Stochastic Fermi Acceleration and the Dissipation of Astrophysical Magnetic Turbulence

by

Robert Selkowitz

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Supervised by Professor Eric G. Blackman Department of Physics and Astronomy The College Arts and Sciences

University of Rochester Rochester, New York

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Curriculum Vitae

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This work is dedicated to the memory of my mother.
Abstract

This thesis comprises a study of the role of stochastic Fermi Acceleration (STFA) in the dissipation of magnetic turbulence in astrophysical plasmas. The STFA process is first examined, and is found to produce electron spectra which bear a strong dependence on scattering processes which maintain pitch angle isotropy among electrons. The STFA process is then applied to the case of impulsive solar flares, where it is found that the shock reprocessing model, which relies on STFA as a first stage acceleration mechanism followed by second stage acceleration at a fast shock, can reproduce many important features of solar flare emission. A series of observational tests is discussed. We also develop a second application of STFA, in which the dissipation of interstellar turbulence is explored. We find that the dissipation length scale is sufficiently large to be inconsistent with turbulence-driven models of radio scintillation.
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Chapter 1

Introduction

Magnetohydrodynamic (MHD) turbulence is a common feature in astrophysical plasmas. An understanding of the dissipation of turbulence is therefore vital to unraveling many astrophysical puzzles. In particular, there is a strong correlation between sites of MHD turbulence and particle acceleration. This thesis comprises a study of the effects of one dissipative process, stochastic Fermi acceleration (STFA), on two particular systems: impulsive solar flares and the Reynolds layer of the interstellar medium. In the former, STFA transfers energy from the turbulent cascade into a power law distribution of non-thermal electrons. In the latter, STFA provides a source of heat and determines the truncation scale of the cascade. In both cases, we find that the eventual electron energy spectrum is governed by pitch angle scattering, which is necessary to sustain STFA.

The remainder of this chapter comprises an overview of relevant aspects of turbulence, solar flares, and radio scintillation. In chapter 2 we explore the process of STFA with applications to non-thermal electron acceleration in impulsive solar flares. Chapter 3 presents a new model for interpreting the measurements of pulse strengths and arrival times in terms of the combined, sequential action of STFA and shock Fermi acceleration. In chapter 4 we present an observational test of the model proposed in chapter 3. In chapter 5 we explore STFA in the interstellar medium (ISM), and show that the inner scale of turbulence is not consistent with turbulence driven models of radio scintillation.
1.1 MHD turbulence

Turbulence is ubiquitous in fluids on all scales. The process begins when an effectively random set of eddies are introduced into the fluid at some largest scale, the outer scale. These eddies interact, shredding each other into ever smaller eddies. This cascade often proceeds over many orders of magnitude in length. Eventually, the eddies become sufficiently small that viscosity in the fluid dissipates the energy more rapidly than it can be transferred into smaller scale eddies. The dissipation length scale is referred to as the inner scale. Turbulence is most easily studied under two assumptions: steady state, in which the input power on the outer scale is uniform over time, and the mean properties at all length scales are time independent; and adiabaticity, where the power contained in the cascade is conserved across all length scales above the inner scale, where it is rapidly dissipated.

Working under these two assumptions, Kolmogorov developed the first predictive theory of turbulence. In Kolmogorov turbulence, the power is input at some outer length scale \( L \). By the adiabaticity assumption, the flux \( F \) through any other length scale \( \lambda \), down to the dissipation scale \( \lambda_{\text{min}} \) is equal to the input power per unit mass, \( P \). Kolmogorov further assumed that the energy per unit mass \( E(k) \) in eddies of a length \( L = k^{-1} \) depends only on the wavenumber \( k \) and \( P \). Using dimensional analysis, Obukhov (1941) found the same result

\[
E(k) = k^a P^b, \tag{1.1}
\]

where \([E] = L^3 T^{-2}, [k] = L^{-1}, \) and \([P] = L^2 T^{-3}, \) such that \( a = -5/3 \) and \( b = 2/3 \), yielding the Kolmogorov scaling law

\[
E(k) \sim k^{-5/3} P^{2/3}, \tag{1.2}
\]

which is observed in incompressible hydrodynamic turbulence.

It was realized that the presence of magnetic fields, and therefore a magnetic energy term, alters the energy density in MHD turbulence. The first treatment, by Ironshikov (1963); Kraichnan (1965) (IK) found a scaling law with \( E(k) \sim k^{-3/2} \). IK turbulence is inherently isotropic, ignoring the influence of a steady mean magnetic field, and is most useful only in cases with a very weak external magnetic field. Inherently anisotropic treatments, such as that by Goldreich and Sridhar (1997) (GS), produce a
spectrum approaching $k^{-5/3}$ in a strong external magnetic field. For weaker external fields, the spectral index ranges between the IK and GS values.

Interestingly, the dissipation mechanism does not enter into the Kolmogorov scaling framework, except to argue for the existence of an inner scale for the cascade. In this regard, it is not vital for the scaling law that the inner scale be set by viscosity (for hydrodynamic turbulence) or resistivity (for MHD turbulence). Instead, any process which drains energy from eddies on a length scale $\lambda$ more rapidly than the energy proceeds along the cascade to the next smaller scale will dissipate the turbulence. Arguments similar to the one used to derive equation 1.2 can be used to show that the eddy turnover time, the characteristic time for energy to travel along the cascade, is given by $\tau_E = \lambda / v$ where $v$ is the characteristic velocity of the turbulence. Physically, $\lambda / v$ represents the eddy crossing time, the time over which a disturbance can cross an eddy of size $\lambda$, and thus the time in which one eddy can fully destroy another of comparable size. This leads to the condition for the inner scale: the dissipative process must act on a timescale $\tau_D < \tau_E$. In chapter 2 we explore STFA, and determine the inner scale of MHD turbulence as set by this process.

1.2 Impulsive solar flares

Solar flares are explosive events in the solar corona which can produce radio, microwave, x-ray, and gamma ray emission. Large flares are typically accompanied by coronal mass ejections (CMEs), in which large numbers of high energy ions are expelled from the corona. CMEs, conversely, are not always associated with large solar flares. In the prototypical large flare, the radiation initially varies impulsively, with many rapid bursts, and subsequently enters a gradual, or steady emission phase.

The only available energy source of sufficient magnitude to power solar flares is magnetic reconnection. Reconnection occurs when oppositely directed magnetic field lines are brought sufficiently close together that the local magnetic field gradient becomes large. This creates a current sheet, in which large currents flow. The small, but nevertheless non-zero, resistivity in the sheet results in a rapid dissipation of magnetic energy and the destruction of field lines near the current sheet. The large scale field reorders itself into a lower energy configuration, while the released magnetic energy is transferred into bulk flow of plasma out of the reconnection region. In flares, the
reconnection most likely occurs in an x-point configuration, with twin outflows: one upward into the corona, the other downward toward the chromosphere. The upward flow is associated with CMEs, while the downward flow produces the solar flare. A diagram of the reconnection region, as well as the downward flow is shown in figure 1.1.

At the top of the depicted region is the reconnection zone, or current sheet (solid black). In this area, the inflow magnetic field (oppositely directed field lines forming the outer border) are drawn together, resulting in reconnection. Below the reconnection zone is the downward bulk outflow. The twin upward flow is not shown. If the downflow speed is sufficiently large shear near the slow shock which bounds the downflow may drive hydrodynamic turbulence, resulting in reseeding of the magnetic field. At low downflow speeds, the bulk flow remains laminar. At the base of the downflow, there may be a loop-top hard X-ray source (grey rectangle). This source is visible in a fraction of impulsive phase flares. The loop-top region may also be associated with a fast MHD shock, again depending on downflow speed. At the very bottom of the figure are the twin footpoint hard X-ray sources (grey rectangles). These are produced by thick target Bremsstrahlung when high energy \( E > 1\text{keV} \) electrons precipitate onto the relatively high density material in the solar chromosphere. Finally, a soft X-ray loop is seen reaching upward from both footpoints, and meeting below the loop-top emission site. This loop traces the shape of the relaxed post-reconnection magnetic field. The material filling the loop is heated as a result of the bulk flow striking the dense footpoint region; the soft X-ray emission is thermal.

It should be noted that figure 1.1 represents a vertical cross-section of the flare. The x-point sits along a line of x-points, which extends into the page. The flare loop is thus a slice of an arcade. Over time, the reconnection process proceeds in two directions. There is a "zipper" evolution in which reconnection proceeds along the line of x-points. There is also a rising motion in which the x-point of a particular cross-section moves upward into the corona.

Of particular interest for this thesis are compact-impulsive (CI) flares, which pass through an impulsive phase in which they emit hard x-rays in short bursts before passing into a more quiescent gradual phase. A typical impulsive phase burst last for \( \sim 1\text{s} \) and produces \( \sim 10^{26}\text{erg} \) of x-rays via Bremsstrahlung at the flare footpoints. It
is with the acceleration of the electrons which produce this emission that we will be primarily concerned.

1.3 Radio scintillation

The characteristics of radio scintillation are familiar to anyone who has ever observed a twinkling star. Starlight, when traversing the atmosphere, encounters random density fluctuations, and thus small index of refraction variations, on its way to the observer. Initially adjacent rays may encounter a different series of perturbations, arriving at the observer non-parallel and non-adjacent, resulting in the spreading of a point source, such as a star. Since the atmospheric fluctuations vary over time, so does the spreading, resulting in the familiar twinkle.

Radio scintillation is analogous: radio waves propagating through the interstellar medium encounter regions of enhanced electron density, and thus pass through varied indices of refraction. Initially parallel adjacent rays emerge non-parallel and non-adjacent. Effective point sources, such as pulsars, are thus observed in the radio with sizes set by the density characteristics of the ISM. While this is traditionally considered a detriment for radio observation - particularly of extended objects, where it can limit resolution - radio scintillation can, with appropriate modeling, be used as a probe of the ISM.

Most models of radio scintillation are built on the premise that the density fluctuations driving the process are the result of a turbulent cascade on the galactic scale. Supperbubbles, the collective blast shells generated when a collection of coeval O and B stars enter the supernova stage nearly simultaneously, are thought to seed this turbulence on scales as great as 100pc. These models share the assumption that the cascade continues down to some small inner length scale, where it is cut off by microphysical processes such as resistivity, viscosity, or resonant damping. The models then attempt to infer this inner scale, as well as the spectral index of the turbulent cascade, from the spreading of point sources. These models, while all fundamentally similar, nevertheless infer a large range of inner scales, from as small as $10^6$cm up to $10^{10}$ cm (Rickett 1990; Moran, et.al. 1990; Molnar, et.al. 1995).

A fundamentally different model has recently been proposed (Boldyrev & Gwinn 2003, 2005; Boldyrev & Konigl 2005; Boldyrev et al. 2002), in which the density fluc-
tuations responsible for radio scintillation are the photo-ionized surfaces of randomly distributed HII clouds, the distribution of which is not connected to the galactic scale turbulent cascade. In chapter 5 we apply our model of STFA to the Reynolds layer of the ISM and find that the predicted turbulent inner scale is $> 10^{12}$ cm, greater than all inferred inner scales in the turbulence models of radio scintillation. This in turn implies that radio scintillation is not associated with galactic scale turbulence, supporting instead models which employ scatterers that are not part of the cascade.

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Figure 1.1 The basic model of solar flare structure Blackman (1997). Reproduced by permission of the AAS.
Chapter 2

Stochastic Fermi Acceleration of Sub-relativistic Electrons and its Role in Impulsive Solar Flares.
We reexamine stochastic Fermi acceleration (STFA) in the low energy (Newtonian) regime in the context of solar flares. The particle energization rate depends on a dispersive term and a coherent gain term. The energy dependence of pitch angle scattering is important for determining the electron energy spectrum. For scattering by whistler wave turbulence, STFA produces a quasi-thermal spectrum. A second well-constrained scattering mechanism is needed for STFA to match the observed 10 – 100keV non-thermal spectrum. We suggest that STFA most plausibly acts as phase one of a two phase particle acceleration engine in impulsive flares: STFA can match the thermal spectrum below 10kev, and possibly the power law spectrum between 10 and 100keV, given the proper pitch angle scattering. However, a second phase, such as shock acceleration at loop tops, is likely required to match the spectrum above the observed knee at 100keV. Understanding this knee, if it survives further observations, is tricky.

2.1 Introduction

Fermi acceleration was first proposed as a mechanism for cosmic ray acceleration (Fermi 1949; 1954). In the original model, compressive perturbations in the Galactic magnetic field, associated with molecular clouds, reflect charged particles. If these clouds converge, the particles gain energy over time. If they diverge, the particles lose energy. Later, it was realized (Bell 1978; Axford, Leer, & Skadron 1978; Krymskii 1977; Blandford & Eichler 1987), that shock fronts are another site of Fermi Acceleration. In the shock acceleration model, charged particles stream into magnetic perturbations in the post-shock region, reflect, and are scattered back across the shock by pre-shock Alfven waves. Repeated reflections steadily accelerate particles to a power law distribution. This has been studied extensively (e.g. Jones & Ellison 1991). If the shock thickness is determined by the ion gyroradius then ions are picked up out of the thermal population but electrons must be injected at energies at or above the thermal energy by a factor of the ratio of proton to electron mass, \( m_p/m_e \), to incur power law acceleration. The injection process is a critical outstanding problem for many applications of shock acceleration. Shock Fermi acceleration is commonly referred to as first order Fermi Acceleration because the sign of the energy gain after each cycle.
is positive and \( dE/dt \propto v_c/v \), where \( E \) and \( v \) are the electron energy and speed, and \( v_c \) the velocity of the magnetic compression.

Fermi acceleration can also take place in a fully turbulent plasma. There, the mirroring sites are turbulent perturbations, typically fast mode magnetohydrodynamic waves, randomly distributed throughout the plasma (e.g. Achterberg (1984)). Electrons encounter these perturbations such that there is a stochastic distribution of energy gaining and energy losing reflections. As is demonstrated below, there is a net dissipation of turbulent energy into high energy electrons. Because the energy gaining and energy losing reflections are equal to first order, STFA is often referred to as second order Fermi acceleration and proceeds more slowly than the first order process. For STFA, \( dE/dt \propto (v_c/v)^2 \).

STFA has been considered extensively as the acceleration mechanism in impulsive solar flares, (e.g. LaRosa et al. (1996)). Observations of these flares show hard X-ray emission with a downward breaking power law spectrum extending from 10keV to at least 0.5MeV with the break energy narrowly distributed around \( E_{br} = 100 \text{keV} \) and a thermal distribution at energies below 10keV (Dulk et.al. 1992; Krucker, & Lin 2002). The time structure of the emission shows distinct spikes of duration \( 1 \text{s} \) and typical energy \( 10^{36} \text{erg} \) (Aschwanden, et.al. 1995). Non-thermal emission occurs principally at the footpoints of the soft X-ray loop, and to a lesser extent at a loop-top hard X-ray source (Tsuneta 1996; Masuda, et.al. 1996). Brown (1971) has shown that the emission at a dense target is consistent with Bremsstrahlung radiation by electrons accelerated to a power law energy distribution at some height above the target; in impulsive flares the acceleration site can be associated with the loop-top region, while the thick target is associated with the footpoints. We demonstrate that STFA is possibly responsible for the acceleration of electrons below 10keV, while first order Fermi processes at the loop-top fast shock may produce the highest energy electrons. In this picture STFA also provides power law distributed electrons in the range \( 10\text{keV} < E < 100\text{keV} \), thus satisfying the shock injection criterion and producing the observed spectral break at 100keV. It is shown that in order to produce this spectrum the pitch angle scattering must obey the restriction that the scattering distance, \( \lambda_p \), is inversely proportional to the energy of the scattered electron in the \( 10 - 100\text{keV} \) range. Matching the knee is difficult, requiring either a cutoff in the secondary pitch
scattering at 100keV, or the sudden onset of yet another pitch scattering agent with a much shorter wavelength. While both are possible, these requirements provide a serious challenge to STFA models of electron acceleration.

It is important to note that the downward break is not observed in all impulsive solar flares. Indeed, Dulk et al. (1992) observed a spread in break energies and spectral indexes. Some of their flares exhibited no discernible break, or even some upward breaks (ankles). In this paper we address the fundamental process of STFA, and show how it can accommodate the downward breaking spectra observed in a subset of impulsive flares. The general form of the electron spectra we model is illustrated in figure 2.1. We assume a thick target Bremstrahlung emission model where the electron spectral index $\delta$ and the photon spectral index $\gamma$ are related by $\delta = \gamma + 1$ (Brown 1971; Tandberg-Hanssen & Emslie 1988). The data shown are mean values taken from the flares studied by Dulk et al. (1992): $E_{br} = 100$keV, $\delta = 4$ below $E_{br}$, and $\delta = 5.25$ above $E_{br}$.

Furthermore, not all flares are observed to be dominated by electron acceleration. A recent flare observed by Hurford et al. (2003) clearly shows regions of emission which are dominated by X-ray emission from electrons as well as regions which are dominated by gyrosynchrotron emission from MeV/nucleon ions. Proton and ion emission appears to be associated primarily associated with larger flare loops. Miller and Roberts (1995); Miller, Emslie, and Brown (2004) propose that this can be explained by a two stage process for ion acceleration. First, ions are accelerated via gyroresonance to speeds of roughly $v_A$ by Alfvén waves, and subsequently are accelerated preferentially over electrons by magnetosonic waves. They argue convincingly for the gyroresonant acceleration by Alfvén waves on the basis of relative ion abundances. However, it is unclear that the second stage acceleration by magnetosonic modes must be resonant. It appears that the second stage is consistent with STFA. In any event, proton acceleration in long flare loops presents an interesting problem for acceleration models, in that super-Alfvénic protons must be preferentially accelerated over electrons, but electron acceleration still must be dominant in shorter loops. In this work, we presume that the loops are sufficiently short that protons remain sub-Alfvénic. Longer loops, and the effects of proton acceleration on the shaping of the observed Bremstrahlung X-ray spectra from high energy electrons, require further study.
STFA is found to depend on two competing effects which we refer to as the steady and diffusive acceleration rates. The steady rate represents the net acceleration of electrons due to the slight advantage of head-on or energy gaining reflections over catch-up or energy losing reflections. The diffusive term represents the spreading of the electron distribution function as a result of the stochastic nature of the reflections. Longair (1994) treats the two effects together using the Fokker-Planck equation. Likewise, Park and Petrosian (1995) discuss general solutions of a simplified Fokker-Planck equation for STFA. In a follow-up work, Park, Petrosian, and Schwartz (1997) apply their solution to solar flares. While they produce spectra consistent with observations, they do not discuss the physics of electron escape. Furthermore, they focus mainly on the regime above 100keV. Some past treatments of STFA in impulsive flares focused exclusively on the diffusive term. LaRosa et al. (1996) derives the diffusive acceleration rate using simple physical arguments. Chandran (2003) derives the diffusive term using both phenomenological arguments and quasi-linear theory. The latter also includes Coulomb losses as a small correction; this is of note since the Coulomb loss term is mathematically similar to a negative steady acceleration term. Herein we derive the steady term and compare it to the diffusive term, finding the steady acceleration to be dominant in the non-relativistic regime for impulsive flares. As will be seen, our steady term differs slightly from that of some previous calculations such as that in Longair (1994) in its dependence on the turbulent magnetic fluctuation strength. This arises because we average only over the pitch angle phase space for which STFA operates in magnetic mirroring, whereas Longair (1994) averaged over all pitch angles in considering a more generic form of Fermi acceleration. A similar averaging over all pitch angles is performed in Skilling (1975); Webb (1983).

We first review the basic Fermi process using a test particle approach. We then derive an expression for the mean acceleration of electrons in a turbulent plasma via STFA, and compare this to the diffusive STFA derived by LaRosa et al. (1996). Finally, we discuss the trapping of electrons in the turbulent accelerating region and show that at non-relativistic energies, the electron spectrum depends strongly on the energy dependence of pitch angle scattering. For scattering by whistler wave turbulence, the emerging spectrum is quasi-thermal. In order to produce the 10–100keV power law in solar flares, we show that there must be an additional source of pitch angle scattering.
which has a length scale inversely dependent on electron energy; this mechanism is tightly constrained. The existence of such a pitch angle scattering mechanism is yet to be determined. This brings to the fore the most pressing difficulty with STFA models of electron acceleration; the pitch angle scattering requirements are stringent and might not be possible to meet.

2.2 The Fermi Acceleration Process

Consider a particle of charge $q$ traveling with gyroradius $r_G$ in a magnetic field of strength $B$ (Fermi 1949; Spitzer 1956). The charge follows a helical path, orbiting the field line while also moving parallel (or anti-parallel) to the field line. Taking the condition for circular motion and the Lorentz force

$$F = qv \pm B = \frac{mv^2}{r_G},$$

where $q$ is the electron charge, and applying conservation laws for angular momentum and kinetic energy ($r_G v_\perp$ and $v^2$ constant) yields

$$\frac{qL}{mE} = \frac{\sin^2 \theta}{B},$$

where $L$ is the angular momentum of the charge, $E$ the kinetic energy, and $v \sin \theta = v_\perp$ relates the total velocity to the component perpendicular to the field line. The pitch angle, $\theta$, is the angle between the field line and the velocity vector. As the charge enters a region of increasing $B$, such as a magnetic compression, the pitch angle evolves according to

$$\frac{\sin^2 \theta_1}{B + \delta B} = \frac{\sin^2 \theta}{B},$$

where $\delta B$ is the increase in field strength and $\theta_1$ is the pitch angle at field strength $B + \delta B$. When $\sin \theta_1 = 1$, the charge cannot penetrate further into the compression, and reflects. This process, known as magnetic mirroring, is commonly used to confine laboratory plasmas (Dendy 1990). It follows immediately that mirroring will not occur at a given compression unless the initial pitch angle satisfies

$$\sin^2 \theta \geq \frac{B}{B + \delta B}.$$
Fermi (1949, 1954) showed that moving magnetic mirrors, in particular molecular clouds, can accelerate charges. In the cloud's frame of reference (primed), mirroring results in only a change in the sign of \( v'_{\parallel} \), the component of the initial velocity of the charge parallel to the field line in the compression's rest frame. Let us work for the moment in the limit where the compression speed and the particle speed are both \( \ll c \). Transforming to the lab frame, \( \delta v_{\parallel} = \pm 2v_c \), where \( v_c \) is the drift velocity of the cloud. The positive (negative) sign is for head-on (catch-up) reflections between the charge and cloud. Catch-up reflections are defined as those where the components of the compression and charge velocities parallel to the field line have the same sign. Head on reflections are those where the parallel components have opposite signs. The net change in energy from a reflection is given by

\[
\delta E_{\pm} = \frac{m}{2} \left( v_f^2 - v_0^2 \right) = \frac{m}{2} \left( 2\delta v_{\parallel} v_0 \cos(\theta) + (\delta v_{\parallel})^2 \right) = \left( 2m \right) (\pm v_c v_{\parallel} + v_c^2),
\]

where \( v_f \) and \( v_0 \) are the speeds after and before reflection, \( m \) is the mass of the charge, and \( v_{\parallel} = v_0 \cos(\theta) \). Head-on reflections result in a positive energy change, while catch-up reflections can result in a negative energy change when \( v_{\parallel} > v_c \). In both types of reflection, there is the positive term proportional to \( v_c^2 \).

