 - mx D + x)}{sin \Theta}} - 1 \right]. \tag{A-12}
\]

In region 1 we must integrate \( \tilde{I}(x) \) over the distance \( d \) from corner of rectangle to \( d = D \cos \Theta \)

\[
I_1(\Theta) = \int_0^D \tilde{I}(x) \, \delta d. \tag{A-13}
\]

We change variables again:

\[
x = d / \cos \Theta, \quad \delta d = dx \cos \Theta, \tag{A-14}
\]

and substitute (A-12) and (A-14) into (A-13) to obtain

\[
I_1(\Theta) = \frac{K_m}{q_m} \cos \Theta \int_0^D \left[ \exp \frac{g_m (\frac{1}{2} mx^2 - mx D + x)}{sin \Theta} - 1 \right] \, dx. \tag{A-15}
\]

To solve (A-15), we make change of variables and complete the square in the exponent (assume \( m \) positive).
Let \( x' = \sqrt{\frac{g_m m}{2 \sin \Theta}} \left( x + \frac{1}{m} - \frac{D}{2} \right) \)
\[
\text{dx}' = \sqrt{\frac{g_m m}{2 \sin \Theta}} \, \text{dx}
\]
\[
(x')^2 = \frac{g_m m}{\sin \Theta} \left[ \frac{x^2}{2} + x \left( \frac{1}{m} - \frac{D}{2} \right) + \frac{1}{4} \left( \frac{1}{m} - \frac{D}{2} \right)^2 \right].
\]

Making these substitutions, the result for region 1 is
\[
I_1(\Theta) = \frac{K_m}{g_m} \cos \Theta \left\{ \sqrt{\frac{2 \sin \Theta}{g_m m}} e^{-x^2} \int_{\alpha}^{\beta} e^{x^2} \, \text{dx} \right\}
\]
with
\[
\alpha = \sqrt{\frac{g_m m}{2 \sin \Theta}} \left( \frac{1}{m} - \frac{D}{2} \right)
\]
\[
\beta = \sqrt{\frac{g_m m}{2 \sin \Theta}} \left( \frac{1}{m} + \frac{D}{2} \right)
\]
\[0 < \alpha < \beta\).

Region 2 is identical to Region 1 in geometry - the only difference is that the gain profile is reversed in space. Using the symmetry theorem and reversing the sign on the slope \( m \), we obtain from (A-17)
\[
I_2(\Theta) = \frac{K_m}{g_m} \cos \Theta \left\{ \sqrt{\frac{2 \sin \Theta}{g_m m}} e^{\beta^2} \int_{\alpha}^{\beta} e^{-x^2} \, \text{dx} \right\}.
\]

Region 3 contributes uniformly across its width; the result is simply
\[
I_3(\Theta) = \frac{K_m}{g_m} (L \sin \Theta - D \cos \Theta)(e^{g_m D / \sin \Theta} - 1).
\]
A.2.2 Analysis for $\tan \theta \leq D/L$

The analysis for $\tan \theta \leq D/L$ also incorporates three geometrical regions shown in figure A-2. The amplification length $l(x, \theta)$ in direction $\hat{s}$ is defined similarly to the previous case. We make the same change of variables as for (A-11) and obtain the intensity integrated along the length $l(x, \theta)$

$$\tilde{I}(x, \theta) \equiv \tilde{I}(x) = \frac{K_m}{g_m} \left[ e^{\frac{g_m}{\sin^2 \theta} \left( \frac{1}{2} x^2 - \frac{D}{2} x + x \right)} - 1 \right].$$  \hspace{1cm} (A-20)

For region 1 we proceed as in deriving (A-15) and obtain

$$I_1(\theta) = \int_0^{L \tan \theta} \tilde{I}(x) \cos \theta \, dx$$

(A-21)

$$I_1(\theta) = \frac{K_m}{g_m} \cos \theta \int_0^{L \tan \theta} \left[ \exp \frac{g_m}{\sin^2 \theta} \left( \frac{1}{2} x^2 - \frac{D}{2} x + x \right) - 1 \right] dx.$$  

