Rayleigh-Taylor-Induced Electromagnetic Fields in Laser-Produced Plasmas

by

Mario J.-E. Manuel

S.M. Aeronautical and Astronautical Engineering
Massachusetts Institute of Technology (2008)
B.S. Aeronautical and Astronautical Engineering
B.S. Physics
B.S. Astronomy
University of Washington (2006)

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Abstract

Spontaneous electromagnetic fields can be important to the dynamic evolution of a plasma by directing heat flow as well as providing additional pressures on the conducting fluids through the Lorentz force. Electromagnetic fields are predicted to affect fluid behavior during the core-collapse of supernovae through generation of fields due to hydrodynamic instabilities. In the coronae of stars, self-generated magnetic fields lead to filamentary structure in the hot plasma. Recent experiments by Gregori et al. investigated sources of protogalactic magnetic fields generated by laser-produced shock waves. In inertial confinement fusion experiments, self-generated electromagnetic fields can also play a role and have recently become of great interest to the community. Present day laser facilities provide a unique opportunity to study spontaneous field-generation in these extreme environments under controlled conditions.

Instability-induced electromagnetic fields were investigated using a novel monoenergetic-proton radiography system. Fusion protons generated by an ‘exploding-pusher’ implosion were used to probe laser-irradiated plastic foils with various preimposed surface perturbations. Imaging protons are sensitive to electromagnetic fields and density modulations in the plasma through the Lorentz force and Coulomb collisions, respectively. Corresponding x-ray radiographs of these targets provided mass density distributions and Coulomb effects on protons were assessed using a Monte Carlo code written using the Geant4 framework. Proton fluence distributions were recorded on CR-39 detectors and Fourier analyzed to infer path-integrated field strengths.

Rayleigh-Taylor (RT) growth of preimposed surface perturbations generated magnetic fields by the RT-induced Biermann battery and were measured for the first time. Good data were obtained during linear growth and when compared to ideal calculations, demonstrated that field diffusion near the source played an important role. At later times in the plasma evolution, 3-D cellular structures were observed for all foil types. These features were found to be analogous to previously observed filamentary field structures by Séguin et al. in laser-driven spherical targets. Face-on images of these field structures provided good data to quantitatively analyze the size of these features, not previously attainable due to the complexity of the 3-D spherical data. Work presented here demonstrates that these field structures are likely caused by the magnetothermal instability in the underdense corona.

Thesis Supervisor: Dr. Richard D. Petrasso
Title: Division Head, High Energy Density Physics

Thesis Reader: Dr. Jeffrey P. Freidberg
Title: KEPCO Professor Emeritus of Nuclear Science and Engineering
Acknowledgments

My arrival into the field of HED physics emerged through a series of serendipitous events, concluding with a random stop at a physics poster session at MIT in the spring of 2006. At this poster session I met James Ryan Rigg, who introduced me to the HEDP Division at MIT, led by Dr. Petrasso. I sincerely thank Ryan for introducing me to this field of research and for all of the invaluable conversations and help in accustoming myself to a new and exciting field of research.

During my time here at MIT, I have had the opportunity to work closely with all of the senior scientists in the HED Division. I thank Dr. Petrasso for the ‘extreme’ encouragement given to me regularly, and for always pushing me to achieve the current goal while keeping the next question in mind. I am grateful to Dr. Frenje for his in-depth knowledge of the field, theoretically and experimentally. I am very happy to have worked with Dr. Séguin in the capacity of programming, simulations, and data analysis. His knowledge and understanding of CR-39 analysis procedures and characteristics was extremely useful. Without the experimental work and support of Dr. Li, many of my accomplishments would not have been possible. Special thanks also go to Jocelyn Schaeffer, Irina Cashen, Robert Frankel, and Ernie Doeg for all of their help in etching and scanning of the CR-39 data.

None of this research would have been possible without the collaborations with our colleagues at the Laboratory for Laser Energetics. Specifically, Dr. McCrory and Dr. Sangster supported us in splitting a joint NLUF shot day (for the first time) into separate shot days on OMEGA and OMEGA-EP, thus allowing us to perform the necessary experiments to finish my thesis work. I thank Dr. Soures and the NLUF program for providing the opportunity to acquire the data used for my thesis. I also thank Dr. Meyerhoffer, Dr. Goncharov, and Dr. Betti for their support and constructive criticism over the last six years. I greatly appreciate all of the help from Sam Morse, Steve Stagnitto, Dave Canning and everyone from the OMEGA operations crew for their help in the experimental design, preparation, and execution of these experiments. A special thanks goes to Sam Roberts, Michelle Burke, Joe Katz, and Andrew Sorce for all of their assistance and support in the setup and execution required to perform the experiments discussed in this thesis.

One’s graduate school experience is grossly dependent on other graduate students sitting in the same situation. I sincerely thank Dan Casey and Nareg Sinenian for always being there to impart their knowledge in plasma and nuclear physics, computer programming techniques, and life in general. I have spent the better part of a decade sharing more than just office space with these two, struggles, achievements, frustrations, joys, pains, and more. Without them, this experience would not have been the same and I look forward to working with them in the future. Finally, I also thank Hans Rinderknecht, Mike Rosenberg, Alex Zylstra, Caleb Waugh, and Hong Sio for collaborations on numerous projects and many conversations on related physics topics.

Throughout my educational endeavors here at MIT, I have required continual support. I can not thank enough my wonderful, loving wife, Eleonora, for her constant reinforcement through this adventure and our ‘relaxation’ time with our amazing border collie, Kepler. Naturally, I wouldn’t be where I am today without the love and support of my parents, Paul Manuel and Rebecca Purdy, and I thank them for everything they have given me, and for their continued encouragement for what I am doing.

Finally, I am extremely thankful to my supervisory thesis committee: Dr. Richard Petrasso, Dr. Chikang Li, Dr. Jeffrey Freidberg, and Dr. Anne White, for guidance and help in making my thesis the best it could be!
Contributions

While in the high energy density (HED) Physics Division at MIT, I have worked on many projects, some not discussed in this thesis, that have provided experience in a variety of research areas in HED science. This, of course, includes close work with senior scientists and fellow graduate students. Many of these projects involved designing and executing experiments on the Linear Electrostatic Ion Accelerator (LEIA) to characterize CR-39 properties and calibrate detectors, as well as contribute to maintaining the lab and improving the accelerator system. I have also supported senior scientists in the design and execution of multiple experimental campaigns at the Omega laser facility. Of specific interest to my thesis work, the experimental investigation into magnetic fields generated by the Rayleigh-Taylor (RT) instability using monoenergetic proton radiography. Much of the work I have done with the HED Physics Division was in collaboration with scientists at Laboratory for Laser Energetics (LLE), Lawrence Livermore National Laboratory (LLNL), Sandia National Laboratory (SNL), and Los Alamos National Laboratory (LANL) and has resulted in original research and subsequent publications in prestigious journals in plasma physics.

1st Author Publications

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<td>$P_a$</td>
<td>ablation pressure</td>
</tr>
<tr>
<td>$\beta_{RT}$</td>
<td>ablative stabilization coefficient</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>adiabatic index</td>
</tr>
<tr>
<td>$\alpha_{B/E}$</td>
<td>amplitude modulation due to sinusoidal electromagnetic fields</td>
</tr>
<tr>
<td>$\alpha_{mass}$</td>
<td>amplitude modulation due to inhomogeneous mass distributions</td>
</tr>
<tr>
<td>$\alpha_{rms}$</td>
<td>measured amplitude modulation in proton-fluence radiographs caused by a combination of field and mass effects. The rms amplitude modulation of a single frequency sine wave is smaller than the sinusoidal amplitude by $\sqrt{2}$</td>
</tr>
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<td>$\rho L$</td>
<td>areal density: density integrated along a path $L$</td>
</tr>
<tr>
<td>$\rho R$</td>
<td>areal density: radially integrated density of an inertial confinement fusion (ICF) capsule</td>
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<td>$\langle \rho L \rangle_{rms}$</td>
<td>areal density modulation measured from x-ray radiographs. The rms amplitude modulation of a single frequency sine wave is smaller than the sinusoidal amplitude by $\sqrt{2}$</td>
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<tr>
<td>$A_t$</td>
<td>Atwood number</td>
</tr>
<tr>
<td>$A(x, y)$</td>
<td>autocorrelation coefficients used in the analysis of cellular features in radiographs</td>
</tr>
<tr>
<td>$B_\perp$</td>
<td>B field perpendicular to the proton trajectory in radiography experiments</td>
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<tr>
<td>$\beta$</td>
<td>beta is the ratio of fluid to magnetic energy-density (pressure) in a plasma</td>
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<td>$\langle B L \rangle_{rms}$</td>
<td>path-integrated B field modulation inferred from proton radiographs</td>
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<td>$a_0$</td>
<td>classical Bohr radius</td>
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<td>$k_B$</td>
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<td>$\sigma_{rms}$</td>
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<td>$\Phi$</td>
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<td>$H_B$</td>
<td>burn parameter</td>
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<tr>
<td>$R_u$</td>
<td>collisional friction force from interspecies collisions with different flow velocities</td>
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<tr>
<td>$R_T$</td>
<td>collisional thermal force from collisions in a region with a temperature gradient</td>
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<td>$Q_\Delta$</td>
<td>collisional heat flow between electrons and ions at different temperatures</td>
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<td>$Q_u$</td>
<td>collisional heating: work done on electrons by both frictional and thermal forces</td>
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<tr>
<td>$c/\omega_{pe}$</td>
<td>collisionless skin depth</td>
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<tr>
<td>$\sigma_C$</td>
<td>Coulomb cross section</td>
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<td>$n_{cr}$</td>
<td>critical density</td>
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Symbolic Notation

$I$ stalk current: In Section 5, this is the total current defining the axisymmetric B field

$\lambda_D$ Debye length

$\langle \theta \rangle_{\text{rms}}$ deflection angle modulation used to model proton deflections due to sinusoidal fields

DD Deuterium-Deuterium: Used in reference to mixtures and fusion reactions

$D^3\text{He}$ Deuterium-Helium-3: Used in reference to mixtures and fusion reactions

$D\text{T}$ Deuterium-Tritium: Used in reference to mixtures and fusion reactions

$D_m$ diffusion coefficient of a resistive plasma

$\tau_{\text{diff}}$ diffusion time for B fields in a resistive plasma

$E_\perp$ E field perpendicular to the proton trajectory in radiography experiments

$e_0$ elementary charge: Defined as a positive quantity

$\langle EL \rangle_{\text{rms}}$ path-integrated E field modulation inferred from proton radiographs

$\omega_{ee}$ electron cyclotron frequency

$\omega_{pe}$ electron plasma frequency

$\epsilon_0$ emissivity of free space

$v_t$ etch rate of the bulk CR-39

$v_t$ etch rate of damage trails (tracks) in CR-39

$\theta'$ exit angle: resultant deflection angle from a single binary Coulomb collision in the center-of-mass (CoM) reference frame

$C(k_x, k_y)$ Fourier transform coefficients used in the analysis of cellular features in radiographs

$\eta_\alpha$ fractional energy deposition by fusion-α particles in the hot-spot

$G$ gain

$\Gamma$ coupling parameter or particle fluence

$\gamma_{KH}$ growth rate for the Kelvin-Helmholtz (KH) instability

$\gamma_{RM}$ growth rate for the Richtmyer-Meshkov (RM) instability

$\gamma_{RT}$ growth rate for the RT instability

$\chi$ Hall parameter

$b$ impact parameter: distance between the two particles in a Coulomb collision

$b_{90}$ 90° impact parameter

$L_S/\ell$ inductance per unit length

$\lambda$ context dependent: In Chapter 6 regarding RT experiments, this is the perturbation wavelength. In Chapter 5 regarding stalk experiments, this is the linear charge density defining the axisymmetric E field

$L_\rho$ density scale length

$Re_m$ magnetic Reynolds number is a dimensionless parameter that compares B-field advection to diffusion. Advection dominates at high values of $Re_m$

$m_a$ mass ablation rate
Symbolic Notation

$m_\mu$  reduced mass

$\lambda_{mfpr}$  mean free path

$\langle \theta^2 \rangle$  mean square deflection angle

$\gamma_{MTI}$ magnetothermal instability (MTI) growth rate that is fastest using the classic theory as discussed in Section 7.6.2

$\lambda_{MTI}$ MTI wavelength with the fastest growth rate using the classic theory as discussed in Section 7.6.2

$\mu_0$  permittivity of free space

$\lambda_{AC}$ characteristic wavelength of cellular features observed in late-time proton radiographs

$\Delta \phi_L$ potential drop due to stalk inductance up to the radiograph measurement location

$\Delta \phi_R$ potential drop due to stalk resistance up to the radiograph measurement location

$\phi_{stalk}$ potential on the stalk at the radiograph measurement location

$\phi_{target}$ potential on the target

$r_{1/e}$  $1/e$-radius: This is the distance at which point a radial profile reaches $1/e$ ($\sim 37\%$) of the peak value

$R_s/\ell$  resistance per unit length

$Re$  Reynolds number is a dimensionless parameter that compares inertial to viscous forces in a fluid. The flow is considered laminar when $Re \leq 2300$ and is considered turbulent when $Re \geq 4000$

$(\frac{d\sigma}{d\Omega})$ Rutherford cross section: It is also standard notation for a general angular differential cross section, but in this thesis typically refers to the Rutherford cross section (RCX) unless otherwise noted

$\langle \sigma v \rangle$ average reactivity: Unless otherwise specified, a Maxwellian distribution is assumed

$c_T$ isothermal sound speed

$\eta$  Spitzer resistivity

$\frac{dE}{dt}$ stopping power

$\Pi$ off-diagonal stress tensor: contains viscosity effects

$\kappa$ total thermal conduction coefficient

$\kappa_\parallel$ thermal conduction coefficient parallel to the magnetic field

$\kappa_\perp$ thermal conduction coefficient perpendicular to the magnetic field

$\tau_{ei}$ collision time: Characteristic collision time of electrons with ions

$\Delta_{DFT}$ uncertainty in $\alpha_{rms}$ due to analysis variation across the lineout envelope

$\Delta_N$ uncertainty in $\alpha_{rms}$ due to statistical variation in the lineout envelope

$\Delta \alpha_{rms}$ total uncertainty in $\alpha_{rms}$ due to statistics and analysis variation

$\mu_U$ uranium conversion factor used to calculate areal density modulation from optical depth measurements
Symbolic Notation

\[ \Sigma \]  \textbf{global variance}: long-scale variability in proton fluence from the capsule backlighter

\[ \sigma \]  \textbf{local variance}: short-scale variability in proton fluence from the capsule backlighter

\[ v_a \]  \textbf{ablation velocity}

\[ v_{\text{imp}} \]  \textbf{implosion velocity}

\[ v_{\text{rel}} \]  \textbf{relative velocity}

\[ V_{\text{adv}} \]  \textbf{advection velocity} of B fields in a plasma including flow and the \textbf{Nernst effect}

\[ V_{\text{Nernst}} \]  \textbf{Nernst velocity} is the velocity B fields are advected due to the \textbf{Nernst effect}

\[ \xi \]  \textbf{fluid vorticity}
Abbreviations

AC autocorrelation
BPS Brown, Preston, and Singleton
CLW collisionless Weibel
CM cold-matter
CoM center-of-mass
CPS charged-particle spectrometer
CW collisional Weibel
DFT discrete Fourier transform
DPP distributed phase plate
DPR distributed polarization rotator
EM electromagnetic
EOS equation-of-state
ETI electrothermal instability
ETP equivalent target-plane
FoV field of view
HED high energy density
ICF inertial confinement fusion
IFE inertial fusion energy
IR infrared
ISM interstellar medium
ITF ignition threshold factor
KH Kelvin-Helmholtz
KO knock-on
LANL Los Alamos National Laboratory
LASER Light Amplification by Stimulated Emission of Radiation
Abbreviations

LEIA  Linear Electrostatic Ion Accelerator
LLE  Laboratory for Laser Energetics
LLNL Lawrence Livermore National Laboratory
LMJ Laser Mégajoule
LP  Li-Petrasso
LPI laser-plasma interactions
LTE local thermal equilibrium

MHD magnetohydrodynamic
MTI magnetothermal instability

NDI nuclear diagnostic inserter
NEC National Electrostatics Corporation
NIC National Ignition Campaign
NIF National Ignition Facility
NRC Nuclear Regulatory Commission

P-V peak-to-valley
PTD proton temporal diagnostic

RC radiochromic
RCX Rutherford cross section
REL restricted energy loss
RF radio frequency
RM Richtmyer-Meshkov
rms root mean square
RT Rayleigh-Taylor

SBD surface barrier detector
SBS stimulated Brillouin scattering
SG super-Gaussian
SN supernova
SNL Sandia National Laboratory
SNR supernova remnant
SRIM Stopping and Range of Ions in Matter
SRS stimulated Raman scattering
SSD smoothing by spectral dispersion

TCC target chamber center
TFD Thomas-Fermi-Dirac
TIM ten-inch manipulator
TNSA target-normal sheath acceleration
TPD two-plasmon decay
Abbreviations

TPS  target positioner system
UV   ultraviolet
WRF  wedge range filter
Prologue

John William Strutt, the third Baron Rayleigh, was the first to publish work on the equilibrium of incompressible fluids of variable density\(^1\) in 1883. Prior to this work, in 1873 Lord Rayleigh was elected as a fellow of the Royal Society and left his 7000 acre estate in Witham, Essex to devote his time to science in 1876. He followed James Clerk Maxwell in 1879 as the second Cavendish Professor of Physics at the University of Cambridge. His research covered a wide range of physics topics including sound, wave theory, color vision, electromagnetism, light scattering, hydrodynamics, and many others. Lord Rayleigh was the President of the Royal Society from 1905 to 1908 and received the Nobel Prize in 1904 for work done in the density of gases and the discovery of Argon. The theoretical work done by Lord Rayleigh concerning the equilibrium of the interface of two fluids was only one of his many great contributions to the broader field of physics.\(^2\) However, he focused mainly on the stability of such systems, not the instability.

Sir Geoffrey Taylor investigated the detailed nature of the instability of liquid surfaces in the presence of an acceleration field\(^3\) and published his work in 1950. Leading up to this work, Sir Taylor was stationed in the Royal Aircraft Factory during World War I because of his work in turbulent air flows. Afterwards, he worked on applications of turbulence in oceanography until 1923 when he was appointed to the Royal Society as a Yarrow Research Professor. During this period, Sir Taylor worked on a variety of research topics related to deformation of solids and fluid mechanics. In 1938, Sir Taylor was appointed as a member of the Civil Defence Research Committee and was already a member of the Physics of Explosives Committee (Physex). His research during World War II involved the detonation of explosives and subsequent propagation of blast waves in fluids. This inevitably aligned Taylor with the Manhattan Project, traveling to Los Alamos twice during the summers of 1944 and 1945. “He pointed out that in many of the applications being considered for the implosion phase of an atomic bomb explosion there was in fact a ‘Taylor instability’ which would start the nuclear explosion too soon.”\(^4\) Taylor delayed publication of his classic work on this instability until 1950\(^3\) when one of his research students, D. J. Lewis, was able to accompany Taylor’s theoretical work with experimental verification.\(^5\)

The ‘Taylor instability’ had become widely known in many physical systems by 1950 and was later entitled the Rayleigh-Taylor instability in recognition of the contributions from both Lord Rayleigh and Sir Taylor. Of particular interest to this work, the Rayleigh-Taylor instability has been found to play important roles in inertial confinement fusion capsule implosions and in many astrophysical phenomena.
Chapter 1

Introduction

Rayleigh-Taylor (RT) is a well-known, heavily studied hydrodynamic instability. In its most basic form, the instability is manifested by stratified fluids of different densities in the presence of gravity.\(^1,3\) If the heavy fluid is supporting the lighter fluid, a perturbation of the interface between them will fluctuate as a standard harmonic oscillator where the fundamental frequency is determined by the densities of the two fluids and the relative acceleration between them. However, if the lighter fluid is supporting the heavier, a perturbation of the interface will grow exponentially and the system is considered RT unstable. This instability is the fundamental physics concept explored in this thesis.

It is not necessary for the RT-unstable system to be composed of a lighter and a heavier fluid. Simply put, a hydrodynamic system where the density gradient opposes the acceleration field will be RT unstable. Of specific interest to work discussed herein is the existence of the Rayleigh-Taylor instability in plasmas. This type of environment occurs in natural astrophysical phenomena and in laboratory plasmas as illustrated in Figure 1-1.
1.1 Rayleigh-Taylor in Plasmas

Two phenomena related to supernovae (SNe), where the RT instability occurs and plays a role in the dynamics, are illustrated in Figure 1-1a-b. In Figure 1-1a, simulations done by Kifonidis et al.\(^6\) demonstrated the role RT plays in the mixing dynamics during core-collapse of supernovae. The inevitable consequence of a supernova is the expansion of material into the interstellar medium (ISM). The composite Hubble\(^7\) image of the Crab Nebula in Figure 1-1b is an example of a supernova remnant (SNR). The expansion of supernova material into the ISM causes the RT instability and generates localized concentrations of matter. These dense clusters can be seen in the image shown in Figure 1-1b and can become the birth places of new stars!

RT also plays a role in mixing of laboratory plasmas, specifically during the implosion of capsules used in inertial confinement fusion (ICF) experiments. During the acceleration and deceleration phases of the implosion process, RT growth of surface perturbations occur as illustrated by the 3-D simulation\(^8\) results shown in Figure 1-1c. These spikes grow on both the inner and outer surfaces and can compromise the target integrity. The shell must remain intact to compress the fuel within the capsule. The additional complexities of RT dynamics in a high-energy-density environment, as seen in inertially confined plasmas, is the main thrust of this thesis.