One can repeat this derivation using Lorentz transformations in place of the Galilean transformations to generalize this result to particles of any energy scattered by compressions which are still restricted to non-relativistic velocities. For a derivation see Longair (1994); we simply cite the result:

\[
\delta E_{\pm R} = 2E \left[ \pm \frac{v_c v_{\parallel}}{c^2} + \frac{v_c^2}{c^2} \right],
\]

where \( c \) is the speed of light in vacuum. Over time, charges trapped between converging magnetic compressions are subject to only head-on reflections, and are accelerated to higher energies.

Notice that the change in momentum of a Fermi accelerated electron is solely in the component parallel to the mean field. This corresponds to an increase in the electron's pitch angle. As an extreme example, consider an electron of initial energy \( E_0 \) with pitch angle in the mean field \( B \) approaching \( \pi/2 \). Upon doubling the electron's energy via Fermi acceleration, the pitch angle in the mean field is reduced to \( \pi/4 \). Clearly, acceleration to high energy must be accompanied by some additional scattering agent.
which isotropizes electron pitch angles on a short time scale, otherwise Fermi acceleration shuts off after small accelerations as pitch angles evolve out of the range given by (2.4). The well known problem of pitch angle scattering remains largely unsolved (e.g. Achterberg (1981); Melrose (1974); LaRosa et al. (1996)).

Fermi acceleration is distinct from the transit time damping (TTD) treated, for example, in Miller, Larosa, & Moore (1996). TTD is the magnetic analog of Landau Damping. In TTD, electrons (or ions) which are near gyroresonance with waves of wavenumber $k$ are pushed towards the resonance by field gradients in the wave which alter the parallel component of the velocity. Gradients in the electron velocity spectrum result in a net damping or enhancement of the waves. In the presence of a spectrum of waves, an electron can drift from resonance at $k$ to $k \pm \delta k$ and so forth, eventually reaching high energies. Fermi Acceleration, however, is a non-resonant interaction. Electrons will mirror at a compression regardless of energy provided that the pitch angle is sufficiently large. TTD is often referred to as resonant Fermi Acceleration because the two processes rely on similar physics. Table 2.1 lists the relevant length and time scales for STFA in impulsive flares.

2.3 Stochastic Fermi Acceleration

We now consider the behavior of charges in a turbulent magnetic plasma where magnetoionic modes provide the sites of magnetic mirroring. This scenario differs from first order Fermi acceleration by shocks in two ways. 1) Consecutive mirroring events are not coherent, but rather stochastically distributed between head-on and catch-up. 2) The turbulent cascade governs the acceleration efficiency; the system picks out a scale where acceleration competes with the cascade. In the solar corona plasma, $v_c$, the velocity of the magnetic compressions, is the phase speed of the magnetoionic modes, which is roughly the Alfvén speed for the fast mode and the sound speed ($c_s$) for the slow mode. Typically, thermal electrons in the corona are super-Alfvénic and non-relativistic, $v_A \ll v_0 \ll c$, and $\beta \sim 0.05$; we will solve the STFA problem in this regime.

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2.3.1 Determination of the Steady Acceleration Rate

Recall that the energy gain from a typical reflection is given in Eq.(2.5) to be $\delta E_{\pm} = 2m(\pm v_{\parallel} v_c + v_c^2)$. We define three parameters: $R$, the total rate of reflections; $R_+$, the rate of head-on reflections; and $R_-$, the rate of catch-up reflections. The relation $R = R_+ + R_-$ is automatically satisfied by this definition as all reflections must be of either the head-on or catch-up type. This allows us to write the approximate acceleration rate as the sum of a coherent term and an incoherent term:

$$\left(\frac{dE}{dt}\right)_S = 2m[(R_+ - R_-)v_{\parallel} v_c + (R_+ + R_-)v_c^2]$$

where the subscript $S$ is used to distinguish our derived acceleration rate from that of LaRosa et al. (1996). The first term, proportional to $(R_+ - R_-)$ represents the mean acceleration due to the offset in the rates of the two types of reflection. The second term, proportional to $(R_+ + R_-) = R$, the total reflection rate, represents the coherent term. This expression gives a full description of the mean acceleration of charges by the non-relativistic STFA process. To evaluate it in a particular plasma requires the determination of the head-on and catch-up reflection rates $R_+$ and $R_-$. 

In order to obtain $R_+$ and $R_-$, consider the path a charge takes to encounter a mirror. Note that there is a distinction between an encounter and a mirroring because of the pitch angle condition for reflection. We take the fraction of encounters which reflect to be $F$ and assume that this fraction is the same for both head-on and catch-up encounters: $F = F_+ = F_-$. This assumption is often taken in the regime where $v \gg v_c$ (LaRosa et al. 1996). While this assumption is not strictly true, the effects of relaxing it are negligible. In Appendix B we repeat this calculation without assuming $F_+ = F_-$. There is a well defined mean distance between encounters, $\lambda_T$ as well as a relative velocity between the particle and the compression $v_{\pm} = v_{\parallel} \pm v_c$. The small difference in the head-on and catch-up speeds is responsible for the offset in rates. For each type of encounter, the mean separation is $2\lambda_T$ and the rate of reflections of any sort is specified by the general relation $R = F v_{rel} \lambda$, with $v_{rel}$ the relative velocity between the particle and compression, yielding

$$R_+ = F \frac{v_{\parallel} + v_c}{2\lambda_T}$$
The offset in rates is thus

\[ R_+ - R_- = F \frac{v_e}{\lambda_T}, \quad (2.8) \]

and we can rewrite the average acceleration rate as

\[ \left( \frac{dE}{dt} \right)_S = 2mF \frac{v_{\parallel}v_e^2}{\lambda_T} + 2mF \frac{v_{\parallel}v_e^2}{\lambda_T} = 4mF \frac{v_{\parallel}v_e^2}{\lambda_T}, \quad (2.10) \]

with the associated acceleration time scale \( \tau_S = E / (\frac{dE}{dt})_S \). We thus see that the acceleration due to the offset in head-on and catch-up reflection rates and the acceleration from the coherent term in \( v_e \) are equal.

To simplify the derivation of the electron spectrum, we recast (2.10) as

\[ \left( \frac{dE}{dt} \right)_S = 4mv_e^2 R = \frac{dE}{dM} \frac{dM}{dt}, \quad (2.11) \]

where we used (2.8) and define \( M \) as the total number of reflections experienced by an electron, \( dM/dt = R \), and

\[ \frac{dE}{dM} = 4mv_e^2. \quad (2.12) \]

The quantity \( dE/dM \), the mean acceleration of an electron per reflection, will be of particular use when examining electron escape in section 2.4.

### 2.3.2 Comparison to the Diffusive Acceleration Rate

A different approach was taken by LaRosa et al. (1996). They set \( R_+ = R_- \) in the \( v \gg v_e \) regime and studied the diffusion of particles through energy space via random walk. The starting point in their calculation of the electron acceleration was the timescale for the e-folding of a charged particle's energy in the turbulent plasma

\[ \frac{1}{\tau_D} = \frac{1}{E} \left( \frac{dE}{dt} \right)_D = \frac{F}{N \delta t}, \quad (2.13) \]

where \( N \) is the number of mirrorings required to double the particle's energy and \( \delta t \) is the time between encounters. They set \( R_+ - R_- = 0 \), and also dropped the last term.
in $v_c^2$ in Eq 2.6. From (2.7) and (2.10) it is clear that if one of their two assumptions is valid the other must also apply, and $(dE/dt)_S = 0$ in that limit. Under these assumptions $\delta E_+$ and $\delta E_-$ are equal in magnitude, and from the standard solution of an evenly weighted random walk $N = (E/\delta E)^2$. The acceleration rate is then

$$\left(\frac{dE}{dt}\right)_D = \frac{E}{\tau_D} = \frac{F\delta E^2 v_{\parallel}}{E\lambda_T} = \frac{8F m v_c^2 v_{\parallel}^3}{\lambda_T v_0^2},$$

(2.14)

where $v_0$ is the total initial speed and the subscript $D$ is used to denote LaRosa et al's diffusive acceleration rate.

To complete the calculation of the acceleration rates, we must obtain $F$ and average over pitch angles. The minimum accessible pitch angle for reflection is related to $F$ by

$$F = \cos(\theta_{\min}) = \left(\frac{\delta B}{B}\right)^{1/2},$$

(2.15)

where we have applied the reflection condition from (2.4). In the regime where the compression ratio $\delta B/B << 1$, which will apply to the plasma of interest, and taking the assumption that pitch angles are isotropic gives $\langle \cos(\theta) \rangle = \cos(\theta_{\min})/2$ and $\langle \cos^3(\theta) \rangle = \cos^3(\theta_{\min})/4$. We now write the averaged acceleration rates as

$$\left(\frac{dE}{dt}\right)_S = 4mF \left(\frac{v_0}{v_c}\right) v_c^2 \frac{(\delta B)}{B},$$

(2.16)

and

$$\left(\frac{dE}{dt}\right)_D = 8F m v_c^2 \left(\frac{v_{\parallel}^2}{v_0^2}\right) \lambda_T^2 \left(\frac{\delta B}{B}\right)^2.$$

(2.17)

What do these two acceleration rates represent? $(dE/dt)_S$ is the steady growth of the mean kinetic energy due to the drain of turbulence by the combined effects the slightly non-zero $(R_+ - R_-)$ and the coherent $v_c^2$ term: a shift of the mean electron energy to higher energy. On the other hand, $(dE/dt)_D$ represents the diffusion of energies away from the mean via random walk: a spreading of the distribution. The relative importance of the two is fixed by their ratio

$$\zeta = \frac{(dE/dt)_S}{(dE/dt)_D} = \frac{B}{\delta B}$$

(2.18)

The combined result of the action of both processes on an initially narrow Gaussian energy distribution is shown in Fig.2.2 where we have chosen $\zeta = 65$. and assume that...
electrons do not escape. Thus we can examine the evolution of electron energy spectra solely due to the influence of the two acceleration rates. As $\zeta$ is increased, the steady (mean growth) term becomes increasingly dominant over the diffusive (distribution widening) term. To understand STFA in a particular plasma, both acceleration rates must be calculated. In the event that the diffusive rate is very small compared to the steady growth rate, it can be ignored. The steady growth rate is always faster than the diffusive growth rate for $a > 1$. As will be shown later, $\zeta \sim 100$ in flare plasmas, and the diffusive term is negligible.

It is very important to note that our result differs from the standard for Fermi Acceleration, in which both the diffusive and steady terms depend on the same power of $\delta B/B$. This difference arises as a result of the averaging over pitch angles. To correctly obtain $\langle \Delta E \rangle$, one must only average over those encounters which result in a reflection. For traditional STFA, the range of pitch angles which reflect is ultimately determined by the turbulent magnetic field strength. One factor of $\cos(\theta)$ in the expression to be averaged results in one factor of $(\delta B/B)^{1/2}$ in the acceleration rate. In other treatments, such as that of Longair (1994); Webb (1983); Skilling (1975), the acceleration mechanism is assumed to act at all pitch angles. In this case, the averaging over $\cos(\theta)$ while maintaining the assumption of pitch angle isotropy, yields a numerical value with no $\delta B$ dependence. In this regime, the diffusive and steady acceleration terms have the same relative strength at all levels of turbulence.

2.3.3 Specification of the Turbulent Cascade

This leaves $\delta B/B$ as the remaining parameter to be determined. It is related to the turbulent length scale $\lambda_T$ through the cascade law. Magnetohydrodynamic (MHD) turbulence proceeds by the shredding of like sized eddies and subsequent formation of smaller eddies; energy input into eddies on a large (outer) length scale, $L_T$, cascades rapidly to smaller length scales on the eddy turnover time, $\tau_{ed}$, and finally dissipates at $\lambda_r$, the dissipation scale. If the cascade obeys Komolgorov’s steady state assumption then the energy flow through all length scales is constant, and independent of the scale. This results in an inertial range between the scales $L_T$ and $\lambda_r$ where the turbulent energy density has a power law dependence on eddy size. The draining of turbulence at eddy size $\lambda_r$ is usually determined by a micro-physical process, such as resistivity.
When STFA is active, the turbulence can instead be drained by pumping energy into electrons. This sets another condition for STFA to proceed in a plasma: there must be some $\lambda_{SF}$ which is greater than the resistive length scale at which the STFA timescale is shorter than the cascade time, otherwise the turbulence will drain at the resistive scale before STFA can produce an appreciable electron acceleration.

There are three major MHD wavemodes: Alfvén waves, and the fast and slow magnetosonic waves. Alfvén waves are purely transverse, and thus do not compress the magnetic field; they cannot participate in STFA. Both the fast and slow modes are compressive, and are in principle capable of Fermi acceleration. It has been argued that in low $\beta$ plasmas such as the solar corona, the slow mode is rapidly dissipated via Landau damping (Achterberg 1981). However, more recent studies of MHD turbulence indicate that the cascade time for GS turbulence is significantly shorter than the electron damping time, and slow mode damping by electrons can be ignored in turbulent flare plasmas (Lithwick and Goldreich 2001). A key difference in the two analyses is that Lithwick and Goldreich (2001) treats MHD turbulence as inherently anisotropic, whereas Achterberg (1981) assumes isotropy. Furthermore, Maron (private communication) has shown in simulations which neglect damping that the slow mode may be driven with much higher total energy content than the fast mode at low $\beta$. A definitive resolution of the issue is beyond the scope of this paper. However, we should point out that up to this point, the calculation is independent of the choice of wave mode. There is one significant difference between the two, however: slow modes propagate at roughly the sound speed ($v_c \sim c_s$) while fast modes propagate at roughly the Alfvén speed ($v_c \sim v_A$). It will be shown that due to the influence of pitch angle scattering, STFA is likely dominated by the fast mode.

MHD turbulence is in general anisotropic; the direction of any large scale mean magnetic field defines a preferred axis. Also, even if the turbulence is isotropic on large scales, smaller scales may see the larger scale turbulent structures as an effective mean field. Goldreich and Sridhar (1997) (hereafter, GS) modified the the Komolgorov assumption for the cascade of slow and Alfvén modes of MHD turbulence under the condition that the turbulence is anisotropic with scale $\lambda_{||}$ along the field line and $\lambda_{\perp}$ perpendicular to the field line. The two directions are found to obey different cascade laws, with $\lambda_{||}$ cascading more weakly than $\lambda_{\perp}$. The parallel direction is of
more importance to STFA, as it represents the distance along the field line between reflection sites. The GS power law for the parallel scale is (Goldreich and Sridhar 1997; Lithwick and Goldreich 2001)

$$\frac{\delta B}{B} = \left(\frac{\lambda_T}{L_T}\right)^{1/2},$$

(2.19)

where $B$ is the mean magnetic field strength, and $\delta B$ is the turbulent field strength at parallel length scale $\lambda_T$.

The exact power law of MHD turbulence remains the subject of some debate, so for now we assume a general power law of form

$$\frac{\delta B}{B} = \left(\frac{\lambda_T}{L_T}\right)^{1/a},$$

(2.20)

where $a > 1$ is an arbitrary index. Using the turbulent power spectrum and substituting for $F$ from (2.15) we can rewrite the acceleration rates as

$$\left(\frac{dE}{dt}\right)_S = \frac{2m}{\lambda_T} v_A^2 v \left(\frac{\lambda_T}{L_T}\right)^{1/a} = \frac{2}{L} m v_A^2 v \left(\frac{\lambda_T}{L_T}\right)^{(1/a)-1}$$

so that $\tau_S = \frac{1}{4} \frac{v L_T}{v_A} \left(\frac{\lambda_T}{L_T}\right)^{1-(1/a)}$

$$\left(\frac{dE}{dt}\right)_D = \frac{2m}{\lambda_T} v_A^2 v \left(\frac{\lambda_T}{L_T}\right)^{2/a} = \frac{m}{L} v_A^2 v \left(\frac{\lambda_T}{L_T}\right)^{(2/a)-1}$$

so that $\tau_D = \frac{1}{4} \frac{v L_T}{v_A} \left(\frac{\lambda_T}{L_T}\right)^{1-(2/a)}$.

For a typical turbulent cascade, where $\lambda_T < L_T$ and $a > 0$, $\tau_S < \tau_D$.

### 2.3.4 $\lambda_{SF}$ and the Role of Pitch Angle Scattering

There remains the final step of determining the particular dissipation scale $\lambda_{SF}$ at which the energy drain takes place. In order for STFA to overcome the cascade of MHD turbulence, it must drain energy at a rate equal to the input rate at the outer scale. If turbulent compressions are shredded and cascade faster than electrons can draw out energy via reflections, then the cascade continues to smaller length scales. At smaller scales, STFA is more rapid. STFA becomes competitive with the cascade at
a scale determined by \( (dE/dt)_S = dE_T/dt \). \( (dE/dt)_S \) is acceleration rate of electrons and \( dE_T/dt \) is the cascade rate of turbulent energy, \( n m_p v^3 / L_T \). The STFA scale \( \lambda_{SF} \) is different for acceleration by the two compressive MHD modes.

For slow mode turbulence, \( v_c = c_s \), and the balance is

\[
\frac{2n m_e e^2 v}{L_T} \left( \frac{\lambda_T}{L_T} \right)^{(1/a)-1} = \frac{n m_p v_A^3}{L_T}.
\]

Solving for \( \lambda_T / L_T \) and associating this particular \( \lambda_T \) with \( \lambda_{SF} \) gives

\[
\frac{\lambda_{SF}}{L_T} = \left[ \frac{1 m_p v_A^3}{2 m_e e^2 v} \right]^\frac{1}{1-a}.
\]

In solar flares and a GS turbulent cascade \( (a = 2) \), \( v_0 = 1.2 \times 10^6 \text{cm/s} \), \( c_s = 3 \times 10^7 \text{cm/s} \) and \( v_A = 1 \times 10^8 \text{cm/s} \) (LaRosa et al. 1996), this gives \( \lambda_{SF}/L_T \sim 10^{-6} \). We have taken the initial electron velocity to be the mean velocity of the thermal background plasma. The cascade will proceed down the inertial range to this length scale where STFA then acts as the micro-physical damping agent, rapidly draining the energy from turbulence into particles.

In the case of the fast mode, where \( v_c = v_A \), the rate balance is

\[
\frac{2n m_e e^2 v}{L_T} v_A^2 \left( \frac{\lambda_T}{L_T} \right)^{(1/a)-1} = \frac{n m_p v_A^3}{L_T},
\]

and the STFA length scale is then given by

\[
\frac{\lambda_{SF}}{L_T} = \left[ \frac{1 m_p v_A}{2 m_e e v} \right]^\frac{1}{1-a}.
\]

For solar flare conditions, and a GS cascade \( (a = 2) \), \( \lambda_{SF}/L_T = 10^{-4} \). We have tacitly assumed that the length scale for pitch angle isotropization is roughly equal to \( \lambda_T \). If it is not, the acceleration rate is retarded significantly, and STFA can be shut off. To understand this we must further explore the role of pitch angle scattering.

As discussed above, pitch angle scattering is necessary during acceleration to maintain a population of electrons which satisfy the pitch angle condition for reflection. The strength of the pitch angle scattering strongly regulates the rate of acceleration. We consider three cases: \( \lambda_p \gg \lambda_{SF}, \lambda_p \ll \lambda_{SF}, \) and \( \lambda_p \sim \lambda_{SF} \) where eddy \( \lambda_p \) is the typical distance over which pitch angles are isotropized. In the first case, \( \lambda_p \gg \lambda_{SF}, \)
electrons reflect a few times and quickly leave the pitch angle range in which they can reflect. They then must stream a distance of order $\lambda_p$ before they can scatter again. Thus the rate of reflections and the acceleration rate are both decreased by a factor of $\lambda_{SF}/\lambda_p$. Since the acceleration rate and cascade rate are not in balance, the cascade continues down to smaller scales $\lambda_T < \lambda_{SF}$. The nominal acceleration rate (eq 2.22) is proportional to $\lambda_T^{-1/2}$, while the retardation factor is proportional to $\lambda_T$. The combined effect is a net acceleration rate which is proportional to $\lambda_T^{1/2}$; smaller scale turbulence is actually less efficient as an accelerator. As a result, STFA never turns on in this regime. In the second case, $\lambda_p \ll \lambda_{SF}$, the pitch angle scattering is far more rapid than acceleration. Since pitch scattering can take an electron through $\mu = 0$, very strongly pitch scattered electrons traverse the plasma by random walking in steps of length $\lambda_p$, again reducing the rate of reflection, this time by a factor of $(\lambda_p/\lambda_{SF})^2$. Unlike the previous case, this is not a problem for STFA; the retarding factor tends towards unity as the cascade continues to scales $\lambda_T < \lambda_{SF}$. The net acceleration rate is proportional to $\lambda_T^{-5/2}$, and STFA turns on as the cascade proceeds to a sufficiently small scale. In case three, where $\lambda_p \sim \lambda_S$, pitch angle scattering and reflections proceed at the same rate. Thus, electrons are capable of streaming freely from compression to compression, while they maintain a nearly isotropic pitch angle distribution. This is the simplest pitch angle scattering regime for STFA.

The identity of the accelerating wavemode is now easy to determine. In section 2.4.1 we show that whistler wave turbulence is a plausible source of pitch angle scattering, at least for the lower energy quasi-thermal component of the spectrum. At 3keV, $\lambda_{wh}/L_T$ is roughly $10^{-4}$. This places slow mode turbulence ($\lambda_{SF}/L_T \sim 10^{-6}$) well in the first regime. Slow modes do not participate in STFA in these flares. Fast modes, however, have $\lambda_{SF}/L_T \sim 10^{-4}$ and therefore are in the nearly ideal range for acceleration. Furthermore, both $\lambda_{SF}$ and $\lambda_p$ grow linearly with electron energy, so as electrons undergo STFA by fast modes in the presence of whistler wave turbulence, they remain in the same pitch angle scattering regime throughout.

### 2.4 The post-acceleration spectrum

The simplest case of STFA is the steady state, where we assume that electrons are injected into a turbulent region at energy $E_0$ at a rate equal to that at which accelerated
electrons escape. The turbulent energy supply is continuously replenished at a large scale. We are concerned with the energy spectrum, $N(E)$, of electrons escaping the region. Note that this is in general different from the spectrum of the electrons within the turbulent region. We define $N_t(E)$ to be the total number of electrons reaching energy at least $E$ before escaping, such that

$$N(E) \propto -dN_t(E)/dE.$$  \hspace{1cm} (2.26)

Initially, we consider the case of strongly relativistic electrons; a full derivation of this regime is presented in the Appendix.