We solve (A-21) for $\tan \theta > D/L$ (A-15) by making a change of variables and completing the square in the exponent (A-16) (assuming slope $m$ is positive). We obtain

$$I_1(\theta) = \frac{K_m \cos \theta}{g_m} \left\{ \sqrt{\frac{2\sin^2 \theta}{g_m^m}} e^{-\alpha^2} \int_0^{\alpha + \gamma} e^{x^2} \, dx - L \tan \theta \right\}$$

(A-22)

with $\gamma \equiv \sqrt{\frac{g_m}{2\sin^2 \theta}} L \tan \theta$.

Region 2 is identical to region 1 in geometry. However, the slope of the gain profile is reversed. So, reversing the sign on the slope $m$ and using the symmetry of the integral under a change in the sign of integration variable $x$, we obtain
Figure A-2

GEOMETRICAL REGIONS
\[ \tan \theta \leq D/L \]

\[ \bar{s} \]

\[ l(\theta) \]

\[ L \tan \theta \]

\[ d \]

\[ \delta d \]

\[ \ell(x, \theta) \]

\[ \ell(x-x_0, \theta) \]

\[ \theta \]

\[ D \cos \theta - L \sin \theta \]

\[ L \sin \theta \]

\[ x_0 \]

\[ x \]

\[ y \]

\[ L \]

\[ D \]
\[ I_2(\theta) = \frac{K_m \cos \theta}{g_m} \left\{ \sqrt{\frac{2}{g_m} \frac{\sin \theta}{g_m}} e^{\beta^2} \int_{\beta - \gamma}^{\beta} e^{-x^2} \, dx - \text{Ltan}\theta \right\}. \]  \hspace{1cm} (A-23)

Region 3 does not contribute uniformly across its width (contrasting with the case \( \tan \theta \geq D/L \)). The intensity integrated in the direction \( \tilde{s} \) beginning at a point \( x_0 \) to point \( x_0 + \text{Ltan}\theta \) (figure A-2) is

\[ \tilde{I}(x_0, \theta) = \frac{K_m}{g_m} \left[ \frac{g_m}{\text{Lsin}\theta} \left( l \frac{2 - m \frac{D}{2}}{x + x_0} \right) \right] \left[ x_0 + \text{Ltan}\theta \right] \]

evaluating

\[ \tilde{I}(x_0, \theta) = \frac{K_m}{g_m} \left[ \frac{g_m}{\text{Lcos}\theta} \left[ mLx_0 + \frac{L}{2} \tan\theta + (1 - m \frac{D}{2})L \right] \right] \left[ x_0 + \text{Ltan}\theta \right] \]  \hspace{1cm} (A-24)

To obtain the radiant intensity \( I_3(\theta) \), we integrate (A-24) over the distance \( d \) across the width of region 3

\[ I_3(\theta) = \int_{L\text{cos}\theta - L\text{sin}\theta}^{L\text{tan}\theta} \tilde{I}(x_0, \theta) \, dx_0 \]

\[ = \int_{L\text{tan}\theta}^{L\text{cos}\theta} \tilde{I}(x_0, \theta) \cos \theta \, dx_0 \]

\[ = \frac{K_m}{g_m} \cos \theta \left\{ g_m/\cos \theta \left[ l \frac{2 - m \frac{D}{2}}{x + x_0} \right] \right\} \int_{L\text{tan}\theta}^{L\text{cos}\theta} \exp \left( \frac{g_m mLx_0}{\text{cos}\theta} \right) \, dx_0 \]

\[ - \int_{x_0}^{L\text{tan}\theta} \, dx_0 \right\}. \]  \hspace{1cm} (A-25)