Figure 1-1: Images of some sample physical systems where the Rayleigh-Taylor instability occurs in plasma: (a) during core-collapse of supernovae, (b) within supernova remnants, and (c) in inertial confinement fusion targets.
1.2 High Energy Density Physics

A physical system whose energy density (pressure) is greater than 1 Mbar ($10^5$ J/cm$^3$ or $10^{11}$ Pa) is considered to be in the high energy density (HED) physics regime. Physical phenomena in these environments exist naturally in the universe within solar and gas-giant cores, supernovae, neutron stars, black hole accretion disks, molecular clouds, planetary nebulae, etc. or in man-made systems such as ICF plasmas and high-intensity-laser-produced plasmas. Figure 1-2 illustrates the variety of astrophysical phenomena that exist in the HED regime and the capabilities of current ICF facilities to probe this parameter space (this figure was adapted from the Nuclear Regulatory Commission (NRC) Report, Frontiers in High Energy Density Physics: The X-Games of Contemporary Science that was published in 2003). It provides a good overview of relevant physical phenomena in the HED regime and the specific areas in temperature-density parameter space that current experimental facilities can achieve.

Energy densities of this magnitude were not available to experimentally investigate until the early 1900s. The advent of the particle accelerator in the 1930s gave physicists
the hardware needed to energize and collimate particle beams. By focusing high energy particle beams onto stationary targets, the HED regime was opened to be experimentally investigated. This led to the concept of beam fusion, which was found to be an extremely inefficient means to achieving fusion energy because of large particle losses and minimal fusion reactions. Subsequently in 1960 the first Light Amplification by Stimulated Emission of Radiation (LASER) was demonstrated by Theodore Maiman at Hughes Research Laboratories. The arrival of lasers to the field of experimental physics led to higher achievable energy densities and the first concept for ICF. However, the lasers of the period were not powerful enough to achieve the necessary conditions for efficient energy production.

Laser technology has since developed to the point of achieving extremely high energy fluxes ($\sim$kJ/mm$^2 = 10^5$ J/cm$^2$ on the OMEGA laser, $\sim$MJ/mm$^2 = 10^8$ J/cm$^2$ at the National Ignition Facility (NIF)) and short pulse durations ($\sim$ns $= 10^{-9}$ s). These high-power laser facilities seek to compress ICF fuel capsules to high densities and temperatures whereby fusion reactions occur and release copious energy. In addition to OMEGA and the NIF, many other lower energy ($\sim$10 J) laser systems around the world can create environments in which HED phenomena can be studied by utilizing ultra short ($\sim$fs $= 10^{-15}$ s) pulses with small ($\sim$100 $\mu$m) spot sizes. In present day laser systems, one talks in Terawatts ($10^{12}$ W) or Petawatts ($10^{15}$ W) of power because of the high energies delivered in such short timescales. These high intensity lasers provide the capabilities to experimentally investigate a vast field of unexplored phenomena in HED environments relevant to the astrophysics and ICF communities.
1.3 Thesis Outline

The RT instability has been a topic of intense research as it pertains to ICF and astrophysical phenomena. In these environments the fluid undergoing this hydrodynamic instability is not charge-neutral, but a plasma consisting of separate populations of ions and electrons. These distinct populations give rise to electromagnetic fields due to charge separation and currents generated by separate fluid flows. Detailed numeric simulations for a number of different environments and physical conditions have predicted the magnitude and structure of RT-induced electromagnetic fields, though no experimental evidence of these fields has been shown. This thesis presents the first experimental measurements of these illusive fields in plasmas and compares them to numerical results.

Chapter 1 qualitatively describes the Rayleigh-Taylor instability and illustrates some physical environments where it plays a role in the system dynamics. A brief introduction to the HED physics regime is given and the technological advances that have granted the experimental capabilities to investigate these environments.

Chapter 2 covers introductory material for ICF in HED research. Top level physics concepts relevant to ignition in ICF are covered and the capabilities of major experimental facilities are discussed.

Chapter 3 introduces some basic plasma physics concepts. An overview of the fluid equations and magnetohydrodynamics is given. The Coulomb interaction between particles is thoroughly discussed and its implications on energy loss and scattering. Basic transport phenomena is covered as it pertains to experiments described in this thesis. Electromagnetic field generation mechanisms are derived from the two fluid equations. A brief overview of plasma instabilities is given for both laser-plasma interactions and basic hydrodynamic instabilities. This section ends with a comprehensive review of the Rayleigh-Taylor instability and its role in ablative systems.

Chapter 4 describes the monoenergetic proton radiography system in detail. A brief overview of short-pulse proton radiography is given and contrasted to the approach used in this work. CR-39 as a proton detector is discussed along with characterization studies done on the MIT Linear Electrostatic Ion Accelerator (LEIA). The validity of the cold-matter approximation in proton radiography is thoroughly discussed. Finally, this section concludes by outlining the Geant4 framework and illustrates its effectiveness as a modeling tool for proton radiography experiments.

Chapter 5 illustrates the use of monoenergetic proton radiography to measure a ‘simple’ field topology and demonstrates the effectiveness of the Geant4 simulation. The support stalks of irradiated capsules were radiographed to measure charge and current distributions during and after the pulse. These measurements were used to quantify the circuit properties of the stalk for the first time. Geant4 is demonstrated as a useful modeling tool for interpretation of proton radiographs and experimental results are discussed.

Chapter 6 covers proton radiography of laser-driven RT experiments. The experimental configuration and diagnostic techniques are explained in detail. The hydrodynamic code DRACO was used to model fluid evolution of the laser-foil interaction and post-processed to calculate magnetic and electric field distributions. Experimental results of path-integrated magnetic-field measurements during linear growth are covered and compared with numerical calculations done in the collisionless limit.

Chapter 7 discusses unexpected results from late-time proton radiographs of planar foils. Coherent cellular field structure was observed at the same time for foils of various preimposed surface conditions. These fields were found to be analogous to previously observed...
filamentary fields around directly-driven spherical targets. Analysis given here demonstrates that the likely source of these coronal field structures is the magnetothermal instability.

Chapter 8 summarizes the multitude of experimental results covered in this thesis. Appendix A lists all of the shot numbers and summarizes important data for all experiments done on OMEGA that were discussed in this thesis. Appendix B gives a detailed description of the analysis used in vacuum studies discussed in Section 4.3.3. In these experiments, the effects on proton response in CR-39 due to prolonged exposure to high vacuum were characterized. Appendix C provides a short overview of the general workflow of the Geant4 code. A description of how to run the simulation for proton radiography experiments is given with a description of how to add a new radiography object in the current code. Appendix D contains a list of useful dimensionless parameters in laser-produced plasmas. Typical parameter profiles are given for a sample plasma environment and results are summarized in a table. Appendix E contains a short description of plasma bubbles generated by laser-foil interactions. The two possible proton radiographic geometries are described and differences between these configurations are discussed. Appendix F provides some practical information regarding laser spots on OMEGA. The laser beam configuration used in experiments discussed in Chapters 6 and 7 is covered.

References


8. LLNL. “Target Physics” (accessed 06/2012).


Chapter 2

High Energy Density Science

The breadth of research in the high energy density (HED) scientific community has dramatically broadened because the development of many new experimental facilities and it is continually growing. A sense of this field can be seen by the variety of research disciplines in HED science: the study of complex interactions between lasers, plasma, and ion/electron beams; the equation-of-state of materials in HED environments; high-current discharges and pulsed power; radiation-matter interactions; HED hydrodynamics; plasma nuclear science; HED astrophysics; and the study of inertially confined fusion energy sources. Current and planned HED facilities provide a unique opportunity to experimentally explore these complex physical systems that were previously only available through observation and/or theory. This chapter covers the basic physics concepts necessary to discuss fusion reactions in Section 2.1. The inertial confinement method for achieving fusion is reviewed in Section 2.2 and this chapter concludes with the capabilities of current experimental facilities.


2.1 Fusion Energy

Net energy output derived from a fusion power plant is a goal of many countries including the United States. Nuclear energy from a fusion reactor does not suffer from the same problems as those from a fission reactor, fusion fuel is readily available and does not generate dangerous radioactive isotopes like those created from the fission of $^{235}\text{U}$. However, unlike fission, fusion reactions require overcoming the Coulombic barrier between the reacting nuclei, necessitating high reactant energies. Therefore, the method of fuel confinement is of paramount interest since extreme temperatures, of order millions of degrees, are required for these reactions to occur. There are two primary schools of thought when it comes to confinement: magnetic and/or inertial. In the former, since the hot fuel is ionized, large magnetic fields ($\sim$ a few Tesla) are used to control and compress the fuel in steady-state operation, continually producing fusion reactions and energy output. Various magnetic topologies are under research with the primary contender being that of the Tokamak, a toroidally shaped magnetic bottle. Initial heating of the fuel is attained through high-power radio frequency (RF) waves and maintained by ohmic heating from the plasma current and alpha-heating from fusion reactions. For more information on magnetic confinement fusion, the reader is encouraged to see the book by Freidberg. The latter school of thought, that will be discussed thoroughly throughout this thesis, is inertial confinement fusion (ICF).

In 1972 John Nuckolls et al. sparked the novel idea of fusion energy and ignition through laser-compression of a fuel capsule to thousands of times liquid density. The ICF approach to fusion energy is initiated by ablation of the surface of a spherical fuel capsule filled with cryogenic Deuterium-Tritium (DT) fuel. Outer surface material is rapidly heated and expands outwards, resulting in a spherical rocket implosion of the fuel capsule. The term ignition refers to an implosion whose DT-fusion alphas are used to heat the remainder of the fuel in an outward propagating burn wave without the need of further external power input (see Section 2.2.2). Nuckolls’ initial estimate of the energy needed to achieve ignition was insufficient due to the presence and prominence of instabilities, due to hydrodynamics (see Section 3.5.2) and laser-plasma interactions (LPI) (see Section 3.5.1) during the implosion. Most research in ICF, since conception in 1972, has been focused on the understanding and mitigation of these instabilities in the search of a functioning fusion reactor design. The two main approaches to an ignited ICF implosion that will be discussed here are termed direct-drive and indirect-drive.

Both ICF drive concepts involve energy deposition by high powered laser systems. However, the method in which the energy is deposited to the fuel varies. Direct-drive implosions involve direct irradiation of the spherical target surface by laser light. On the other hand, indirect-drive involves lasers that are incident upon the inside of a high-Z (high atomic number), cylindrical can called a hohlraum. The irradiation of the inner hohlraum wall converts the laser energy into black body emission x rays that ablate the spherical target surface and drives the fuel inwards in a similar fashion as direct-drive. By using the black body x rays, short wavelength nonuniformities are smoothed out when compared to direct illumination. Each method has its advantages and disadvantages, but both seek a ‘hot-spot’ fuel configuration. This fuel-mass structure is realized by a shell of cold fuel that is dense enough to stop DT-fusion alphas surrounding the hot, sparse, core. It is this ‘hot-spot’ in the core that must produce enough DT fusion reactions to spark a burn wave through the cold fuel; resulting in net energy production.
2.1. FUSION ENERGY

Figure 2-1: The binding energy per nucleon of different elements is plotted against their respective mass number. At small nuclei (low A) the binding energy per nucleon increases up to A = 56 (Iron). After Iron, the binding energy per nucleon slowly decreases with increasing A. It is clear that for a reaction to release energy, the reactants must fuse for A < 56 and fissure for A > 56. (Plot adapted from Atzeni)

2.1.1 The Basics

Nuclear fusion is the process by which nuclei combine together to form heavier elements. This is the process used by stars to create energy. The amount of energy released by a single nuclear reaction is calculated through Einstein’s most famous formula

\[
E = \Delta mc^2 ,
\]

\[
\Delta m = \sum m_{\text{reactants}} - \sum m_{\text{products}} .
\]

Energy is released because the mass of the products is less than that of the reactants, and thus the binding energy of product nuclei is greater than that of the original reactants. Because energy released in a nuclear reaction is a result of the change in nuclear binding energy, it is useful to note how the binding energy of the elements change as a function of mass number A. Figure 2-1 illustrates how the binding energy per nucleon changes as a function of mass number. Fission energy is created by heavy nuclei because the binding energy increases as the material fissures and becomes lighter (A > 56), whereas fusion energy is released as lighter elements fuse to become heavier (A ≤ 56). For this reason, fusion energy research focuses on using small nuclei, namely the hydrogenic isotopes Deuterium and Tritium. The net energy released from a fusion reaction is called the Q-value and is calculated by the difference in binding energy as

\[
Q = \sum B_{\text{products}} - \sum B_{\text{reactants}} ,
\]

where B is the binding energy of the specified reactants or products. Table 2.1 shows the fusion reactions that are discussed in this thesis where the Q-values have been divided between the products using the conservation of momentum.
Table 2.1: Three relevant fusion reactions to work discussed in this thesis. DT is the fuel used in ignition capsules and hence the primary reaction of concern in fusion energy. The mirror reaction to this is the D$^3$He which is utilized in monoenergetic proton radiography discussed in Chapter 4. The DD reaction will take place in capsules of either fuel fill because there are plenty of D nuclei present that will interact. The branching ratio for the two DD reactions is 50% at reactant energies of interest.

<table>
<thead>
<tr>
<th>Reactants</th>
<th>Products</th>
<th>Q-value [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D + T</td>
<td>$\alpha$ (3.5 MeV) + p (14.1 MeV)</td>
<td>17.6</td>
</tr>
<tr>
<td>D + $^3$He</td>
<td>$\alpha$ (3.6 MeV) + p (14.7 MeV)</td>
<td>18.4</td>
</tr>
<tr>
<td>D + D</td>
<td>T (1.0 MeV) + p (3.0 MeV)</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>$^3$He (0.8 MeV) + n (2.5 MeV)</td>
<td>3.3</td>
</tr>
</tbody>
</table>

In order to fuse two positively charged nuclei, a particle must overcome the repulsive Coulomb barrier produced by the other. The electric potential caused by a field particle of charge $Z_f e_0$ has the form

$$V_f = \frac{Z_f e_0}{4\pi\epsilon_0 r},$$

where $\epsilon_0$ is the permittivity of free space, $r$ is the distance from the field particle. For a test particle with charge $Z_t e_0$ to classically interact with the nucleus of the field particle, it would have to have a kinetic energy greater than its potential energy in the Coulomb field $U = Z_t V_f$. Using the approximate size of the nucleus ($r_{nuc} \approx 1.4A^{1/3} \times 10^{-15}$ m, where $A$ is the mass number) and assuming the fusion reaction of Deuterium and Tritium, the kinetic energy required is $\sim 500$ keV. Luckily, the quantum behavior at these scales allows for particles to tunnel through the potential barrier; allowing nuclei at much lower energies to interact. The likelihood for a test particle of a specified energy to interact with a field particle is quantified by the cross section of the interaction. The detailed theory behind fusion cross sections is a complex topic, involving quantum mechanics and resonance theory, and is beyond the scope of this thesis; for more detailed information, the reader is encouraged to see Chapter 1 in the book by Atzeni. For the purposes of work discussed herein, the fusion cross section will be considered a quantity that one can look up in a number of tables for the reactions of interest.

Nuclear interactions in ICF or stellar media are typically characterized in terms of the volumetric reaction rate ($R$). For two-body fusion reactions with ion number densities $n_1$ and $n_2$ it may be defined as

$$R_{12} = \frac{n_1 n_2}{1 + \delta_{12}} \langle \sigma v \rangle_{12},$$

$$\langle \sigma v \rangle_{12} = \int \sigma(|\vec{v}|)|\vec{v}|f(\vec{v})d\vec{v},$$

where $\langle \sigma v \rangle_{12}$ is the average reactivity, $v$ is the relative speed, and $f(\vec{v})$ is the normalized
2.1. FUSION ENERGY

relative velocity distribution function, such that

\[ n \equiv \int \vec{\nu} f(\vec{\nu}) d\vec{\nu} . \]  

(2.7)

The Kronecker δ symbol accounts for reactants that are of the same species, so that a particle cannot fuse with itself; δ = 1 for reactants of the same species and 0 otherwise. Equation 2.6 can also be rewritten in terms of the center-of-mass energy by making the proper substitution of \(|\vec{\nu}| = v = \sqrt{2m_r/E}\), where \(m_r = \frac{m_1m_2}{m_1+m_2}\) is the reduced mass. In many cases, it is a good approximation to assume that the reactant particles have reached thermal equilibrium, and therefore are represented well by a single temperature Maxwellian distribution function,

\[ f(\vec{\nu}) = \left( \frac{m_r}{2\pi k_B T} \right)^{3/2} e^{-\frac{m_r \nu^2}{2k_B T}} , \]  

(2.8)

where \(k_B\) is Boltzmann’s constant and \(T\) is the single temperature of the system. The average reactivities of the reactions listed in Table 2.1 are shown in Figure 2-2 assuming a single temperature Maxwellian distribution. The volumetric reaction rate, in general, is a function of space and time so that the total yield \(Y_{12}\) is the integral over all relevant space and time,

\[ Y_{12} = \int_t \int_{\vec{r}} R_{12}(\vec{r}) d\vec{r} dt . \]  

(2.9)

In many instances it is convenient to approximate this double integral using average quantities over the actual burn duration, such that

\[ Y_{12} = \bar{n}_1 \bar{n}_2 (\bar{\sigma} v)_{12} V \bar{t} , \]  

(2.10)

where the bar indicates the quantity averaged over the burn. The total fusion energy yield is calculated by multiplying the reaction yield by the Q-value of the specific fusion reaction. The gain \(G\) of the fusion system is defined as the fusion energy output over the laser energy input. In ICF, the energy introduced to the system is the total laser energy on target and the energy output is the total fusion yield. The primary method researchers are pursuing to achieve \(G > 1\) in ICF is through a specific fuel-mass configuration known as hot-spot ignition.
Figure 2-2: Maxwellian reactivities of the fusion reactions listed in Table 2.1. The D(d,p)T (dotted) and D(d,n)$^3$He (dashed) branches are shown, though the branching ratio is very close to 50% at temperatures less than $\sim 10$ keV. Reactivities are calculated using Equation 2.6 and assuming that the two reactant species are in thermal equilibrium such that their distribution function is characterized by a single temperature Maxwellian. (Parameterizations used from Atzeni\textsuperscript{5})
2.2 Inertial Confinement Fusion

The ICF program seeks to achieve high energy gain through compression and ignition of a capsule filled with cryogenic DT fuel. A successful ignition implosion will consist of a series of precisely timed shocks that coalesce and converge at the right time relative to peak compression. In the direct-drive design, strong shocks are launched into the fuel through a series of pickets preceding the main laser pulse as seen in Figure 2-3. Spherical convergence of these shocks spark the fusion burn wave as illustrated in Figure 2-4. Shock fronts travel at the local speed of sound, defined by the ratio of the bulk modulus \( K \) and density \( \rho \)

\[
c^2 = \frac{K}{\rho},
\]

where for an adiabatic process, \( K \) is approximated as

\[
K = \gamma p.
\]

For a neutral fluid of pressure \( p \) and adiabatic index \( \gamma \), the adiabatic sound speed is

\[
c_s = \sqrt{\frac{\gamma p}{\rho}},
\]

and for a plasma with ion mass \( m_i \) and average charge state \( Z \), this becomes

\[
c_s = \sqrt{\frac{\gamma Z k_B T_e}{m_i}}.
\]

The fuel is isentropically compressed as each shock passes through and accelerates the bulk fuel inward during the main drive. Efficient fuel compression is characterized by the

![Figure 2-3: A generic triple picket laser pulse is plotted versus time. Precise timing between pickets is required for the shocks to coalesce at the proper time for ignition. The main pulse begins after the third picket to compress and accelerate the fuel inwards. For ignition, shock coalescence and peak compression are timed to initiate a spark at the hot-spot.](image-url)
adiabat ($\alpha$) of the implosion

$$\alpha = \frac{p_{\text{fuel}}}{p_{\text{Fermi}}},$$

where $p_{\text{fuel}}$ is the local pressure of the fuel and $p_{\text{Fermi}}$ is the Fermi-degenerate pressure. Laser pulses are specifically designed to minimize this parameter within the fuel using pickets. The spikes in laser intensity launch shocks prior to the main drive as shown in Figure 2-3. Every shock travels faster than the preceding one because of the increase in temperature behind the shock. This means that at a definitive distance into the cold fuel, the shocks will catch up with one another and combine to a single strong shock wave. The coalesced shock breaks out of the cold fuel, intensifying as it spherically converges and produces a small number of fusion reactions during shock-burn. The shock rebounds at the center, and propagates outwards to meet and decelerate the incoming cold, dense DT fuel. The incoming fuel creates a spherical cavity within which the shock reverberates until fuel stagnation. During this time, persistent shock propagation through the DT vapor and subsequent compression by the incoming fuel, heat it to very high temperatures $\sim$5 keV at densities $\sim$1000 g/cm$^3$; resulting in many fusion reactions during compression-burn.

Though shocks are launched by intensity pickets, acceleration of the fuel requires a continuous inward force maintained by the laser pulse. Radiation is not incident on to the DT fuel itself, but on the outer shell (ablator) that contains the fuel as illustrated by the outer orange circle in Figure 2-4a. Deposition of energy by the main pulse continually ablates material from the outer shell that expands outward into the vacuum. It is this ablation pressure that predominantly accelerates the remaining ablator and shock-compressed fuel inwards, not the radiation pressure. The dense fuel becomes a spherically converging piston driven by the ejection of mass from the ablating shell.