To appreciate the calculational differences between the non-relativistic regime of interest to solar flares and the more commonly studied relativistic regime, we begin with the latter. Following the approach used by Bell (1978) for Shock Fermi acceleration, and writing $dN_t/dM = -p_{esc}N_t$, where $p_{esc}$ is the mean probability of an electron escaping from the acceleration region, gives

$$\frac{dN_t}{dE} = - \frac{dN_t}{dM} \frac{dM}{dE} = -p_{esc}N_t \frac{dM}{dE} = -p_{esc}N_t \frac{1}{\alpha E},$$  \hspace{1cm} (2.27)

where on the right hand side we have for the moment taken the strongly relativistic limit: $dE/dM = \alpha E$ and assumed $p_{esc}$ to be constant. This treatment of the highly relativistic limit follows Fermi (1949). One can solve for $N_t(E)$ by separating variables, integrating both sides and inverting the logarithms, resulting in the familiar power law (see e.g. Fermi (1949); Longair (1994); Jones (1994))

$$N_t(E) = N \left( \frac{E}{E_0} \right)^{-\frac{p_{esc}}{\alpha}},$$  \hspace{1cm} (2.28)

where $N$ is the total number of electrons. From Eq. (2.26), one obtains

$$N(E) = N_0 \left( \frac{E}{E_0} \right)^{-(1+\frac{p_{esc}}{\alpha})},$$  \hspace{1cm} (2.29)

where $N_0dE$ is the number density of escaped electrons with $E = E_0$. Notice that the logarithmic integrals in both $N_t$ and $E$ are vital to producing the power law.

For STFA by fast mode waves, the computation is more complicated because $p_{esc}$ is energy dependent. We solve for a general $p_{esc}$ in the non-relativistic regime, leaving the specification of the trapping for later discussion. In the non-relativistic regime,
the acceleration rate is not proportional to the kinetic energy as it is in the strongly relativistic regime. Instead, one has \( \frac{dE}{dM} = 4mv_A^2 \). We thus write, using (2.11),

\[
\frac{dN_t}{dE} = \frac{Ntp_{\text{esc}}}{4mv_A^2},
\]

which can be rewritten as

\[
\frac{dN_t}{N_t} = \frac{p_{\text{esc}}}{4mv_A^2} dE.
\]

We have assumed that \( \frac{dN_T}{dM} = p_{\text{esc}}N_t \). This is reasonable as long as the electrons can be treated as statistically independent and collisionless. In this case, it is reasonable to assume that \( p_{\text{esc}} \) carries no inherent dependence on \( N_t \) and the escape rate is simply given by the product of the number of electrons in the volume and the mean escape probability. Taking \( p_{\text{esc}} = p_0(E/E_0)^{-1} \) allows us to solve for a particularly interesting \( N(E) \). We see immediately that

\[
\frac{dN_t}{N_t} = -\frac{E_0}{E} \frac{p_0}{4mv_A^2} dE,
\]

and \( N(E) \) is again a power law energy distribution:

\[
N(E) = N_0 \left( \frac{E}{E_0} \right)^{-(1+\delta)},
\]

where \( \delta = p_0E_0/4mv_A^2 \). However, in any other case, STFA does not produce a simple power law. The importance of the trapping mechanism is now clear; the combined energy dependence of the acceleration and escape must be \( E^{-1} \) to produce a power law spectrum. Such a spectrum relies on the coincidental logarithmic integrals over both \( N \) and \( E \).

### 2.4.1 Calculation of \( p_{\text{esc}} \)

Let us now calculate \( p_{\text{esc}} \) for non-relativistic electrons within the turbulent volume, and consequently the energy spectrum of electrons. For simplicity, let us take the turbulent region to be rectangular, with the long axis, \( z \), parallel to the direction of the bulk flow, with \( z = 0 \) and \( z = L_F \) fixed to the downstream and upstream boundaries of the region respectively. \( L_F \) is taken to be the extent of the region of turbulent flow,
which is presumed to be the entire distance between the reconnection sheet and the
top of the soft X-ray loop. This distance is typically of size $10^{10}$ cm for solar flares
(Tsuneta 1996). The largest eddy size in the turbulence, $L_T$ is set by the width of
the outflow, typically $10^8$ cm. Thus the turbulent volume consists of a number of cells,
each of which flows downward from the reconnection point towards the loop-top. An
electron escapes the acceleration region only when it reaches the X-ray loop at the base
of the turbulent region. These individual cells may be associated with single bursts
or fragments of X-ray emission, and thus are responsible for the temporal structure
of impulsive flares. In order to escape the region with energy $E(M)$, an electron
must stream from its location in the region at some height $z$ to the boundary at
$z = 0$ after the $M$th reflection without further reflection. We will assume that the
electrons are contained in the region in the $x-y$ plane by gyration around large scale
field lines. To further simplify the problem, we shall assume that the electron density
remains uniform throughout the turbulent region. We also neglect the bulk flow speed,
$v_f = 8 \times 10^7$ cm s$^{-1}$ (Tsuneta 1996) since the length of the downflow region is roughly
$10^{10}$ cm. This gives a flow time from the reconnection region to the loop-top of 100 s.
The acceleration process is fixed to the much shorter 1 s time scale by the temporal
size of the observed energy release fragments and the MHD eddy turnover time. Thus,
bulk flow into the flare loop is not likely to be a dominant process in cutting off the
acceleration.

Take the mean $z$-component of the distance streamed between reflections to be $\lambda_z$;
$\lambda_z$ carries an energy dependence inherited from the energy dependence of the pitch
scattering. The probability of escaping at $z = 0$ after the $M$th reflection from a point
at height $z$ is given by

$$p_{esc}(z) = \frac{1}{2} e^{-z/\lambda_z}, \quad (2.34)$$

and the mean escape probability of electrons distributed uniformly across the length
of the region is

$$p_{esc} = \frac{1}{L_F} \int_0^{L_F} p_{esc}(z) dz = \frac{\lambda_p}{L_F} (1 - e^{-2L_F/\lambda_p}), \quad (2.35)$$

where we have taken $\lambda_z = \lambda_p/2$ from the isotropy in pitch angles, with $\lambda_p$ the pitch.
angle scattering length scale. To obtain the spectrum of solar flare electrons requires specification of the pitch angle scattering.

Both Miller, Larosa, & Moore (1996) and LaRosa et al. (1996) assume strong scattering, and suggest that the scattering agent above 1 keV is resonant interaction with lower hybrid (LH) turbulence, or circularly polarized electromagnetic waves, such as whistler waves. Below 1 keV, Coulomb interactions are thought to be sufficiently strong to isotropize the electrons. It should be noted that Melrose (1974) sets the threshold for the whistler mode resonance at 25 keV for flare plasmas, while Miller & Steinacker (1992) argue that the resonances extend down to 1 keV. We assume the latter. In a recent study, Luo, et al. (2003) considered whether LH wave turbulence is the primary mode of electron acceleration in solar flares. They concluded that the pitch angle scattering is too inefficient to maintain isotropy. Thus, we assume that LH wave turbulence cannot supply sufficient pitch angle scattering to sustain STFA either. Whistler waves are more promising.

Melrose (1974) associates the frequency of pitch angle scattering $\nu$ with the pitch angle diffusion coefficient in the quasi-linear equation. Thus, $1/\nu$ is the characteristic time scale for effective pitch angle isotropization of the electron distribution. It should be noted that, in general, some small anisotropy is likely to remain in the distribution, and that this anisotropy could be responsible for the generation of the whistler waves. However, the source of these waves is still uncertain. From Melrose (1974), we have that

$$\nu = \frac{\epsilon(\omega_R)}{\gamma_e \Omega_e n_e m_e c^2},$$

(2.36)

where $\gamma_e \sim 1$ is the Lorentz factor of the electron, $\Omega_e$ is the electron gyrofrequency, $\omega_p$ is the plasma oscillation frequency, and

$$\epsilon(\omega_R) = \frac{m_p}{2} n_p \delta B^2,$$

(2.37)

is the energy density of the turbulence at the resonant wavelength. This allows us to rewrite (2.36) as

$$\nu = 5 \times 10^7 B_{100} n_{10}^{-1/2} \left( \frac{\lambda_T}{L_T} \right)^{1/2},$$

(2.38)
where we have used $v_F^2 = B^2/4\pi n_p$, $\Omega_e = 1.8 \times 10^8 B_{100}$, $\omega_p = 5.7 \times 10^9 n_{10}^{1/2}$, and the dimensionless parameters $B_{100} = B/100\text{G}$ and $n_{10} = n_e/10^{10}\text{cm}^{-3}$.

We must compare $\nu$ to the growth time for pitch angle anisotropy due to STFA. Bearing in mind that for STFA, $dv_{\perp}/dt = 0$,

$$\frac{dE}{dt} = m v_{\parallel} \frac{dv_{\parallel}}{dt}, \tag{2.39}$$

and the pitch angle evolves according to

$$\frac{d(\cos \theta)}{dt} = \frac{1}{m v^2 \cos \theta} \frac{dE}{dt}. \tag{2.40}$$

Substituting in from equation (21) for $dE/dt$ and assuming a GS cascade ($a = 2$), gives

$$\nu_{SF} = \frac{d(\cos \theta)}{dt} = \frac{2}{L_T} \left( \frac{\lambda_T}{L_T} \right)^{(1/2)} \frac{v_F^2}{\nu} \frac{1}{\nu \cos \theta}. \tag{2.41}$$

For impulsive flares, $E_0 = 0.3\text{keV}$, $B_{100} = 2$, and $n_{10} = 1$. Taking $\lambda_T = \lambda_{SF}$ results in $\nu = 1 \times 10^6 \text{s}^{-1}$ and

$$\nu_{SF} = 2 \times 10^2 \left( \frac{E_0}{E} \right)^{1/2} \text{s}^{-1}, \tag{2.42}$$

where $\nu_{SF}$ is evaluated at the threshold pitch angle for reflection. To maintain pitch angle isotropy, scattering by whistler waves must occur on a time scale shorter than pitch angle evolution by STFA. Thus, as long as $\nu_{SF} < \nu$, isotropy can be maintained. This condition is met for all $E > E_0$. Whistler modes, if present, are capable of providing sufficient pitch angle scattering to maintain isotropy.

In addition to maintaining pitch angle isotropy, the scattering mechanism must also operate at a length scale which traps electrons in the volume; $\lambda_{wh} \ll L_T = 10^6\text{cm}$ must be satisfied, or else electrons rapidly leave the acceleration region and STFA shuts off. We obtain the pitch scattering length scale for whistler waves, $\lambda_{wh}$,

$$\lambda_{wh} = \frac{\nu}{\nu} = 2 \times 10^3 \left( \frac{E}{E_0} \right)^{1/2} \text{cm}. \tag{2.43}$$

The required condition is satisfied for energies below 100kev.
2.4.2 The Electron Spectrum and Constraints on the Secondary Pitch Angle Scattering

In order to obtain the spectrum of the escaped electrons, we can now substitute the functional form for $p_{esc}$ from (2.35) into (2.31)

$$\frac{dN_t}{N_t} = \frac{p_{esc}}{4mv_A^2} dE = \frac{\lambda_p}{L_F} (1 - e^{-L_F/\lambda_p}) \frac{1}{4mv_A^2} dE. \quad (2.44)$$

Next, by choosing $\lambda_p = \lambda_{wh}$, we use $\lambda_p/L_F = AE^{1/2}$, with $E$ in units of keV, and rearranging, obtain

$$N(E) = \frac{dN_t}{dE} = AE^{1/2}(1 - e^{-1/AE^{1/2}}) \frac{1}{4mv_A^2} N_t. \quad (2.45)$$

The resulting spectrum, $N(E)$, is plotted in figure 2.4. This is consistent with the thermal component to the flare spectrum observed using RHESSI (Krucker, & Lin 2002).

In addition to the thermal component, RHESSI observations show a clear power law region extending from roughly 10keV up to at least 50keV, above which the data are uncertain, but consistent with a continuing power law. Previous observations using the ISEE3/ICE instrument also show a power law throughout the range of the instrument, 25 – 300keV; the spectrum typically breaks downward at 100keV (Dulk et.al. 1992). More recent observations with RHESSI could push the low energy threshold for the power law as high as 35keV (Holman et. al. 2003). The spectral index below the break is $\sim 3$, while above the break it is $\sim 4$. Elsewhere (Blackman (1997); Selkowitz and Blackman (2004) in preparation), we discuss first order acceleration at the loop-top fast shock. Fast shocks are well known to accelerate super-thermal particles to power law energy spectra, even in the non-relativistic limit (Bell 1978).

However, in order to be accelerated, electrons must satisfy the requirement that $E \geq (m_p/m_e)v_s^2 = 10$keV in solar flare plasmas, where $v_s = 10^8$cm s$^{-1}$ is the inflow speed of the plasma at the shock (Blackman & Field 1994). This places the injection energy at roughly 100keV. The correspondence of the shock injection energy and the observed break energy is noteworthy. A possible mechanism to reproduce the observations is for STFA to produce a power law spectrum in the 10 – 100keV regime which then satisfies the injection criterion for loop-top fast shocks. Since STFA by magneto sonic
turbulence in the presence of whistler wave turbulence pitch scattering is insufficient to produce the power law component. However, it is possible that a second pitch scattering agent exists which produces the power law in the 10 - 100 keV range. We examine the constraints imposed on this scattering.

To produce a power law spectrum, non-relativistic STFA requires $p_{\text{esc}} \propto E^{-1}$. From (2.35), we see that this is true only if $\lambda_p/L_p = \Gamma/E$, where $2E/\Gamma \gg 1$ and the exponential term is small. Here $\Gamma$ is a constant parameter which fixes the strength of the pitch scattering. While the physics of the pitch angle scattering is not well understood, this constrains the scattering mechanisms available to STFA. We define $\lambda_C = L_T \Gamma/E = 2 \times 10^7 (E_0/E) \text{cm}$ to be the pitch scattering length scale of the constrained mechanism, and the electron spectrum is given by

$$N(E) = N_0 \left( \frac{E}{E_0} \right)^{-\left(1+\Gamma/(4mv_0^2)\right)}.$$  \hspace{1cm} (2.46)

$\Gamma$ is constrained by the observed X-ray spectral index of $\gamma = 3$. It is a standard prediction of flare models (Brown 1971; Stepanov and Tsap 2002; Kiplinger, et al. 1984) that the spectrum of electrons accelerated above the loop-top is steeper than the spectrum of the thick target Bremstrahlung X-rays emitted at the footpoints in solar flares. We assume that to match the RHESSI X-ray data requires an index $\sim 4$ for the electrons, or

$$\Gamma = 12mv_0^2 = 0.072 \text{keV} = 0.17E_0.$$  \hspace{1cm} (2.47)

The exponential term in $p_{\text{esc}}$ is indeed small as $2E/\Gamma = 270$ at $E = 10 \text{keV}$. The electron spectral index could conceivably be as high as 6, in which case $\Gamma = 0.28E_0$, and the exponential can still be safely neglected.

It is insufficient to merely produce the proper power law. The scattering agent must also be able to reproduce the transition from thermal to power law spectrum at the correct energy, $E_C$. We recall $\lambda_{wh}$ to be the length scale of pitch angle scattering associated with whistler wave turbulence. In impulsive flares, $\lambda_p = \lambda_{wh}$ below $E_C$. Above $E_C$, $\lambda_p = \lambda_C$. To obtain $E_C$ one sets $\lambda_{wh} = \lambda_C$. $E_C = 23 \text{keV}$, which is consistent with the observed threshold of $\sim 10 \text{keV}$.

There is one more important constraint imposed by the observations: the knee at 100 keV. Dulk et al. (1992) demonstrate a distinct downward break in the power law
spectrum at roughly $E_{br} = 100\text{keV}$. The break energy varies somewhat from flare to flare, but is consistently observed in all of the impulsive flares in their sample. Unlike downward breaks, upward breaks are easily explained by the meshing of two acceleration mechanisms, as the shallow component which dominates above the break emerges naturally from beneath the steeper power law which dominates below. For upward breaks, $E_{br}$ is the naturally occurring crossover point. The matching problem is much more difficult for knees in the absence of significant cooling on timescales of interest. Since the steep component is above the break energy and the shallow component below it, both must be truncated at the break energy. If either one extended beyond the break, then that one would overrun the other, and there would be no break at all. The most natural solution for a knee is a single acceleration mechanism which undergoes some transition at the break energy. One such example is the knee found by Bell (1978b) in the spectra of shock accelerated electrons at roughly $1\text{GeV}$. This knee results from the transition from the non-relativistic to the relativistic regime. There is no apparent natural transition for STFA of electrons at $100\text{keV}$. However, there is a well defined low energy cutoff for a power law spectrum at $100\text{keV}$, the shock injection energy. The shock injection threshold is not only at the right energy, but is also a variable cutoff, depending on the ion temperature and local magnetic field strength, consistent with the variability in the observed $E_{br}$.

Bell (1978b) has shown that shock acceleration does not change the spectrum of electrons if the pre-shock spectrum is shallower than the post-shock spectrum which obtains from a steep pre-shock spectrum. Shock Fermi acceleration cannot steepen a power law spectrum; it can only make it shallower. This is another difficulty for knee matching. The STFA spectrum must have a sharp cutoff at $E_{br}$ in order to match the knee. This may not be an impossible condition to meet, especially as $E_{br}$ may be greater than the injection energy, not precisely equal to it.

One natural cutoff occurs when $\lambda_G = \lambda_r$; acceleration will shut off when the cascade reaches the resistive scale. For slow modes in impulsive flares, $\lambda_r = 10^3\text{cm}$ (LaRosa et al. 1996; Chandran 2003; Lithwick and Goldreich 2001), which for $\Gamma = 0.073\text{keV}$ gives a cutoff energy of $7 \times 10^5\text{keV}$, which is both too high and far outside of the non-relativistic regime. A possible solution is that the constrained pitch scattering has a maximum resonant threshold at $E_{br}$. Another possible solution is that yet another
very strong pitch scattering mechanism has a threshold energy of $E_{br}$ and a length scale $< \lambda_r$. Both of these solutions are presently ad hoc. This underscores the need for a more thorough understanding of pitch angle scattering in astrophysical plasmas. It also illustrates the limitations of STFA as an acceleration mechanism in solar flares; if STFA alone were to account for the spectrum from $10 - 100$ keV, the tight constraints on the pitch angle scattering mechanism that we have identified are required.

2.5 Summary and Discussion

STFA in the non-relativistic limit behaves differently from highly relativistic STFA. At the core of these differences is the energy dependence of the electron velocity at low energies. Thus, unlike the relativistic case, both the rate of reflections and the probability of escaping the acceleration region at an energy $E$ vary. Using a test particle approach, we have examined this behavior and derived the spectrum of post-acceleration electrons in a plasma under impulsive solar flare conditions.

For traditional STFA, where there is a minimum pitch angle constraint which determines whether an individual encounter results in reflection, it is seen that the steady acceleration rate can dominate over the diffusive acceleration rate. This arises from the averaging over pitch angles to evaluate $\langle \Delta E \rangle$. Some previous treatments of the generalized Fermi acceleration problem do not have such reflection conditions, and thus do not retain factors of the turbulent field strength, $\delta B/B$, when averaging. In those treatments, such as Longair (1994); Skilling (1975); Webb (1983), the steady and diffusive terms typically are seen to be of the same order. For some processes this is appropriate, however non-resonant STFA is not one of them. Thus, the phase space conditions for scattering by the acceleration mechanism can play a very significant role, even in cases where pitch angle isotropy is maintained.

The nature of the pitch angle scattering turns out to be the dominant factor in determining electron escape, and therefore the shape of the spectrum. We find that whistler wave turbulence, which is well studied in solar flares (Melrose 1974; Miller & Steinacker 1992), is an excellent source of pitch angle scattering which allows STFA to produce a quasi-thermal electron distribution that peaks at $E \approx 5$keV. This matches the lowest energy portion of the observed X-ray emission very well. However, to produce the power law spectrum observed in the range $\sim 10 - 100$keV by STFA
requires at least a second scattering mechanism. Matching the spectral index and the
transition energy from quasi-thermal to power law spectrum requires an undetermined
scattering mechanism which satisfies $\lambda_C/L_P = \Gamma/E$ with $\Gamma = 0.073\text{keV}$, and naturally
becomes the dominant pitch angle scatterer at roughly 20keV.

If the constrained pitch angle scattering mechanism is discovered, it implies that
the acceleration of electrons in solar flares is at least a two stage process. The first
stage, STFA in the downflow region, produces both the quasi-thermal spectrum below
$\sim 10\text{keV}$ and the lower half of the power law spectrum up to 100keV. To produce the
highest energy electrons, as well as the spectral break at $E = 100\text{keV}$ requires a second
acceleration mechanism at the top of the soft X-ray loop. We are further exploring the
possibility that first order acceleration at a weak fast shock, formed as the downflow
impacts the top of the closed flare loop, is responsible for electron acceleration to the
highest observed energies. Acceleration at fast shocks is known to have an injection
energy of roughly 100keV, and varies with temperature. This coincides with the break
in the power law spectrum at 100keV, and is consistent with the variability observed
by Dulk et al. (1992) in $E_{br}$.

Recently, Chandran (2003) concluded, using quasi-linear theory, that STFA for
slow modes is not viable in the 10-100keV regime. While we also find slow modes to
be ineffective, differences between our paper and Chandran (2003) must be kept in
mind. Chandran (2003) assumed that $dp/dt \propto p$ for STFA. While this is true in the
strongly relativistic limit for STFA, we do not assume that this is true in the lower
energy regime (see eq. 22). Second, unlike Chandran (2003) we do not assume herein
that $P_{esc}$ has to be energy independent. These two assumptions play a significant role
in shaping electron spectra.

Another concern which can be raised about the effectiveness of STFA as the elec­
tron acceleration engine in impulsive flares is the total energetics of the process. Since
STFA, as developed above, only is efficient in a short length scale regime where one
also has $\delta B/B \ll 1$ it might seem that only a small fraction of the released flare energy
is available for electron acceleration. This is not the case. The total energy contained
in single turbulent cell is given by $(1/2) m_p v_A^2 n L^3 \sim 10^{26}\text{erg}$, where $n = 10^{10}\text{cm}^{-3}$ is
the electron number density in the flare plasma. The energy in a single turbulent cell
is similar to the energy contained in one X-ray emission fragment. Although only a
fraction of the energy in one turbulent cell is ever at $\lambda_p$ at one time, it does all cascade down to $\lambda_p$ over an eddy turnover time. Thus, while $\delta B/B$ is always small, the energy throughput can still be high enough to accelerate the electrons. The similarity in total energy between a single turbulent cell and an individual impulsive X-ray fragment strongly suggests that the two are related.