Performing the integration in (A-25)

\[ I_3(\theta) = \frac{K_m}{g_m} \cos \theta \left\{ g_m/\cos \theta \left[ l \frac{2 - m \frac{D}{2}}{x + x_0} \right] \right\} \]

\[ x \]
\[
\left[ e^{-\frac{g_m L^2 \tan \theta}{\cos \theta} - mL} \right] \frac{\cos \theta}{g_m mL} - (D-L \tan \theta) \right]\];
\]

then regrouping and using the substitution \( \sinh x = \frac{e^x - e^{-x}}{2} \)

\[
I_3(\theta) = \frac{k_m}{g_m} \cos \theta \left\{ e^{\frac{g_m L}{\cos \theta}} \frac{2 \cos \theta}{g_m mL} \sinh \left[ \frac{g_m L}{2 \cos \theta} (D-L \tan \theta) \right] - (D-L \tan \theta) \right\}. \quad (A-26)
\]

The analytic solutions for the angular distribution are collected in Table A.1.

A.3 Integration of Radiation Transport Equation in Two-Region ASE Model

The geometry of the two regions is shown in figure 6-2.

Each region is of length \( L \) and width \( D \) with a fluorescence rate \( k_F \) in the thermal region and \( k_G \) in the amplifying region. The gain coefficient is \( g \) in the amplifying region.

The ASE angular distribution is found by integrating the radiation transport equation (2-2), repeated here

\[
\frac{d\tilde{I}(\vec{s})}{ds} = g \tilde{I}(\vec{s}) + k. \quad (A-27)
\]

We may integrate (A-27) in the direction \( \vec{s} \) over the length \( l(x) \) in the thermal region and \( l'(x) \) in the amplifying region (\( x \) is measured normal to \( \vec{s} \)) to obtain

\[
\tilde{I}(\vec{s}) = \tilde{I}_F e^{g l'(x)} + \frac{k_G}{g} (e^{g l'(x)} - 1). \quad (A-28)
\]

The first term is the fluorescence from the thermal region, \( \tilde{I}_F \), amplified by the gain \( e^{g l'(x)} \) in the amplifying region. The second term yields the ASE from the amplifying region only, this was derived in section 2.3.2, equations (2-7) and (2-8).
Table A.1

Summary of Analytic Solutions for Angular Distribution

\[ I(\theta) = I_1(\theta) + I_2(\theta) + I_3(\theta) \]

\( \tan \theta > \frac{D}{L} \)

\[ I_1(\theta) = \frac{K_m}{g_m} \cos \theta \left\{ \sqrt{\frac{2 \sin \theta}{g_m}} e^{-\alpha^2} \int_{\alpha}^{\beta} e^{\gamma^2} \, dx - D \right\} \quad (A-17) \]

\[ I_2(\theta) = \frac{K_m}{g_m} \cos \theta \left\{ \sqrt{\frac{2 \sin \theta}{g_m}} e^{\beta^2} \int_{\alpha}^{\beta} e^{-x^2} \, dx - D \right\} \quad (A-18) \]

\[ I_3(\theta) = \frac{K_m}{g_m} \left( L \sin \theta - D \cos \theta \right) \left( e^{\frac{g_m D}{\sin \theta}} - 1 \right) \quad (A-19) \]

\( \tan \theta \leq \frac{D}{L} \)

\[ I_1(\theta) = \frac{K_m}{g_m} \cos \theta \left\{ \sqrt{\frac{2 \sin \theta}{g_m}} e^{-\alpha^2} \int_{\alpha}^{\alpha+\gamma} e^{\gamma^2} \, dx - L \tan \theta \right\} \quad (A-22) \]

\[ I_2(\theta) = \frac{K_m}{g_m} \cos \theta \left\{ \sqrt{\frac{2 \sin \theta}{g_m}} e^{\beta^2} \int_{\beta-\gamma}^{\beta} e^{-x^2} \, dx - L \tan \theta \right\} \quad (A-23) \]

\[ I_3(\theta) = \frac{K_m}{g_m} \cos \left\{ e^{\frac{g_m L}{\cos \theta}} \frac{2 \cos \theta}{g_m L} \sinh \left[ \frac{g_m L}{2 \cos \theta} \left( D - L \tan \theta \right) \right] \right\} \quad (A-26) \]