Figure 2-4: An overview of the isobaric hot-spot ignition concept. (a) Radiation is incident on a cryogenic DT capsule. (b) The outer plastic shell is ablated away while a series of strong shocks are sent through the fuel compressing it to $\sim$1000 g/cm$^3$. (c) The shocks coalesce and converge at the center of the capsule during peak compression forming an isobaric fuel configuration: a hot-spot in the center surrounded by a shell of cold, dense DT fuel. (d) Fusion reactions in the hot-spot spark a burn wave that propagates through the cold fuel producing more and more fusion reactions as it penetrates outwards.
2.2.1 The Ablation Process

An ablatively-driven target acts as a rocket where the payload and rocket fuel are one-in-the-same. Consider a slab target with initial mass per unit area \( m_0 \), which is ablating (ejecting) areal mass density at a rate \( \dot{m}_a \), such that the mass per unit area remaining \( m \) in the target may be expressed

\[
m = m_0 - \dot{m}_a t .
\]

(2.15)

Mass ejection is caused by the applied ablation pressure \( P_a \) and the resulting force balance on the remaining mass dictates

\[
m \frac{dv}{dt} = P_a .
\]

Integration of this equation leads to the ideal rocket equation

\[
v = \frac{P_a}{\dot{m}_a} \ln \left( \frac{m_0}{m} \right) ,
\]

(2.16)

where \( v \) is the velocity of the remaining mass and the exhaust (blow-off) velocity \( (v_{bo}) \) has been written as the ratio of ablation pressure to mass ejection rate. In an ICF capsule, the velocity \( v \) of the remaining mass corresponds to the implosion velocity \( v_{imp} \) of the cold DT fuel. The implosion velocity is extremely important in the formulation of the ignition threshold factor (ITF) as discussed in Section 2.2.2. For a fixed remaining mass, only the ablation pressure and resultant ablated mass flow rate play a role in the final velocity of the remaining target.

Much work has been done to develop the theory behind indirectly- and directly-driven targets and the reader is encouraged to see the summaries given by Lindl et al.\(^8,9\) for a global overview and other references on specific topics. In the directly-driven scenario, initial incident radiation ionizes the target and the resulting plasma expands into the vacuum. For the duration of the drive, lasers are interacting with a plasma and laser energy is absorbed or scattered through various processes as briefly discussed in Section 3.5.1. Energy is predominantly absorbed by plasma electrons, whether through wave-particle damping or through plasmon-particle damping. Energy is distributed to the plasma ions through collisions with electrons. Furthermore, it is the plasma electrons that must conduct heat to the target to continue ablating material away from the shell. It was observed early-on that a classical treatment would over estimate the heat flux and a phenomenological, ‘flux-limited’ form is implemented in many cases, in 1-D as

\[
q = \min \left\{ -\kappa T_e^{5/2} \frac{dT_e}{dx} , -5 \phi \rho c_T^2 \frac{dT_e}{[|dT_e/dx|]} , \right\} ,
\]

(2.17)

where \( \kappa \approx 3/(Ze_0^4 \ln \Lambda \sqrt{32\pi m_e}) \) is the classical heat conductivity coefficient, \( \rho \) is the fluid density, and \( c_T = \sqrt{p/\rho} \) is the isothermal sound speed. The minimum of the two expressions is typically taken, though in some calculations\(^10\) a harmonic mean of the two is used. The flux limit \( \phi \) is a constant defined for a particular laser-plasma interaction. However, flux-
limited heat flow has another popular definition,

\[ q_{\text{flux-limit}} = -f_n e_m \left( \frac{T_e}{m_e} \right)^{3/2} \frac{dT_e}{dx} \frac{dT_e}{dx}, \tag{2.18} \]

where the so-called ‘flux-limiter’ \( f \) defines the level of inhibited heat flow and is related to the flux limit \( \phi \) by

\[ f = \sqrt{\frac{25 m_e (Z + 1)^{3/2}}{Z A^{1/2}} \phi}. \tag{2.19} \]

The form shown in Equation 2.18 and the bottom expression in Equation 2.17 are identical and both forms are used regularly in the literature. Furthermore, the most common colloquial term used when discussing heat flow inhibition is the ‘flux-limiter’ and this typically means \( f \), not \( \phi \). In simulations of plastic (CH) plasmas with laser intensities below \( 5 \times 10^{14} \) W/cm\(^2\), as in the experiments discussed in Chapters 6 and 7, a constant flux-limiter of \( f = 0.06 \) (\( \phi = 0.45 \) in these CH plasmas) has been shown\(^{11} \) to reproduce experimental results well. The physical mechanisms causing the reduction in heat transport include large magnetic fields\(^{12-16} \) and ion acoustic turbulence,\(^{17,18} \) resulting in lateral transport and plasma instabilities. For some of the early work done on inhibited electron transport, the reader is recommended to see work done by Fabbro, Max, and Fabre \textit{et al.}\(^{19} \)

The ablation pressure \( P_a \) and mass flow \( \dot{m}_a \) are dependent on the laser and plasma conditions. Fabbro \textit{et al.}\(^{19} \) showed that when heat conduction is treated classically in a 1-D slab geometry, the ablation pressure can be expressed

\[ P_a \approx 12 \left( \frac{I_{14}}{\lambda_\mu} \right)^{2/3} \left( \frac{A}{2Z} \right)^{1/3} [\text{Mbar}], \tag{2.20} \]

and the mass ablation rate per unit area as

\[ \dot{m}_a \approx 1.5 \times 10^5 \left( \frac{I_{14}}{\lambda_\mu} \right)^{1/3} \left( \frac{A}{2Z} \right)^{2/3} [\text{g/cm}^2/\text{s}], \tag{2.21} \]

where \( I_{14} \) is the absorbed laser intensity in \( 10^{14} \) W/cm\(^2\), \( \lambda_\mu \) is the vacuum wavelength of the laser light in microns, \( A \) is the ion mass number, and \( Z \) is the effective charge state. The scalings and numeric coefficients in these equations are essentially unchanged\(^{19} \) whether the corona is treated adiabatically or isothermally. However, if the flux-limited heat conduction model is used, additional factors of \( (\phi/0.6)^{1/3} \) and \( (\phi/0.6)^{2/3} \) are needed in Equations 2.20 and 2.21, respectively. The resultant blow off velocity can then be written

\[ v_{b_o,\text{classic}} \approx 800 \left( I_{14} \lambda_\mu^2 \right)^{1/3} \left( \frac{2Z}{A} \right)^{1/3} [\mu\text{m/\text{ns}}], \tag{2.22} \]

or in the flux-limited regime,

\[ v_{b_o,\text{limited}} \approx 800 \left( I_{14} \lambda_\mu^2 \right)^{1/3} \left( \frac{2Z}{A} \right)^{1/3} \left( \frac{\phi}{0.6} \right)^{-1/3} [\mu\text{m/\text{ns}}], \tag{2.23} \]
where in these expressions $\phi$ has been normalized to a value of 0.6, which for a fully ionized CH(1:1.38) plasma corresponds to a flux-limiter of $f \approx 0.08$. These relations demonstrate how the ablation process depends on the incident laser parameters and plasma conditions. One caveat in this description is that $I_{14}$ is the absorbed, not incident, intensity which is a complex topic in plasma physics. The reader is encouraged to see the book by Kruer\textsuperscript{20} for detailed analytic treatments of various absorption and scattering mechanisms relevant to laser-plasma interactions in ICF. Nevertheless, high-intensity lasers heat and ablate mass off of the irradiated capsule, where the reacting forces drive the remaining mass and shock-compressed fuel to spherical convergence and fusion in the hot-spot.

2.2.2 Basic Ignition Physics in ICF

To ignite a propagating burn wave through the cold dense DT fuel after stagnation of the incoming fuel, a significant portion of DT-\(\alpha\) energy must be deposited in the hot-spot. Fusion products deposit their energy to the hot-spot through Coulomb collisions; the physics of collisions and stopping power are discussed in Section 3.2. For a particle of a given energy, in a specific material, the stopping power goes up with the areal density, path-integrated density, \((\rho R)\). The threshold for self-heating in the-hot-spot is determined through the energy balance and depends on $\rho R$ and ion temperature $T_i$.

Self-Heating Criterion

The hot-spot will self-heat when the power deposited by the fusion $\alpha$ particles ($P_{\alpha,\text{dep}}$) is greater than the power lost by radiation ($P_{\text{rad}}$), thermal conduction ($P_{\text{cond}}$), and mechanical work done by the hot-spot ($P_{\text{mech}}$),

$$P_{\alpha,\text{dep}} > P_{\text{rad}} + P_{\text{cond}} + P_{\text{mech}}.$$  \hspace{1cm} (2.24)

The dominant radiative loss comes from Bremsstrahlung radiation. In current designs, ignition capsules are not optically thick to these x-rays and therefore they serve as an energy-loss mechanism for the hot-spot. Volumetric energy loss from Bremsstrahlung radiation is

$$P_{\text{rad}} = C_{\text{rad}}\rho_{\text{hs}}^2 T_{\text{hs}}^{1/2} \left[ \frac{\text{W}}{\text{cm}^3} \right],$$  \hspace{1cm} (2.25)

where $\rho_{\text{hs}}$ is the density of the hot-spot in g/cm$^3$, $T_{\text{hs}}$ is the ion temperature in keV, and $C_{\text{rad}} = 3.05 \times 10^{16}$. Energy is also removed from the hot-spot through thermal conduction down the temperature gradient. Classical heat flux is defined by thermal diffusion

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{q} = 0,$$  \hspace{1cm} (2.26)

where $U$ is the thermal energy per unit volume and $\mathbf{q}$ is the heat flux

$$\mathbf{q} = -\kappa_e T_e^{5/2} \nabla T_e,$$  \hspace{1cm} (2.27)

and $\kappa_e$, the thermal conductivity coefficient for electrons is

$$\kappa_e \approx \frac{C_{e,\text{cond}}}{\ln A},$$  \hspace{1cm} (2.28)
where \( \ln \Lambda \) is the Coulomb logarithm. Total losses come from both ions and electrons, though thermal conduction due to ions is typically neglected because \( \kappa_i \) is a factor of \( \sqrt{m_e/m_i} \) smaller than \( \kappa_e \). Integrating Equation 2.26 over the volume of the hot-spot (\( V \)), and applying Gauss’ theorem,

\[
\oint_V \frac{\partial U}{\partial t} \, dV = -\oint_S \nabla \cdot \mathbf{q} \, dV = -\oint_S \mathbf{q} \cdot d\mathbf{S} ,
\]

so the power lost by a spherically symmetric hot-spot due to conduction is

\[
P_{\text{cond}} \approx \frac{S}{V} C_{e,\text{cond}} T_e^{5/2} \frac{\ln \Lambda}{\nabla T_e} ,
\]

where \( S \) is the surface area of the hot-spot. Now let the temperature gradient be approximated by the temperature and radius of the hot-spot, so

\[
P_{\text{cond}} \approx \frac{3C_{e,\text{cond}} T_{hs}^{7/2}}{\ln \Lambda R_{hs}^2} \left[ \frac{W}{\text{cm}^3} \right] ,
\]

where \( C_{e,\text{cond}} = 9.5 \times 10^{12} \), \( T_{hs} \) is the electron temperature in keV, and \( R_{hs} \) is the radius of the hot-spot in cm. Energy will also be lost from the central hot-spot if it performs mechanical work on the surrounding cold dense fuel. The standard definition for work done by a fluid is

\[
E_{\text{mech}} = p_{hs} \, dV ,
\]

and the mechanical power density is

\[
P_{\text{mech}} = \frac{1}{V} \frac{dE_{\text{mech}}}{dt} .
\]

Combining these equations and using the volume of the hot-spot results in

\[
P_{\text{mech}} = \frac{3p_{hs} \, dR_{hs}}{R_{hs} \, dt} .
\]

For an isobaric fuel configuration where the pressure in the hot-spot is equal to the pressure in the cold fuel, \( \frac{dR_{hs}}{dt} = 0 \) and the hot-spot does not lose any energy to mechanical work during ignition.

The only source of energy to compensate for these losses is the energy deposited by the initial DT-fusion burn. Because of the large mean free path in the hot-spot, energy deposition from 14.1 MeV DT-neutrons is negligible; however, energy deposition in the cold fuel will be higher. The 3.5 MeV alpha particles, on the other hand, deposit their energy through Coulomb collisions with ions and electrons in the hot-spot to self-heat the core. Volumetric power deposition by fusion alphas can be expressed in terms of the total ion density \( n_i \), the energy of each fusion \( \alpha (E_\alpha) \), and the atomic fractions of Deuterium \((f_D)\) and Tritium \((f_T)\) in the hot-spot,

\[
P_{\alpha,\text{dep}} = \eta_\alpha f_D f_T n_i^2 (\sigma v)_{DT} E_\alpha .
\]
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Figure 2-5: Criterion for DT-$\alpha$ particles to self-heat the hot-spot in an ICF capsule implosion. Points to the right of the short-dashed (orange) line are where alpha heating is higher than the conduction losses. Points above the long-dashed (red) line have alpha heating dominating over radiation losses. The thick solid (black) line is the threshold for self-heating the hot-spot as defined in Equation 2.37, where alpha heating is greater than radiation losses and the residual power still compensates for heat conducted out of the hot-spot.

The fractional deposition of the alpha energy $\eta_\alpha$ is a function of the hot-spot conditions, such that

$$ P_{\alpha,\text{dep}} = \eta_\alpha C_\alpha (\rho R)_{hs} \langle \sigma v \rangle_{DT} \left[ \frac{W}{\text{cm}^3} \right], \quad (2.36) $$

where $C_\alpha = 8.02 \times 10^{33}$ for a 50-50 DT fuel mixture, $\eta_\alpha = \eta_\alpha (\rho_{hs} R_{hs}, T_{hs}, \ln \Lambda)$, the DT reactivity $\langle \sigma v \rangle_{DT}$ is a function of only temperature in cm$^3$/s, and $\rho_{hs}$ is the density of the hot-spot in g/cm$^3$.

Assuming isobaric ignition and inserting Equations 2.25, 2.31, and 2.36 into Equation 2.24, the self-heating criterion for ICF is

$$ (\rho R)_{hs} > \left[ \frac{(3C_{e,\text{cond}}/\ln \Lambda) T_{hs}^{7/2}}{\eta_\alpha C_\alpha \langle \sigma v \rangle_{DT} - C_{rad} T_{hs}^{1/2}} \right]^{1/2}. \quad (2.37) $$

Using the Li-Petrasso stopping power$^{21}$ for ions in ICF plasmas, the fraction of energy deposited by 3.5 MeV $\alpha$ particles was found to be approximately linear to $(\rho R)_{hs}$ and inversely proportional to the temperature $T_{hs}$,

$$ \eta_\alpha \approx C_\alpha \frac{(\rho R)_{hs}}{T_{hs}^{1.14}}, \quad (2.38) $$

where $C_\alpha = 59.5$, $(\rho R)_{hs}$ is in g/cm$^2$, and $T_{hs}$ is in keV. In this analysis, the hot-spot density
and \ln \Lambda were set to \sim 0.3 \text{ g/cm}^3 \text{ and } 10, \text{ respectively; } \eta_\alpha \text{ deviations of } < 10\% \text{ were observed for densities between 0.1 and 1 g/cm}^3 \text{ at hot-spot temperatures of 1-20 keV. Using this approximation for fractional } \alpha \text{-energy deposition in the hot-spot, the solid line in Figure 2-5 was derived. Points in the } T_{hs}(\rho R)_{hs} \text{ parameter space that are to the right of this line represent hot-spot environments where fusion } \alpha \text{ particles deposit enough energy to overcome radiation and conduction losses. It is in this regime when ignition may occur.}^{1}

**Ignition Threshold Factor**

The Lawson-like criterion discussed in the previous section was derived using a simple 0-dimensional energy balance for the necessary conditions to self-heat the hot-spot with fusion alpha particles. A detailed derivation of the generalized Lawson Criterion has been defined by Betti et al.\textsuperscript{22} for ICF implosions to compare achievements in ICF with magnetic confinement devices. However, another useful treatment of the ignition threshold problem has been summarized by Haan et al.\textsuperscript{23} where they derived an expression for the so-called ignition-threshold-factor (ITF):

\[
ITF = I_0 \left( \frac{M_{DT}}{M_0} \right) \left( \frac{v}{v_0} \right)^8 \left( \frac{\alpha}{\alpha_0} \right)^{-4} \left( 1 - 1.2 \frac{\Delta R_k^{R_{hotspot}}}{R_{hotspot}} \right)^4 \left( \frac{M_{clean}}{M_{DT}} \right)^{0.5} \left( 1 - P_{HS} \right), \tag{2.39}
\]

where the ITF is a dimensionless parameter normalized to unity for marginal ignition and the 0-subscripted parameters are the nominal values as determined from many 1-D, 2-D, and 3-D implosion simulations as discussed by Haan.\textsuperscript{23} The first three parameter ratios in parentheses result from an ideal analysis: \( M_{DT} \) is the fuel mass, \( v \) is the volume-averaged fuel velocity, and \( \alpha \) is the implosion adiabat. The last three factors within parentheses in Equation 2.39 relate to realistic 3-D effects: \( \Delta R_k^{R_{hotspot}} \) is a weighted RMS deviation of the hot-spot radius from its mean value of \( R_{hotspot} \), \( M_{clean}/M_{DT} \) corrects for fuel lost to mixing with the outer ablating material, and \( P_{HS} \) is a measure of the purity of the hot-spot. The detailed derivation and definitions of these parameters are beyond the scope of this thesis, however, it is important to note the high dependence on implosion velocity. The kinetic energy of the impoding fuel is converted into thermal energy in the hot-spot, which directly relates the implosion velocity to reaching ignition conditions in the hot-spot.

**Burn Fraction**

As deposited \( \alpha \)-energy heats the hot-spot, more fusion reactions occur, and a runaway thermal reaction, ‘ignition’, begins. The inner layers of cold, dense DT fuel ablates away into the hot-spot supplying more fuel to burn. In this manner, the burn wave propagates through the cold DT fuel. An energy gain greater than one is achievable because no further energy input is required to effectively burn the fuel. The efficiency of an ICF implosion can

\(^{1}\text{It is also possible to reach ignition, as determined by simulations, outside of this region in } \rho R\text{-T parameter space. This occurs at higher temperatures where the thermal burn wave may be initiated at lower areal densities and will reach the self-heating regime after the burn wave begins.}\)
be defined in terms of the fraction ($\Phi$) of fuel that was burned in fusion reactions,

$$
\Phi = \frac{n_{DT}^{(i)} - n_{DT}^{(f)}}{n_{DT}^{(i)}},
$$

(2.40)

where $n_{DT}^{(i)}$ and $n_{DT}^{(f)}$ are the initial and final number densities of DT pairs respectively. For simplicity, assume a solid sphere of 50-50 DT fuel with volume $V$ and that the only loss channel for D or T is through the DT fusion reaction. The number density of DT pairs at any time during the burn can then be calculated by the following rate equation

$$
\frac{dn_{DT}}{dt} = -n_{DT}^2 \langle \sigma v \rangle_{DT}.
$$

(2.41)

Performing the necessary integrations results in

$$
n_{DT}^{(f)} = \frac{n_{DT}^{(i)}}{I + 1},
$$

(2.42)

$$
I = n_{DT}^{(i)} \tau_{con} \int_0^{\tau_{con}} \langle \sigma v \rangle_{DT} dt,
$$

(2.43)

where $\tau_{con}$ is the inertial confinement time of the fuel; it is noted that this is different than the energy confinement time. Fuel confinement is determined by the time it takes for a sound wave to propagate the distance between the center and the outer edge of the sphere,

$$
\tau_{con} = \frac{R^{(i)}}{c_s}.
$$

(2.44)

The spherical burn wave propagates at the sound speed during confinement at a radius

$$
R(t) = R^{(i)} - c_s t,
$$

(2.45)

such that for a spherically converging volume,

$$
\int_0^{\tau_{con}} \frac{V(t)}{V^{(i)}} dt = \int_0^{\tau_{con}} \left(1 - \frac{c_s t}{R^{(i)}}\right)^3 dt = \frac{R^{(i)}}{4c_s}.
$$

(2.46)

It is clear that the actual confinement time for a spherical fuel configuration is $1/4$ that of the one dimensional case shown in Equation 2.44. Now substituting Equation 2.42 into 2.40 and writing the burn fraction $\Phi$ in its typical form,

$$
\Phi = \frac{(\rho R)^{(i)}}{(\rho R)^{(i)} + H_B},
$$

(2.47)

with the burn parameter $H_B$ defined as

$$
H_B = \frac{2m_{DT} R^{(i)}}{\langle \sigma v \rangle_{DT} \tau_{con}} = \frac{8m_{DT} c_s}{\langle \sigma v \rangle_{DT}}.
$$

(2.48)
Figure 2-6: The burn parameter as a function of ion temperature for fusion reactions shown in Table 2.1. A lower burn parameter means a higher burn fraction for a given areal density. The areal density of an implosion is a function of the compression of the capsule. Therefore, for a given amount of compression, the most burn-efficient fuel is DT. The minimum burn parameter for DT fusion is $\sim 6.7 \, \text{g/cm}^2$ at a temperature of $\sim 38 \, \text{keV}$ which provides the highest burn fraction for a given areal density.

where $\langle \sigma v \rangle_{DT}$ is the DT reactivity evaluated at the burn averaged ion temperature, $m_{DT}$ is the average ion mass of D and T. It is clear from Equation 2.47 that the amount of fuel burned goes up with areal density, and approaches one at high $\rho R$ or low burn parameter. Because the burn parameter is inversely proportional to the reactivity, there can be a significant dependence on ion temperature. Figure 2-6 illustrates the temperature dependence of the burn parameter for the fusion reactions shown in Table 2.1. It is obvious, that for a given fuel areal density, the most burn will occur in DT; this is of course because the DT reactivity is higher than that of the other reactions. The burn parameter for DT fusion has a minimum of $\sim 6.7 \, \text{g/cm}^2$ at an ion temperature of $\sim 38 \, \text{keV}$. As the temperature increases past the minimum, $H_B$ only slowly increases, but for ion temperatures less than $\sim 20 \, \text{keV}$ the burn parameter is a strong function of ion temperature. To achieve the high temperatures required for a high burn fraction, self-heating by fusion products as discussed in Section 2.2.2 is obligatory. A gain higher than 1 requires ignition!