Miller, et.al. (1997) estimates that as much as $\approx 94\%$ of the magnetic energy in a flare is available in the turbulence, which is sufficient to produce the high energy electrons inferred from the observed X-rays, but raises concerns about the efficiency of STFA, particularly in competition with other sources of dissipation. While we have not fully studied other dissipation mechanisms which might compete with STFA for this energy, three significant ones can be ruled out: proton acceleration by STFA, Landau damping, and resistive dissipation of the turbulence. The latter two have already been discussed. Proton acceleration is a significant concern since Fermi acceleration of protons and heavy ions was in fact the very problem Fermi intended to solve. Thermal protons in coronal flare plasmas are sub-Alfvenic and thus cannot meet the condition for mirroring (LaRosa et al. 1996; Blackman 1999). However, Miller and Roberts (1995) argues convincingly that gyroresonant interaction of protons and heavy ions with Alfven waves can accelerate them to velocities above $v_A$ on a relatively short time scale. Within their model, the ions then are accelerated by compressive magnetosonic waves at the expense of electron acceleration. Recent RHESSI observations (Hurford et al. 2003) indicate that the emission signatures of ions and electrons are spatially separated, with the ion emission associated with longer loops. Miller, Emslie, and Brown (2004) concluded that these observations are consistent with the gyroresonance model of ion acceleration; as the loops grow longer, protons are more likely to reach super-Alfvenic speeds, and thus can be accelerated by the magnetosonic waves. This second phase of acceleration need not be gyroresonant. While it appears promising, further study is required to determine if STFA models can accomodate these results.

The strong dependence of the post-acceleration electron spectrum on the pitch angle scattering agent is both a positive and negative feature. It leaves STFA considerable flexibility in matching various characteristics of solar flare X-rays which fall outside of the simple scenario studied in this paper. For example, Lin et al. (1981) first observed a superhot component in a solar flare, which has since been supported by
RHESSI observations Krucker, et.al. (2003). This thermal, or nearly thermal, spectral component is seen at energies of up to 35keV. Within our STFA framework, the super-hot emission can easily be explained by an enhancement of pitch angle scattering at lower energies, either by increased whistler wave turbulence, or some other scattering agent. While this flexibility naturally allows for the wide range of flare characteristics observed, it does not yet definitively solve the flare acceleration problem. Instead, it shifts the focus exclusively to a well constrained, but largely unspecified, array of pitch angle scattering mechanisms. This is the single greatest obstacle to STFA models of acceleration.

In short, STFA can naturally account for the thermal spectrum below 10keV, and somewhat less naturally for the non-thermal spectrum between 10keV and 100keV. There we have shown that \( \lambda_p \) must depend inversely on particle energy, in contrast to that of pitch angle scattering by whistler waves below \( \sim 10 \) kev, which is proportional to the particle energy. Above 100keV, shock acceleration is a natural possibility; the needed injection of super-thermal electrons may be provided by STFA operating at energies below \( E_{br} \). The knee at 100keV remains the most difficult spectral feature to accommodate, and we have explained the difficult requirements to pitch angle scattering that this demands.

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2.6 Appendices

2.6.1 Appendix A: The Power Law Spectrum of Relativistic STFA

Notice that the spectrum we obtain for STFA is different from the power law result of Jones (1994). This is a matter of regime; we discussed in the text the acceleration of non-relativistic particles in a region of non-relativistic turbulence, here we show that when the particles are relativistic, a power law spectrum emerges.

Recall that (2.5) for fully relativistic electrons in a region of non-relativistic turb-
Bulence is given by (2.6)

\[ \delta E_{\pm R} = 2E \left[ \pm \frac{\mathbf{v}_A \mathbf{v}_{\parallel}}{c^2} + \frac{\mathbf{v}_A^2}{c^2} \right], \]

where \( E \) is the total energy, kinetic plus rest, of the electron before reflection. Notice that if we the low velocity limit, \( v \ll c \) where \( E = mc^2 \), the expression reduces to (2.5). The relative velocity between the compression and electron for head-on and catch-up type interactions are still given by (2.8) so the steady acceleration rate is given by

\[ \left( \frac{dE}{dt} \right)_S = 2E \left[ (R_+ - R_-) \frac{\mathbf{v}_A \mathbf{v}_{\parallel}}{c^2} + R_\times \frac{\mathbf{v}_A^2}{c^2} \right] = 4F \frac{\mathbf{v}_A^2 \mathbf{v}_{\parallel}}{c^2} E. \quad (2.48) \]

Alternatively, we can find the mean acceleration per reflection by multiplying equation 2.48 by \( R^{-1} \)

\[ \left( \frac{dE}{dl} \right)_S = 4\frac{\mathbf{v}_A^2}{c^2} E. \quad (2.49) \]

In the highly relativistic limit, \( E \) is just the kinetic energy, and we recover the familiar result (Jones 1994) that \( dE/dt \propto E \). This proportionality is expected to produce a power law. We derive the power law spectrum for STFA of highly relativistic electrons by following the approach of Bell (1978) and assume that \( p_{\text{esc}} \) is a constant in flare plasmas, independent of electron energy. We start by integrating \( dE/dl \) to obtain \( E(l) \)

\[ l = \frac{1}{A} \ln \left( \frac{E}{E_0} \right), \quad (2.50) \]

where \( A = 4\frac{\mathbf{v}_A^2}{c^2} \).

The probability of an electron remaining in the acceleration region for at least \( l \) reflections is given by

\[ P(l+) = (1 - p_{\text{esc}})^l. \quad (2.51) \]

Taking the logarithm and substituting in for \( l \) from equation 2.51, gives

\[ \ln P(E+) = l \ln (1 - p_{\text{esc}}) = \frac{1}{A} \ln \left( \frac{E}{E_0} \right) \ln (1 - p_{\text{esc}}) = \ln \left( \frac{E}{E_0} \right)^{(-p_{\text{esc}}/A)^{-1}}. \quad (2.52) \]
where we used the approximation \( \ln(1 - p_{\text{esc}}) = -p_{\text{esc}} \) for \( p_{\text{esc}} \ll 1 \) in obtaining the expression to the right of the final equals sign. Differentiating with respect to \( E \), results in

\[
P(E) = E_0 \left( \frac{E}{E_0} \right)^{(-p_{\text{esc}}/A)}
\]

where \( P(E)dE \) is the unnormalized probability of a post-acceleration electron having the energy \( E \). In the limit, where \( p_{\text{esc}} \) is extremely small, the relativistic STFA spectrum has power law index \( \sim 1 \). In a plasma where \( p_{\text{esc}} \sim A \), the power law index can grow larger, and the index is very sensitive to \( p_{\text{esc}} \). In the third regime, where \( p_{\text{esc}} \gg A \), electrons stream out of the turbulent volume quickly, do not experience much acceleration, and have a very steep power law energy distribution with virtually no very high energy electrons \( (E \gg E_0) \).

### 2.6.2 Appendix B: Derivation of Steady Acceleration Rate with \( F_+ \neq F_- \)

In section 3.1 we derived the steady acceleration rate for electrons in a low \( \beta \) turbulent magnetic plasma. This derivation was contingent on the assumption that \( F_+ = F_- = F \), which is not strictly valid. Blackman (1999) calculates \( F \) for Fermi acceleration. By resetting the limits of the integral in his eq (12), and renormalizing for the smaller phase space, one arrives at

\[
F_\pm = \cos \phi_m \left[ \pm \frac{v_A}{v} + \left( 1 - \left( \frac{v_A}{v} \right)^2 \left( 1 - \cos^2 \phi_m \right) \right)^{1/2} \right],
\]

where \( \cos \phi_m \) is the minimum pitch angle at which an electron will reflect and \( v_A/v \) is the ratio of the Alfvén speed to the electron speed. We rename these quantities \( A \) and \( B \) respectively; both are small quantities. By taking a series expansion of eq (2.54) and truncating it at second order in \( B \), it can be simplified to

\[
F_\pm = A \left[ 1 \pm B - \frac{1}{2} B^2 \right].
\]

Recall that from (2.8),

\[
R_\pm = F_\pm \left( \frac{\sqrt{1 + \frac{v_A}{v}}}{2\lambda} \right) = A \left[ 1 \pm B - \frac{1}{2} B^2 \right] \frac{v}{\lambda}.
\]
From this one easily obtains

\[ R = (R_+ + R_-) = \frac{v}{\lambda} \left[ A^2 \left( 1 - \frac{1}{2}B^2 \right) + AB^2 \right], \quad (2.57) \]

and

\[ (R_+ - R_-) = \frac{v}{\lambda} \left[ A^2 B + AB \left( 1 - \frac{1}{2}B^2 \right) \right]. \quad (2.58) \]

This gives us all of the ingredients for calculating the steady acceleration from (2.7)

\[ \left( \frac{dE}{dt} \right)_S = 2m[(R_+ - R_-)v_A + (R_+ + R_-)v_{A^2}]. \]

The resulting acceleration rate is

\[ \left( \frac{dE}{dt} \right)_{S_h} = \frac{2mv^3}{\lambda} AB^2 \left[ 2A + A^2 - \frac{1}{2}B^2 - \frac{1}{2}AB^2 + B^2 \right], \quad (2.59) \]

where we have added the additional subscript \( b \) to indicate the distinction from the previously calculated rate. The steady acceleration rate found in (2.10) from the assumption \( F_+ = F_0 = F \) is

\[ \left( \frac{dE}{dt} \right)_S = \frac{4mv^3}{\lambda} A^2 B^2. \quad (2.60) \]

Note that provided \( A > 4B \) this is the largest term in (2.59). Indeed, for coronal flare plasma, \( B \sim 0.1 \) at electron energy \( E_0 \) and decreases with increasing energy while \( A \sim 0.1 \) as well at \( E_0 \), but is largely insensitive to electron energy. At the onset of the power law regime, \( E = 10keV = 30E_0 \), and \( B \sim 0.01 \); all terms of order \( B^3 \) or higher can be neglected, as can the term in \( A^3 \). Thus we can safely use the assumption \( F_+ = F_- = F \) in this regime, and (2.11) is reasonable.

2.7 Bibliography

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<table>
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<td>Turbulent outer scale</td>
<td>$\tau_S$</td>
<td>Steady STFA acceleration time</td>
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<td>$\lambda_T$</td>
<td>Turbulent eddy scale</td>
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Table 2.1 Table of length and time scales relevant to STFA in impulsive flares.
Figure 2.1 The typical downward broken power law of an impulsive flare, as observed by Dulk et al. (1992). The left panel shows the hard X-ray spectrum, with $E_{br} = 100\text{keV}$, and spectral indices above and below $E_{br}$ of 4.25 and 3 respectively. The right hand panel shows the electron spectrum in the emission region inferred from the given photon spectrum using a thick target Bremstrahlung model for the emission (Brown 1971; Tandberg-Hanssen & Emslie 1988). Again, $E_{br} = 100\text{keV}$ and the spectral indices above and below $E_{br}$ are 4 and 5.25.
Figure 2.2 The evolution of a sample electron energy distribution with an initially Gaussian velocity distribution. The peak of the distribution is evolved from the flare thermal energy, 0.2 keV to the post-STFA mean energy at 16 keV. A) The initial distribution function. B) The same distribution after being evolved only by the steady process. C) The distribution evolved through both the steady and diffusive processes. Note that the relative width of the electron energy distribution, $\Delta E / E_m$, decreases with increasing mean energy $E_m$. 

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Figure 2.3 From (Blackman 1997). Sketch of a typical impulsive solar flare. Note that the x-point reconnection occurs in the filled region at the top of the diagram. Only the downward half of the outflow is shown. Reproduced by permission of the AAS.
Figure 2.4 The spectrum of non-relativistic STFA under impulsive flare conditions with whistler wave turbulence as the only pitch angle scatterer. $E_0 = 0.3\text{keV}$.
Figure 2.5 The spectrum of non-relativistic STFA under impulsive flare conditions with whistler wave turbulence and a second source of pitch angle scattering. The second pitch angle scattering source obeys the constraints required to produce a power law. $E_0 = 0.3\text{keV}$. 
Chapter 3

The Shock Reprocessing Model of Electron Acceleration in Solar Flares
We propose a new two-stage model for acceleration of electrons in solar flares. In the first stage, electrons are accelerated stochastically in a post-reconnection turbulent downflow. The second stage is the reprocessing of a subset of these electrons as they pass through a weakly compressive fast shock above the apex of the closed flare loop on their way to the chromosphere. We call this the "shock reprocessing" model. The model reproduces the sign and magnitude of the energy dependent arrival time delays for both the pulsed and smooth component of impulsive solar flare x-rays, but requires either enhanced cooling or the presence of a loop-top trap to explain the concavity of the observed time delay energy relation for the smooth component. The model also predicts an emission site above the loop-top, as seen in the Masuda flare. The loop-top source distinguishes the shock reprocessing model from previous models. The model makes testable predictions for the energy dependence of footpoint pulse strengths and the location and spectrum of the loop-top emission, and can account for the observed soft-hard-soft trend in the spectral evolution of footpoint emission. The model also highlights the concept that magnetic reconnection provides an environment which permits multiple acceleration processes. Which combination of processes operate within a particular flare may depend on the initial conditions that determine, for example, whether the reconnection downflow is turbulent or laminar. The shock reprocessing model comprises one such combination.

3.1 Introduction

An important observational constraint on electron acceleration and transport processes in solar flares is imposed by X-ray arrival time delay measurements (Aschwanden, et.al. 1995; Aschwanden et al. 1996a,b, 1997, 1998; Aschwanden 1998; Aschwanden et al. 1999). It is observed that the non-thermal hard X-ray emission consists of two distinct components, a smooth, slowly varying background, and a pulsed, rapidly varying modulation. Detailed studies of the temporal structure of the two components show that they possess several opposite characteristics. Typically, the variations in the smooth component are first observed at lower energies, then at increasingly higher energy. The arrival time delay between 25keV and 150 keV electrons is of order 0.5 – 10s (Aschwanden et al. 1997). Conversely, the pulses are observed first in the highest energies, with lower energy photons appearing later; the lag time for 25keV photons
relative to 150keV photons is of order 50ms. The trap-precipitation model of Melrose & Brown (1976) offers one possible explanation for the time delay measurements. In this model, electrons are accelerated to high energy above the loop in the turbulent downflow beneath the flare's coronal reconnection site and are then injected into the closed flare loop on the pulse time scale. Some electrons are trapped between magnetic compressions in the loop before precipitating onto the chromosphere, while the remainder precipitate directly. The two populations produce the smooth and pulsed components respectively.

Although the trap precipitation model may plausibly account for the time delay observations in some flares, some significant uncertainties remain. While the needed injection may arise naturally as a feature of many acceleration models which are situated in the reconnection current sheet (i.e. Litvinenko (2000)), it is not clear how injection would take place in models which employ acceleration in the turbulent downflow region which may form between the reconnection sheet and the closed soft x-ray loops. In addition, LaRosa & Shore (1998) argue that the pulsed structure can be produced by fluctuations in spectral hardness instead of a pulsed injection of non-thermal electrons. Furthermore, the model predicts that coronal X-ray emission emanates from within the trap region. The trap precipitation model places the trap, and thus this emission site, within the closed flare loop itself. This is seemingly at odds with at least some observations, such as those of of Sui et al. (2004); Masuda, et.al. (1996); Alexander & Metcalf (1997) which reveal a compact non-thermal source above the loop-top during the main phase of some flares. Another disadvantage of the trap precipitation model is that it does not retain temporal information of the acceleration process; such information is lost at the injection point. A different scenario in which the properties of the acceleration mechanism are more closely linked to the emission may provide more opportunity to test both the time delay mechanism and the acceleration mechanism observationally. We propose a new model called the “shock reprocessing model” as such an alternative; it circumvents the loop injection problem, places the coronal emission above the loop-top, and retains some details of the acceleration mechanism in the footpoint emission.

The shock reprocessing model employs both second order stochastic Fermi acceleration (STFA) and first order acceleration at fast shocks. Electrons are accelerated via
STFA in the turbulent downflow region located below the reconnection point. Due to the fairly long acceleration time, typically a few seconds, variations on the eddy time, $\sim 1\text{s}$, are smoothed out in the temporal spectrum of the STFA accelerated population. STFA, operating on a time scale of $\sim 0.5 - 10\text{s}$, produces the smooth component of the X-ray emission described in Aschwanden, et.al. (1995). As the flow impinges upon the flare loop, typical flow speeds are marginally super-magnetosonic (e.g. Blackman & Field 1994). A weakly compressive fast shock forms. Because the flow is only marginally super-magnetosonic, the shock can disappear and reform as the flow speed fluctuates on the eddy time. This is a key feature of the shock reprocessing model. First order Fermi acceleration occurs at the fast shock and the transient nature of the shock can be responsible for the pulsed emission structure observed in impulsive solar flares. Furthermore, the shock is strongest above the apex of the loop, and diminishes toward the wings. The shock formation condition is typically met over a portion of the loop; as a result, only a fraction of the STFA accelerated electrons undergo the second phase of acceleration.

We discuss the details of the acceleration mechanisms in section 3.2. In section 3.3 we determine the range of shock parameters required by the shock reprocessing model, and develop a simple model of shock formation. In particular, we calculate the compression ratio and spatial filling fraction of the shocks. We also consider a hybrid model in which shock reprocessing exists concurrently with loop-top trapping. In section 3.4 we discuss the role of cooling in the STFA region in matching the observed smooth component time delays and the presence of loop-top emission sites in a small fraction of Yohkoh flares, such as the Masuda flare (Masuda et al. 1994). The distinct presence of these X-ray sources just above and exterior to the loop-top is a distinguishable feature of the shock reprocessing model when compared to the trap precipitation model. We conclude in section 3.5.

### 3.2 Flare properties and acceleration mechanisms

Before examining the shock reprocessing model in depth, we briefly review models of flare morphology and some properties of STFA and shock Fermi acceleration which will later be employed.
3.2.1 Flare properties

It is well understood that solar flares are driven by magnetic reconnection in the solar corona (Priest & Forbes 2002). The reconnected field lines relax into a closed loop structure evidenced by the soft x-ray loops which are filled with hot plasma. Hard x-rays are observed at the chromospheric footpoints of the closed loops as well coronal hard x-ray sources in some flare. The acceleration mechanisms responsible for the hard-xray producing electrons are not yet fully known; candidates include DC field acceleration (Litvinenko 2000), stochastic acceleration (Miller, LaRosa, & Moore 1996; Chandran 2004; Selkowitz & Blackman 2004), and shock acceleration (Tsuneta & Naito 1998), or combinations of these processes (Blackman & Field 1994; Blackman 1997; Somov & Kosugi 1997). Further complicating the issue is that the downflow plasma between the reconnection point and the relaxed loops may either be laminar or turbulent. Laminar flow would favor stationary (Melrose & Brown 1976), and collapsing (Karlický & Kosugi 2004) trap precipitation models of electron acceleration, which require an ordered large scale field structure. Turbulent downflow would favor stochastic acceleration models, which require turbulent field structure.

In addition, there are two possible directions of motion to consider for the evolution of the reconnection point. The standard model (i.e. Priest & Forbes (2002)) focuses on the vertical direction of motion, in which the reconnection point moves upward through the corona over time. As a result, the footpoint separation and height of the soft x-ray loop increases with time. However, recent observations with RHESSI indicate that many flares have a “zipper” morphology; the reconnection evolves laterally along an arcade structure at near constant height, with the soft x-ray loop following the reconnection point along the arcade at near constant footpoint separation and loop height (Grigis & Benz 2005). A 2-D cross section of flare morphology is shown in figure ???. The region labeled outflow may be either turbulent or laminar, depending on the local Alfvén speed and the initial gradient in the downflow across the downflow cross section (Chieuh & Zweibel 1987). The shock reprocessing model focuses primarily on vertical flare evolution in the presence of turbulent downflows.

It is important to note that there is also a variety of coronal hard x-ray emission characteristics observed. The discovery event, dubbed the Masuda flare (Masuda et al. 1994), shows hard x-ray emission above the closed loop with a non-thermal kernel.
(Alexander & Metcalf 1997) as well as a very hot thermal region. On the other hand, many coronal sources appear to be wholly thermal (Emslie 2003). Some are located inside the soft x-ray loops, not above (Veronig & Brown 2004). The variety of x-ray emission properties and reconnection morphologies implies that even if all solar flares result from a basic reconnection environment and a soft x-ray loop, they cannot all be explained within a single scenario of specific processes such as trap precipitation or shock reprocessing. Key elements, such as whether the reconnection downflow is laminar or turbulent may result in the differences between some classes of flares. A necessary condition for turbulence to form in the reconnection outflow, super-Alfvénic downflow speeds, is also an approximate result of reconnection outflows (Blackman & Field 1994; Karlický & Kosugi 2004). Ultimately, models which consider flares with and without turbulent downflows need to be comparatively explored.

### 3.2.2 Power law acceleration solely by STFA

STFA is capable of accelerating solar flare electrons out of a thermal population (Miller, LaRosa, & Moore 1996; LaRosa et al. 1996; Chandran 2004; Selkowitz & Blackman 2004) and up to high energies with a spectral shape that depends mainly on the mechanism which traps electrons in the acceleration region. In the reconnection outflow, trapping is provided by wave particle interactions which also serve as the pitch angle scattering necessary to sustain acceleration (Achterberg 1981). Below a transition energy $E_t \sim 100$keV, pitch angle scattering in solar flares may occur through interactions with whistler waves Hamilton & Petrosian (1992); LaRosa et al. (1996); Selkowitz & Blackman (2004). In the presence of these waves, STFA produces a quasi-thermal electron spectrum below $E_t$. Above $E_t$, the scattering mode required to produce a power law spectrum is constrained but undetermined. This remains a key unresolved issue for STFA. In order to produce a power law, the pitch angle scattering length must be inversely proportional to the electron energy $E$ (Selkowitz & Blackman 2004)

$$\frac{\lambda_p}{L} = \frac{\Gamma}{E},$$

where $\lambda_p$ is the pitch angle scattering length scale, $L$ is the linear size of the acceleration region, and $\Gamma$ is a constant fixed by the scattering physics. The resulting spectrum
has the form

\[ N(E) \propto E^{-\gamma_{st}}, \]  

(3.2)

where \( N(E) \) is the number of electrons with kinetic energy \( E \), and the spectral index \( \gamma_{st} \) is given by

\[ \gamma_{st} = 1 + \frac{\Gamma}{4m_pv_A^2}, \]  

(3.3)

where \( m_p \) is the proton mass and \( v_A \) is the local mean Alfvén speed in the plasma. Variations in \( \Gamma \), resulting from local changes in the pitch angle scattering length scale, produce the observed range of spectral indexes among flares. Typically, STFA electron (X-ray) spectra must produce a mean index of 4.5(3.5) to match observations (Bromund, McTiernan & Kane 1995).

### 3.2.3 Power law acceleration solely by fast shocks

Shock Fermi acceleration has also been considered as a means of producing the power law electrons required to explain flare X-rays, for example by Blackman (1997) and Tsuneta & Naito (1998). A stationary fast shock can form at the point where the turbulent reconnection outflow plows into the loop. This shock, located just above the top of the loop, can accelerate electrons into a power law distribution.

Shock Fermi acceleration is well studied (Bell 1978; Bell 1978b; Blandford & Eichler 1987). In the standard theory, charged particles gain energy from repeated transits across the shock. Since there is a net energy gain in each cycle, the process is first order and results in a rapid acceleration. The energy spectrum produced by shock Fermi acceleration in impulsive solar flares in the non-relativistic regime is a power law of the form

\[ N(E) \propto E^{-\delta_r}, \]  

(3.4)

where \( \delta_r = 3/(2r - 2) \), and \( r \) is the density compression ratio across the shock (Bell 1978b). Note that this differs from the result of \( \delta_r = (r + 2)/(r - 1) \) which applies for strongly relativistic electrons (Bell 1978). Furthermore, the shock compression ratio is related to the downflow Mach number \( \mathcal{M} \equiv v_f/c_s \).
where $\gamma$ is the adiabatic index of the plasma. From simple theoretical estimates, it is expected that the shocks are weakly compressive (Blackman & Field 1994; Blackman 1997). To fit the observationally inferred electron spectrum (Brown 1971; Bromund, McTiernan & Kane 1995) of $(\delta \nu) = 4.5$ requires a compression ratio $r = 4/3$, significantly less than the maximum of $r = 4$ for a non-cooling shock in a monatomic gas, and consistent with theoretical predictions that the shocks are weak.