\[ - \left( D - L \tan \theta \right) \}

\[ \alpha = \sqrt{\frac{g_m}{2 \sin \theta}} \left( \frac{1}{m} - \frac{D}{L} \right) \]

\[ \beta = \sqrt{\frac{g_m}{2 \sin \theta}} \left( \frac{1}{m} + \frac{D}{L} \right) \]

\[ \gamma = \sqrt{\frac{g_m}{2 \sin \theta}} \cdot \frac{L}{\tan \theta} \]
To obtain the radiant intensity, $I(\vec{s})$, from the entire volume we must integrate (A-28) over the transverse coordinate $x$. Using the value for the fluorescence $I_F = K_F l(x)$ we obtain

$$ I(\vec{s}) = \int K_F l(x) \, e^{g_1'(x)} \, dx + \text{ASE term.} \quad (A-29) $$

To integrate equation (A-29) we must consider three cases (where $\phi = \pi - \Theta$ as shown in figure 6-2):

$$ \tan \phi \leq \frac{L}{2D}, \quad \frac{L}{2D} \leq \tan \phi \leq \frac{L}{D}, \quad \text{and} \quad \tan \phi \geq \frac{L}{D}. \quad \text{This is necessitated by the change in geometry at these angles.} $$

**Case A** $\tan \phi \leq \frac{L}{2D}$

We consider five distinct regions contributing to the ASE radiant intensity; these regions are shown in figure A-3. Region 1 contributes only fluorescence; region 2 exhibits uniform fluorescence amplified by a non-uniform gain; region 3 contributes uniform fluorescence and gain; region 4 is a triangular region of fluorescence with uniform gain; and region 5 produces the ASE derived in (2-7) and (2-8).

**Region 1**

This region is a triangle with area $\frac{D^2}{2} \tan \phi$, so the radiant intensity is simply

$$ I_1(\phi) = K_F \frac{D^2}{2} \tan \phi. \quad (A-30) $$
Figure A-3

GEOMETRICAL REGIONS
$tan \phi \leq L/2D$

1

$D/cos \phi$

2

$K_F$

3

$Lcos \phi - 2Dsin \phi$

4

$\phi$

5

$K_G$

g

$x$

$\ell(x)$

$d\ell(x)$

$s$

$l(\phi)$

$L$

$D$

$D$
Region 2

The gain-length product increases linearly from zero to \( g \frac{D}{\cos \phi} \) when traversing this region. An integration is performed over the local transverse coordinate \( x \):

\[
I_2(\phi) = \int_0^{D \sin \phi} K_F \frac{D}{\cos \phi} e^{g \int_0^x (l(x)) \, dx}
\]

where: \( l(x) = \frac{x}{\sin \phi \cos \phi} \)

\[
I_2(\phi) = K_F \frac{D}{g} \sin \phi \left[ e^{gD/\cos \phi} - 1 \right] .
\] (A-31)

Region 3

The emission from this region is found by taking the product of the gain, \( e^{gD/\cos \phi} \), and the fluorescence, \( K_F \frac{D}{\cos \phi} \left[ L \cos \phi - 2D \sin \phi \right] \):

\[
I_3(\phi) = K_F D \left[ L - 2D \tan \phi \right] e^{gD/\cos \phi} .
\] (A-32)

Region 4

The fluorescence is the same as region 1, but is amplified, so:

\[
I_4(\phi) = K_F \frac{D^2}{2} \tan \phi \ e^{gD/\cos \phi} .
\] (A-33)

Region 5

The ASE from a homogeneous region was derived in (2-8) \((\phi = \pi - \theta)\)
\[ I_1(\phi) = \frac{1}{2} K_F D^2 \tan \phi \]  

**(Region 1)**

The fluorescence is simply:

\[ I_1(\phi) = \frac{1}{2} K_F D^2 \tan \phi \]  