### 2.2.3 Exploding Pushers

Unlike the ignition style targets discussed in Section 2.2, exploding-pusher targets are at ambient temperature and the fuel in the capsule is therefore a gas, not a solid. Exploding-pusher targets were widely used in early direct-drive ICF experiments. The attractive features of exploding-pusher targets included the insensitivity to instabilities of typical ignition capsules such as the Rayleigh-Taylor (RT) and electron-preheat instabilities. However, because of its intrinsically different dynamic structure, the exploding pusher could never be used to reach ignition conditions. The fuel density will never become high enough to sustain the propagation of a burn wave as schematically shown in Figure 2-7. Exploding pushers also have thin capsule walls so that the dominant heating and compression
2.2. INERTIAL CONFINEMENT FUSION

Figure 2-7: An overview of the exploding-pusher capsule concept. (a) Lasers are incident on
an ambient temperature ICF capsule. (b) The outer shell is almost completely ablated away
and a shock is launched, heating the fuel as it spherically converges. (c) The shock converges
at the center and fusion products are emitted. Ideally there is not much mechanical work
done by the shell, so most of the reactions take place during shock-burn.

The mechanism is shock convergence, not mechanical work. Because of the lower fuel mass in
exploding-pusher capsules, energy is deposited to the fuel by the converging shock and will
raise the ion temperature much faster than in the cryogenic ignition targets. High tempera-
tures produced in exploding pushers do, however, provide a well characterized source of
fusion products with simpler implosion dynamics than ignition-style targets.

Exploding pushers are used to calibrate nuclear diagnostics and, in the case of proton
radiography, as a backlighting source. Because of the low total areal density, charged particle
fusion products are not significantly ranged-down and the dominant source of broadening
of the fusion products is due to the Doppler shift of reacting fuel ions. In these cases, the
energy spectra of fusion products are very well approximated as Gaussian

\[ f(E) = \frac{n}{\sqrt{2\pi}\sigma_E} e^{-\frac{(E-E_1)^2}{2\sigma^2_E}}, \quad (2.49) \]

with the standard deviation (\(\sigma_E\)) dependent on the reacting nuclei and temperature (\(T_i\)).

\[ \sigma^2_E = 2E_1 \frac{m_2}{m_1} T_i, \quad (2.50) \]

where \(E_1\) and \(m_1\) are the mean energy and mass of the fusion product in question, respec-
tively, and \(m_2\) is the mass of the other fusion product. If fusion burn takes place during the
laser pulse, charged-particle acceleration and additional energy broadening can take place
because of capsule charging.\(^{26,27}\) However, if burn occurs after the laser pulse and the shell
is mostly ablated away, exploding pushers provide a very well characterized monoenergetic
charged-particle source that can be used to calibrate a variety of nuclear diagnostics.\(^{26,28}\)

Irrespective of the timing of the burn relative to the laser pulse, exploding pushers still
provide a monoenergetic source that emits particles in a quasi-isotropic\(^{29}\) fashion. To take
advantage of this fact, exploding pushers have been used to develop a novel backlighting
technique using D\(^3\)He fusion protons. These capsules and the backlighting technique are
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Figure 2-8: A schematic detailing the use of a high-Z hohlraum for indirect drive ICF. (a) Lasers are incident onto the inner walls generating a high-Z plasma. (b) The laser-plasma interactions generate an x-ray “oven” to irradiate the capsule. Correct shock timing and compression of the capsule is still required for ignition, but the radiation has simply moved from the ultraviolet to the x-ray regime.

discussed in great detail in Section 4.2. However, proton backlighting of this type requires laser beams to implode the exploding pusher capsule in addition to beams irradiating the target to be imaged. Today’s ICF facilities consist of many laser beams with a variety of driving options, providing an environment that supports irradiation of multiple targets.

2.2.4 HED Facilities

High energy density environments are created in laboratories across the world. Thanks to dropping costs and readily available technology, the high intensity lasers typically used to produce these systems have been becoming more popular to use at the university level for research. These smaller systems typically consist of a few beams (or less) and can not create the conditions needed for ICF ignition in the lab, however they fulfill a necessary role in diagnostic development and basic HED science. Larger facilities with many beams provide the necessary power and illumination uniformity needed to implode an ignition style target. The Laser Mégajoule (LMJ) facility is currently under construction in France and will have the capabilities of imploding targets to ignition-relevant conditions. The only facility currently operational with this capability is the National Ignition Facility (NIF).

The National Ignition Facility

Located at Lawrence Livermore National Laboratories, the NIF facility ground was broke in 1997 and the 192 beam system was completed in 2009. The NIF is a neodymium glass laser system whereby a single ytterbium flash in the infrared (IR) is used to initialize the laser pulse with a wavelength of $\lambda \sim 1.053 \, \mu m$. The pulse is temporally shaped and split into 192 individual beams. Each beam is amplified many times before being frequency tripled, such that every beam entering the NIF target chamber is now in the ultraviolet (UV) with a wavelength $\lambda \sim 0.351 \, \mu m$ and carries up to $\sim 9.3$ kJ of energy for a total of $\lesssim 1.8$ MJ with all beams accounted for. The beams are arranged in a symmetric hemispherical pattern with 96 beams distributed in each hemisphere. The reason for this beam configuration lies in the indirect-drive point design of the NIF.

For indirect-drive ICF, lasers are incident on the inside walls of a cylindrical object known as a hohlraum. The hohlraum is composed of a complex cocktail of high-Z ma-
terials in order to efficiently create soft x-rays inside as illustrated in Figure 2-8. The x-ray “oven” created in the hohlraum is the irradiation mechanism, however, proper shock timing and adiabatic fuel compression is still required for ignition. The benefit of the indirect-drive approach is that the illumination of the capsule surface is devoid of small scale non-uniformities because the hohlraum acts as a blackbody radiation source with a temperature of $\sim 300 \text{ eV}$ ($3.5 \times 10^6 \degree \text{C}$). Soft x-rays in this temperature range have wavelengths of order $\sim 0.1 \text{ fm}$ ($0.1 \times 10^{-15} \text{ m}$) effectively reducing the irradiation wavelength by 9 orders of magnitude. Shorter wavelengths are beneficial for some plasma instabilities and allow for direct deposition of radiation energy into the ablating shell. However, only a fraction of the incident laser light is converted to soft x-rays; serving as a loss mechanism that is irrelevant in direct-drive ICF. Also, correctly launching shocks through the capsule fuel is more complex due to the dynamic laser-plasma environment inside of the hohlraum. The first ignition attempts, however, will take place using the indirect-drive approach during the National Ignition Campaign (NIC) which is currently underway. Though, the success of the NIC program is dependent on technological and diagnostic development done at the Omega laser facility.

The Omega Laser Facility

Not only does the Omega laser facility serve as a platform for fielding, testing, and developing new diagnostics necessary for the NIC program, it is an unparalleled system to perform a broad range of basic science experiments. This facility is comprised of two independent systems, OMEGA-60 and OMEGA-EP (extended-performance). Both systems use neodymium amplification with pulse shaping capabilities. These systems can run different experimental campaigns in parallel because each laser system is electrically independent and beams are propagated into individual vacuum chambers with separate diagnostic suites. Or, in ‘joint’ operation, a single short-pulse beam from OMEGA-EP can propagate into the OMEGA-60 chamber to accommodate experiments requiring such a capability. A wide variety of experiments can be done at the Omega laser facility because the two systems provide very different environments to perform research in HED science.

OMEGA-60 is composed of 60 individual beams that are frequency tripled upon entrance to the vacuum chamber, at a wavelength of $\lambda \sim 0.351 \text{ \mu m}$. Each laser beam has a nominal energy of 500 J (for a total of 30 kJ in all beams) and can be pointed to any location within 1 cm of target chamber center (TCC)$^\text{ii}$. Beams enter the spherical target chamber in a soccer ball pattern and can be split into three different ‘legs’ that can be run by two separate drivers. An Aitoff projection of the OMEGA-60 target chamber layout can be seen in Figure 2-9 for orientation of beam legs and diagnostic ports. One leg can be driven separately from the other two such that each driver may have its own pulse shape. These laser driving capabilities allows for some lasers to be used for backlighting purposes and others used to irradiate a target, with the ability to change the relative timing between drives before every shot. Depending on the complexity of the experiment, the shot cycle is $\sim 1/\text{hr}$, where the minimum is set by the cooling time of the optics, $\sim 45 \text{ min}$.

The current system layout was completed and qualified in 1995, so operations and diagnostics have benefitted from many years of experience. The diagnostic suites available for experimental performance are wide ranging. There are many fixed diagnostic for neutronics, x-rays, and charged particles as seen in Figure 2-9, but the ability for a generic diagnostic

\textsuperscript{ii}TCC will be used throughout to refer to the center of any target chamber in context.
payload is made available through the ten-inch manipulator (TIM) system. Users can design and develop their own diagnostics that can be fielded using any of the TIM ports. Most primary diagnostics used for experiments discussed in this thesis were TIM-based, along with system diagnostics for measuring laser timing, spot size, and intensity uniformity. The OMEGA-60 facility has devoted much research into elimination of non-uniformities in the laser spot through development of distributed phase plates (DPPs)\textsuperscript{31} and smoothing by spectral dispersion (SSD)\textsuperscript{32} technologies, resulting in broadband irradiation deviations of $\sigma_{\text{rms}} \sim 12\%$. Many of the technologies developed for OMEGA-60 were also implemented on the newly constructed OMEGA-EP system.

Completed in 2008, the OMEGA-EP laser has shown great promise as an HED science facility. This laser system offers four individually driven beams, two of which are capable of only ‘long’-pulse operation and the other two that, in addition to ‘long’, can also be used for ‘short’-pulses. Here, ‘long’ refers to laser pulses 0.1-10 ns and ‘short’ to pulses 1-100 ps in duration. When beams are compressed to short time scales, frequency tripling is not performed, such that short-pulses have $\lambda \sim 1.053 \mu\text{m}$, whereas long-pulse beams are incident with $\lambda \sim 0.351 \mu\text{m}$. In long-pulse mode, beams can have 0.25-5 kJ of energy each providing intensities up to $\sim 3 \times 10^{16}$ W/cm$^2$; longer pulses allow for more energy. In contrast, short-pulse beams may carry 0.8-2.6 kJ over shorter time scales and in smaller spots achieving intensities up to $\sim 2 \times 10^{20}$ W/cm$^2$. Typically, the long-pulses are used to drive targets for a variety of HED experiments and short-pulses for backlighting in either the OMEGA-EP or OMEGA-60 target chambers. Diagnostics available on the OMEGA-EP system are not as numerous as OMEGA-60, however TIM-based diagnostics are also implemented. Previous experience with OMEGA-60 has helped with the implementation of DPP and SSD\textsuperscript{iii} technologies on OMEGA-EP. The system is still quite new and more experimental and laser-

\textsuperscript{iii}SSD implementation on the OMEGA-EP system is still in progress at this time.
system diagnostics are under development. The Omega facility is capable of performing large-scale ICF and HED experiments that are impossible elsewhere, though can not reach ignition conditions in capsule implosions. Therefore, in addition to basic science experiments, the Omega facility serves as a technologic and diagnostic developmental platform for the NIF, however, much research and development for diagnostic implementation at Omega takes place at smaller laser facilities.

**Other Laser Facilities**

Over the last decade, many new smaller laser facilities have been constructed. Table 2.2 provides information regarding the capabilities of some of these smaller lasers, new and old. Moreover, these facilities have the ability to perform smaller-scale physics experiments and are extremely important for diagnostic development and training of students.

Table 2.2: Other laser facilities in the United States used for High Energy Density science. Compiled by the Omega Laser Users Group student/post-doc panel 2011.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>UM</td>
<td>L-cubed</td>
<td>0.01</td>
<td>0.03</td>
<td>3×10⁹</td>
<td>500 Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T-cubed</td>
<td>8</td>
<td>0.4</td>
<td>4×10¹⁹ (18)</td>
<td>10⁻⁵⁻⁹</td>
<td></td>
</tr>
<tr>
<td>LLNL</td>
<td>Titan</td>
<td>150–350</td>
<td>0.7–9+</td>
<td>2×10²⁰</td>
<td>8/10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Janus</td>
<td>1000</td>
<td>10⁵</td>
<td>10⁻⁸</td>
<td>5-7/day</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Callisto</td>
<td>12</td>
<td>0.060</td>
<td>10²²</td>
<td>10 Hz (0.2 J)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Comet</td>
<td>15</td>
<td>0.150</td>
<td>3×10¹⁹</td>
<td>4/hr</td>
<td></td>
</tr>
<tr>
<td>LLE</td>
<td>MTW</td>
<td>10</td>
<td>0.6–100</td>
<td>2×10¹⁹</td>
<td>10/day</td>
<td></td>
</tr>
<tr>
<td>UT</td>
<td>TPW</td>
<td>180</td>
<td>0.170</td>
<td>2×10²¹</td>
<td>1/hr</td>
<td></td>
</tr>
<tr>
<td>LANL</td>
<td>Trident</td>
<td>80</td>
<td>0.5</td>
<td>10²⁰</td>
<td>8-10/day</td>
<td></td>
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**References**


**For up-to-date information on the capabilities of the OMEGA-60 and OMEGA-EP systems, the reader is encouraged to visit the Omega Facility Documentation website.**


Chapter 3

Introductory Plasma Physics

A plasma is a gas which has been heated to temperatures so high that bound electrons are stripped from their nuclei resulting in a fluid with separate populations of ions and electrons. Because these fluids are charged, in addition to following the laws of fluid mechanics, they are bound by Maxwell’s equations. Though separate, these fluids are intimately connected through the Lorentz force and each can influence the fluid behavior of the other. This ‘collective’ behavior is used to describe the plasma as a whole, though it is composed of multiple component fluids. The typical definition of a plasma consists of three criteria:

I  Collective Effects: \( n \frac{4}{3} \pi \lambda_D^3 > 1 \)

For a plasma to act with collective behavior, a single charged particle must interact with many others; not just the nearest one. For this to hold, there must be many charged particles within a particle’s sphere of influence. The radius of this sphere is the Debye length (\( \lambda_D \)) which determines the characteristic distance of electrical screening within the plasma.

II  Bulk Effects: \( \lambda_D << L_{\text{plasma}} \)

Bulk plasma effects dominate over edge interactions when the Debye length is much less than the physical size of the plasma. The plasma is considered quasineutral when this holds true.

III  Interaction Time: \( \omega_{pe} > \nu_{en} \)

The response time for plasma electrons, characterized by the electron plasma frequency (\( \omega_{pe} \)), must be shorter than that of neutral particle interactions. For electrostatic dynamics to dominate neutral gas kinetics, the electron plasma frequency must be larger than the electron-neutral collision time. This means that the plasma response to electric fields is faster than any neutral interactions.

The quantities listed above, \( \lambda_D \) and \( \omega_{pe} \), are basic plasma parameters used to assess the screening capabilities and response times of a plasma, respectively. Because electrons are much more responsive than ions, electron conditions are used to define these plasma parameters. The definitions of these quantities, as derived in many introductory plasma physics
books,\textsuperscript{1–4} are

\[
\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{e_0 n_e}}, \tag{3.1}
\]

\[
\omega_{pe} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}, \tag{3.2}
\]

where \(T_e, n_e,\) and \(m_e\) are the electron temperature, number density, and mass, respectively. The elementary charge is \(e_0,\) \(\epsilon_0\) is the emissivity of free space, and \(k_B\) is Boltzmann’s constant. A derivation from a simplified fluid momentum equation for electrons could be shown, but instead a qualitative physical description of what these parameters mean is given to provide better insight to the plasma dynamics.

The Debye length represents the \(1/e\) screening distance of an ion of charge \(q_i\) in a quasi-neutral plasma. In other words, the electric potential \(V_i\) created by an ion in a plasma does not fall-off inversely with distance \(r\) as expected by a standard Coulombic potential. Instead, the ion is shielded by plasma electrons that reduce the effective potential exponentially as a function of distance,

\[
V_i = \frac{q_i}{4\pi\epsilon_0 r} \exp{-r/\lambda_D}. \tag{3.3}
\]

The interpretation of how plasma parameters affect the shielding are clear from the definition in 3.1. As the number of electrons per unit volume increase, the amount of negative charge felt near the ion has ‘increased’, which in turn enhances the shielding. However, if the temperature increases, electrons speed by the ion faster and the effective negative charge felt near the ion has ‘decreased’, thereby increasing \(\lambda_D\) and reducing the electron screening effect. The strength of this screening effect determines the \textit{plasma coupling}.

The number of electrons in a Debye sphere \(N_D\) is closely related to the Coulomb (or plasma) parameter \(\Gamma.\) This \textit{coupling parameter} is defined by the ratio of the Coulomb potential energy of a particle to it’s random thermal energy:

\[
\Gamma \equiv \frac{\langle PE\rangle_C}{\langle KE\rangle_T} = \frac{e^3}{4\pi\epsilon_0^{3/2} Z^{1/2}} \frac{n_e^{1/2}}{T_e^{3/2}}, \tag{3.4}
\]

\[
\approx 6 \times 10^{-5} \frac{Z n_e [10^{20} \text{ cm}^{-3}]^{1/2}}{T_e [\text{keV}]^{3/2}}, \tag{3.5}
\]

where \(Z\) is the charge state of the ion. A simple manipulation shows that \(\Gamma\) and \(N_D\) are inversely proportional, \(\Gamma = (3N_D)^{-1}.\) The plasma is weakly coupled when \(\Gamma<<1\) and the Coulombic potential energy is much less than the thermal energy. Under these conditions there are many particles in a Debye sphere and the collective electrostatic behavior dominates over single binary collisions. Conversely, when \(\Gamma>>1\) on average there is less than one particle in a Debye sphere and particle dynamics are dominated by binary collisions. In \textit{inertial confinement fusion (ICF) and some laser-matter interactions, areas in the plasma may be moderately (\(\Gamma\sim 1\)) or strongly (\(\Gamma>>1\)) coupled}. In these cases, physical parameters that are based on collective behavior are not strictly valid. General plasma phenomena discussed in this thesis will fit within the weakly coupled regime and exceptions will be dealt with as needed. Where the Debye length describes the spatial scale for electrostatic effects, the electron plasma frequency characterizes the temporal response.
To understand the fastest response of a plasma, consider an infinite, homogeneous, quasi-neutral plasma with stationary ions and ‘cold’ electrons.\textsuperscript{1} If the electrons were displaced from an otherwise steady state configuration, they will return to their position due to the resulting Coulombic force. This restoring force will pull the electrons past their equilibrium and the electron density will fluctuate in time with the frequency described in 3.2.

The resulting standing wave frequency is firstly a result of finite electron inertia. In many cases, electron inertia is neglected, $m_e \to 0$ and $\omega_{pe} \to \infty$, this means that the electrons can react instantly; which is obviously not true, but when viewed on typical ion-relevant time scales can provide extremely useful insight. The response frequency is also enhanced with increasing electron number density; if there are more electrons per unit volume, the restoring force will be stronger, thereby decreasing the reaction time (increasing the frequency). The electron plasma frequency represents the shortest time scale that is important for a given plasma. Because of the very broad range of spatial and temporal scales in plasma physics, it is important to distinguish what limits are important to the phenomena of particular interest and proceed accordingly. For convenience, Equations 3.1 and 3.2 are given in relevant units:

\begin{align}
\lambda_D & \approx 2.35 \times 10^{-2} \sqrt{\frac{T_e[\text{keV}]}{n_e[10^{20}\text{cm}^{-3}]} \text{[\mu m]}}, \\
\omega_{pe} & \approx 5.64 \times 10^5 \sqrt{n_e[10^{20}\text{cm}^{-3}]} \left[\text{1 ns}\right].
\end{align}

Qualitative descriptions of important plasma parameters have been given and their impact on spatial and temporal scales. Many other basic plasma phenomena necessary for understanding the physics mechanisms in laser-produced plasmas are given in the succeeding sections. This chapter begins in Section 3.1 with the basic two-fluid magnetohydrodynamic (MHD) equations. Section 3.2 covers Coulomb interactions in detail and the associated energy loss and scattering effects of $\sim$MeV protons in plasma and cold matter. A short summary of relevant transport phenomena is given in Section 3.3. Specific attention is paid to magnetic and electric field generation in plasmas in Section 3.4. Next, a summary of plasma instabilities is given in Section 3.5 for relevant high energy density (HED) environments. This chapter concludes in Section 3.6 with a detailed analysis of basic Rayleigh-Taylor (RT) physics in the classic stratified fluid problem and in an ablatively driven system.