Weak fast shocks are expected, but standard shock Fermi acceleration also requires injection of high energy electrons which satisfy two conditions: their gyro-radius must be larger than the shock thickness (roughly the thermal proton gyro-radius) to see the shock as thin, and the electrons must be energetic enough to scatter off of upstream Alfvén waves (Bell 1978; Blandford & Eichler 1987). The threshold energy, $E_{\text{in}}$ for injection at impulsive solar flare shocks is found to depend on the angle of obliquity between the normal to the shock surface and magnetic field lines, and ranges from 2keV at 85° to 10keV at 0° (Tsuneta & Naito 1998). In the following we propose that STFA, in the presence of pitch scattering by whistler waves, is the shock acceleration injection mechanism; it accelerates electrons from the background plasma to energies above the proton thermal energy. Note that this scenario differs from that of Tsuneta & Naito (1998) in which whistler waves are also called upon as a scattering mechanism, but the acceleration is single stage; all electron acceleration takes place at the shock in their model.

The electron spectrum from fast shock acceleration following STFA in the presence of whistler waves can be reproduced in some STFA models without shock acceleration, although the spectrum from shocks depends only on the compression ratio. However, in the case with shocks, the transition to a power law spectrum occurs at the shock injection energy, $E_{\text{in}}$. A similarity between STFA and shocks is that both have difficulty in producing downward spectral breaks from an initially steep spectrum but can produce upward breaks (hardening). It has been established by Bell (1978b) for example, that processing of an electron population by a shock can harden an already existing power law distribution, with spectral index $\gamma_{\text{in}}$, or reduce the index below $\gamma_{\text{in}}$, but cannot soften it, or raise the index above $\gamma_{\text{in}}$. If there is a power law spec-
trum downstream of a shock with a low energy cutoff below the injection energy and the shock is sufficiently strong, the upstream spectrum will be harder above $E_{in}$ but remain unchanged below $E_{in}$.

3.3 The shock reprocessing model

In the following section, we present the shock reprocessing model and compare it to the trap precipitation model of Melrose and Brown (1976).

3.3.1 Overview of the model

Observations of non-thermal X-ray emission at the footpoints of solar flares indicate that the observed emission can be divided into two components, a smoothly varying part and a rapidly varying set of pulses (Aschwanden, et.al. 1995). Observations in multiple energy bands show that for the smooth component, variations in X-ray intensity arrive first at lower energies, and later at higher energies. The pulses exhibit an opposite time delay structure; variations are seen first in higher energy bands. The series of papers by Aschwanden and collaborators (Aschwanden, et.al. 1995; Aschwanden et al. 1996a,b, 1997, 1998; Aschwanden 1998; Aschwanden et al. 1999) models the X-ray arrival time observations in solar flares within the trap precipitation model. We propose the shock reprocessing model as an alternative.

In the trap precipitation model (Fig. ??), reconnection occurs high above the flare loop. Electrons are then accelerated to high energies above the loop over a short time scale, resulting in a pool of non-thermal electrons in a near equilibrium distribution. An injection event loads a pulse of electrons onto the loop with an energy spectrum matching that of the pool above the loop-top. Injection occurs over a very short time, 50ms, matching the observed X-ray pulse durations. Injected electrons travel down the flare loop, where they encounter a magnetic mirror as field lines converge. Electrons with sufficiently high pitch angles relative to the local field are trapped at the mirror, while those with small pitch angles stream through. This divides the electrons into two populations: directly precipitating, and trap precipitating electrons. The directly precipitating electrons do not reflect at the mirror, and proceed directly to the dense footpoint region. There they emit X-rays via thick target bremsstrahlung;
these electrons retain the short pulse time scale of the injection, and thus produce the pulsed emission component. Since the pulse electrons are injected simultaneously at all energies, the main time dispersion of a pulse results from time of flight down the loop, with higher energy (faster) electrons arriving earlier than lower energy (slower) electrons, consistent with the pulse observations. The second population, trap precipitated electrons, is mirrored at the magnetic field compressions due to their large pitch angles. These electrons remain trapped between the mirrors on opposite sides of the flare loop until such time as their pitch angles scatter to sufficiently small values that they can pass through the mirrors. Aschwanden et al. (1997) find that the slow time delays are consistent with loop-top trapping, where escape from the trap is governed by the collisional scattering time which varies as $E^{3/2}$. Because the trapping time is significantly longer than the injection time, electrons from many injections escape together, creating a smooth component. As a result of the positive energy dependence of the trapping time, lower energy electrons escape the trap earlier than higher energy electrons, and the smooth emission component exhibits variations at lower energies first. The trap precipitation model offers an explanation for the two component emission, as well as the time delay data.

The shock reprocessing model is an alternative to trap precipitation. An outline of the model is shown in Fig. ??.. As in trap precipitation, the shock reprocessing model begins with reconnection high above the flare loop, followed by acceleration above the loop-top. However, in the shock reprocessing model the initial acceleration is slow and consistent with STFA in the reconnection outflow, occurring on a time scale of $0.5 - 10s$ (Chandran 2004; Selkowitz & Blackman 2004). Following STFA, the electrons flow into the region just above the loop-top. A weakly compressive fast shock forms (Blackman & Field 1994; Tsuneta 1996) at, or just above, the loop-top. The shock can vary in spatial extent and compression ratio. A fraction of the STFA accelerated electrons also pass through the shock and is accelerated a second time (reprocessed). The remainder of the electrons pass directly to the flare footpoints.

Like the trap precipitation model, the shock reprocessing model involves two populations of electrons, but in this case they are the directly streaming (STFA only) electrons and the shock reprocessed electrons. The directly streaming electrons produce the smooth X-ray component. Since there is no trapping or loop injection within
this model, variability in the produced emission results solely from the acceleration process. The $\sim 0.5 - 10$ s acceleration time for STFA (Selkowitz & Blackman 2004; Chandran 2004) is consistent with the delay time observed between 25keV and 150keV electrons in flare observations (Aschwanden, et.al. 1995; Aschwanden et al. 1996a,b, 1997, 1998; Aschwanden 1998; Aschwanden et al. 1999). The reprocessed electrons are those which pass through the shock and undergo a second, rapid acceleration. Subsequently, they travel without coronal trapping to the footpoints. Because the shock acceleration time scale is extremely short, reprocessed electrons can be considered effectively injected at the loop-top simultaneously across all energies. The number of electrons which are reprocessed is determined by the spatial extent of the shock. We proceed by developing a phenomenological procedure for determining pulse strengths, discussing shock formation and geometry as further constraints on the model, and predicting the location of the coronal emission site coincident with the STFA and cooling in a region above the loop-top.

The ability to produce the smooth delay component without a loop-top, or above the loop-top trap is a key point of distinction of the shock reprocessing model from trap precipitation models. Bellan (2003) presents some theoretical arguments as to why a loop should maintain a uniform cross section to explain observations of static loops which often appear nearly axially symmetric from footpoint to apex. If the loops that form in the trap precipitation model have time to relax before the non-thermal electrons can reach the trapping region, effectively becoming static loops, then these calculations may imply the absence of the trapping required by the trap-precipitation model. However, if the loops do not have time to relax during the flare, then the steady-state Bellan argument would not necessarily challenge the trap demanded by the trap precipitation model. Expansion of the closed loop is often assumed to occur near the loop-top and is a requirement of the trap precipitation model to create the trap in the form of a magnetic mirror. It should be noted that the shock reprocessing model can still operate in the presence of a trap. In this case, electrons incident on the footpoints are distinguished by two properties, shocked vs. unshocked and trapped vs. untrapped, resulting in four populations. Below we parameterize the model in the absence of trapping, and consider the impact of trapping on the resulting x-ray spectrum in section 3.3.6.
3.3.2 Parameterizing the model

The filling fraction of the shock, $F$, can be understood as both the fraction of the downflow cross section which encounters the shock, and as the fraction of non-thermal electrons which is reprocessed. Note that this sets the firm upper bound $F = 1$. The physics relating $F$ to shock formation is deferred to section 3.3.4. Here, we constrain $F$ from observations and power law models of the electron and x-ray spectra. We begin by assuming that the electron spectrum resulting from STFA is a single power law, given by

$$N_s = N_{0s} \left( \frac{E}{E_{0s}} \right)^{-\delta_s}, \quad (3.6)$$

where $N_s$ is the total number of electrons per unit energy per unit time passing through a slab of unit surface area coplanar with the shock (henceforth we refer to similar quantities as areal fluxes), and is therefore the total non-thermal electron population, as a function of energy. $N_{0s}$ is the areal flux of electrons at energy $E_{0s}$, the lower threshold energy for the onset of the STFA power law spectrum, and $\delta_s$ is the spectral index of the STFA electrons. The total rate of non-thermal electrons is then given by

$$N_T = A \int_{E_{0s}}^{\infty} N_s dE = \frac{A E_{0s} N_{0s}}{\delta_s - 1}, \quad (3.7)$$

where $A$ is the area of the downflow. Because of the injection criterion, shock reprocessing does not increase the total number of non-thermal electrons. As a result, the total number of reprocessed non-thermal electrons, $N_p$, can be written as

$$N_p = F A \int_{E_{or}}^{\infty} N_s dE = F A E_{or} N_{0s} \left( \frac{E_{or}}{E_{0s}} \right)^{-\delta_s} \left( \frac{E_{or}}{E_{0s}} \right)^{-1} = F E_{or} \left( \frac{E_{or}}{E_{0s}} \right)^{-1} N_T, \quad (3.8)$$

where we have taken into account only those electrons with energy above the shock injection threshold, $E_{or}$, and assume $E_{or} > E_{0s}$.

We can obtain a second expression for $N_p$ by assuming a power law of similar form to Eq. (3.6) for the reprocessed electrons,

$$N_r = N_{or} \left( \frac{E}{E_{0s}} \right)^{-\delta_r}, \quad (3.9)$$
where quantities with the subscript \( r \) are defined similarly to those with an \( s \) in Eq. (3.6), but for the shock reprocessed population. The total number of electrons in the reprocessed population, \( N_p \), is obtained analogously to Eq. (3.7) by integrating over energy

\[
N_p = FA \int_{E_{0r}}^{\infty} N_r dE = F \frac{A E_{0r} N_{0r}}{\delta_r - 1}.
\]  

(3.10)

Setting Eqs. (3.8) and (3.10) equal yields

\[
N_{0r} = N_{0s} \frac{\delta_r - 1}{\delta_s - 1} \left( \frac{E_{0r}}{E_{0s}} \right)^{-(\delta_s - 1)}. \]  

(3.11)

For thick target bremsstrahlung at the footpoints, the photon number spectrum is likewise a power law

\[
M_x = M_{0x} \left( \frac{E}{E_{0x}} \right)^{-\alpha_x},
\]  

(3.12)

where \( x \) can be either \( r \) or \( s \) for the shock and STFA components, \( M_x \) is the areal photon flux at energy \( E \), and \( M_{0x} \) is the areal photon flux at energy \( E_{0x} \). The spectral index for the photons is related to that of the electrons by \( \alpha_x = \delta_x - 1 \) (Brown 1971). Combining this with Eqs. (3.6) and (3.9) yields the additional constraint

\[
(\delta_s - 1) \frac{M_{0s}}{N_{0s}} = (\delta_r - 1) \frac{M_{0r}}{N_{0r}}.
\]  

(3.13)

Integrating both the STFA and reprocessed spectra of Eq. (3.12) from the minimum observed energy \( E_{\text{min}} \), where we assume \( E_{\text{min}} > E_{0r} > E_{0s} \), gives the areal photon flux for STFA and shock reprocessed acceleration

\[
M_T = \int_{E_{\text{min}}}^{\infty} M_x dE = \frac{M_{0s} E_{0s}}{\delta_s - 2} \left( \frac{E_{\text{min}}}{E_{0s}} \right)^{-(\delta_s - 2)},
\]  

(3.14)

and

\[
M_p = \int_{E_{\text{min}}}^{\infty} M_x dE = \frac{M_{0r} E_{0r}}{\delta_r - 2} \left( \frac{E_{\text{min}}}{E_{0r}} \right)^{-(\delta_r - 2)},
\]  

(3.15)

respectively. Using Eqs. (3.11), (3.13), (3.14), and (3.15) we can rewrite \( M_p \) in terms of \( M_T \)

\[
M_p = \left( \frac{\delta_s - 2}{\delta_r - 2} \right) E_{0r}^{\delta_r - \delta_s} E_{\text{min}}^{\delta_s - \delta_r} M_T.
\]  

(3.16)
Given the above, we can construct photon fluxes for the pulsed and smooth X-ray components given values of the parameters of the model: \( F, \delta_r, \) and \( \delta_s. \) We define the smooth and pulsed emission as follows. The smooth emission is the total rate of photon counts at the footpoints in the absence of pulses, which is simply given by Eq (3.14). The pulse emission, \( M_p, \) is the enhancement above the smooth emission, which we take at the peak of the pulse, and is given by

\[
M_p = F M_p - F M_T = F \left( \frac{\delta_s - 2}{\delta_r - 2} \right) E_{br}^{\delta_r - \delta_s} E_{\text{min}}^{-\delta_r - 1} M_T,
\]

which is the total emission by the reprocessed electrons, \( F M_p, \) less the emission that would have been produced by those electrons if they were not reprocessed, \( F M_T. \) For further simplicity, we choose to measure energy in units of \( E_{br}. \) Thus, we can write

\[
M = \frac{M_p}{M_T} = F \left[ \left( \frac{\delta_s - 2}{\delta_r - 2} \right) E_{\text{min}}^{-\delta_r - 1} \right] .
\]

The model produces pulses of magnitude \( M = M_p/M_T \) determined by four parameters: the shock filling fraction \( F, \) the STFA electron spectral index \( \delta_s, \) the reprocessed electron spectral index \( \delta_r, \) and the minimum observed energy \( E_{\text{min}}. \)

An important limitation of the above derivation is that the integral in equation 3.15 is divergent for any \( \delta_r \leq 2. \) At these spectral hardnesses, one must take into consideration the upper cutoff energy of the electron spectrum which arises from the limited energy budget of the flare. We disregard this regime both because the shocks are predicted to be weakly compressive, and because we show below that the constraints on shock formation and pulse strength place limitations on flares prohibiting such spectra.

### 3.3.3 Fitting the observed pulse strengths

Fig. ?? is a set of curves of constant pulse strength \( M = 0.1, 0.2, 0.4, 0.6 \) in \( \delta_r \) vs \( F \) space. We have taken \( E_{\text{min}} = 5E_{br}/3 \) and \( \delta_s = 4, \) consistent with the low energy observational cutoff of \( E_{\text{min}} = 25keV \) in Aschwanden, et.al. (1995) and a presumed non-thermal cutoff energy of \( \sim 15keV. \) Since \( \delta_r \) is determined by the shock compression ratio, pulsing (as opposed to steady enhancement) of the reprocessed component requires variation of the shock compression ratio, or equivalently the downflow Mach number, on the pulse time scale. The curves diverge asymptotically toward \( F = \infty \) as
\( \delta_r \) approaches \( \delta_s \), corresponding to idea that an absence of spectral hardening (when \( \delta_s = \delta_r \)) corresponds to no actual pulsing. This is expected since Bell (1978b) found that a shock can harden, but not soften, an electron spectrum. Therefore, the shock reprocessing model can only produce pulsing via reacceleration at the shock if \( \delta_r < \delta_s \), and furthermore a larger value of \( F \) is needed as the difference \( \delta_s - \delta_r \) grows small. This imposes an upper limit on \( \delta_r \) for any given observed \( M \), which is set by the \( F = 1 \) line. By definition, \( F > 1 \) represents an unphysical solution, where the number of reprocessed electrons exceeds the total number of non-thermal electrons. The shock reprocessing model thus can constrain parameter space. Any choice of \( M \) and \( \delta_r \) which predicts \( F > 1 \) represents a physically unrealizable state. For example, Fig. ?? shows that pulses stronger than \( M = 0.6 \) cannot be formed by shocks with \( \delta_r > 3.4 \), while pulses of strength \( M = 0.4 \) can be formed out to \( \delta_r = 3.6 \). Generally, the stronger the pulse, the harder the limiting shock spectrum. Finally, notice that all of the curves converge toward \( F = 0 \) at \( \delta_r = 2 \); this results from the divergence noted in the previous section.

Fig. ?? shows the effects of varying the value of \( E_{\text{min}} \) on the curves of constant \( M \) in \( F \) and \( \delta_r \) space. We have taken \( \delta_s = 4 \) for each of these plots. As we raise \( E_{\text{min}} \), the curves shift toward smaller values of \( F \) for any given \( \delta_r \). The shift results from moving the observational cutoff energy further from the onset of the reprocessing power law. The difference between the two power law spectra, \( M_r - M_s \), is greater at higher energies. Moving the lowest energy of observation further away from the shock injection energy results in stronger pulses. Therefore, the model predicts that observed pulses will be stronger in higher energy bands for any given flare. We explore the ramifications of this prediction in section 3.4. Likewise, Fig. ?? shows the effects of varying \( \delta_s \) while keeping \( E_{\text{min}} \) fixed. In this set of plots, \( E_{\text{min}} = 5E_{\text{cr}}/3 \), and \( \delta_s \) takes the values 3.5, 4, 4.5, and 5. The asymptotic divergence of \( F \) always occurs at \( \delta_r = \delta_s \). Increasing \( \delta_s \) results in the formation of stronger pulses at any given values of \( \delta_r \) and \( F \).

3.3.4 Constraints of shock formation

An additional constraint on the parameter space available for shock reprocessing can be obtained from studying shock formation. A detailed study of this problem is beyond
the scope of the current work, but a simplified treatment can provide some insight.

Fig. ?? shows a schematic model of the shock forming region. We assume that the
flow forms a shock at any point along the loop-top where the component of the flow
speed normal to the loop surface is super-fast-magnetosonic \( v_f > c_f \). Furthermore,
we take the shock to be planar, and compute an averaged compression ratio along
the shock. The loop geometry is taken to be elliptical, consistent with the time
of flight (TOF) measurements of Aschwanden et al. (1996a). Although the TOF
measurements were analyzed within the trap-precipitation framework, they remain
valid within the shock reprocessing model as well. In both cases, the pulsed emission
represents electrons which are effectively injected simultaneously at all energies at the
loop-top, and therefore the arrival time energy dispersion in the pulses represents the
TOF in both models.

The tangent line to the loop surface has a slope

\[ s = \frac{x h^2}{\sqrt{h^2 - x^2 k^2}} \quad (3.19) \]

where \( h \) is the loop height, \( 2w \) is the footpoint separation, and \( x \) is the distance from
the loop center. The component of the flow normal to the loop \( v_\perp \) is given by

\[ v_\perp = v_f \left( \frac{1}{1 + s^2} \right)^{1/2} \quad (3.20) \]

where \( v_f \) is the downflow speed. Setting Eq. (3.20) equal to the fast-magnetosonic
speed, \( c_f \), substituting for \( s \) from Eq. (3.19), and solving for \( x \) gives the critical
distance, \( x_{\text{max}} \), for shock formation. The covering fraction of the shock is given by
\( F = x_{\text{max}}/w \). The effective averaged compression ratio of the shock is given by
evaluating Eq. (3.5) at each point in the range \( 0 < x < x_{\text{max}} \) and averaging. The
resulting averaged spectral index \( \delta_r \) of electrons accelerated at the shock obtains
from Eq. (3.4). Solutions of these equations are plotted in Fig. ?? for various values
of \( h/w \). For all loop heights, \( F \) approaches 1 as the flow approaches the strong shock
limit of \( \delta_r = 1/2 \). As \( \delta_r \) grows large, the flow velocity decreases, and \( F = 0 \) at the
limit of \( v_f \leq c_f \), or \( \delta_r = \infty \). Notice that as the loop gets more elongated (larger \( h \)), \( F \)
decreases.

We overlay Fig. ?? onto panel (c) of Fig. ?? to produce panel (a) of Fig. ??, which
incorporates the two sets of constraints: the pulse strengths from the shock reprocessing model, and the shock formation physics. The physically reasonable region, \( F < 1 \) and \( \delta_r < 4 \), is delineated by the box. Selecting values for the two direct observables, loop height \( h \) and pulse strength \( M \) picks out a pair of curves, one from each set. These curves have a single intersection, fixing values of \( F \) and \( \delta_r \). Panel (b) of Fig. ?? repeats panel (a) with \( \delta_r = 5 \). Observations indicate that typically \( 0.1 < M < 0.6 \) and \( h/w \sim 1 \) (Aschwanden et al. 1996a). This places \( 3 < \delta_r < 5 \), consistent with the downstream fast shock spectral indexes calculated in Blackman & Field (1994) and the observed X-ray spectral indexes.

3.3.5 Loop-top emission and smooth component time delays

An above-the-loop-top hard X-ray source has been observed in some impulsive phase flares, most notably the Masuda flare (Masuda, et.al. 1996; Tsuneta 1996; Sui et al. 2004), but not in others. During the main phase of the Masuda flare, a kernel within the thermal source is clearly observed to be non-thermal (Alexander & Metcalf 1997). It has been argued (Blackman 1997; Tsuneta & Naito 1998) that the loop-top source is associated with trapping and acceleration of electrons near the stationary fast shock. In this scenario, the conditions for shock formation are met in a small fraction of flares, and then only marginally, resulting in weakly compressive downstream shocks in some flares, and no downstream shocks in others (Tsuneta 1996). The low shock formation rate may explain the infrequent detection of non-thermal loop-top emission. It has also been argued (Petrosian et al. 2002) that the non-thermal loop-top source is actually prevalent in impulsive flares, but usually very dim, with typical intensity less than one tenth that of the footpoint sources. The contrast limit of Yohkoh is roughly 10; observations are limited to unusually bright loop-top sources or limb flares with obscured footpoint emission. We show that the shock reprocessing model can produce these broad features. That being said, there are recent RHESSI observations of two flares which are dominated by coronal emission that is cospatial with the soft x-ray loop (Veronig & Brown 2004). We consider this latter issue further in section 3.5.