**(Region 2)**

The gain-length product increases linearly from zero to \( g \left[ \frac{L}{\sin \phi} - \frac{D}{\cos \phi} \right] \) while traversing the region. In a homologous manner to case A, we integrate over the region:

\[ I_2(\phi) = \int_0^{\frac{L}{\cos \phi} - \frac{D}{\cos \phi}} K_F \frac{D}{\cos \phi} e^{g x / \sin \phi \cos \phi} \, dx \]

Thus:

\[ I_2(\phi) = K_F \frac{D}{g} \sin \phi \left[ e^{\left( \frac{L}{\sin \phi} - \frac{D}{\cos \phi} \right)} - 1 \right] \]  

**(Region 3)**

The integration in this region is complicated by the linear variation of both the gain-length product and the fluorescence. We begin by writing the general form of the amplified fluorescence:
Figure A-4

GEOMETRICAL REGIONS
$L/2D \leq \tan\phi \leq L/D$

\[ K_f \]

\[ L/\sin\phi - D/cos\phi \]

\[ 2D\sin\phi - L\cos\phi \]

\[ L\cos\phi - D\sin\phi \]

\[ L - x \]

\[ L - D\tan\phi \]

\[ K_G, g \]
\[ I_3(\phi) = \int_0^{2D\sin\phi - L\cos\phi} K_F l(x) e^{g \left[ \frac{L}{\sin\phi} - l(x) \right]} \, dx \]  
(A-37)

where:

\[ l(x) = \frac{D}{\cos\phi} - \frac{x}{\sin\phi \cos\phi} \]

from figure A-4. By substituting for \( l(x) \), making the change of variables

\[ x' = \frac{gx}{\sin\phi \cos\phi}, \quad dx = dx' \sin\phi \cos\phi/g, \]

and then integrating, we obtain

\[ I_3(\phi) = \frac{K_F}{g} \left[ e^{g \left( \frac{L}{\sin\phi} - \frac{D}{\cos\phi} \right)} \left( -\frac{\sin\phi \cos\phi}{g} - D\sin\phi \right) \right. \]

\[ + e^{gD/\cos\phi} \left( L\cos\phi + \frac{\sin\phi \cos\phi}{g} - D\sin\phi \right) \]  
(A-38)

**Region 4**

The gain is uniform over this region, so the amplified fluorescence is simply:

\[ I_4(\phi) = K_F (L - D\tan\phi)^2 \frac{1}{2\tan\phi} e^{gD/\cos\phi}. \]  
(A-39)

**Region 5**

The ASE from this region is described by the same equation (A-34) as in case A.
Case C \quad \tan \phi \geq \frac{L}{D}

Region 1

The fluorescence from region 1, (see figure A-5) is easily shown to be

\[ I_1(\phi) = K_F \left( DL - \frac{L^2}{2\tan \phi} \right) \quad (A-40) \]

Region 2

The fluorescence and gain-length product both vary linearly with transverse coordinate $x$. We proceed as for Case B, Region 2, and write:

\[ I_2(\phi) = \int_0^L K_F l(x) e^{\left[ \frac{L}{\sin \phi} - l(x) \right]} \, dx \quad (A-41) \]

where $l(x) = \frac{x}{\sin \phi \cos \phi}$ as shown in figure A-7. Let $x' = -\frac{gx}{\sin \phi \cos \phi}$ and integrate (A-41) to obtain

\[ I_2(\phi) = \frac{K_F}{g} \left[ \sin \phi \cos \phi \left( e^{gL/\sin \phi} - 1 \right) - L \cos \phi \right] \quad (A-42) \]

Region 3

The ASE from this region was derived in equation (2-7)

with $\phi = \pi - \Theta$

\[ I_3(\phi) = K_G \left( \frac{L}{g} \left[ e^{gL/\sin \phi} - 1 \right] \left[ \frac{2}{gL} \sin \phi \cos \phi + \frac{D}{L} \sin \phi \cos \phi \right] - 2 \cos \phi \right) \quad (A-43) \]
GEOMETRICAL REGIONS
\[ \tan \phi \geq L/D \]