\textsuperscript{1}Cold refers to a state where there is no random motion of the electrons, but where they are still free to move as a collective fluid.
3.1 Magnetohydrodynamics

There are many different forms of the MHD equations governing plasma evolution. We begin with the most fundamental kinetic description of a distribution of particles \((f_j)\) from basic statistical mechanics, the Boltzmann equation:

\[
\frac{\partial f_j}{\partial t} + v_j \cdot \nabla r f_j + a_j \cdot \nabla v f_j = \sum_k \left( \frac{\partial f}{\partial t} \right)_{j,k} .
\]  

(3.8)

The distribution function \(f_j\) describes the particles in a six-dimensional phase-space (3 space and 3 velocity) with interactions between particles of all species \((k)\) are described by the collision operator on the right-hand-side. In the case of a plasma, the acceleration field \(a\) felt by the particles is dominated by the Lorentz force caused by electric field \(E\) and magnetic field \(B\),

\[
a_j = \frac{q_j}{m_j} (E + v_j \times B) ,
\]

(3.9)

resulting in the plasma Fokker-Planck equation

\[
\frac{\partial f_j}{\partial t} + v_j \cdot \nabla r f_j + \frac{q_j}{m_j} (E + v_j \times B) \cdot \nabla v f_j = \sum_k \left( \frac{\partial f}{\partial t} \right)_{j,k} ,
\]

(3.10)

when neglecting collisions, this becomes the so-called Vlasov equation

\[
\frac{\partial f_j}{\partial t} + v_j \cdot \nabla r f_j + \frac{q_j}{m_j} (E + v_j \times B) \cdot \nabla v f_j = 0 .
\]

(3.11)

Treatment of the collision operator is a rich and complex topic in plasma physics. For purposes of this thesis, only elastic Coulomb collisions are considered and are discussed in Section 3.2. However, inelastic particle interactions, i.e. charge exchange, ionization, dissociation, etc. could be included in the collision operator, but these topics go beyond the scope needed here. When considering only elastic collisions, it should be noted that the collision operator will conserve number, momentum, and energy for a single species \((j)\):

\[
\int \left( \frac{\partial f}{\partial t} \right)_{j,k} d^3 v_j = 0 ,
\]

(3.12)

\[
\int m_j v_j \left( \frac{\partial f}{\partial t} \right)_{j,j} d^3 v_j = 0 ,
\]

(3.13)

\[
\int \frac{m_j v_j^2}{2} \left( \frac{\partial f}{\partial t} \right)_{j,j} d^3 v_j = 0 .
\]

(3.14)

Momentum exchanged through elastic collisions between species is conserved,

\[
\int m_j v_j \left( \frac{\partial f}{\partial t} \right)_{j,k} d^3 v_j = -\int m_k v_k \left( \frac{\partial f}{\partial t} \right)_{k,j} d^3 v_k ,
\]

(3.15)
as well as energy,
\[
\int \frac{m_j v_j^2}{2} \left( \frac{\partial f}{\partial t} \right)_j \, d^3v_j = - \int \frac{m_k v_k^2}{2} \left( \frac{\partial f}{\partial t} \right)_{k,j} \, d^3v_k .
\] (3.16)

Taking these same moments of the entire Fokker-Plank equation (3.10) produces the typical fluid description of a plasma. For a detailed explanation of this procedure the reader is encouraged to see Braginskii’s chapter in Reviews of Plasma Physics\(^2\) and the book by Helander and Sigmar,\(^5\) though some results are summarized here.

A particle population undergoing random collisions will drive the population to local thermal equilibrium (LTE) resulting in a Maxwellian distribution with number density \(n_j(r,t)\), temperature \(T_j(r,t)\), and mean velocity \(V_j(r,t)\),
\[
f_{0j} = n_j \left( \frac{m_j}{2\pi k_B T_j} \right)^{3/2} e^{-\frac{m_j (v_j - V_j)^2}{2k_B T_j}} .
\] (3.17)

Deviations from LTE result in higher order terms of the distribution function,
\[
f_j(v_j) = f_{0j} + f_{1j} + ... \tag{3.18}
\]

Braginskii\(^2\) calculated the first correction term \(f_{1j}\) to the zeroth-order Maxwellian distribution. This correction is necessary to characterize some phenomena in the fluid description, though it will not be discussed in detail. Taking the zeroth moment of the Fokker-Planck equation yields the continuity equation,
\[
\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j V_j) = 0 ,
\] (3.19)

and the first velocity moment yields the conservation of momentum,
\[
m_j n_j \left( \frac{\partial V_j}{\partial t} + V_j \cdot \nabla V_j \right) = -\nabla p_j - \nabla \cdot \Pi_j + q_j n_j (E + V_j \times B) + R_j ,
\] (3.20)

where \(p_j = n_j T_j\) is the isotropic fluid pressure, \(\Pi_j\) is the off-diagonal stress tensor
\[
\Pi_j = \int m_j (v_j - V_j)(v_j - V_j) f_j(v_j) d^3v_j - p_j I .
\] (3.21)

The isotropic pressure components are removed from the tensor to be consistent with Equation 3.20 and plasma viscosity (off-diagonal terms) may be calculated from higher order terms of the distribution function. The viscosity tensor is thoroughly discussed by Braginskii\(^2\) and Helander,\(^5\) but will not be discussed in detail here. One important point however, is that because the ion mass is so much greater than that of the electron, viscous effects are always dominated by ions and are typically neglected in the electron momentum equation.

Collisional momentum transfer effects with all species are contained in \(R_j\) by
\[
R_j = \sum_k \int m_j (v_j - V_j) \left( \frac{\partial f}{\partial t} \right)_{j,k} \, d^3v_j ,
\] (3.22)
CHAPTER 3. INTRODUCTORY PLASMA PHYSICS

and will be discussed in more detail in Section 3.3.2. The second velocity moment of the Fokker-Planck equation yields the energy equation,

\[
\frac{3}{2} \left( \frac{\partial p_j}{\partial t} + \nabla \cdot (p_j \mathbf{V}_j) \right) = -p_j \mathbf{V}_j \cdot \nabla \mathbf{V}_j - (\Pi_j \cdot \nabla) \cdot \mathbf{V}_j - \nabla \cdot \mathbf{q}_j + Q_j,
\]

where \( \mathbf{q}_j \) is the ‘classical’ heat flux density in the rest frame of the fluid

\[
\mathbf{q}_j = \int \frac{m_j}{2} (\mathbf{V}_j - \mathbf{v}_j)^2 \mathbf{v}_j f_j(\mathbf{v}_j) d^3 \mathbf{v}_j.
\]

The classical heat flux is a fundamental quantity for thermal transport and will be a topic of discussion in Chapter 6. The integrals must be performed on higher order terms in of the distribution function. These calculations are beyond the scope needed, so some results of specific interest from Braginskii are summarized here.

The electron heat flux can be broken into two distinct pieces \( \mathbf{q}_e = \mathbf{q}_e^u + \mathbf{q}_e^T \): one caused by friction (\( \mathbf{q}_e^u \)) between electrons and ions and another due to temperature gradients (\( \mathbf{q}_e^T \)).

\[
\mathbf{q}_e^u = \beta \parallel \mathbf{u} \parallel + \beta \perp \mathbf{u} \perp + \beta \wedge \mathbf{b} \times \mathbf{u},
\]

\[
\mathbf{q}_e^T = -\kappa \parallel \nabla T_e - \kappa \perp \nabla T_e - \kappa \wedge \mathbf{b} \times \nabla T_e,
\]

where \( \parallel, \perp, \) and \( \wedge \) indicate parallel, perpendicular, and diamagnetic directions relative to the magnetic field, respectively. The relative velocity between electrons and ions is \( \mathbf{u} \) and the frictional coefficients \( \beta \) are defined by

\[
\beta \parallel = n_e T_e \beta_0, \quad \beta \perp = n_e T_e \frac{\beta_1 \chi^2 + \beta_0'}{\chi^4 + \delta_1 \chi^2 + \delta_0}, \quad \beta \wedge = n_e T_e \frac{\chi (\beta_1' \chi^2 + \beta_0'')}{\chi^4 + \delta_1 \chi^2 + \delta_0},
\]

and the thermal coefficients \( \kappa \) are defined by

\[
\kappa \parallel = \frac{n_e T_e \tau_{ei}}{m_e} \gamma_0, \quad \kappa \perp = \frac{n_e T_e \tau_{ei}}{m_e} \frac{\gamma_1' \chi^2 + \gamma_0'}{\chi^4 + \delta_1 \chi^2 + \delta_0}, \quad \kappa \wedge = \frac{n_e T_e \tau_{ei}}{m_e} \frac{\chi (\gamma_1' \chi^2 + \gamma_0'')}{\chi^4 + \delta_1 \chi^2 + \delta_0},
\]

where \( \tau_{ei} \) is the electron-ion collision time (discussed in Section 3.3.1), and \( \chi \) is the so-called Hall parameter. This quantity provides a metric for the effect on transport phenomena due to particle gyrations around magnetic field lines and is defined by

\[
\chi \equiv \omega_{ce} \tau_{ei},
\]

where \( \omega_{ce} = e_0 B/m_e \) is the electron cyclotron, or gyro, frequency. The Hall parameter is a measure of the level of electron magnetization. If \( \chi >> 1 \), then the electrons are strongly magnetized and a typical electron will make many gyrations about a magnetic field line before a characteristic collision occurs. Conversely, when \( \chi << 1 \), the electrons are weakly magnetized and transport occurs near the classical limit (as if the magnetic field was not present). The values of the other coefficients in Equations 3.27 and 3.28 are given by Braginskii and this information is reproduced at the end of this section for reference. The last term in the energy equation \( Q_j \) is due to heat generated by collisions.
and will be discussed in more detail in Section 3.3.2, but can be written

\[ Q_j = \sum_k \int \frac{m_j}{2} (\mathbf{v}_j - \mathbf{V}_j)^2 \left( \frac{\partial f}{\partial t} \right)_{j,k} d^3 \mathbf{v}_j. \]  

(3.30)

In addition to the conservation equations, the MHD description includes Maxwell’s equations to govern electrodynamics in these conducting fluids.

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \text{(Faraday’s Law)} \]  

(3.31)

\[ \nabla \times \mathbf{B} = \mu_0 j + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}, \text{(Ampere’s Law)} \]  

(3.32)

\[ \nabla \cdot \mathbf{E} = -\varepsilon_0 \varepsilon_0 \left( n_e - \sum_{j \neq e} Z_j n_j \right), \text{(Gauss’ Law)} \]  

(3.33)

\[ \nabla \cdot \mathbf{B} = 0, \]  

(3.34)

where \( j = -\varepsilon_0 n_e \mathbf{u} \) is the current density, \( \varepsilon_0 \) is the emissivity of free space, \( \mu_0 \) is the permittivity of free space, and \( \varepsilon_0 \) is the elementary charge. The local current density is

\[ j = -\varepsilon_0 n_e \mathbf{V}_e + \sum_{j \neq e} \varepsilon_0 Z_j n_j \mathbf{V}_j, \]  

(3.35)

such that \( \mathbf{u} \equiv \mathbf{V}_e - \sum \mathbf{V}_i \) because of charge neutrality. The net charge density in 3.33 and current density in 3.35 have been written explicitly for a multi-component plasma. The last equation not included thus far is the equation-of-state (EOS). This is also an extremely rich and diverse topic in HED physics, however, for analytic purposes discussed herein, the EOS for each species is typically assumed to be that of a polytropic gas,

\[ p_j = n_j k_B T_j, \]  

(3.36)

thus, for adiabatic processes,

\[ p_j \propto n_j^\gamma, \]  

(3.37)

where \( \gamma \) is the polytrope (also known as the adiabatic- or polytropic- index). This assumption is accurate for fully ionized gases, for which \( \gamma = 5/3 \), and when the population is in thermal equilibrium. These conditions do not always hold, but are a good starting point for many analytic calculations. In many simulations used in HED research, EOS tables are implemented and this is discussed further in Section 6.2 for a specific relevant case.

In general, a plasma may be comprised of multiple ion species in a variety of different ionization states. To correctly describe a multi-component plasma, Equations 3.19–3.30 must be written for each individual species. It is easily seen that such an approach becomes quite complex very quickly. To alleviate some of these intricacies, the two-fluid approach is used. In this model, the plasma consists of a population of electrons (\( f_e \)) and a single population of ions (\( f_i \)). For multicomponent plasmas, effective quantities are used to model
multiple ion populations as one, such that

\[ m_i = m_{eff} = \frac{\sum_j n_j m_j}{\sum_j n_j}, \quad (3.38) \]

\[ Z_i = Z_{eff} = \frac{\sum_j n_j Z_j^2}{\sum_j n_j Z_j}. \quad (3.39) \]

In this manner, the set of MHD equations is reduced to a maximum of six conservation equations, four Maxwell equations, and an EOS for each species. More assumptions can be made to further simplify these equations and will be addressed as they arise in context.

---

\[ ii \] Recent developments in HED research, specifically experiments by Casey et al., have shown that multi-fluid effects, such as species separation, are important in ICF implosions. The single-ion-species approximation, by definition, neglects these effects. Related theoretical explanations of multiple-ion-species effects have recently become another important topic in the HED community.
Table 3.1: Adaptation of Table 2 from Braginskii\textsuperscript{2} that provides coefficient values used in the electron transport equations. For convenience, coefficients for the fully ionized CH(1:1.38) plasma relevant to experiments discussed in Chapters 6 and 7 were calculated through linear interpolation. The exact solutions in the limit of $B=0$ are given in parentheses for $Z = 1$ and $Z = \infty$ for the first three coefficients.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$Z = 1$</th>
<th>$Z = 2$</th>
<th>$Z = 3$</th>
<th>CH(1:1.38)</th>
<th>$Z = 4$</th>
<th>$Z = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0 = 1 - (\alpha'_0/\delta_0)$</td>
<td>0.5129 (0.5063)</td>
<td>0.4408</td>
<td>0.3965</td>
<td>0.3944</td>
<td>0.3752</td>
<td>0.2949 (3π/32)</td>
</tr>
<tr>
<td>$\beta_0 = \beta'_0/\delta_0$</td>
<td>0.711 (0.7033)</td>
<td>0.9052</td>
<td>1.016</td>
<td>1.0234</td>
<td>1.09</td>
<td>1.521 (3/2)</td>
</tr>
<tr>
<td>$\gamma_0 = \gamma'_0/\delta_0$</td>
<td>3.1616 (3.203)</td>
<td>4.89</td>
<td>6.064</td>
<td>6.15</td>
<td>6.92</td>
<td>12.47 (128/3π)</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>3.7703</td>
<td>1.0465</td>
<td>0.5814</td>
<td>0.5643</td>
<td>0.4106</td>
<td>0.0961</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>14.79</td>
<td>10.8</td>
<td>9.618</td>
<td>9.562</td>
<td>9.055</td>
<td>7.482</td>
</tr>
<tr>
<td>$\alpha'_1$</td>
<td>6.416</td>
<td>5.523</td>
<td>5.226</td>
<td>5.211</td>
<td>5.077</td>
<td>4.63</td>
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<tr>
<td>$\alpha'_2$</td>
<td>1.837</td>
<td>0.5956</td>
<td>0.3515</td>
<td>0.342</td>
<td>0.2566</td>
<td>0.0678</td>
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<td>$\gamma'_1$</td>
<td>1.704</td>
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<td>1.704</td>
<td>1.704</td>
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<tr>
<td>$\gamma'_2$</td>
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<td>0.2358</td>
<td>0.1957</td>
<td>0.094</td>
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<td>$\beta'_1$</td>
<td>5.101</td>
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<td>4.233</td>
<td>4.222</td>
<td>4.124</td>
<td>3.798</td>
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<tr>
<td>$\beta'_2$</td>
<td>2.681</td>
<td>0.9473</td>
<td>0.5905</td>
<td>0.5762</td>
<td>0.4478</td>
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<td>$\beta''_1$</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\beta''_2$</td>
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<td>1.784</td>
<td>1.442</td>
<td>1.426</td>
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<td>$\gamma'_1$</td>
<td>4.664</td>
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<tr>
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<td>2.5</td>
<td>2.5</td>
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<td>13.53</td>
<td>13.44</td>
<td>12.65</td>
<td>10.23</td>
</tr>
</tbody>
</table>
3.2 Coulomb Interactions

The dominant collisional mechanism in plasma is due to long-range Coulomb interactions. These collisions allow for the transfer of momentum and energy between particles, hence thermalization and friction may occur. Because this interaction is vital to momentum and energy transport, we will work through the basic physics of the non-relativistic 2-body problem. As a test particle ($t$) with charge $Z_te_0$ interacts with a single field particle ($f$) with charge $Z_fe_0$, it feels a force

$$
F_C = \frac{Z_tZ_f e_0^2}{4\pi\epsilon_0 r^2} \hat{r},
$$

(3.40)

where $r$ is the relative distance between the particles, and $\hat{r}$ is the unit vector pointing from the field to the test particle. The analysis begins by setting the problem in the center-of-mass (CoM) reference frame of the interacting particles, such that the 2-particle problem is reduced to a single particle with reduced mass $m_\mu$ that is deflected through the standard Coulomb force originating at the CoM. Particle quantities in the CoM frame are related to the lab quantities by

$$
m_\mu = \frac{m_t m_f}{m_t + m_f},
$$

(3.41)

$$
v_{rel} = v_t - v_f,
$$

(3.42)

$$
V_{CoM} = \frac{m_t v_t + m_f v_f}{m_t + m_f}.
$$

(3.43)

A schematic drawing of the Coulomb interaction in relative coordinates is shown in Figure 3-1. By approaching the problem in this manner, the initial analysis becomes simpler. The problem is now confined to the plane which contains both $v_{rel}$ and $r$ and is axially symmetric. Furthermore, the angular momentum and energy of the reduced mass particle is conserved. In this simple system with known charges and masses, there is a one-to-one correspondence between the exit angle $\theta'$ and the initial impact parameter $b$. Through the use of the previously mentioned conservation laws and some trigonometry, the expression for $\theta'$ becomes:

$$
\theta' = \pi - 2b \int_{r_{min}}^{\infty} \frac{dr}{r^2 \sqrt{1 - \frac{b^2}{r^2} - \frac{E_U}{E_K}}},
$$

(3.44)

where $r_{min}$ is the distance at closest approach, $E_U$ and $E_K$ are the Coulomb potential and kinetic energy of the test particle, respectively, given by

$$
E_U = \frac{Z_tZ_f e_0^2}{4\pi\epsilon_0 r},
$$

(3.45)

$$
E_K = \frac{1}{2} m_\mu v_{rel}^2.
$$

(3.46)

After performing the integral in Equation 3.44 one obtains an equation for $\theta'$,

$$
\theta' = 2 \tan^{-1} \left( \frac{b_{90}}{b} \right),
$$

(3.47)
in terms of the impact parameter $b$ and the $90^\circ$ impact parameter $b_{90}$ that causes a $90^\circ$ deflection of the test particle,

$$b_{90} = \frac{Z_t Z_f e_0^2}{4\pi\varepsilon_0} \frac{1}{m_\mu v_{\text{rel}}^2}. \quad (3.48)$$

This result gives the exit angle for a given impact parameter and covers the range $0 < \theta' < \pi$ for impact parameters $0 < b < \infty$. Because this is an elastic collision, $|v_{\text{rel}}| = v_{\text{rel}}$ is constant during the interaction and the final velocities of the interacting particles in the CoM are

$$v'_{t} = v_{\text{rel}} \frac{m_\mu}{m_t} (\cos \theta' \hat{x} + \sin \theta' \hat{y}), \quad (3.49)$$

$$v'_{f} = -v_{\text{rel}} \frac{m_\mu}{m_f} (\cos \theta' \hat{x} + \sin \theta' \hat{y}). \quad (3.50)$$

To convert back into the lab frame, we simply add in the CoM velocity,

$$v'_{t} = \frac{m_\mu}{m_f} v_{t,0} + \frac{m_\mu}{m_t} v_{f,0} + v_{\text{rel}} \frac{m_\mu}{m_t} (\cos \theta' \hat{x} + \sin \theta' \hat{y}), \quad (3.51)$$

$$v'_{f} = \frac{m_\mu}{m_f} v_{t,0} + \frac{m_\mu}{m_t} v_{f,0} - v_{\text{rel}} \frac{m_\mu}{m_f} (\cos \theta' \hat{x} + \sin \theta' \hat{y}). \quad (3.52)$$

Now, letting the original trajectory to be aligned with the x-axis, the initial velocities are

$$v_{t,0} = v_{t,0} \hat{x}, \quad (3.53)$$

$$v_{f,0} = -v_{f,0} \hat{x}. \quad (3.54)$$
and the final velocities are related to the initial velocities by
\[
\frac{v'_t}{v_{t,0}} = \frac{m_{\mu}}{m_t} \left( \frac{m_t}{m_f} + \cos \theta' - \frac{m_{\mu}}{m_t} \left( 1 + \cos \theta' \right) \right) \hat{x} + \left( 1 - \frac{v_{f,0}}{v_{t,0}} \right) \sin \theta' \hat{y}, \tag{3.55}
\]
\[
\frac{v'_f}{v_{t,0}} = \frac{m_{\mu}}{m_f} \left( 1 - \cos \theta' - \frac{m_{\mu}}{m_t} \left( \frac{m_f}{m_t} - \cos \theta' \right) \right) \hat{x} + \left( \frac{v_{f,0}}{v_{t,0}} - 1 \right) \sin \theta' \hat{y}. \tag{3.56}
\]

Up to this point, no assumptions about the test or field particles have been made; this is the true analytic solution to the 2-body Coulombic problem. In proton radiography, Coulomb collisions can play a significant role in particle motion through energy loss and scattering. These processes are caused by many collisions due to the long ranging Coulomb potential. Therefore, to truly capture these effects, a Monte Carlo method is typically implemented.