Although the Masuda flare exhibits loop-top pulses, they are longer in duration than the footpoint pulses. Loop-top pulses are typically of duration \( \geq 10s \) (Mrozek & Tomczak 2004), as opposed to the sub-second pulse structure in the footpoint emis-
sion. The pulsed component which we attribute to shock reprocessing is the sub-second pulse structure in the footpoint emission, not the much longer pulses which may come from a globally bursty reconnection. When the sub-second pulses are subtracted from the emission profile the remaining component comprises the smooth profile. In this respect, in addition to explaining the sporadic appearance of above the loop-top non-thermal sources, cooled STFA also produces the proper time delays for the smooth emission component. Aschwanden et al. (1997) analyze the arrival time delays for the smoothly varying X-ray component of 78 flares. From these, they obtain plasma densities in the soft X-ray loops for 44 events, rejecting the remaining 34. Of the rejected events, 29 were unsuitable to the analysis technique because the fast varying component could not be sufficiently deconvolved from the smooth component. The remaining 5 rejections were due to poor convergence of their fitting model. The remaining 44 flares have smooth component time delays consistent with the trap precipitation model, matching the predicted trapping time \( \tau(E) \propto E^{3/2} \).

A time delay effect is also predicted by the shock reprocessing model. Here, the pre-shock STFA rate determines the smooth component time delays. Variations in the smooth component are the result of changes in the STFA acceleration region, and appear at high energy on a time scale equal to the STFA acceleration time to that energy. The time delay curve can thus be obtained from the STFA acceleration rate:

\[
\frac{dE}{dt} = Q(E),
\]

where \( Q(E) \) is prescribed by STFA. We define

\[
\tau = -\int_{E_0}^{E} \frac{dE}{Q(E)}.
\]

For non-relativistic STFA \( Q(E) \) is energy independent (Selkowitz & Blackman 2004) and \( \tau \propto E \). For statistical acceleration in the quasi-linear regime, Chandran (2004) finds: \( Q(E) \propto E \), and \( \tau \propto e^{E/E_0} \). Neither of these forms is consistent with a good fit to \( \tau \propto E^{3/2} \). Cooling in the acceleration region, which is neglected by Selkowitz & Blackman (2004) can be invoked to reduce the acceleration rate.

We first consider the case where STFA with \textit{in situ} cooling by Bremsstrahlung occurs in a region above the loop-top shock, and show that this is not a sufficient
cooling mechanism. The power loss due to bremsstrahlung for a single electron is given by (Rybicki & Lightman 1979)

\[ P_B = 4.5 \times 10^{-5} n_{10} E_1^{1/2} \text{keV s}^{-1}, \]  

(3.23)

where the dimensionless electron density \( n_{10} = n/(10^{10}\text{cm}^{-3}) \), \( E_1 = E/1\text{keV} \), and we have assumed a constant Gaunt factor \( G_{ff} = 1 \). Notice the linear density dependence for the Bremsstrahlung rate, which arises because we consider the mean cooling rate for a single electron in a background of protons at density \( n_{10} \) (not the emission from an ensemble of electrons in a background of protons).

It has previously been noted (Brown 1971; Schatzman 1965) that Coulomb losses due to electron-electron scattering are typically greater than the Bremsstrahlung cooling rate at hard x-ray energies in flare plasmas. While this is true, within the STFA acceleration region, the electron spectrum is nearly thermal. The non-thermal post-acceleration spectrum results from the escape rate of electrons from the acceleration region (Selkowitz & Blackman 2004) and thus the escaped electrons. Coulomb scattering among electrons within the region does not strongly affect a given electron’s acceleration time; on average, each electron receives as much as much energy due to Coulomb scattering as it loses. In the surrounding area, which is filled with ambient plasma, STFA accelerated electrons which have escaped the acceleration region do interact with the surrounding cooler electrons, resulting in cooling of the accelerated electrons, reshaping of the spectrum, and heating of the ambient plasma. This heated plasma may emit the thermal loop-top X-ray component observed in a number of flares.

Unlike Coloumb scattering among electrons, Coulomb scattering of electrons with protons is a potential cooling mechanism, since solar flare protons are sub-Alfvenic and do not participate in STFA. The Coulomb loss rate from electron-proton collisions is given by (Spitzer 1956)

\[ \left( \frac{dE}{dt} \right)_{ep} = 2.5 \times 10^{-2} n_{10} E_1^{-1/2}. \]  

(3.24)

The electron-proton loss rate is significantly greater than the bremsstrahlung loss rate below \( E = 1000\text{keV} \), and electron cooling within the limited STFA acceleration region is dominated by the former process. Nevertheless, we find that it is not sufficient
to produce time delays which match observations. Taking electron-proton Coloumb
interactions as the primary source of cooling, we can subtract the cooling term from
\( Q(E) \) to get

\[
Q^*(E) = Q(E) - P_C = \left( \frac{dE}{dt} \right)_{\text{STFA}} - P_C,
\]

where \( Q^*(E) \) is the net power input and \( P_C \) is the power lost due to Coloumb cooling.
We have adopted the STFA acceleration rate of Selkowitz & Blackman (2004). This
gives

\[
Q^*(E) = 100n_{10}^{3/2} - 2.5 \times 10^{-2}n_{10}E_1^{-1/2}\text{keV},
\]

in the energy range \( 1\text{keV} < E < 200\text{keV} \). Taking the flare electron density to be \( n_{10} = 1.7 \) and performing the integral in equation 3.22 yields no significant effect on time
delay due to cooling. This remains true for any density in the range \( 0.1 < n_{10} < 10 \).
In figure ?? we plot \( \tau(E) \) for uncooled STFA as well as \( \tau \propto E^{3/2} \) (Aschwanden et
al. 1997). The \( E^{3/2} \) functional form of the delay is directly observed in the Yohkoh
data. We have selected a typical magnitude for \( \tau \) based on the observations reported
in Aschwanden et al. (1997), and a reasonable trap region density of \( n_{10} = 20 \). The
figure shows this in comparison to the shock reporcessing model with no cooling above
the trap. The density predicted by shock reprocessing, \( n_{10} \propto 1.7 \), is lower than that of
the trap-precipitation model, which is reasonable since the trap is expected to be in the
higher density loop region, while STFA occurs well above the loop in a lower density
region. Although weakly cooled or uncooled STFA is consistent with the magnitude
of time delay measurements of Aschwanden, et.al. (1995); Aschwanden et al. (1996a,b,
1997, 1998); Aschwanden (1998); Aschwanden et al. (1999) over the observed energy
range of \( 10 - 200\text{keV} \), it clearly cannot reproduce the curvature. But STFA does
provide acceleration on the correct time scale.

It is important to note that while a strong cooling mechanism is not yet demon­
strated in the acceleration region, The presence of weak non-thermal loop-top emis­
ion regions observed in Yohkoh studies of flares (Masuda, et.al. 1996; Tsuneta 1996;
Petrosian et al. 2002), implies that an additional source of cooling, such as a wave
instability, may be present. Stronger cooling would simultaneously provide an expla­
nation for this emission site as well as increase the acceleration time, and may thus
improve the fit of the STFA time curve to the observed data. Alternatively, a hybrid of trap-precipitation and shock reprocessing discussed in section 3.3.6 may also eliminate the need for enhanced cooling, but would have difficulty explaining the above-the-loop emission region.

3.3.6 The role of loop-top trapping

Because the trap-precipitation model relies on acceleration above the trap, and the shock reprocessing model relies on the formation of a shock above the closed loop, it is possible for both processes to exist in tandem in a particular flare. Downflow electrons can be loaded into a loop-top trap region after passing through the above the loop-top shock. Introduction of a trap below the shock will affect the energy spectrum and arrival time profile of the non-thermal electrons. The non-thermal electrons in the downflow region can be divided into the four populations based on whether electrons are reprocessed, trapped, both, or neither. The regimes are illustrated in table 3.1.

When considering the temporal spectrum of each population, it is important to recall that the shock reprocessing precedes trapping at the loop-top. We thus label the populations (1) Reprocessed-untrapped (henceforth RU), (2) Reprocessed-trapped (RT), (3) STFA-only-Trapped (ST), and (4) STFA-only-Untrapped (SU). Of these four, we shall see that only RU electrons produce fast pulses at the footpoints.

Consider first shock reprocessed electrons. If there is a far upstream variation in the reconnection downflow, either in the reconnection rate or the flow speed, this will result in a variation in the throughput of electrons, and thus a brightness variation in the footpoint emission. RU electrons will proceed directly to the footpoint as in the untrapped shock reprocessing model, exhibiting short time scale pulses. RT electrons are injected into the loop-top trap, as in the traditional trap-precipitation model. Indeed, in the absence of an STFA-only population (the $F = 1$ limit), shock-reprocessing with trapping reduces to a variant of trap-precipitation where the combination of STFA and fast shock acceleration fills the role of the above the trap acceleration mechanism. Our treatment of shock formation implies that impulsive flares operate sufficiently far from the $F = 1$ limit that the non-reprocessed populations of ST and SU electrons are significant. Non-reprocessed electrons are incident on the trapping region with an energy spectrum and time spectrum already consistent with smooth component
footpoint emission. The ST population resides in the trap for a time proportional to $E^{3/2}$ and emerges with a very similar spectrum to the RT population. The SU population emerges unaltered by the trap.

In this discussion, we have taken the simplifying assumption that there is negligible cooling of electrons in the trap.

The overall spectral behavior is then determined by two parameters: the filling fraction $F$ and the trapping fraction $T$, where the latter is defined as the fraction of non-thermal electrons in the downflow which become trapped at the loop-top. In figure ?? we show the filling fraction required to produce a pulse of given strength $M$ in a downflow with STFA spectral index $\delta_0$ for the full range of $T$ values. Overlaid on the graph are the curves from shock formation constraints for $h/w = 1, 1.5, 2$. The model produces physically reasonable $F$ values. The data set used in Aschwanden et al. (1999) provides values of $T$ (their $q_t$) for a set of flares within the pure trap-precipitation model within a range of $0.4 < T < 0.8$. We would expect to find slightly lower values of $T$ when shock reprocessing and loop-top trapping are both active because of the contribution of the SU population to the smooth emission component.

While the problem of determining $T$ for particular flare observations from the combined reprocessing-trap model is deferred to later work, this preliminary examination implies that the two processes may be concurrently active in solar flares.

### 3.4 Discussion of observational implications and predictions

Three major observational constraints on any model of electron acceleration in impulsive solar flares are the production of proper pulse strengths, the energy dependent arrival time delays, and the appearance of loop-top emission sites in only a fraction of observed flares. The shock reprocessing model accounts for all three of these features through a two stage acceleration process. In the first phase, electrons are accelerated via STFA with appreciable thin target bremsstrahlung cooling in the turbulent downflow region above the flare loop. The cooling of trapped electrons produces the loop-top source, which is often too dim relative to the footpoints to be observed by Yohkoh. Subsequently, a portion of the electron population undergoes diffusive shock
acceleration at a weakly compressive stationary fast shock, splitting the electrons into two populations: the shock reprocessed pulse population, and the unshocked STFA population. Sub-second pulses result from modulation of the shock strength on short time scales. Since the predicted shocks are only weakly compressive, small changes in the reconnection outflow can remove or reestablish the shock.

The presence of the shock below the loop-top emission site implies that the pulse structure observed in footpoint X-rays, and generated by variations in the shock compression ratio, are not present in loop-top sources. This prediction of the shock reprocessing model needs to be tested, and should be feasible over time as the RHESSI dataset grows to include large numbers of flares with detectable loop-top sources. Sui et al. (2004) find evidence of smoothly varying loop-top emission in three RHESSI observed flares. There is no apparent pulse structure in these sources. However, Sui et al. (2004) argue that these coronal sources are distinct from Masuda flare type loop-top emission sources. The observations of Mrozek & Tomczak (2004) show a two component emission structure, with both pulsed and smooth emission, superficially similar to the footpoints. However, in these sources, the pulse time scale is typically 10s or larger, not sub-second as in the footpoints. Such coronal pulses can be attributed to modulations in either the reconnection or downflow environment. They are not inconsistent with the shock reprocessing model. Still, larger sampling statistics on variability in loop-top emission sources would be of great interest in determining the applicability of the shock reprocessing model to impulsive solar flares.

An additional concern for any model of solar flares is the electron supply problem. For example, the hard x-rays emitted in the Masuda flare require a non-thermal electron throughput of $2 \times 10^{35}$ s$^{-1}$ (Masuda et al. 1994). The reconnection downflow for the same flare contained $\sim 5 \times 10^{35}$ electrons s$^{-1}$ (Tsuneta & Naito 1998). Thus the electron acceleration mechanism must either be highly efficient, accelerating a fraction of the downflow electrons of order unity, or a secondary supply of electrons must be available. STFA falls into the first category; the process is highly efficient, so the problem of supplying a sufficient flux of electrons is alleviated. Since the bulk plasma flows downward to the footpoints, including both the electron and proton populations, charge neutrality is maintained without the need to rely upon a return current.

Shock reprocessing also makes predictions regarding the pulsed emission. Because
the pulse component of the emission is caused by the harder shock reprocessed spectrum, the observed pulse magnitude is dependent on the energy bin, where higher energy bins show larger pulse strengths. From Eq. (3.18), $M \propto E_{\text{min}}^{\delta_x - \delta_r}$, where $M$ is the relative pulse strength, and $E_{\text{min}}$ is the minimum energy of the observations. Notice that raising $E_{\text{min}}$ increases the pulse strength. Shock reprocessing predicts that observations in a sequence of energy bins would show stronger pulses at higher energies. Obtaining such data is within the capability of current instrumentation. The magnitude of the effect can be calculated by evaluating Eq. 3.18 multiple times, with $E_{\text{min}}$ taken to be the lower energy limit of each detection band. This provides the pulse strength for all x-ray emission at photon energy $E > E_{\text{min}}$. To carry out the observational test, one may reformulate the definition of $M$ slightly by reevaluating the integrals in Eqs. (3.14) and (3.15) with the upper limit $E_{\text{max}}$ set to the high energy end of the bin. Alternatively one may bin all photons with energy $E > E_{\text{min}}$ for successively increasing values of $E_{\text{min}}$. For simplicity, we perform the latter procedure here, assuming energy bins with $E_{\text{min}}$ taken in 20keV increments from 10keV up to 150keV. For all bins, $E_{\text{max}} = \infty$. The results are shown in table 3.2. The range of pulse strengths predicted for energy bins in observations of a single flare is very strongly dependent on the difference $\delta_x - \delta_r$. Even in the case of $\delta_r = 3.8$, a large range of pulse strengths is expected $0.07 < M < 0.54$. This effect can be measured, and can be used to test the shock reprocessing model and constrain $\delta_r$, even for moderately small values of $F$.

Furthermore, the strong pulses at high energies are consistent with the soft-hard-soft (SHS) spectral pattern observed in many flare spectra, even down to subsecond time scales (Grigis & Benz 2004, 2005). Typically, the SHS pattern traces total luminosity. As flare emission cycles through pulses, the spectrum starts out soft at the low luminosity onset of the pulse, steadily increases up to a maximal hardness at the pulse peak, then decays back to a softer spectrum as the luminosity returns to the pre-pulse minimum. The shock reprocessing model generates pulses via strengthening of the shock compression, which corresponds to hardening of the reprocessed spectrum, as illustrated in Table 3.2. Small changes in the super-magnetosonic Mach number of weakly compressive shocks can produce significant changes in the compression ratio. For pulse strengths in excess of 1, the footpoint X-ray spectrum will have index $\gamma_r$. 

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the shock spectral index, since the emission will be dominated by the reprocessed population. As the pulse progresses toward the peak luminosity, the shock compression ratio peaks as well, and so does the spectral hardness. During the decay phase, the shock compression decreases, and the spectrum softens, resulting in a SHS pattern. Moderate pulses, especially those with large $F$ will still produce a weak SHS pattern simply due to the large number of reprocessed electrons.

3.5 Conclusion

Magnetic reconnection is an environment for particle acceleration in solar flares. Within the reconnection region and ensuing outflows, a combination of acceleration mechanisms may operate. We have proposed the shock reprocessing model as one scenario of interest for solar flares when the downflows from reconnection sites are turbulent.

Four major features of the shock reprocessing model developed in this paper are as follows:

1. The model posits a two stage acceleration process: STFA in a turbulent reconnection downflow followed by first order acceleration at a loop-top fast shock. Hard X-ray emission sources are predicted in the STFA region, as well as at the foot-points.
2. Pulsed emission is produced by variations in the compression ratio of the fast shock, and thus in the post-shock electron spectrum. The shock does not fill the entire cross section of the downflow, and is weakly compressive. It is seen that relative pulse strengths produced within the shock reprocessing model are reasonable for realistic values of the local plasma parameters, and a simple model of shock formation. Because the above-the-loop emission region occurs above the shock in our model, the generation of pulses at the shock predicts that the above-the-loop-top source does not exhibit the subsecond pulse structure observed at footpoints.
3. Both the fast (pulse) and slow (smooth) time delay measurements (Aschwanden, et.al. 1995; Aschwanden et al. 1996a,b, 1998; Aschwanden 1998; Aschwanden et al. 1999) can be explained within the shock reprocessing model. The fast time delays in the pulsed component are time of flight dispersion of the shock accelerated electrons. The slow time delays in the smooth component reflect the STFA acceleration rate, producing the proper time scale. Cooling is required to correctly match the curvature, although partial trapping at the loop top may also resolve this issue.
(4) The shock reprocessing model predicts an increase in relative pulse strength at higher energies. This prediction remains to be tested, and can distinguish between the shock reprocessing and trap precipitation models.

The shock reprocessing model is not meant to be an exclusive solution to electron acceleration in all solar flares. It seems unlikely that the wide variety of flare phenomena can be explained if all reconnection regions always have the same relative contributions from the combination of acceleration mechanisms therein.

Generic features such as magnetic reconnection, soft x-ray loops, and footpoint hard x-rays, do appear to be common to the vast majority of flares. However, coronal x-ray emission sources appear sometimes with thermal and other times with non-thermal spectral characteristics, sometimes above the soft x-ray loop, or sometimes within it, and other times not at all. Also, reconnection morphology, while apparently similar in any 2-D snapshot along the plane of the soft x-ray loops, can have either vertical or lateral ("zipper") dynamical evolution.

One particularly difficult observation to explain within the shock reprocessing model (without the addition of loop-top trapping) is the appearance, in a small number of flares, of coronal hard x-ray emission emanating from within the closed soft x-ray loop (Veronig & Brown 2004). These sources are non-thermal, and hard x-ray emission in the flares is dominated by coronal, rather than footpoint sources. Coronal sources could be produced within the trap-precipitation model in the presence of an abnormally dense loop; the trapped electron population within the loop would emit via thick target Bremsstrahlung. Conversely, the shock reprocessing model does not load the electrons onto the closed flare loop, and thus cannot produce these sources. This class of loop-top emission has only been observed in a few events. As the total number of coronal sources within loops grows, it would be of great interest to determine which, if any, environmental parameters such as laminar vs. turbulent downflows correlate with the location of the coronal source.

The phenomenological array of flares can likely be understood by a relatively small number of acceleration scenarios which each operate within the basic paradigm of magnetic reconnection and outflow. The power source driving all flares is reconnection high in the corona, which launches a downflow. Within the outflow, particle acceleration occurs, followed by x-ray, gamma ray, and radio emission in the lower corona and chro-
mosphere. The particular acceleration scenario is determined by local environmental parameters in the reconnection, downflow, and emission regions. Shock reprocessing and trap precipitation are two scenarios within this framework, which, as discussed in section 3.3.6 may not be mutually exclusive, even within single flare events. A third scenario is likely required to explain proton dominated flares (Hurford et.al. 2003; Miller and Roberts 1995; Miller, Emslie, and Brown 2004).

In summary: the shock reprocessing model, whereby a fraction of stochastically accelerated electrons also passes through a weakly compressive stationary shock as they stream toward chromospheric footpoints, provides a scenario for explaining a variety of features in impulsive solar flares within the more general reconnection and outflow framework. The model produces pulsed and smooth spectral X-ray emission components consistent with the time delay observations of Aschwanden, et.al. (1995); Aschwanden et al. (1996a,b, 1997, 1998); Aschwanden (1998); Aschwanden et al. (1999). Shock reprocessing also predicts the appearance of above the loop-top coronal emission sites observed in a fraction of flares. Furthermore, the model makes a sequence of testable predictions regarding the pulse strength as a function of energy and the SHS emission pattern. We have discussed how it is observationally feasible to test the shock reprocessing model.

3.6 Bibliography

References


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Jones F. C. & Ellison D. C. 1991, Space Science Reviews, 58, 259

<table>
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Table 3.1 The four populations of non-thermal electrons.
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Table 3.2 Pulse strengths $M$ for increasing low energy cutoff and a range of $\delta_r$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 3.1 The basic model of solar flare structure Blackman (1997). Reproduced by permission of the AAS.
Figure 3.2 A flowchart of the trap precipitation model. $\alpha$ is the electron pitch angle cosine and $\alpha_0$ is the critical pitch angle cosine for trapping in the loop.
Figure 3.3 A flowchart of the shock reprocessing model.
Figure 3.4 Curves of constant $M$. From top to bottom: $M = 0.6, 0.4, 0.2, 0.1$. $E_{\text{min}} = 5/3$. $\delta_s = 4$ Only $F \leq 1$ corresponds to physically realizable states.
Figure 3.5 Curves of constant $M$ in $\delta_F - F$ space as $E_{\text{min}}$ is varied; $\delta_s = 4$. (a) $E_{\text{min}} = 3/3$. (b) $E_{\text{min}} = 4/3$. (c) $E_{\text{min}} = 5/3$. (d) $E_{\text{min}} = 6/3$. $E_{\text{min}}$ is normalized to 15keV. In each panel, curves from left to right are $M = 0.6, 0.4, 0.2, 0.1$. Only $F \leq 1$ corresponds to physically realizable states. Increasing $E_{\text{min}}$ results in a lower required $F$ for a given $M$. 

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Figure 3.6 Curves of constant \( M \) in \( \delta_s - F \) space as \( \delta_s \) is varied. (a) \( \delta_s = 3 \). (b) \( \delta_s = 4 \), (c) \( \delta_s = 5 \). (d) \( \delta_s = 6 \). \( E_{\text{min}} = 25\text{keV} \) in all four panels. In each panel, curves from left to right are \( M = 0.6, 0.4, 0.2, 0.1 \). Only \( F \leq 1 \) corresponds to physically realizable states. Increasing \( \delta_s \) implies a higher \( F \) is required to produce a given \( M \).
Figure 3.7 A schematic of the shock formation model. $v_f$ is the downflow speed, $v_n$ is the component of the speed normal to the loop, $s$ is the slope of the loop, $w$ and $h$ the loop width and height, and $x$ the distance of the point at which $s$ is evaluated from the mid-line of the loop. A fast shock forms above the loop-top for all values of $x$ where $v_n$ is super-fast-magnetosonic. In the reconnection downflow, the fast-magnetosonic speed is effectively equal to the sound speed and we can write the shock formation condition as $v_n > c_f$. 
Figure 3.8 The shock formation model in $F - \delta_r$ space for $h = 1$ (top), 1.5, 2, 2.5, 3, 4 (bottom)
Figure 3.9 Overlaid plots of the dual constraints of the shock reprocessing model for pulse strengths and the shock formation model. Left $\delta_s = 4$, Right: $\delta_s = 5$. The inner box represents the physically allowed region.
Figure 3.10 $\tau$ vs. $E$ for uncooled STFA (solid line) and the trap-precipitation model (dotted line) (Aschwanden et al. 1997). The STFA model is with density $n_{10} = 1.7$ while the trap-precipitation model is shown at density $n_{10} = 20$. 