There are many numerical options available for calculating scattering and energy loss of ions in matter. One popular simulator for ions in cold matter is from Ziegler. The code is called \textbf{Stopping and Range of Ions in Matter (SRIM)}\textsuperscript{10} and accounts for many physics caveats not discussed in the succeeding Sections 3.2.1 and 3.2.2; shielded electric potential, stopping due to electrons and ions, etc. SRIM has a simple user interface and performs calculations for ions with energies up to \(\sim 1\) GeV in arbitrary materials. Nonetheless, SRIM does calculations in slab geometries only; for more complex problems which arise in typical proton radiography experiments, a Geant4 (\textbf{Ge}ometry and \textbf{t}racking)\textsuperscript{11,12} simulation code has been written.

Geant4 is an object-oriented toolkit written in C++ used to simulate particle physics experiments in 3-dimensional complex geometries. It is developed extensively for use at CERN and the SLAC National Accelerator Laboratory. Geant4 is an open-source library of classes where users write their own simulations using available physics packages or create their own physics classes. A simulation has been developed using the Geant4 toolkit to track protons through matter and electromagnetic fields and to create synthetic radiographs, as discussed in Section 4.4. Despite the robust architecture of Geant4, the current implementation of charged particle stopping is still in the cold matter regime, as in SRIM. To verify the validity of the cold-matter (CM) approximation in these calculations, an analytic approach to the problem is first discussed and comparisons of results under CM and plasma conditions are made. When put into physical context, some basic limitations are exposed and standard assumptions discussed.

### 3.2.1 Scattering

Scattering of charged particles in matter due to Coulomb collisions is quantified by the \textbf{Rutherford cross section (RCX)}. For convenience, the prime denoting the final exit angle of the test particle (as in Section 3.2) has been removed, however \(\theta\) is still the exit angle of the 2-body problem in the CoM. The RCX is the probability of a test particle with an impact parameter between \(b\) and \(b + db\), to scatter into a solid angle \(d\Omega\). Figure 3-2 illustrates how the conservation of particles is used to geometrically define the initial small area on the impact parameter ring and set it equal to the scattering area (probability) per Steradian multiplied by an infinitesimal solid angle:

\[
bdbd\varphi = \left( \frac{d\sigma}{d\Omega} \right) \sin \theta dBd\varphi \tag{3.57}
\]
3.2. COULOMB INTERACTIONS

Figure 3-2: Schematic used in deriving the Rutherford Cross Section. Particles incident through the impact parameter ring on the left must exit the scattered ring on the right. The RCX defines the probability for a particle to end up at a given solid angle. To calculate a total cross section the RCX must be integrated over all $4\pi$ Steradians; also note that the prime has been dropped from the exit angle $\theta$.

where $bdbd\varphi$ is the small area with impact parameters between $b$ and $b + db$, and $\sin \theta d\theta d\varphi$ is the small solid angle into which the particles are scattered. This equation can be solved for the RCX $(d\sigma/d\Omega)$ as a function of $b_{90}$ and $\theta$,

$$
\left( \frac{d\sigma}{d\Omega} \right) = \left| \frac{b_{90}}{\sin \theta d\theta} \right| .
$$

(3.58)

The standard RCX definition is found by simply combining Equations 3.47 and 3.58. The probability of a test particle to scatter into a solid angle $d\Omega$ may then be written,

$$
\left( \frac{d\sigma}{d\Omega} \right) = \frac{b_{90}^2}{4} \frac{1}{\sin^4 \left( \theta/2 \right)} .
$$

(3.59)

It is clear from this expression that small angle scattering will dominate Coulomb collisions; again this is due to the long range of the potential well. It is also noted that as the relative speed between particles increases, the RCX decreases, indicating that at higher energies, particles interact less. Equation 3.59 can be integrated to compute the total Coulomb cross section $(\sigma_C)$ as

$$
\sigma_C = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \left( \frac{d\sigma}{d\Omega} \right) d\Omega ,
$$

(3.60)

$$
\approx \frac{\pi b_{90}^2}{\theta_{\text{min}}^2} ,
$$

(3.61)

where $\theta_{\text{max}}$ is the maximum deflection angle corresponding to the smallest impact parameter, and $\theta_{\text{min}}$ is the minimum deflection angle corresponding to the largest impact parameter. In this formulation, limits of integration must be determined by the physics of the system because the integrand in Equation 3.60 diverges.
The Coulomb collision mean free path ($\lambda_{mfp}$) of a test particle amidst a field of particles ($n_f$) is simply calculated from the total cross section $\sigma_C$ as

$$\lambda_{mfp} = \frac{1}{n_f \sigma_C}. \quad (3.62)$$

Furthermore, the mean square scattering angle for a single collision may be directly calculated from the RCX as

$$\langle \theta^2 \rangle_1 = \frac{\int (\frac{d\sigma}{d\Omega}) \theta^2 d\Omega}{\int (\frac{d\sigma}{d\Omega}) d\Omega}, \quad (3.63)$$

where the subscript indicates that the calculation is for a single interaction and integrals are done with the same limits ($\theta_{\text{min}}$ to $\theta_{\text{max}}$) as defined for the cross section. Integration of the above equation leads directly to

$$\langle \theta^2 \rangle_1 \approx 2\theta_{\text{min}}^2 \ln \left( \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \right), \quad (3.64)$$

where integration limits for a single binary collision have still not been defined. In practice, incident particles interact with many field particles as they traverse a material of length $L$ and number density $n_f$ such that the overall exit angle from the system is due to many successive independent events. In this case, the central limit theorem applies and the probability distribution of exit angles is approximated by a Gaussian centered on the forward direction. For purposes of estimating deflections angles and specifically for comparisons between CM and plasma conditions, the metric used is the total mean square deflection angle $\langle \theta^2 \rangle$,

$$\langle \theta^2 \rangle = n_f L \sigma_C \langle \theta^2 \rangle_1 \approx 2\pi b^2 n_f L \ln \left( \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \right). \quad (3.65)$$

Proton radiography discussed in this thesis focuses mainly on imaging with fusion protons with energies of $\sim 3$ MeV and $\sim 15$ MeV. Images are blurred by scattering with plasma ions, not electrons, because similar mass particles exchange momentum more efficiently (deflect more) than unlike-mass particles as will be discussed in Section 3.3. With this in mind, Equation 3.65 is evaluated for high energy ($\sim$MeV) imaging protons traversing a material with ion mass $m_i \approx A_i m_p$ and charge $Z_i$ and density $\rho \approx n_i m_i$ resulting in

$$\theta_{\text{rms}} = \sqrt{\langle \theta^2 \rangle} \approx \left( \frac{Z_i e_0^2}{4\pi \varepsilon_0 m_p v^2_{\text{rel}}} \right) (2\pi n_i L \ln \Lambda)^{1/2},$$

$$\approx \left( \frac{e_0^2}{\sqrt{32\pi m_p e_0}} \right) \left( \frac{Z_i (1 + A_i)}{E_p (1 - v_i/v_p)^2} \right) \left( \frac{\rho L}{A_i} \ln \Lambda \right)^{1/2} \quad (3.66)$$

where the Coulomb logarithm has been defined by the limits of integration $\Lambda \equiv \theta_{\text{max}}/\theta_{\text{min}}$. From Equation 3.66, it is clear that proton scattering scales as the square root of the areal density ($\rho L$), is linearly proportional to the field ion charge $Z_i$, and is inversely proportional
3.2. COULOMB INTERACTIONS

The Coulomb interactions are dependent on the incident proton energy. For convenience, Equation 3.66 is evaluated in relevant units

\[ \theta_{\text{rms}} \approx 0.25 \left( \frac{Z_i(1 + A_i)}{E_p[\text{MeV]}(1 - v_i/v_p)^2} \right) \left( \frac{\rho L[\text{mg/cm}^2]}{A_i} \right)^{1/2} \ln \Lambda \text{ [degrees]} . \]  

(3.67)

**Scattering in Cold Matter**

Up to this point, the state of the field material has not been defined. Previous calculations were concerned only with binary interactions and left integration limits arbitrary, but finite. The exact limits chosen depend on the physical situation being modeled and may vary on the reference used. The model adopted by Jackson\textsuperscript{13} is implemented here and adapted for comparisons between CM and plasma conditions. The minimum and maximum deflection angle limits in the CM approximation are defined as

\[ \theta_{\text{max,CM}} = \frac{\hbar}{m \mu v_{\text{rel}}} R , \]  

(3.68)

\[ \theta_{\text{min,CM}} = \max \left( \frac{Z_i e^2}{2 \pi \epsilon_0 m \mu v_{\text{rel}}^2 a}, \frac{\hbar}{m \mu v_{\text{rel}} a} \right) , \]  

(3.69)

where \( R \approx 1.4 \times 10^{-15} A_i^{1/3} \text{ m} \) is the nuclear radius and represents the minimum impact parameter and \( a = 1.4 \times a_0 Z_i^{-1/3} \) is the approximate shielding radius for an atom and represents the maximum impact parameter in terms of the classical Bohr radius \( (a_0 \approx 5.3 \times 10^{-11} \text{ m}) \). The two options for \( \theta_{\text{min}} \) result because there is a quantum limit on the minimum momentum exchanged between the two particles. If \( v_{\text{rel}}/c \equiv \beta \gtrsim 0.015 Z_i \), then the first option in Equation 3.69 is lower than this quantum value and the quantum limit must be used. For protons used in radiography discussed herein, \( \beta_{15 \text{MeV}} \approx 0.18 \) and \( \beta_{3 \text{MeV}} \approx 0.08 \) and \( Z_i \approx 3-4 \), therefore the quantum limit on \( \theta_{\text{min}} \) is the most relevant value in this work. The maximum deflection angle, which results from the smallest impact parameter, has a quantum limit associated with \( R \) and this is always used. Using the classical \( \theta_{\text{min}} \) value in Equation 3.61, the total Coulomb cross section in cold matter is

\[ \sigma_{C,\text{CM}} \approx \frac{\pi a^2}{4} , \]  

(3.70)

\[ \approx 4.3 \times 10^{-17} Z_i^{-2/3} \text{ [cm}^2 \text{]} , \]

or, in the more relevant quantum limit

\[ \sigma_{C,\text{CM}} \approx \left( \frac{1.4 a_0 e^2 \sqrt{\pi m_p/2}}{4 \pi \epsilon_0 \hbar} \right)^2 \frac{Z_i^{4/3}}{E_p} , \]  

(3.71)

\[ \approx 4.4 \times 10^{-18} Z_i^{4/3} \frac{E_p}{[\text{cm}^2]} , \]

where constants have been evaluated, substitutions for \( v_{\text{rel}} \) and \( m \mu \) have been made, and relations using relevant units are shown for convenience with \( E_p \) is in MeV. In the classical limit under this formulation (Equation 3.70), it is noted that the Coulomb cross section is dependent only on the field particle’s atomic radius, but is only relevant for low energy protons. For high energy protons, as used in proton radiography, the cross section decreases...
with increasing proton energy. Furthermore, the argument of the Coulomb logarithm in the quantum limit reduces to \( \Lambda \approx 5.3 \times 10^4/(A_f Z_f)^{1/3} \) and the rms deflection angle in the CM approximation is expressed as

\[
\theta_{\text{rms,CM}} \approx \left( \frac{Z_i(1 + A_i)}{E_p} \right) \sqrt{\frac{\rho L}{A_i} \ln \left( \frac{5.3 \times 10^4}{(A_f Z_f)^{1/3}} \right)}
\]

\[
\approx 0.25 \left( \frac{Z_i(1 + A_i)}{E_p} \right) \sqrt{\frac{\rho L}{A_i} \ln \left( \frac{5.3 \times 10^4}{(A_f Z_f)^{1/3}} \right)} \text{[degrees]},
\]

where \( E_p \) is given in MeV and the areal density \( \rho L \) is given in mg/cm\(^2\) in the second expression. Next, the same calculation is performed for protons traversing a plasma.

### Scattering in Plasma

Under the formulation discussed in this section, the primary difference between scattering in plasma and that in cold matter arises in the definition of the \( \theta \) integration limits. The maximum scattering angle is unchanged in a plasma because the smallest impact parameter is still dependent on the nuclear size of the field ion. However, where the atomic shielding radius is used as the maximum impact parameter in cold matter, the proper shielding distance in a plasma is set by the Debye length \( \lambda_D \). Making this substitution, Equation 3.69 becomes

\[
\theta_{\text{min}, \text{plasma}} = \max \left( \frac{Z_i e_0^2}{2 \pi \epsilon_0 m \mu v_{\text{rel}}^2 \lambda_D}, \frac{\hbar}{m \mu v_{\text{rel}} \lambda_D} \right),
\]

\[
= \max \left( \frac{Z_i e_0^3}{2 \pi \epsilon_0^{3/2} m \mu v_{\text{rel}}^2} \sqrt{\frac{n_e}{n_e}}, \frac{\hbar e_0}{m \mu v_{\text{rel}} \sqrt{n_e k_B T_e}} \right).
\]

Furthermore, the relative velocity between field ions and incident protons is approximated for thermalized ions using the most probable speed, such that \( v_{\text{rel}} \approx v_p \left( 1 + \frac{T_i}{A_i E_p} \right) \), however this temperature correction is very small for \( \sim \)MeV protons and will be neglected. Again, in cases relevant to proton radiography discussed here, the quantum limit on \( \theta_{\text{min}} \) is appropriate and the Coulomb logarithm argument is reduced to \( \Lambda \approx \lambda_D/R \), where \( R \) is again the nuclear radius. For completeness, the classical and quantum values for the total Coulomb cross section in a plasma are

\[
\sigma_{C, \text{plasma}} \approx \frac{\pi \lambda_D^2}{4},
\]

\[
\approx 4.3 \times 10^{-12} \frac{T_e}{n_e} \text{[cm}^2\text{]},
\]

and in the more relevant quantum limit

\[
\sigma_{C, \text{plasma}} \approx \left( \frac{m_p Z_i^2 e_0^2}{32 \pi \epsilon_0 \hbar^2} \right) \left( \frac{1}{n_e} \right) \left( \frac{T_e}{E_p} \right),
\]

\[
\approx 4.3 \times 10^{-13} \frac{Z_i^2 T_e}{n_e E_p} \text{[cm}^2\text{]},
\]
where the proton energy $E_p$ is given in MeV, the electron number density $n_e$ is in units of $10^{20}$ cm$^{-3}$, and the plasma electron temperature $T_e$ is in keV. Under these same conditions, the rms deflection angle for protons traversing a thermalized plasma can then be written

$$\theta_{\text{rms,plasma}} \approx \left( \frac{e^2}{\sqrt{32\pi m_p\epsilon_0}} \right) \left( \frac{Z_i(1 + A_i)}{E_p} \right) \sqrt{\frac{\rho L}{A_i}} \ln \left( \frac{\lambda_D}{R} \right),$$

(3.76)

or in relevant units as

$$\approx 0.25 \left( \frac{Z_i(1 + A_i)}{E_p} \right) \sqrt{\frac{\rho L}{A_i}} \ln \left( \frac{1.7 \times 10^7 T_e^{1/2}}{n_e^{1/2} A_i^{1/3}} \right) \text{[degrees]},$$

where the areal density $\rho L$ is again given in mg/cm$^2$ and $T_e$ is in keV in the second expression. When comparing the results of Equation 3.72 and Equation 3.76, it is clear that the integration limits can change drastically over many orders of magnitude between cold matter and plasma. However, the rms deflection angle scales (very weakly) as the square root of the logarithm of the limit ratio so that this effect will be relevant only in certain regimes.

The validity of the cold matter approximation is addressed for a specific experimental case relevant to proton radiography in Section 4.4.1 for both proton scattering and energy loss.

### 3.2.2 Energy Loss

Where scattering of ions in matter was dominated by interactions with heavy nuclei, in the regimes of interest here ion energy loss is dominated by interactions with electrons. The calculation shown below is done for cold matter to illustrate the main points of the derivation and emphasize the physics. In the CM limit, the only field particles of concern are ‘cold’ electrons and in the case of $\sim$MeV ions $v_t >> v_f$. After simplifying Equation 3.55, the test particle’s velocity after a collision can now be expressed as

$$v_t' = \frac{m_p}{m_t} v_{t,0} \left( \left( \frac{m_t}{m_f} + \cos \theta' \right) \hat{x} + \sin \theta' \hat{y} \right),$$

(3.77)

then substituting Equation 3.47 into Equation 3.77, this leads to

$$\frac{E_t'}{E_{t,0}} = \frac{1 + \bar{b}^2 - 4 m_p^2 m_t}{1 + \bar{b}^2},$$

(3.78)

where $E_{t,0}$ and $E_t'$ are the test particle energy before and after the collision respectively and $\bar{b} = b / b_{90}$. The total change in energy ($dE_t$) of a test particle traveling through a small volume ($dV$) of field particles with density $n_f$ is found through the following relation

$$dE_t = n_f(E_t - E_t') dV = n_f(E_t - E_t') b \, db \, dl \, d\varphi.$$

(3.79)

This differential energy loss per unit length (stopping power) is found by integrating over an annulus in impact parameter space such that,

$$\frac{dE_t}{dl} = 2\pi n_f \int_{b_{\text{min}}}^{b_{\text{max}}} (E_t - E_t') b \, db.$$
then, using Equation 3.78, substitute in for $E'_t$

$$E'_t = -E_t \pi n_f b_{90}^2 \frac{8m_n^2}{m_t m_f} \int_{b_{min}}^{b_{max}} \frac{\tilde{b} \tilde{d} b}{1 + \tilde{b}^2},$$

(3.80)

and perform the integration to arrive at,

$$\frac{dE_t}{dl} = -\pi n_f m_t \frac{E_t}{E_t} \left( \frac{Z_t Z_f e_0^2}{4\pi \epsilon_0} \right)^2 \ln \left( \frac{1 + \tilde{b}_{max}^2}{1 + \tilde{b}_{min}^2} \right),$$

(3.81)

where $\tilde{b}_{max}$ and $\tilde{b}_{min}$ are the maximum and minimum impact parameters, respectively, normalized to $b_{90}$. Recall that the limits of integration for the scattering calculation were done over $\theta$, where the relationship between $\theta$ and $b$ is given by the solution to the 2-body problem in Equation 3.47.

It is instructive to point out the proportionality of $\frac{dE}{dl}$ to the mass ratio observed in 3.81. In general, ions will be slowed down by both nuclei and electrons in the material. However, it is clear that ions will lose most of their energy to electrons ($m_f = m_e$) in the material because of the mass ratio $m_t/m_f$, justifying the assumption at the beginning of this section in the CM limit. Now setting $m_t \approx A_t m_p$ for an arbitrary heavy ion slowing down, the stopping power is

$$\frac{dE_t}{dl} = -\pi n_e A_t m_p \frac{Z_t e_0^2}{4\pi \epsilon_0} \ln \left( \frac{4m_e E_t}{A_t m_p I} \right),$$

(3.82)

The details of charged particle energy loss in cold matter is a problem that has been extensively studied. The short derivation shown above illustrates a simplified solution based on the framework of the 2-body problem. As in the discussion of scattering, complications arise when defining the limits of integration. The formalism typically used is that from Bethe with the non-relativistic stopping power equation given as

$$\frac{dE_t}{dl} = -\pi n_e A_t m_p \left( \frac{Z_t e_0^2}{4\pi \epsilon_0} \right)^2 \left( \frac{4m_e E_t}{A_t m_p I} \right)^2 \ln \left( \frac{4m_e E_t}{A_t m_p I} \right),$$

(3.83)

where the maximum impact parameter limit $b_{max}$ is related to $I$, the mean ionization energy of atoms in the material. Though tables of $I$ can be used for accurate calculations, Bloch used the assumption that $I \approx Z \times 10$ eV, resulting in the so-called Bethe-Bloch formula:

$$\frac{dE_t}{dl} = -\pi n_e A_t m_p \left( \frac{Z t e_0^2}{4\pi \epsilon_0} \right)^2 \ln \left( \frac{4m_e E_t}{A_t m_p Z f \times 10 \text{ eV}} \right),$$

(3.84)

The total energy loss of a particle is not solvable in closed form, but it is easily integrated numerically with known values for the material and test ion. As a first order approximation, this equation is good for calculating stopping power of ions in cold matter and shows the dependencies on ion energy and material parameters. However, numeric simulations can account for the statistical nature of these interactions and can more accurately calculate the stopping power in materials.

The simplest way to accommodate for energy loss in a plasma is to adjust the impact parameter limits, as was done in the scattering calculation shown in Section 3.2.1. As
previously discussed, in a plasma, charges are shielded at distances greater than \( \lambda_D \), such that an appropriate upper limit on the impact parameter is \( b_{\text{max}} \approx \lambda_D \). Implementing this limit, the stopping power equation may be written

\[
\frac{dE_t}{dl} = -\frac{2\pi n_e A_t m_p}{E_t} \frac{Z_t^2 e_0^2}{4\pi \epsilon_0^2} \ln \Lambda,
\]

(3.85)

where \( \ln \Lambda \) is the Coulomb logarithm now defined classically for a plasma as

\[
\ln \Lambda = \ln \left( \frac{\lambda_D}{b_{90}} \right) \quad \text{(3.86)}
\]

\[
= \ln \left( \frac{\epsilon_0 k_B T_e}{e_0^2 n_e} \frac{8\pi \epsilon_0 m_e E_t (1 + v_e/v_t)^2}{Z_f Z_t A_t m_p e_0^2} \right) \quad \text{(3.87)}
\]

\[
\approx 9.8 - \ln \left( \frac{Z_f Z_t A_t n_e^{1/2}}{T_e^{1/2} E_t (1 + v_e/v_t)^2} \right) \quad \text{(3.88)}
\]

where \( n_e \) is expressed in \( 10^{20} \text{ cm}^{-3} \), \( T_e \) is in keV, and \( E_t \) is in MeV. This simplistic approach, however, is insufficient to describe relevant physics for ion stopping in arbitrary plasmas. Though collisions with electrons are still the dominant loss channel in the plasma (over ions), incident ions may no longer be much faster than the electrons. Plasma electrons can have temperatures in the \( \sim \text{keV} \) range, and because of their lighter mass, can have a comparable (or faster) speed than \( \sim \text{MeV} \) ions. Furthermore, in addition to energy lost to binary collisions in a plasma, incident ions also transfer energy to bulk plasma oscillations. The Li-Petrasso (LP) \(^{16}\) stopping power treatment accounts for energy loss to both particle and wave channels for \( \ln \Lambda \gtrsim 2 \) as well as quantum corrections to the Coulomb logarithm. A very comprehensive description of energy loss in plasma is given by Brown, Preston, and Singleton (BPS). \(^{17}\) In the BPS derivation, equations were expanded using the coupling parameter \( \Gamma \) as the expansion parameter and solved exactly to \( O(\Gamma^2) \). However, in Section 4.4.1, the LP model will be used for comparisons of proton stopping in cold matter versus plasma under typical laser-produced plasma conditions.
3.3 Energy and Momentum Transport

In any physical plasma, there are at least two particle components, ions and electrons. In many plasmas relevant to laser-matter interactions, there are multiple ion species which may, or may not, be thermalized (Maxwellian) in addition to a single non-Maxwellian (or multi-temperature Maxwellian) population of electrons. Plasma particles interact through Coulomb collisions as discussed in Section 3.2 and it is through this mechanism that energy and momentum may be exchanged between individual particles and between different particle species. The detailed study of these exchanges is analyzed through transport theory and is yet another rich and complex topic in plasma physics.

The details truly needed to quantitatively discuss transport theory in plasmas goes back to the collision operator briefly discussed in Section 3.1. An in depth quantitative treatment of this theory, however, is beyond the scope needed here to grasp some typical time scales and physical phenomena relevant to plasmas of interest in this thesis. Some results will be shown from the classic transport chapter in Reviews of Plasma Physics from Braginskii \(^2\) and the book by Helander \(^3\) as well as the plasma physics text book from Freidberg; \(^3\) the reader is encouraged to reference these texts for details on the derivations of these quantities. In this section, some simple scalings for useful time scales are discussed and their implications for other processes.

3.3.1 Collision Time Scales

The two-body Coulomb collision was treated in detail in Section 3.2 and the solution in the CoM reference frame was the Rutherford differential cross section (Equation 3.59). The mean square deflection angle after many collisions experienced was derived in Equation 3.65. Let the characteristic collision time \(\tau\) of a test particle moving with speed \(v_t\) be estimated by setting the distance travelled, \(L\) in Equation 3.65, to \(v_t\tau\) such that

\[
\langle \theta^2 \rangle \approx 2\pi b_9^2 n_f v_t \tau \ln \Lambda.
\]

Next, let the characteristic deflection be \(\sim \pi/4\) and solve for the collision time,

\[
\tau_{ei} \approx \frac{\pi^3 \varepsilon_0^2 m_e^2 v_{rel}^2 v_e}{2 e_0^4 n_i Z_i^2 \ln \Lambda},
\]

where the notation has been chosen for an electron losing momentum to the ion species. Now, let the velocity of all particles be approximated by the mean speed of a thermal distribution \(v_T^2 \approx \frac{8T}{\pi m}\),

\[
\tau_{ei} \approx \frac{16\pi^{3/2} \varepsilon_0^2 m_e^{1/2} T_e^{3/2}}{2^{1/2} e_0^4 n_i Z_i^2 \ln \Lambda}.
\]
At this point, it is important to note a slight numerical difference in Equation 3.90 with those listed by others.\(^2,3,5\) Using a much more detailed treatment of the collision operator, one obtains

\[
\tau_{ei} \approx \frac{12\pi^{3/2} e_0^2}{2^{1/2} e_0^4} \frac{m_i^{1/2} T_e^{3/2}}{n_i Z_i^2 \ln \Lambda},
\]

\[
\approx 100 \frac{T_e^{3/2}}{n_i Z_i^2 \ln \Lambda} \text{ [ps]},
\]

where, in the second expression for \(\tau_{ei}\), \(T_e\) is in keV and \(n_i\) is in \(10^{20}\) cm\(^{-3}\). For purposes of estimating time scales, a difference of a factor of order unity does not affect the interpretation of the physics. The characteristic collision frequency is simply defined as the inverse of the collision time, \(\nu_{ei} \approx \tau_{ei}^{-1}\). The electron-ion momentum relaxation time is used as a characteristic measure for other collisional exchange processes.

Energy transfer due to collisions requires a different moment of the collision operator, and will not be discussed in detail here. Suffice it to say, after addressing the proper integrals\(^3\) the question of thermal equilibration may be presented simply

\[
\frac{dT_e}{dt} = \frac{1}{\tau_{eq}} (T_e - T_i),
\]

where the equilibration time \(\tau_{eq}^{ei}\) is assumed to be approximately constant in time,

\[
\tau_{eq}^{ei} \approx \frac{m_i}{m_e} \frac{\tau_{ei}}{2}.
\]

It was just shown that the characteristic collision time \(\tau_{ei}\) is dependent on the temperature. Therefore, under the thermal equilibration problem discussed, \(\tau_{ei}\) must be evaluated at an intermediate temperature between \(T_e\) and \(T_i\). Interspecies thermal equilibration will always take longer than the characteristic relaxation time by a factor of the mass ratio. A summary of relaxation and equilibration time scales is given in Table 3.2.

Table 3.2: Collisional time scales in a plasma as multiples of the characteristic electron-ion relaxation time \(\tau_{ei}\). For convenience of notation, the ion-to-electron mass ratio has been defined as \(m_i/m_e \equiv \mu_{ie} \gg 1\).

<table>
<thead>
<tr>
<th>Interacting Particles</th>
<th>(\tau_{ij} [\tau_{ei}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron-ion ((\tau_{ei}))</td>
<td>1</td>
</tr>
<tr>
<td>ion-electron ((\tau_{ie}))</td>
<td>(\sim \mu_{ie} Z_i^{-1})</td>
</tr>
<tr>
<td>electron-electron ((\tau_{ee}))</td>
<td>(\sim Z_i)</td>
</tr>
<tr>
<td>ion-ion ((\tau_{ii}))</td>
<td>(\sim \mu_{ie}^{1/2} Z_i^{-2})</td>
</tr>
</tbody>
</table>

These collision times provide some insight into the internal dynamics of the plasma and relevant time scales for various processes. Because of their low mass, electrons lose a comparable amount of momentum whether interacting with other electrons or much more massive ions, and \(\tau_{ei} \sim \tau_{ee}\). This randomization of electron momentum tends to thermalize
the electron population relatively quickly. However, the randomization of ion momentum due to electron collisions is minimal as indicated by $\tau_{ie} \gg \tau_{ei}$. Therefore, the ion population will thermalize due only to ion-ion collisions as indicated by $\tau_{ii} \ll \tau_{ie}$. Lastly, the two species will thermalize between themselves, assuming no external heating, on the longest time scale $\tau_{ei}^{eq} \sim \tau_{ie}$. Collisional interactions occurring on the microscale lead to macroscopic effects in the bulk plasma.

3.3.2 Collisional Effects: Frictional and Thermal Forces

The general form of the collisional force was given in Equation 3.22 as the first velocity moment of the collision operator. The detailed treatment of this procedure is presented in detail by Braginskii, but the results will only be qualitatively described here. In a two-species plasma, the only net momentum transfer that occurs is between the two species, there is no net force within a single species. The resultant collisional force can be broken into frictional ($R_u$) and thermal ($R_T$) components and they are equal and opposite for the two species. In the previous section it was shown that the frequency of momentum transfer from ions to electrons ($\nu_{ie}$) was $\sim Z_i m_e / m_i$ lower than that of electrons to ions ($\nu_{ei}$). This momentum-transfer asymmetry is amplified in the presence of a thermal gradient (due to the dependence of $\nu_{ei} \propto v^{-3}$) and causes a net force on the electrons. In a cold region (higher collision frequency), more electron momentum is lost to the ions than in a region that is warmer (lower collision frequency), thereby generating an effective force in the direction opposite the temperature gradient. This is the thermal force due to Coulomb collisions

$$R_T = -\beta_0 n_e \nabla T_e .$$

(3.94)

Furthermore, frictional effects arise from collisions between particles of different species, namely when the mean ion and electron fluid velocities are not equal $\mathbf{u} \equiv \mathbf{V}_e - \mathbf{V}_i \neq 0$. Collisions act to randomize directed motion of either fluid, but if both fluids are moving with the same mean velocity, this effect tends to cancel out on average. Conversely, as is typically the case, the electron fluid moves faster than the ion fluid and a net drag force is felt by the electrons due to ion collisions

$$R_u = -\alpha_0 \frac{m_e n_e}{\tau_{ei}} \mathbf{u} ,$$

(3.95)

where the constants $\beta_0$ and $\alpha_0$ are defined by Braginskii, but in the case of a CH plasma are $\sim 1$ and $\sim 0.4$, respectively. Thus, the total force on electrons due to collisions can now be expressed as

$$R_e = -\alpha_0 \frac{m_e n_e}{\tau_{ei}} \mathbf{u} - \beta_0 n_e \nabla T_e ,$$

(3.96)

The work done on electrons due to these collisional forces ($Q_u$) may be calculated by

$$Q_u = -(R_T + R_u) \cdot \mathbf{u} ,$$

(3.97)

$$= \beta_0 n_e \nabla T_e \cdot \mathbf{u} + \alpha_0 \frac{m_e n_e}{\tau_{ei}} u^2 .$$

(3.98)

Now using the definition of current flow, $\mathbf{j} = -e_0 n_e \mathbf{u}$, the ohmic heating done on the electron
fluid can be written,

$$
R_u \cdot u = \alpha_0 \eta j^2 , \tag{3.99}
$$

where $\eta = m_e/(e_0^2 n_e T_{ei})$ is the Spitzer resistivity

$$
\eta = \frac{(2m_e)^{1/2} e_0^2 Z_i \ln \Lambda}{12 \pi^{3/2} e_0^3 T_e^{3/2}} , \tag{3.100}
$$

$$
\approx 3.3 \times 10^{-9} Z_i \ln \Lambda \frac{T_e^{3/2}}{T_e^{3/2}} \ [\Omega \cdot \text{m}] ,
$$

where in the last expression $T_e$ is given in keV. The charge state $Z_i$ was left in Equation 3.100 because it came from $\tau_{ei}$ and this expression is the classical plasma resistivity for an arbitrary $Z_i$ as derived by Spitzer. Using this convention, the total work done through collisions can be written,

$$
Q_u = -\frac{\beta_0}{e_0} \nabla T_e \cdot j + \alpha_0 \eta j^2 . \tag{3.101}
$$

In general, these heating terms apply to ions as well, however, they are down by a factor of $m_e/m_i$ and are typically neglected in the ion fluid equation. The last term necessary to balance the collisional heat flow ($Q_\Delta$) between the ion and electron populations is from thermal equilibration,

$$
Q_\Delta = -\frac{3}{2} \frac{n_e}{\tau_{ei}} (T_e - T_i) .
$$

As discussed in Section 3.3.1, thermal equilibration between the species, namely collisional energy loss from electrons to ions occurs when $T_e > T_i$, such that,

$$
Q_\Delta \approx -\frac{3m_e n_e}{m_i \tau_{ei}} (T_e - T_i) . \tag{3.102}
$$

Thus, collisional energy terms in the electron and ion fluid equations may be written

$$
Q_e = \beta_0 n_e \nabla T_e \cdot u + \alpha_0 \frac{m_e n_e}{\tau_{ei}} u^2 - \frac{3m_e n_e}{m_i \tau_{ei}} (T_e - T_i) , \tag{3.103}
$$

$$
Q_i = \frac{3m_e n_e}{m_i \tau_{ei}} (T_e - T_i) . \tag{3.104}
$$

The discussion given above did not consider magnetic field effects, but served to qualitatively describe the sources of the various collisional terms. However in the presence of a magnetic fields, as was discussed with respect to heat transfer in Section 3.1, collisional effects may be altered by the gyro motion of the particles around the magnetic field lines. This is covered by Braginskii, and the resultant collisional force $R$ is reproduced here,

$$
R_u = -\alpha_\parallel u_\parallel - \alpha_\perp u_\perp + \alpha_\lambda b \times u , \tag{3.105}
$$

$$
R_T = -\beta_\parallel T_e \nabla T_e - \beta_\perp T_e \nabla T_e - \beta_\lambda b \times \nabla T_e . \tag{3.106}
$$
The frictional force coefficients $\alpha$ are defined by

\[
\alpha_\parallel = \frac{m_e n_e}{\tau_{ei}} \alpha_0, \quad \alpha_\perp = \frac{m_e n_e}{\tau_{ei}} \left( 1 - \frac{\alpha_1' \chi^2 + \alpha_0'}{\chi^4 + \delta_1 \chi^2 + \delta_0} \right), \quad \alpha_\wedge = \frac{m_e n_e}{\tau_{ei}} \frac{\chi(\alpha''_1 \chi^2 + \alpha''_0)}{\chi^4 + \delta_1 \chi^2 + \delta_0}, \quad (3.107)
\]

and the thermal force coefficients $\beta^{\text{uT}}$ are defined by

\[
\beta_{\parallel}^{\text{uT}} = n_e \beta_0, \quad \beta_{\perp}^{\text{uT}} = n_e \frac{\beta_1' \chi^2 + \beta_0'}{\chi^4 + \delta_1 \chi^2 + \delta_0}, \quad \beta_{\wedge}^{\text{uT}} = n_e \frac{\chi(\beta''_1 \chi^2 + \beta''_0)}{\chi^4 + \delta_1 \chi^2 + \delta_0}. \quad (3.108)
\]

The $\parallel$, $\perp$, and $\wedge$ symbols indicate parallel, perpendicular, and diamagnetic directions relative to the magnetic field, respectively. Diamagnetic corresponds to the direction perpendicular to the magnetic field and the vector causing the effect. The coefficients given by Braginskii were reproduced in Table 3.1 of this thesis for reference.

The classic, detailed treatment of frictional and thermal effects in a magnetized plasma is given by Braginskii. The results discussed above were given without consideration to anisotropic effects due to magnetic fields. Braginskii derived the transport coefficients in detail with this in mind, though that level of detail was beyond the scope necessary to understand the underlying physics. For a full description of the collisional treatment, the reader is encouraged to see Braginskii\textsuperscript{2} and Helander.\textsuperscript{5}
3.4 Self-generated Electromagnetic Fields in Plasma

Self-generated, electromagnetic fields have been observed in many laser-produced plasmas. Electric-field generation primarily occurs in response to gradients in the electron pressure, whereas the dominant source of self-generated magnetic fields is related to perpendicular gradients in the electron temperature and density. Such non-collinear temperature and density gradients occur in laser-ablated targets due to Rayleigh-Taylor (RT) growth and will be discussed in Chapter 6. The following sections provide a brief overview of the mechanisms involved in self-generated electromagnetic fields.

3.4.1 Electric Field Generation

Electrical shielding occurs in plasmas due to the high mobility of electrons. In a uniform plasma Debye shielding of positively charged ions screens electric fields with scale size $L_D > \lambda_D$, where $\lambda_D$ is the local, electron Debye length. Debye shielding neutralizes individual charges and characterizes the quasi-neutrality of the plasma. This allows for the collective behavior to dominate over small-scale Coulombic effects.

Long scale-length charge separation, however, can generate electric fields inside plasmas. In typical laser-produced plasmas the Debye length is much smaller ($\sim$nm) than other gradient scale lengths of interest ($\sim$µm). The investigation of electric field generation begins with the electron momentum equation:

$$m_e n_e \left( \frac{\partial V_e}{\partial t} + V_e \cdot \nabla V_e \right) = -\nabla p_e - \nabla \cdot \Pi_e - e_0 n_e (E + V_e \times B) + R_e. \quad (3.109)$$

This equation may be solved for the electric field,

$$E = -\frac{\nabla p_e}{e_0 n_e} - V_e \times B - \frac{m_e}{e_0 n_e} \left( \frac{\partial V_e}{\partial t} + V_e \cdot \nabla V_e \right) + \frac{R_e}{e_0 n_e}. \quad (3.110)$$

Next, electron inertia is neglected on hydrodynamic time scales, such that $m_e \to 0$ and it is recognized that viscosity is dominated by ion motion, so electron viscosity is ignored. This results in the formulation presented by Braginksii,

$$E \approx -\frac{\nabla p_e}{e_0 n_e} - V_e \times B + \frac{R_e}{e_0 n_e}. \quad (3.111)$$

From this equation, the generalized Ohm’s law may be easily derived. If the frictional force is written as $R_u \approx e_0 n_e \eta |j|$, and the thermal force is neglected, then

$$E + V_i \times B \approx \frac{1}{e_0 n_e} (j \times B - \nabla p_e) + \eta j, \quad (3.112)$$

where the relationship between $u$ and $j$ has also been used. In steady state, the pressure gradient balances with the $j \times B$-force and the first term on the right-hand-side of Equation 3.112 is zero. However, in general for laser-plasma interactions, a steady-state pressure equilibrium is not reached. To simply understand the basic generation mechanisms, dissipative effects due to collisions are ignored forming the Hall MHD source of electric fields. It is then clear that electric fields are mainly generated in response to the electron pressure.
with an additional component due to the collisionless Hall effect,

\[ E \approx \frac{1}{e_0 n_e} (j \times B - \nabla p_e) - V_i \times B = -\frac{\nabla p_e}{e_0 n_e} - V_e \times B. \quad (3.113) \]

### 3.4.2 Magnetic Field Generation

Unlike electric fields, magnetic fields are not shielded out by electron screening effects and can dramatically affect plasma dynamics. Magnetic field generation is described by Faraday’s Law (Equation 3.31). Substituting Equation 3.111 into Faraday’s Law produces the equation governing magnetic field evolution,

\[
\frac{\partial B}{\partial t} \approx \nabla \times \left( \frac{\nabla p_e}{e_0 n_e} + V_e \times B - \frac{R_e}{e_0 n_e} \right). \quad (3.114)
\]

In its typical form, the current density \( j = e_0 n_e (V_i - V_e) \) is used in place of the mean electron fluid velocity. Then, neglecting displacement current, Ampere’s Law (3.32) relates the current density \( j \) and magnetic fields. With these substitutions, the general form for magnetic field evolution in a plasma becomes

\[
\frac{\partial B}{\partial t} \approx \nabla \times \left( \frac{\nabla p_e}{e_0 n_e} + V_i \times B - \frac{\nabla \times B}{\mu_0 e_0 n_e} - \frac{R_T + R_U}{e_0 n_e} \right). \quad (3.115)
\]

Each term is described as follows: (a) the Biermann battery\(^{23}\) or thermo-electric term, (b) the dynamo or fluid convection term, (c) the collisionless Hall term, and (d) the collisional terms. Magnetic field generation in plasma is a complex topic that has been investigated by many.\(^{2,20,24,25}\) Within the thermal and frictional force expressions, various diffusion, convection and field generation sources are contrived (including the well known Nernst effect\(^{26}\)) and are thoroughly described by Haines.\(^{20}\) Magnetic field generation, however, is largely dominated by sources due to the gradient of the isotropic electron pressure, which is the foundation for estimating field strengths and structures.

To derive the well known Biermann battery source term, convection, diffusion, and collisional effects are ignored, and the isotropic pressure gradient, (a) in Equation 3.115, is shown to be the primary source of self-generated magnetic fields. Using the standard definition of the electron pressure as \( p_e = n_e T_e \), this thermo-electric source term is clearly shown to be driven by non-collinear temperature and density gradients

\[
\frac{\partial B}{\partial t} \approx \nabla T_e \times \nabla n_e / e_0 n_e. \quad (3.116)
\]

Though this formulation is not an accurate model, it serves to illustrate the primary magnetic field generation mechanism.