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Figure 3.11 $F$ vs. $\delta_r$ for pulse strength $M = 0.2$ at varied values of $T$. 

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Chapter 4

Observational Tests of the Shock Reprocessing Model
In chapter 3, we developed the shock reprocessing model as a candidate for electron acceleration in impulsive solar flares. This model makes specific predictions regarding the strength of pulses, as well as the soft-hard-soft spectral evolution of hard x-ray (HXR) emission during single pulses. In this chapter, we provide further detail and preliminary results of observational tests of the model. We conclude by outlining an extension of the model to ion acceleration in the case of proton dominated flares. These three avenues provide a summary of the direction of further research into the role of STFA and shocks in flare acceleration phenomena.

4.1 HXR Pulse Strength in Impulsive Solar Flares

The shock reprocessing model can be used to make predictions of the strength of the observed short time scale pulses in footpoint X-ray emission from solar flares. Recall that the model, as developed in chapter 2, calculates the spectral index for STFA accelerated electrons, $\delta_s$, as well as for reprocessed electrons, $\delta_r$. From these we were able to obtain formulas for the photon energy spectrum resulting from Bremsstrahlung of the STFA accelerated electrons, $M_T$, as well as the photon energy spectrum resulting from the shock reprocessed electrons, $M_p$, where

$$M_T = \int_{E_{min}}^{\infty} M_s dE = \frac{M_{0s} E_{0s}}{\delta_s - 2} \left( \frac{E_{min}}{E_{0s}} \right)^{-(\delta_s - 2)},$$

(4.1)

and

$$M_p = \int_{E_{min}}^{\infty} M_p dE = \frac{M_{0r} E_{0r}}{\delta_r - 2} \left( \frac{E_{min}}{E_{0r}} \right)^{-(\delta_r - 2)},$$

(4.2)

and $E_{0s}$, is the low energy cutoff of the non-thermal component, $E_{0r}$ is the shock injection energy (and therefore the low energy cutoff for reprocessing), and $E_{min}$ is the low energy cutoff of the detector. The latter quantity is of great importance for any discussion of observations. In the case of the BATSE instrument onboard CGRO, $E_{min} = 25keV$ in channel 1, the low energy detector channel.

From equations (?? and ??, as well as constraints relating $E_{0s}$ and $E_{0r}$, we obtained a pulse strength $M$,

$$M = \frac{M_p}{M_T} = F \left[ \left( \frac{\delta_s - 2}{\delta_r - 2} \right) \frac{E_{0s}}{E_{min}} - 1 \right],$$

(4.3)
where $F$ represents the portion of the total electron beam which undergoes reprocessing, and we have integrated total photon counts above $E_{\text{min}}$ with an infinite upper bound. These calculations indicate that the pulse strength is expected to increase with increasing values of $E_{\text{min}}$ for any given choice of the parameters $F$, $\delta_s$, and $\delta_r$. However, it is impractical to perform the observations in the way suggested above. BATSE collected HXR spectral data in 16 energy channels, the first three of which covered the energy ranges $25 - 50\text{keV}$, $50 - 100\text{keV}$, and $100 - 300\text{keV}$. Data from channels 1 and 2 provide the most valid testing ground for the pulse strength predictions of our model, channel 3 data are less useful because at the upper energy limit of $300\text{keV}$ electron kinematics are unavoidably relativistic.

We model the pulse strengths more appropriately for the energy bins of BATSE by rewriting the limits of integration in equations (4.4) and (4.5):

\[
M_T = \int_{E_{\text{min}}}^{E_{\text{max}}} M_s dE,
\]

and

\[
M_p = \int_{E_{\text{min}}}^{E_{\text{max}}} M_r dE,
\]

where $E_{\text{max}}$ is the high energy boundary of the bin. In the case of channel 1, for example, $E_{\text{min}} = 25\text{keV}$ and $E_{\text{max}} = 50\text{keV}$. Carrying out the integrals,

\[
M_T = \frac{M_0s E_{0s}}{\delta_s - 2} \left( \left( \frac{E_{\text{min}}}{E_{0s}} \right)^{2-\delta_s} - \left( \frac{E_{\text{max}}}{E_{0s}} \right)^{2-\delta_s} \right),
\]

and

\[
M_p = \frac{M_0r E_{0r}}{\delta_r - 2} \left( \left( \frac{E_{\text{min}}}{E_{0r}} \right)^{2-\delta_r} - \left( \frac{E_{\text{max}}}{E_{0r}} \right)^{2-\delta_r} \right).
\]

From this we obtain the relative pulse strength,

\[
M_P = \frac{M_p}{M_T} = F \frac{\delta_r - 2}{\delta_s - 2} \left( \left( \frac{E_{\text{min}}}{E_{0s}} \right)^{2-\delta_s} - \left( \frac{E_{\text{max}}}{E_{0s}} \right)^{2-\delta_s} \right).
\]

where $M_P$ is the relative pulse strength. In the following, we will use $M_{P1}$ for the lowest BATSE energy channel ($25-50\text{keV}$) and $M_{P2}$ for the second energy channel.
(50-100keV). We can now take the ratio of the pulse strengths in BATSE channels 1 and 2, \( R_{21} \)

\[
R_{21} \equiv \frac{M_{P2}}{M_{P1}}.
\]  

Given a particular pair of channels at fixed energies the ratio depends only on 4 parameters: \( E_{0s}, E_{0r}, \delta_s, \) and \( \delta_r, \) but is insensitive to the shock filling fraction \( F. \)

Figure ?? shows the variation of \( R_{21} \) with \( \delta_r \) at three values of \( \delta_s. \) For these we retain the assumption that \( E_{0s} = 15\text{keV} \) and \( E_{0r} = 25\text{keV}. \) Note that even though the pulse strengths tend toward zero at the limit \( \delta_r = \delta_s, \) the ratio of pulse strengths remains larger than 1. Because reprocessing cannot cause spectral softening, the model does not admit pulses for \( \delta_r > \delta_s; \) figure ?? does not extend the ratio curves beyond that limit. Figure ?? shows the pulse strengths in channel 1 for the same values of \( E_{0s}, E_{0r}, \) and \( \delta_s. \) If we take the channel 1 pulse strength as a proxy for the overall pulse strength and apply the constraints found in chapter 3 for the physically plausible regime - \( 0.1 < M < 0.6 \) - we expect that most flares will be in the regime \( \delta_s - 1 < \delta_r < \delta_s, \) and that \( 3.5 < R_{21} < 6. \)

The trap precipitation model makes a far simpler prediction for \( R_{21}. \) In this model, all acceleration takes place prior to the loop-top injection. One thus expects that all directly precipitating electrons -which produce the pulses within this model- have the same HXR emission spectrum. For this reason, the pulses should be equally strong in all energy bins, and \( R_{21} = 1. \)

In order to distinguish between the predictions of the two models, we require two things: access to a collection of high temporal resolution HXR data for a large number of solar flares, and an algorithm for extracting the short time scale pulses from the smoother long time scale variations within these data. The former exists in terms of a solar monitoring project using BATSE and CGRO. The latter is less readily available. Pulsed emission has been deconvolved from smooth emission using time filters by Aschwanden, et.al. (1995); Aschwanden et al. (1996a,b, 1997, 1998); Aschwanden (1998); Aschwanden et al. (1999). They then interpolated a linear model for the smooth flux component between the minima at opposite ends of each peak, and cross-correlated peaks between energy channels. The flux enhancement associated with a peak is taken to be the difference between the total flux and the interpolated smooth component. The goal of their analysis was to determine arrival time differences...
of related peaks in different energy channels. Within the trap-precipitation model, they interpreted the pulsed emission as the arrival of bursts of directly precipitating electrons at the loop footpoints. If the entire burst is assumed to be injected at the loop-top simultaneously, then it can be expected to produce an emission pulse in all energy bins. The pulses are not simultaneous in each bin, however, owing to time-of-flight dispersion of the electrons as they travel from the injection point to the footpoint region. The size of an individual peak is largely unimportant in their analysis, only the times of the onset of the pulse, maximum flux, and end of the pulse matter.

Unfortunately, we are so far limited to one test case. Aschwanden, et.al. (1995) use the flare of 1991, June 15, to illustrate their pulse identification and interpolation methods. In this flare, they identify 20 distinct pulses, and give detailed interpolation information in BATSE channels 1 and 2 for the first five, labeled Structure 0-4 in figure ??, reproduced from Aschwanden, et.al. (1995).

Note particularly that the upper row of the second panel provides, in normalized units, the photon count rates in each channel. Each pulse is normalized individually, to produce a peak flux of $\sim 2.0$. It should be noted that since Aschwanden, et.al. (1995) only was concerned with arrival times, and the current analysis is only concerned with relative strengths of the pulsed and smooth emission, absolute fluxes are not necessary. These normalized flux counts are the quantities obtained in equation ??, and used to predict $R_{21}$. In table ?? we present $M_1$, $M_2$, and $R_{21}$ for each pulse. The pulse strength ratios in each structure of this flare are consistent with the trap-precipitation model, not with the shock reprocessing model.

It is difficult in this case to assess the uncertainty in the pulse strength measurements. We have thus assumed a maximum uncertainty, in the relative units of Aschwanden, et.al. (1995) of 0.05 in all count rate values. This allows us to place approximate maximum and minimum values on the range of $R_{21}$. The uncertainties arise from two sources. First, the evaluation method used was undeniably crude: pulse and smooth component strengths were read directly from the graphs in figure ??, Second, the interpolation method used in Aschwanden, et.al. (1995) is not designed to give fully accurate values for the smooth component count rate at the peak of a pulse. Because the count rates themselves were not of great significance in the analysis therein, it was sufficient for their purposes to use a linear interpolation between the flux minima at
either end of a peak in order to estimate the smooth flux between pulse onset and end. However, a simple visual inspection of the upper panel of figure ?? indicates that the smooth component peaks and decays substantially over the lifetimes of the first 6 pulses. A sequence of linear interpolations between pulse endpoints must provide inaccurate pulse sizes.

Regardless of these concerns, the fairly high robustness of the $R_{21} = 1$ result strongly indicates that this 1991 June 16 flare is not well described by the shock reprocessing model. There are three possible interpretations of this result: the shock reprocessing model is, in a general sense, not a valid description of flare physics; the shock reprocessing is possibly only valid for a subset of impulsive flares of which the 1991 June 15 flare is not a member; the flare of 1991 June 15 is a low probability case wherein the STFA and shock spectral indices, $\delta_s$ and $\delta_r$ are nearly equal. With only one flare analyzed, it is not possible to distinguish among the three scenarios.

### 4.2 Soft-Hard-Soft (SHS) Spectral Evolution

A second prediction of the shock reprocessing model is a soft-hard-soft spectral evolution for the non-thermal flare emission during each pulse. The flux intensity is expected to rise as the electron spectrum hardens ($\delta_r$ decreases). There are two factors which influence this behavior: an energy increase, on average, per electron above the shock injection energy $E_{in}$ and an overall increase in the number of electrons included in the downflow region at higher $\delta_r$. We address both factors and show that the combination of the two is consistent with the observations of ?.

It was established that the effect of a fast shock at the loop-top is to reprocess electrons with an energy spectrum

$$N_T(E) \propto \left( \frac{E}{E_0} \right)^{\delta_s} ,$$

(4.10)

to a harder spectrum

$$N_p(E) \propto \left( \frac{E}{E_0} \right)^{\delta_r} ,$$

(4.11)

provided that the electrons meet the injection criterion $E > E_{\text{min}}$ and the spectral index associated with the shock is harder than that associated with the injected (pre-
sumably STFA accelerated) spectrum, $\delta_r < \delta_s$. Due to the constancy of the total number of non-thermal electrons in the system, this essentially pivots the electron spectrum for all energies $E > E_{in}$ about some unknown pivot point $E_{piv}$. One expects an increased electron flux for energies above the pivot, and a decrease below it.

In addition to the pivot, there is a substantial flux enhancement at high shock spectral index due to the overall increase in bulk plasma flow into the shock region. Recall that the spectral index for shock accelerated electrons is given by

$$\delta_r = \frac{(r + 2)}{r - 1},$$

where the compression ratio, $r$ of the shock depends on the Mach number, $\mathcal{M}$ of the downflow

$$\mathcal{M} = \frac{(\gamma + 1)M^2}{\gamma + 1 + (\gamma + 1)(M^2 - 1)},$$

and $\gamma = 5/3$ is the adiabatic index of the plasma. Combining these, we obtain a relationship between the spectral index and downflow Mach number,

$$\mathcal{M} = \frac{\sqrt{\delta_r + 2}}{\sqrt{\delta_r - 2}}.$$ 

Given the reasonable assumption that the sound speed is constant over the duration of a single pulse, $\mathcal{M}$ is a tracer of the downflow speed, and thus of the total electron flux incident on the emitting region.

Combining the two effects, we construct a formula for the electron flux

$$N(E) \propto \mathcal{M} \left( \frac{E}{E_0} \right)^{\delta_r} = \sqrt{\frac{\delta_r + 2}{\delta_r - 2}} \left( \frac{E}{E_0} \right)^{\delta_r}.$$ 

The appearance of $\mathcal{M}$ as a coefficient corresponds to the velocity dependent variation in total flux associated with changes in the shock compression ratio, while the power law factor reflects the variation in the energy per electron with shock compression ratio.

Quantitative observations of the relationship between the emitted flux at 35keV and the spectral index of X-ray emission were carried out by ?. Their results show a strong correlation between the photon spectral index, $\gamma$ and emitted flux with a large scatter. The data are consistent with the relationship
\[ \gamma = A F_{35}^{-\alpha}, \quad (4.16) \]

where \( A \) and \( \alpha \) are fit parameters and \( F_{35} \) is the photon flux at \( E = 35 \text{ keV} \). Their analysis leaves open the interpretation of this relationship, and does not propose a physical model for the SHS behavior. They do, however, point out that any model which produces variation in \( F_{35} \) solely due to turning around a pivot point is not consistent with the data set taken as a whole, but may be of some validity in treating individual pulses.

Figure ?? compares our model’s prediction to the data set. We follow ? in plotting the data on a logarithmic scale, reversing their axes. As a result, their slope of \( \alpha \approx 0.2 \) appears in our plot (dash-dot line) as a slope \( \alpha^{-1} \approx 5 \). Our prediction (solid line) does deviate from the empirical fit, but is encouragingly close, particularly in light of the scatter evidenced in the data. The shock reprocessing model is preliminarily able to predict the SHS pattern both qualitatively and quantitatively.

### 4.3 Ion acceleration within the shock reprocessing framework

It is evident that there is substantial acceleration of protons and other ions to GeV energies nearly cospatially with electron acceleration in impulsive solar flares (i.e. ?). These protons appear to be accelerated in the same or similar reconnection downflows as electrons, with evidence that proton acceleration dominates over electron acceleration in longer flare loops (??). It is necessary that the shock reprocessing model be able to accommodate these observations, either through direct prediction of the ion acceleration phenomena, or by the recognition of additional acceleration processes which dominate shock reprocessing in long flare loops. We present an overview of the issue, which remains an important direction for future work.

? argues that ions are accelerated by cyclotron resonance with turbulent Alfvén waves in the downflow. This acceleration is typically slower than the acceleration of electrons, but given a loop of sufficient length an appreciable number of protons in the high energy tail of the initial distribution will be accelerated to speeds \( v_p > v_A \), the criterion for STFA. Since STFA time scales are inversely proportional to the
mass of the accelerated particle, acceleration of super-Alfvénic ions dominates that of electrons once the condition is met. Essentially, the cyclotron resonance acts as a pre-accelerator, pushing the tail of the ion distribution up to the velocity threshold for STFA, at which point ion acceleration is sufficiently rapid to preferentially drain the remaining turbulence. The model of \( ? \) capable of explaining the occurrence of ion abundance enhancements relative to the background corona in some higher mass species (i.e. Fe, Ne) as well as the absence of enhancements in lower mass species (i.e. C, He\(^4\)). One species in particular, \( He^3 \) requires an additional acceleration source to explain an anomalous enhancement.

For the shock reprocessing model, the issue of ion abundance is particularly interesting because in the framework of \( ? \), this phenomenon differentiates between purely stochastic and purely shock driven models of acceleration. Since the condition for acceleration at a shock is that the ion gyroradius is comparable to, or larger than the proton gyroradius, thermal ions necessarily meet the injection condition regardless of species. For acceleration at a shock, one thus expects all species to be accelerated equally, and there should be no enhanced abundances. (Indeed, this is observed for gradual type flares, implying a substantial difference in the reconnection downflow and resulting acceleration environment between those flares and the impulsive flares we have considered.) The cyclotron resonance acts when the condition

\[
\frac{u}{m} > -\frac{L}{C}
\]

where \( u \) is the frequency of the wave, \( L \) is an integer, and \( \Omega = qB/m \) is the cyclotron frequency for an ion of charge \( q \) and mass \( m \) in a magnetic field \( B \). From the resonance condition, we see that there is a minimum frequency \( \omega_m \) or maximum wavelength \( \lambda_M \), corresponding to the \( L = 1 \) case. From the cyclotron frequency, we see that \( \lambda_M \) increases proportionally to the mass of the ion. As driven turbulence cascades from an outer scale, higher mass species cross their \( L = 1 \) resonance earlier. Therefore, a larger portion of the thermal tail of large mass species (Fe, Ne) reaches the STFA injection energy than of low mass species (C, He\(^4\)). However, the relatively low overall abundances of these ions allows the turbulent cascade to proceed, only partially damped, down to sufficiently small lengths to accelerate protons, which are abundant enough to truncate the cascade.

While the observations clearly favor a prominent role for purely stochastic acceleration models over purely shock acceleration models, it remains to be determined if a
hybrid model, such as shock reprocessing, can also satisfy the ion acceleration observations. While the details of this model remains for future work, we anticipate some qualitative results. Since the reprocessed spectrum remains tied to the compression ratio of the shock, the gamma ray emission produced by nuclear processes involving the GeV ions should be impulsive, with time structures that are similar to those of the electrons. The SHS pattern of x-ray emission will likewise manifest for gamma ray emission since that too is a function of the downflow rate and shock formation parameters. Finally, it is plausible that the observed ion abundance rates will be matched, provided that shock acceleration works preferentially on the faster moving ions which are preaccelerated in the turbulent downflow. This seems plausible since these faster moving ions will be able to undergo more shock crossings (gaining an energy boost at each crossing) than the slower moving, sub-Alfvénic ions which were not preaccelerated. This would preserve the enhanced ion abundances at high energy produced by the stochastic preacceleration. There are two reasons for this. First, faster ions will travel through a greater total path via random walk in the time that the parcel of gas with which they are associated is carried past the shock region by bulk flow. Second, sub-Alfvénic ions are less likely to scatter off of the downstream Alfvén wave turbulence which is the presumed mechanism by which ions repeatedly cross back to the upstream side of the shock. The two effects in combination increase the number of crossings undergone by each ion.

4.4 Conclusion

Acceleration of electrons in impulsive solar flares remains an open problem in solar physics. We have outlined the shock reprocessing model, whereby electrons are accelerated first via the dissipation of magnetic turbulence and then a second time at a fast shock at the apex of the closed flare loop.

We make two significant predictions regarding the emitted x-ray spectrum. Within the shock reprocessing model, short time scale pulses are larger in higher energy detector bins. The one flare analyzed herein is not consistent with this prediction; it is consistent with the trap-precipitation model. The second prediction is that the short term pulses exhibit a spectral index evolution in which spectral hardness traces total emitted flux. In this aspect, the model is consistent with the correlation between the
emitted flux at 35keV and spectral index observed by ?. To date, a comparison of the same observations with the trap-precipitation model has not been carried out.

Additionally, we stress that solar flares are multi-faceted environments for particle acceleration. Evidence for this is the distinction between electron and proton dominated flares. ? has demonstrated that proton (and ion) acceleration is consistent with stochastic processes in the presence of a preacceleration via cyclotron resonance. As such, long loops are required for protons to reach the threshold for stochastic acceleration. This multi-stage process is consistent with the observed ion abundances in impulsive flares. It is plausible that the shock reprocessing model is consistent with these results as well.

4.5 Bibliography

References

Table 4.1 Observed pulse strengths and ratios from the 1991 June 15 flare. Note that the pulse strengths in Channels 1 and 2 are in scaled, with the maximum count rate of each pulse set separately to 2.0.

<table>
<thead>
<tr>
<th>Structure</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td>0.27</td>
<td>0.65</td>
<td>0.46</td>
<td>0.21</td>
<td>0.41</td>
</tr>
<tr>
<td>Channel 2</td>
<td>0.25</td>
<td>0.88</td>
<td>0.45</td>
<td>0.21</td>
<td>0.50</td>
</tr>
<tr>
<td>Ratio: $R_{21}$</td>
<td>0.93</td>
<td>1.35</td>
<td>0.98</td>
<td>1.00</td>
<td>1.22</td>
</tr>
</tbody>
</table>
Figure 4.1 The shock reprocessing model prediction for $R_{12}$ as a function of $\delta_r$ for $\delta_s = 4.0, 4.5, 5.0$. Each curve is cut off at the limit $\delta_s = \delta_r$. 

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Figure 4.2 Predicted pulse strengths in BATSE channels 1-3 (bottom to top) with $\delta_s = 4.0$ as a function of $\delta_r$. Note that the units are a dimensionless ration of the pulse strength to the smooth component in the same channel.
Figure 4.3 Pulse strengths in multiple BATSE channels for structures 0-4 of the 1991, June 15, flare. Reproduced from Aschwanden, et.al. (1995). The units in the first row of the lower panel are arbitrarily scaled for each pulse, such that the flux count for that pulse peaks at $\sim 2.0$. 

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Figure 4.4 A comparison of the SHS behavior predicted by the shock reprocessing model (solid line) to data (dot dashed line). Total flux is plotted in normalized units vs. $\delta_r$. 

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Chapter 5

On the Role of STFA in Setting the Dissipation Length Scale of Interstellar Turbulence
We consider the dissipation by Fermi acceleration of magnetosonic turbulence in the Reynolds Layer of the interstellar medium. The scale in the cascade at which electron acceleration via stochastic Fermi acceleration (STFA) becomes comparable to further cascade of the turbulence defines the inner scale. For any magnetic turbulent spectra equal to or shallower than Goldreich-Sridhar this turns out to be $\geq 10^{12}$ cm, which is much larger than the shortest length scales observed in radio scintillation measurements. While STFA for such spectra then contradict models of scintillation which appeal directly to an extended, continuous turbulent cascade, such a separation of scales is consistent with the recent work of Boldyrev & Gwinn (2005) and Boldyrev & Konigl (2005) suggesting that interstellar scintillation may result from the passage of radio waves through the galactic distribution of thin ionized boundary surfaces of HII regions, rather than density variations from cascading turbulence. The presence of STFA dissipation also provides a mechanism for the non-ionizing heat source observed in the Reynolds Layer of the interstellar medium (?). STFA accommodates the proper heating power, and the input energy is rapidly thermalized within the low density Reynolds layer plasma.

5.1 Introduction

Radio scintillation has long been associated with interstellar turbulence (e.g. Minter and Spangler 1997)). A major requirement of turbulence-based scintillation models is that the inner scale of the cascade is comparable to the smallest scale of the scintillation. An alternative model has been proposed (Boldyrev & Gwinn 2003, 2005; Boldyrev & Konigl 2005) in which the scintillation instead results from index of refraction variations at the photoionized surfaces of a non-Gaussian distribution of either warm ISM regions or HII clouds. We find that Stochastic Fermi Acceleration (STFA) is a sufficiently efficient process in the Reynolds Layer that it imposes a cutoff at a scale too large to be consistent with radio scintillation, supporting the non-Gaussian cloud model.

Observations of line ratios by ? using the Wisconsin H-alpha Mapper (WHAM) indicate that the Reynolds layer of the Milky Way interstellar medium (ISM) cannot be heated solely by photoionization. Variation with galactic latitude, $|z|$, of $[SII]/H\alpha$ and $[NII]/H\alpha$ both are found to be consistent with increases in electron tempera-
ture, $T_e$, but not in the ionization fractions of either species. Additionally, studies of external galaxies NGC891, NGC 4631, and NGC 3079 (??) show similar evidence of supplemental, non-photoionizing heat sources in the Reynolds layer, the region of low density, strongly ionized gas located $\sim$ 1kpc away from the galactic midplane. These studies supplement the $[SII]/H\alpha$ and $[NII]/H\alpha$ ratios with analysis of $[OIII]/H\beta$ and $[OII]/H\alpha$, which also are consistent with non-ionizing heating. The heating rate in the Milky Way is consistent with being either proportional to electron density, $n_e$, or density independent. The heat source is empirically found to have a power input of $\epsilon = G_1 n_e$, where $G_1 \sim 10^{-25}$ erg s$^{-1}$, or $\epsilon = G_2$, where $G_2 \sim 10^{-27}$ erg s$^{-1}$ cm$^{-3}$. This allows the supplemental heat source to dominate at high $|z|$, where $n_e$ is low and photoionization heating to dominate at low $|z|$ where $n_e$ is high. A number of potential mechanisms are presented by ? as supplemental heat sources, including photoelectric grain heating, Coulomb collisions with cosmic rays, magnetic field reconnection, and dissipation of superbubble driven magnetic turbulence.

We consider the role of STFA in the dissipation of interstellar turbulence, looking primarily for the inner scale of the cascade. ISM turbulence is driven principally by superbubbles, the large blast shells carved out when the members of an OB association reach the supernova stage in a short time period. STFA occurs when electrons traveling in along a magnetic field line encounter moving compressions of the local magnetic field and are reflected. While STFA drains turbulence primarily into ions for thermal pressure dominated plasmas, electrons are the primary energy recipient when the ion speed is sub-Alfvenic (i.e. LaRosa et al. (1996); ?). The latter case is relevant for the present study because the Reynolds layer seems to be a weakly magnetically dominated plasma (?).

The energy change, $\delta E$ from a single STFA mirroring is (??Longair 1994; ?)

$$\left(\frac{2E}{c^2}\right) (v_A^2 \pm v_e v_A),$$

where $E$ is the initial energy of the reflected particle, $c$ is the speed of light, $v_A$ is the Alfven speed, and $v_e$ is the electron speed. Typical Reynolds layer temperatures are $0.6 < T_4 < 1.2$ where $T_4 = T_e/10^4 K$. This corresponds to an energy in the range $0.5 < E < 1.1$ eV. Thus $\delta E$ is significantly below ionization energies, which are tens of eV, and even if the reflection rate is high, STFA of protons is a non-ionizing process.
in the turbulent Reynolds layer of the ISM.

In section 2 we show that STFA provides the correct power and that the energy input is quickly thermalized, consistent with the conditions imposed on the heating source. We then determine the truncation scale for the turbulent cascade. In section 3, we discuss the implications of the inferred STFA dissipation scale for models of interstellar scintillation, finding that STFA truncates the cascade at a long length scale, inconsistent with turbulence models of radio scintillation. We conclude in section 4.

5.2 Stochastic acceleration in the turbulent Reynolds layer

The power available for electron heating in the MHD turbulent cascade, $\epsilon_T$, can be estimated as

$$\epsilon_T = n_e m_p v_A^3 \frac{v_A}{L},$$

where $n_e$ is the electron number density, $v_A$ is the local Alfvén speed, and $L$ is the outer scale of the interstellar turbulence inertial range. From ?: $v_A = 2.3 \times 10^6$ cm s$^{-1}$ and $L = 10^{19}$ cm. ? infer this value for $L$ by analysis of emission measure and rotation measure structure functions. From ?: $n_e = 0.28$ cm$^{-3}$ at galactic latitude $|z| = 1$ kpc. $\epsilon_T = 5 \times 10^{-26}$ erg s$^{-1}$ cm$^{-3}$, consistent with the heating rate called for in ? and calculations of superbubble injected power (?).

? examined STFA in the $v_A \gg c_s$ regime with applications to solar flares, and concluded that the post-acceleration spectrum is dominated by the rate of escape from the acceleration region. The post-escape spectrum is important in flares because the observed x-ray emission is generated via Bremsstrahlung as electrons escape the acceleration region and encounter the dense chromospheric plasma. In the turbulent Reynolds layer, the acceleration region is effectively infinite in extent. Instead of observing emission by escaped electrons, the observed emission comes from electrons which remain in the Reynolds layer. In this case we are concerned with the spectrum of electrons still confined within the acceleration region. Regardless, the acceleration rate found in ? remains valid

$$\left( \frac{dE}{dt} \right)_S = \langle \delta E \rangle R,$$

(5.3)
where
\[
\langle \delta E \rangle = 4m_e v_A^2,
\]
\[\text{(5.4)}\]
is the average energy per reflection and
\[
R = \frac{v_d}{2L} \left( \frac{\lambda_s}{L} \right)^2,
\]
\[\text{(5.5)}\]
is the rate of reflections, such that
\[
\left( \frac{dE}{dt} \right)_s = \frac{2}{L} m_e v_A^2 v_d \left( \frac{\lambda_s}{L} \right)^{-(1-1/a)},
\]
\[\text{(5.6)}\]
and \(\lambda_s\) is the turbulent inner scale (associated with the parallel component of the magnetic field gradient for anisotropic turbulence), \(v_d\) is the effective relative speed of the electrons and compressions within the plasma, \(m_e\) is the electron mass, and \(a\) is the spectral index of the turbulent cascade
\[
\frac{\lambda}{L} = \left( \frac{\delta B}{B} \right)^a.
\]
\[\text{(5.7)}\]

It is assumed throughout our analysis that at length scales comparable to \(L\) the strength of the magnetic fluctuations, \(\delta B\) is comparable to that of the mean magnetic field, \(B\). The magnetic spectrum of turbulence in the ISM is difficult to measure. Between \(10^{19}\)pc and \(0.03\)pc the spectrum appears to be close to Kolmogorov \((a = 3)\) (and flatter on larger scales) (???). There is no direct measure of the magnetic spectrum on scales below \(0.03\)pc.

For the range of magnetic spectra \(2 \leq a \leq \infty\) the result from our calculations to follow, that the cascade truncation scale well exceeds the smallest scintillation scale, will not change. However, we first consider a Goldreich-Sridhar (henceforth GS) turbulent spectrum with \(a = 2\) (Goldreich and Sridhar 1997) (see also ?). Like ?, GS turbulence is fully anisotropic, and is based on an incompressible cascade. It involves a more rapid cascade in the direction perpendicular to the local mean field than parallel to it. Turbulence becomes less and less compressible on smaller scales, and since the Reynolds layer seems to be modestly magnetically dominated (?), GS is plausible on small enough scales: The GS spectrum more closely arises in compressible and incompressible simulations when an initially relative strong field is imposed (e.g. ??, ??, ??), i.e. when the initial ratio of thermal to magnetic pressure, \(\beta < 1\) and the
velocity fluctuations are of order the initial field. Other values for \( a \) should not be ruled out however, because the magnetic turbulent spectrum for galactic ISM conditions is not a solved theoretical problem. In particular, when an initially weak field is imposed, driven incompressible simulations show that the magnetic spectral index may be flatter, and flatter than the velocity spectra. Although it may approach \( 5/3 \) for magnetic Prandtl number of order unity (?) there may be a trend toward further flattening at larger magnetic Prandtl numbers (????). The magnetic spectrum is steeper in the presence of kinetic helicity (?). The compressible driven simulations of ? show that the stronger the initial field, the closer the magnetic and velocity spectra match, and the flatter the magnetic spectra the weaker the initial field. The role of the magnetic Prandtl number is hard to assess in ? since no explicit viscosity or resistivity is used.

Proceeding with \( a = 2 \), we note that for this case, ? find that the parallel cascade law is relevant to STFA, and \( \lambda \) in eq. (??) corresponds to the parallel cascade law. When electrons are able to freely stream from one STFA scattering site to the next without deflection by pitch angle scattering (the free streaming limit) \( v_d \) is equal to the electron speed \( v_e \) and,

\[
\left( \frac{dE}{dt} \right)_f = \frac{2}{\rho} m_e v^2 e \left( \frac{\lambda_f}{L} \right)^{-1/2}.
\]

In order to determine the truncation scale of the cascade, we impose the balance condition that the turbulent power, \( \epsilon_T \) is equal to the STFA acceleration rate. Setting the STFA rate equal to the total power selects a single value of \( \lambda_f \), which is an effective upper limit on the truncation scale. We use the subscript \( f \) to denote the free streaming limit.

In the limit of very strong pitch angle scattering, where the scattering length scale \( \lambda_p \ll \lambda_f \), \( \lambda_p \) can be considered the electron mean free path. Electrons are effectively trapped in regions smaller than \( \lambda_f \), executing a random walk with small drift speed. They encounter magnetic compressions only as the compressions stream past, significantly reducing the acceleration rate below that of Eq ???. The power balance condition between STFA and the cascade power is not met, and the cascade will proceed to scales shorter than \( \lambda_f \). The cascade cannot continue farther than a length scale comparable to \( \lambda_p \). At this point, the system passes back into a free-
streaming regime, with a substantially shorter $\lambda$ than is required to meet the power balance condition. The turbulence then drains rapidly.

From the above two paragraphs, we conclude that the cascade would be truncated at a scale no smaller than the lesser of $\lambda_p$ and $\lambda_f$. To assess the minimum scale of dissipation for the Reynolds layer, we assume that the pitch angle scattering is dominated by electron-electron Coulomb scattering. Spitzer (1956) finds the Coulomb self-collision time, $t_c$, to be

$$t_c = \frac{0.266 T_e^{3/2}}{n_e \ln \Lambda}, \quad (5.9)$$

where $T_e$ is the electron temperature, and $\ln \Lambda \sim 25$ is determined by the effective long range cutoff of the Coulomb force in a plasma. The electron speed is given by $v_e = \sqrt{2kT/m_e}$, and thus

$$\lambda_p = v_e t_c = \sqrt{\frac{2kT}{m_e}} \frac{0.266 T_e^{3/2}}{n_e \ln \Lambda} = 2 \times 10^{13} T_4^2,$$  

where $T_4 = T_e/10^4$. For the range of temperatures observed by $?, 0.6 < T_4 < 1.2$, the electron mean free path falls in the range $8 \times 10^{12} \text{cm} < \lambda_p < 3 \times 10^{13} \text{cm}$. Taking the lower limit of this range fixes the lower bound of the cascade truncation scale.

In the free streaming limit we can find $\lambda_f$ by setting $\epsilon_T = n_e (\frac{dE}{dt})_f$

$$\lambda_f = \left( \frac{2m_e}{m_p} \right)^2 \left( \frac{v_e}{v_A} \right)^2 L = 6 \times 10^{14} T_4^{-1}. \quad (5.11)$$

For the observed temperature range of $0.6 < T_4 < 1.2$, $10^{15} \text{cm} < \lambda_f > 5 \times 10^{14} \text{cm}$. This sets an upper bound on the cascade truncation scale.

It should be noted that the free streaming approximation is invalid in the Reynolds layer as $\lambda_f/\lambda_p = 100$ at $T_4 = 6$. There are, on average 100 pitch angle scattering events per electron per encounter with a compression. Given the low fraction of encounters which result in a reflection, $F$, electrons cannot stream freely from one reflection site to the next. The quantity $F$ is determined by the pitch angle condition for reflection

$$\cos^2 \theta_{\min} < \frac{\delta B}{B}, \quad (5.12)$$

and the turbulent cascade law (e.g. eq. (??) for GS turbulence). All electrons with $\theta > \theta_{\min}$ reflect, and
\[ F = \frac{1}{4\pi} \int_{\theta_{\text{min}}}^{\pi} 2\pi \sin(\theta) d\theta = \frac{1}{2} (\lambda_f/L)^{1/4}, \]

where we have integrated over all angles greater than \( \theta_{\text{min}} \) to find the fraction of phase space which satisfies the pitch angle condition for reflection, and we have used (5.12) for the last equality. Electrons which have too large a component of their momentum in the direction parallel to the field are not stopped by the compressions, and pass through them. The acceleration rate must be retarded significantly. We have bounded the turbulent dissipation scale \( 8 \times 10^{12} \text{cm} < \lambda_s < 10^{15} \text{cm} \) for GS turbulence for a Goldreich-Sridhar, \( a = 2 \), cascade.

In order to produce a dissipation scale consistent with the smallest scales observed in scintillation measurements, the cascade must truncate at the inner scale predicted by scintillation models. Stated values of this scale vary: \( 3.5 \times 10^6 \text{cm} \) in Moran, et al. (1990), \( 3 \times 10^7 \text{cm} \) in ?, and \( 3 \times 10^{10} \text{cm} \) in Rickett (1990). Rewriting eq (5.12) for an arbitrary value of \( a \), we have

\[ \lambda_f = \left( \frac{2m_e}{m_p} \right)^{1-1/a} \left( \frac{v_e}{v_A} \right)^{1-1/a}. \]

In order to produce a value \( \lambda_f = 10^{10} \text{cm} \), the cascade must have a very steep spectrum, with \( a < 1.3 \). The other predictions require even steeper spectra. Such spectra are inconsistent with simulations of interstellar turbulence, as well as the inferred spectral indices of scintillation models, which place the lower limit near a Kolmogorov spectrum, \( a = 3 \) (5.12).

Assuming the Reynolds layer is weakly magnetically dominated (7) implies that we have studied dissipation of fast magnetosonic mode turbulence. On the other hand, ? argues that for thermally dominated plasmas, \( \beta > 1 \), the turbulent density fluctuations are dominated by the slow and entropy mode, and that the fast mode is decoupled from the slow, entropy, and Alfven modes. Of the slow and entropy modes, only the slow mode is important for STFA. While the fast mode could be dominant for STFA in the Reynolds layer, the acceleration by slow modes does achieve the same qualitative result, albeit at a shorter dissipation scale. In the Reynolds layer, where \( \beta \sim 0.1 \), the slow mode velocity is given roughly by the sound speed \( c_s \sim 0.2v_A \). If both modes are present with equal energy density, the fast mode is more efficient for
STFA than the slow mode. To apply eq. (??) to STFA of slow modes in a plasma with $\beta < 1$ requires the replacement of $v_A$ with $c_s$, such that

$$\left(\frac{dE}{dt}\right)_S = \frac{2}{L} m_e c_s^2 v_d \left(\frac{\lambda_s}{L}\right)^{-\frac{1}{1+a}}.$$

(5.15)

For free streaming electrons, there is no change in $v_d = v_e$ and the dissipation scale is only decreased by a factor of $(c_s/v_A)^2 = 0.04$. For trapped electrons, the mean free path continues to place a lower bound on the dissipation. Despite the moderate decrease in $\lambda_s$ in a slow mode dominated cascade, it remains well above the scintillation length scale.

In addition to providing the correct power input and a physically plausible dissipative scale, STFA must supply energy in a manner consistent with heating, as opposed to non-thermal acceleration. ? demonstrated that, in the limit of no self-interaction which is appropriate for solar flare applications above a few keV, the electron energy spectrum within the acceleration region is shaped by two competing effects: a bulk shifting to higher energy, and a diffusive spreading of the spectrum. The resulting spectrum can be quasi-thermal, even when electrons are not truly sharing energy. However, in the case of ISM heating, where Coulomb self-scattering of electrons is presumed to be the dominant source of pitch angle scattering, STFA is accompanied by rapid thermalization of the electron population. Consider the acceleration time, $\tau_{STFA}$

$$\tau_{STFA} = \frac{E}{\left(\frac{dE}{dt}\right)_S} = 3 \times 10^{12} T_4 s,$$

(5.16)

and the thermalization time, $\tau_{eq}$, (Spitzer 1956)

$$\tau_{eq} = 6 \times 10^5 T_4^{3/2} s,$$

(5.17)

where we have taken the thermalization time to be the two species equilibration time, with both species being electrons at a single temperature, $T_4$. When $\tau_{eq} < \tau_{STFA}$, as is the case for the Reynolds layer, then the energy input via STFA is shared among electrons rapidly; the energy spectrum is thermal. The electron-proton thermalization time is greater than $\tau_{eq}$ by a factor of the mass proton to electron ratio $m_p/m_e = 1836,$
which is nevertheless shorter than the acceleration time. The protons (and heavy ions) remain in equilibrium with the electrons as well.

### 5.3 Implications of the turbulent cutoff scale for models of interstellar scintillation

The dominant power source for interstellar turbulence is likely supernovae and superbubble shells (??). Superbubbles are powerful and frequent enough to pass through periodically, and thus reseed turbulence in the entire galactic disk. If the efficiency for converting a single supernova’s mechanical luminosity of $10^{51}$ erg to turbulent energy is $\eta_{SN} = 0.1$, the rate of supernova events in the Milky Way is $R_{SN} = 50$ yr$^{-1}$, then the total turbulent driving from supernovae and superbubbles is estimated to be $3 \times 10^{-26}$ erg s$^{-1}$ (reviews by (??) and references within). This is consistent with the estimated damping rate $\epsilon_T$ from eq (1) as well as the required supplemental heating rate (??).

Although we have considered a GS cascade law because it is inherently anisotropic, and incorporates the magnetic field into the formalism directly, we emphasize again that the GS cascade, with $a = 2$ magnetic spectral index lies at one end of a range of possible cascade power laws (e.g. Goldreich and Sridhar (1997); ??). As discussed, varying $a$ over a wide range does not substantially alter the results of our analysis.

Likewise, it is not certain which magnetosonic wave mode, fast or slow, is dominant in the Reynolds layer. ?? argues for slow modes to dominate in a high $\beta$ plasma. Fast mode turbulence may play a stronger role in $\beta \sim 0.1$ diffuse Reynolds layer plasma. Because the fast mode is a more efficient STFA accelerator, due primarily to its higher phase speed, we emphasized STFA on fast modes, but it was shown in section 2 that the major qualitative result of our study holds for either wave mode: STFA is a sufficient source of heating, and the dissipation scale of MHD turbulence is longer than the short scales observed in radio scintillation measurements.

The correspondence between the turbulent driving power and the inferred supplemental heating rate of ?? is suggestive of a connection. This connection would be more strongly verifiable if the turbulent dissipation scale could be linked to direct observations of the Reynolds layer. This has not yet been done. ?? and ?? considered that
the radio scintillation observations (e.g. Armstrong, Rickett, Spangler 1995) may be consistent with scattering by localized turbulent density fluctuations across the inertial range of Kolmogorov turbulence, but they further suggest that the “fluctiferous” medium is either HII region envelopes, or the higher density portions of the warm ionized medium (WIM). Although possible turbulent dissipation scales (like the Larmor radius) of the scintillating region correspond well with the smallest scales of radio scintillation, no scintillation measurements are analyzed in lines of sight through the low density Reynolds layer. Instead assume that either the Larmor radius or the ion inertial length is the key dissipative scale in the Reynolds layer, just as in denser regions of the ISM, and use this assumption to calculate dissipation and heating rates.

We find that the STFA turbulent dissipation scale is instead likely far larger ($\lambda_s \geq 10^{13}$ cm) in the Reynolds layer, for GS turbulence. The STFA dissipation scale does drop to the observed scintillation scale in the case of a very steep turbulent spectrum at small scales, with index $a \leq 1.3$. Interpretations of the scintillation as turbulence driven, such as $?, \alpha = 3$. Although STFA is seemingly at odds with radio scintillation measurements, it may not be if the source of the scintillation is not a continuous turbulent cascade as is commonly assumed. In particular, recent studies have found that interstellar radio scintillation is in fact consistent with a non-Gaussian distribution of stellar ionization shells Boldyrev & Gwinn (2003, 2005); Boldyrev & Konigl (2005). Local index of refraction variations at the thin photoionized surfaces of molecular clouds are distributed randomly throughout the interstellar medium. Radio waves are deflected through small angles at these fronts, and adjacent rays can be deflected through differing paths. A detailed model employing Levy statistics provides an excellent fit to the temporal spreading of pulses from distant pulsars, which traditional cascade scintillation models have had difficulty explaining. Boldyrev & Gwinn (2005) and Boldyrev & Konigl (2005) conclude that the density fluctuations are more likely to result from randomly distributed thin ionization shells in the ISM than from global turbulence. As a result, the scintillation would no longer provide a constraint on the inner scale of the turbulent cascade. Turbulence driven on very large ($\geq 10$ pc) scales could damp on scales larger than the smallest scintillation scales and dissipation of the turbulence by STFA would not conflict with scintillation measurements.
The lower bound of the dissipation scale found above violates an assumption used in the approach of ?. Given that the measured supplemental heating rate in the low density ISM can be well explained by turbulent dissipation, one may infer that turbulence is driven and dissipated throughout the ISM. The density is higher and temperature lower in the lower $|z|$, and thus $\lambda_p$ is shorter, in the portions of the ISM where the scintillation may occur. STFA is then even more closely tied to the lower bound for $\lambda_s$ set by Coulomb scattering at low $|z|$. If STFA is important, it would then be unlikely that turbulence input by superbubbles cascades to sufficiently small scales in any region of the interstellar medium to account for the scintillation observations. This gives credence to the argument of Boldyrev & Gwinn (2003, 2005); Boldyrev & Konigl (2005), which does not rely on cascading turbulence but concludes that the statistical spatial distributions of the ionized boundaries of molecular clouds better explains scintillation than do models based on turbulence dominated density fluctuations.

5.4 Conclusion

Dissipation of turbulence almost certainly plays a role in the heating of the interstellar medium. The turbulent energy supply from supernovae is sufficient to provide the supplemental heating source required by the observations of ???. We have shown that STFA can act as the damping mechanism in the Reynolds layer of the Milky Way, truncating the turbulent cascade at length scales no shorter than $8 \times 10^{13}$cm for a cascade no steeper than Goldreich-Sridhar. This truncation scale is too large to be consistent with turbulence-based models of interstellar scintillation. Instead, models like those of Boldyrev & Gwinn (2003, 2005); Boldyrev & Konigl (2005), which do not rely on the cascade, are required.

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