The first step to a more tractable model for magnetic field evolution is to note that the collisionless Hall term is second order in \( B \) and can thus be neglected in comparison to other terms. This results in the following,

\[
\frac{\partial B}{\partial t} \approx \nabla \times \left( \frac{\nabla p_e}{e_0 n_e} \right) + \nabla \times (V_i \times B) + \frac{\eta}{\mu_0} \nabla^2 B + \nabla \times B \times \frac{\nabla \eta}{\mu_0} + \nabla \times \left( \frac{R_T}{e_0 n_e} \right). \quad (3.117)
\]
where the substitution $R_u = e_0 n_e j$ has also been made. If the collisionless, Hall MHD, limit is now taken, the magnetic field evolution can be greatly simplified to,

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \times \left( \frac{\nabla p_e}{e_0 n_e} \right) + \nabla \times (\mathbf{V}_i \times \mathbf{B}) . \quad (3.118)$$

It is now convenient to point out the similarity of Equation 3.118 to that of fluid vorticity in an inviscid fluid,

$$\frac{\partial \mathbf{\xi}}{\partial t} \approx -\nabla \times \left( \frac{\nabla p}{\rho} \right) + \nabla \times (\mathbf{V} \times \mathbf{\xi}) , \quad (3.119)$$

where $\mathbf{\xi} = \nabla \times \mathbf{V}$ is the fluid vorticity, $\mathbf{V} = \mathbf{V}_i$ is the fluid velocity, $p = p_e + p_i$ is the total pressure, and $\rho = m_in_i$ is the fluid density. Furthermore, assuming that $T_e \approx T_i$, it is easily shown that the total pressure is related to the electron pressure,

$$p \approx \frac{Z+1}{Z} p_e . \quad (3.120)$$

It is now easily verified that the magnetic field in this case is simply proportional to the fluid vorticity,

$$\mathbf{B} \approx -\frac{m_i}{e_0(Z+1)} \mathbf{\xi} . \quad (3.121)$$

This equation is an exact solution to the magnetic field under the assumptions described. However, in many cases the resistivity is not negligible and in some instances the Nernst effect, caused by the collisional thermal force $R_T$, must also be included. The implications of these terms are discussed as needed in Chapters 6 and 7.
3.5 Plasma Instabilities

Instabilities play an important role in any experimental system. Knowing when and how a system can go unstable allows engineers and scientists to design mitigation schemes and safety protocols. There are two different classes of instabilities, absolute and convective. An absolute instability is one in which the exponential growth of an initial perturbation to the system increases with time but remains near the original location of the instability. If there is a flow associated with the unstable system, and the instability grows exponentially but ‘convects’ away from the instability origin fast enough, after some time, the instability origin is no longer unstable. It is clear that the flow of the system can be altered by the reference frame of the observer, such that in one frame an instability is convective, but under a simple transformation, the same instability could be viewed as absolute. The distinction between these instability types is qualitatively illustrated in Figure 3-3.

In laser-produced plasmas, instabilities can be further categorized by two dominant groups: laser-plasma instabilities that are related to electromagnetic wave interactions with the plasma, and hydrodynamic instabilities that consider the fluid motion of the plasma. A subset under the hydrodynamic instability umbrella is MHD instabilities that relate the conducting fluid with magnetic topologies. MHD instabilities are studied in depth and play significant roles in magnetic confinement devices (tokamaks, z-pinches, etc.), but they will not be discussed in general here. In ICF, the role of hydrodynamic instabilities, specifically Rayleigh-Taylor (RT), was acknowledged and addressed in the first ICF publication by Nuckolls in 1972.\textsuperscript{29} It was understood early-on that the RT instability would determine the minimum thickness allowable for the capsule shell and that if not for ablative stabilization, that inertial fusion energy (IFE) was dead in the water. Another historic ICF publication was the comprehensive review article from Bruckner and Jorna\textsuperscript{30} that covered a number of issues in IFE including many plasma-wave phenomena in the context of laser-plasma interactions relevant to ICF.

Figure 3-3: (a) Diagram of an absolute instability. The perturbation at the origin of the instability grows exponentially in time. (b) Diagram of a convective instability. The instability convects away from its origin, leaving it stable afterwards.
3.5. PLASMA INSTABILITIES

Figure 3-4: A sample laser-foil interaction with an intensity of $\sim 4 \times 10^{14}$ W/cm$^2$ incident onto a $\sim 20$ μm thick CH foil. General terminology for different regions is presented and important surfaces called out.

3.5.1 Laser-plasma Interactions

The interaction of electromagnetic (EM) waves with plasma ions and electrons is yet another rich topic in plasma physics. Because the plasma may dynamically interact with electric and magnetic fields, the strength of many laser-plasma interactions (LPI) is directly related to the intensity of the incoming EM wave. Instabilities driven by laser-plasma interactions are somewhat mitigated by limiting the drive intensity. For this reason, an ICF capsule cannot simply be ‘driven’ harder to reach higher fuel temperatures and ignition conditions. Electromagnetic wave interactions are the mechanisms by which energy is coupled to the plasma and the understanding of these processes is essential. The growth of instabilities discussed here may be calculated directly from individual dispersion relations for specific types of plasma waves. A detailed description and quantitative analysis of these phenomena, however, is beyond the scope of this thesis, the reader is encouraged to see books by Stix$^{31}$ and Kruer$^{32}$ for detailed information on this topic. In this section, some typical plasma waves will be qualitatively discussed to provide the reader with some basic terminology and understanding of the role they play in ablatively driven targets.

Recall that the characteristic response of a plasma was related to the electron plasma frequency $\omega_{pe}$ (Equation 3.2). EM waves with frequencies lower than the local $\omega_{pe}$ will effectively ‘see’ a perfectly conducting wall. If no damping were to occur, the wave would be totally reflected. As an EM wave of frequency $\omega_L$ (and vacuum wavelength $\lambda_L$) traverses a plasma up the density gradient, as illustrated in Figure 3-4, it will eventually reach the ‘critical’ surface where the wave is reflected. This occurs when $\omega_L = \omega_{pe}$, such that the
critical density $n_{cr}$ may be defined as:

$$n_{cr} = \frac{4\pi^2 c^2 \epsilon_0 m_e}{e^2} \frac{1}{\lambda_L^2}, \quad (3.122)$$

and in relevant units

$$n_{cr} \approx 1.1 \times 10^{21} \frac{1}{\lambda_L[\mu m]^2} \text{ cm}^{-3}.$$

Because this is such a fundamental quantity, it will be used as a reference point when talking about various laser-plasma interactions. For the OMEGA laser experiments discussed throughout this thesis, the vacuum wavelength is $\lambda_L = 0.351 \mu m$, resulting in a critical density of $n_{cr} \approx 90 \times 10^{20} \text{ cm}^{-3}$.

A sample plasma environment is shown in Figure 3-4 for an on-target intensity of $\sim 4 \times 10^{14} \text{ W/cm}^2$ incident onto a $\sim 20 \mu m$ CH foil. The profiles shown were predicted by the radiation-hydrodynamics code $^{33,34}$ DRACO for experiments discussed in Chapter 6. The simulated laser pulse was 2 ns and the plasma environment shown is at $\sim 1.5$ ns after laser onset. Figure 3-4 illustrates the different interaction regions where the critical surface separates the plasma into two distinct sections: the under-dense plasma and the over-dense plasma. It also demonstrates the relative length scales involved under the sample laser-target conditions. The laser can not penetrate beyond the critical surface, so the LPI takes place in the under-dense region.

The preferred method of coupling laser energy to thermal plasma energy is through collisional absorption (inverse bremsstrahlung). In this process, coherent motion of electrons in the wave field is randomized through collisions with plasma ions. This interaction couples laser energy directly to local thermal energy which produces a relatively smooth energy deposition profile; although, collisional absorption has been shown $^{35,36}$ to decrease with increasing intensity. Absorption peaks near the critical surface, but occurs along the entire path of the EM wave. Other mechanisms for coupling laser energy to the plasma require first exciting a plasma wave (plasmon) that damps out on plasma electrons, thereby converting the laser energy into thermal plasma energy.

As the EM wave climbs the density gradient towards the critical surface, linear mode coupling may occur by exciting an electron plasma wave (Langmuir wave). This conversion takes place only for so-called ‘p-polarized’ light, which means that some component of the electric field vector is parallel to the plane of incidence formed by the EM wave propagation direction and the density gradient. This component of the incoming wave can stimulate the Langmuir wave near the critical surface. Langmuir waves manifest as perturbations in the electron density whereby the electrons oscillate about their equilibrium position. Wave energy is transferred to electrons through collisionless resonance interactions (Landau damping) with electrons that have velocities near the phase velocity of the Langmuir wave. This resonant absorption $^{37}$ tends to generate electrons hotter than those found in the bulk plasma, thereby creating an extended high energy tail in the electron velocity distribution.

Another class of waves that are excited by incident laser light are classified as three-wave coupling interactions, or parametric instabilities. These interactions may be characterized by the fact that the incoming laser photon decays into two separate waves, and these waves
Table 3.3: List of parametric decay channels relevant to laser-plasma interactions.

<table>
<thead>
<tr>
<th>Process</th>
<th>Wave 1</th>
<th>Wave 2</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion-acoustic Decay</td>
<td>Langmuir</td>
<td>ion-acoustic</td>
<td>$\sim n_{cr}$</td>
</tr>
<tr>
<td>Two-plasmon Decay (TPD)</td>
<td>Langmuir</td>
<td>Langmuir</td>
<td>$\sim \frac{1}{4} n_{cr}$</td>
</tr>
<tr>
<td>Stimulated Raman (SRS)</td>
<td>Scattered Photon</td>
<td>Langmuir</td>
<td>$\leq \frac{1}{4} n_{cr}$</td>
</tr>
<tr>
<td>Stimulated Brillouin (SBS)</td>
<td>Scattered Photon</td>
<td>ion-acoustic</td>
<td>$\leq n_{cr}$</td>
</tr>
</tbody>
</table>

must follow conservation of energy and momentum, such that:

$$\omega_L = \omega_1 + \omega_2 ,$$  \hspace{1cm} (3.123)
$$k_L = k_1 + k_2 ,$$  \hspace{1cm} (3.124)

where subscript $L$ indicates incoming laser quantities and subscripts 1 and 2 indicate the excited waves. A summary of these processes is given in Table 3.3.

The parametric decay channels occur at different locations in the underdense plasma and lead to ion-acoustic waves, Langmuir waves, and scattered photons. The first channel in Table 3.3 is the ion-acoustic decay. In this process, the incoming photon decays into a Langmuir wave and an ion-acoustic wave. The ion-acoustic waves are electrostatic perturbations in both ion and electron densities, but have a much lower frequency than the Langmuir wave ($\omega_i << \omega_{pe}$). Therefore, the match condition for this process occurs only near the critical surface. The next channel is two-plasmon decay (TPD) and it manifests as the incident photon decays into two separate Langmuir waves. Therefore, TPD must occur near the quarter-critical surface for proper match conditions. Because ion-acoustic waves are much lower in frequency, the vast majority of laser energy in these channels is coupled to the Langmuir waves that damp on resonant electrons near the phase velocity.

The so-called ‘stimulated’ processes occur when the incoming photon ‘scatters’ off the plasma, thereby generating a plasma wave and a scattered photon ($\omega'_L$). The stimulated Brillouin scattering (SBS) process corresponds to a stimulated ion-acoustic wave, where the scattered photon carries most of the energy ($\omega'_L \sim \omega_L$). The stimulated Raman scattering (SRS) process is caused by electrons quivering in the incident photon electric field, generating a Langmuir wave, whereby oscillating electrons then emit a scattered photon. The highest energy SRS photon is produced at the quarter-critical surface where $\omega'_L$ is equal to the local plasma frequency. Because these processes have continuous scattered spectra, they may occur up to the critical and quarter-critical surfaces for SBS and SRS, respectively. Scattered photons are frequency down-shifted and therefore have a lower critical density than the original incident photon. Excited Langmuir waves will tend to generate hot electron populations that do not thermalize locally and scattered photons will either resonantly interact with electrons or backscatter and leave the system.

Efficient coupling of laser energy to thermal energy is an intense field of study in plasma physics. Though not discussed explicitly, plasma wave instabilities are calculated from wave dispersion functions. Because these instabilities are linked to interactions with the wave electric field, laser intensity plays an important role. At higher intensities ($I_L \lambda_L^2 \approx 10^{15}$ W $\mu m^2/cm^2$), \textsuperscript{35} ponderomotive acceleration can generate another source of hot electrons. How-
Figure 3-5: (a) Velocity shear in stratified fluids generates the Kelvin-Helmoltz instability. (b) A shock wave incident onto a stratified fluid interface excites the Richtmyer-Meshkov instability. (c) Density gradients opposing the acceleration direction causes the Rayleigh-Taylor instability.

ever, experiments discussed in this thesis do not involve laser-plasma interactions in this regime. Collisional absorption is the preferred method of coupling laser energy to thermal plasma energy, but plasma wave instabilities generate hot electron populations that do not contribute to bulk plasma energy and can scatter light out of the system. Some mitigation of these loss mechanisms is achieved through optimization of laser intensity and various smoothing techniques, such as smoothing by spectral dispersion (SSD),\textsuperscript{38} distributed polarization rotators (DPRs),\textsuperscript{39} and distributed phase plates (DPPs).\textsuperscript{40}

3.5.2 Hydrodynamic Interactions

ICF implosions are essentially spherical rockets driven by mass ejection through laser ablation as was discussed in Section 2.2.1. The previous section discussed various mechanisms by which laser energy is converted to plasma thermal energy in the underdense ablated material. Neglecting details of the laser-plasma interactions, the continuous capsule-plasma system may be considered as a purely hydrodynamic environment consisting of inhomogeneous temperature and density distributions. Like any other hydrodynamic system, the imploding capsule environment is susceptible to numerous instabilities, and these processes can jeopardize capsule integrity during the implosion. Three typical hydrodynamic instabilities are illustrated in Figure 3-5 and are qualitatively discussed below under the classical stratified-fluid scenario.
3.5. PLASMA INSTABILITIES

Kelvin-Helmholtz

Velocity shear between stratified fluids will cause perturbations on the interface to grow due to the Kelvin-Helmholtz (KH) instability. Figure 3-5a illustrates an interface between fluids with a relative velocity difference. The fluid flow need not be in opposite directions, only a net relative velocity parallel to the wave vector is necessary. During linear growth, in the CoM reference frame and with no external acceleration, perturbation amplitudes will increase exponentially ($h \sim h_0 e^{\gamma_{KH}t}$) with a growth rate ($\gamma_{KH}$) of

$$\gamma_{KH} = \frac{\rho_l \rho_l}{\rho_h + \rho_l} |k \cdot \Delta u|,$$  

(3.125)

where $k$ is the perturbation wave vector, $\Delta u$ is the vector difference in fluid velocities, and $\rho_l$ and $\rho_h$ are the low and high density fluids, respectively. Equation 3.125 is the growth rate typically quoted in the literature, however this is only valid in the special case that acceleration is negligible. When this is not the case, but surface tension is still ignored, the growth rate in the CoM reference frame is

$$\gamma_{KH} = \sqrt{\frac{\rho_l \rho_l}{\rho_h + \rho_l} a_k - \frac{\rho_h \rho_l}{\rho_h + \rho_l} (\rho_h + \rho_l)\Delta u^2 \cos^2 \alpha},$$  

(3.126)

where $A_t = (\rho_h - \rho_l)/(\rho_h + \rho_l)$ is the Atwood number and $\alpha$ is the angle between the wave and relative velocity vectors. Under these conditions, in order for instability to occur the quantity under the square root in Equation 3.126 must be less than zero, resulting in a minimum mode number for instability

$$k_{min} = \frac{\rho_h}{\rho_l} \left( 1 - \left( \frac{\rho_l}{\rho_h} \right)^2 \right) \frac{a}{(\Delta u)^2 \cos^2 \alpha}.$$

(3.127)

Only perturbations with modes greater than $k_{min}$ will be KH unstable. It is interesting to note that no matter how small the velocity difference, $\Delta u$, there exists a sufficiently small wavelength perturbation that will be unstable to KH growth. As the amplitude height approaches the perturbation wavelength, the interface becomes asymmetric and the characteristic rolls of the KH instability appear, as schematically shown in Figure 3-5a. This instability, for example, is the mechanism by which wind blowing over a lake generates waves on the water’s surface. In laser-plasma interactions, the KH instability plays a significant role in non-linear Rayleigh-Taylor evolution causing the characteristic ‘mushroom’ top of density spikes and subsequent roll-up and fluid mixing that occurs thereafter.

Richtmyer-Meshkov

When a shock traverses a perturbed fluid interface, as illustrated in Figure 3-5b, perturbations on the interface are susceptible to the Richtmyer-Meshkov (RM) instability. During shock transit, an impulse acceleration, $a_{RM}(t) = |\Delta u| \delta(t-t_0)$, is applied at the transit time $t_0$. Perturbations become imprinted on the reflected and transmitted shock fronts as shown in the bottom portion of Figure 3-5b. After shock transit, the perturbations on the fluid
interface grow linear in time \((h \sim h_0(1 + \gamma_{RM} t))\) with a growth rate \((\gamma_{RM})\) of

\[
\gamma_{RM} = A_t k|\Delta u|, \tag{3.128}
\]

where \(A_t\) is the Atwood number of the post-shocked interface, \(k\) is the wave number, and \(|\Delta u|\) is the representative effect of the impulse caused by the shock. The reason for only linear growth in the RM instability is due to the fact that the driving force behind the instability is transient. After the shock traverses the interface, the instability source is gone and growth proceeds linearly. The RM instability itself is not generally of great concern in ICF because of its slow growth rate. However, RM can generate larger initial perturbation amplitudes for the most concerning hydrodynamic instability in ICF, Rayleigh-Taylor.

**Rayleigh-Taylor**

Hydrodynamic systems where a high-density fluid \((\rho_h)\) is supported by a lower-density fluid \((\rho_l)\), as illustrated in Figure 3-5c, are unstable to Rayleigh-Taylor (RT) growth. An overview of typical equations will be given in this section with detailed explanations and derivations provided in Section 3.6. Small amplitude perturbations on the interface grow exponentially in time \((h \sim h_0 e^{\gamma_{RT} t})\), with a classical linear growth rate \((\gamma_{RT})\) of

\[
\gamma_{RT} = \sqrt{A_t ak} \tag{3.129}
\]

where \(A_t\) is the Attwood number, \(k\) is the wave number of the perturbation, and \(a\) is the acceleration. In laser-matter interactions, as seen in ICF, a continuous density profile is created whereby the ablating mass accelerates into the lighter, expanding plasma, forming an RT-unstable region at the ablation front. The ablative nature of the RT-instability in laser-produced plasmas has been predicted\(^{42-44}\) and verified\(^{45-47}\) to have a stabilizing effect on the linear growth rate. For an ablatively driven target \(A_t \approx 1\) and the linear growth rate is given approximately by:\(^{48}\)

\[
\gamma_{RT} = \sqrt{\frac{ka}{1 + kL_\rho} - \beta_{RT}kv_a}, \tag{3.130}
\]

where \(L_\rho\) is the density scale length, \(\beta_{RT}\) is the ablative stabilization coefficient \((\beta_{RT} \approx 3\) for direct-drive\(^{42,49}\)) and \(v_a\) is the ablation velocity. The ablative, linear growth rate shows that ablation will stabilize perturbations smaller than \(\lambda \approx 2\pi\beta_{RT}^2v_a^2/a\) \((\sim 1-10 \mu m\) for typical parameters). The physics basis for Equations 3.129 and 3.130 are thoroughly discussed in the following section.
3.6 Rayleigh-Taylor Physics

Measurements of magnetic field generation due to RT in laser-produced plasmas is the main thrust of this thesis and as such, special attention to the underlying physics of RT is warranted. Much work has been done on RT in a variety of environments, of particular interest here is work related to ablatively-driven RT and this will be discussed in greater detail in Section 3.6.2. First, a comprehensive overview of the classic stratified fluid problem is given in Section 3.6.1.

3.6.1 Classic Rayleigh-Taylor

The classic RT derivation begins with the mass (Equation 3.19) and momentum (Equation 3.20) conservation equations discussed in Section 3.1:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 ,
\]

\[
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \rho \mathbf{a} - \nabla p ,
\]

where the ion equations have been written with \( \rho = n_i m_i \), the stress tensor \( \Pi \) has been ignored along with collisional losses \( R \), and an additional constant force due to an acceleration \( \mathbf{a} \) has been appended to the right-hand-side of the momentum equation as an externally applied force. These equations are linearized by allowing all quantities to be of the form \( Q = Q_0 + Q_1 \), where 0\(^{th}\)-order quantities are in equilibrium and 1\(^{st}\)-order quantities are small-amplitude perturbations to the equilibrium solution. The resulting 1\(^{st}\)-order conservation equations are written as

\[
\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \mathbf{u}_1 + \rho_1 \mathbf{u}_0) = 0 ,
\]

\[
\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} + (\rho_1 \mathbf{u}_0 + \rho_0 \mathbf{u}_1) \cdot \nabla \mathbf{u}_0 + \rho_0 \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 = \rho_1 \mathbf{a} - \nabla p_1 .
\]
These linearized equations are Fourier analyzed by allowing 1st-order perturbation quantities to vary as

$$Q_1 = \tilde{Q}_1 e^{i(k \cdot r - \omega t)} ,$$

(3.135)

where \(k\) is the wave number and \(\omega\) is the frequency. The goal of the linear analysis is to obtain the dispersion relation \(\omega(k)\) for this system and determine the circumstances under which the system may go unstable and with what growth rate.

The sample system is shown in Figure 3-6 with the axes definitions that are used throughout the derivation. In this scenario, the acceleration is in the x-direction and constant in space and time. The Fourier analysis will comprise modes in the y-z plane, such that \(k = k_y \hat{y} + k_z \hat{z}\) and quantities are allowed to freely vary in the x-direction; \(\tilde{Q}_1 \rightarrow \tilde{Q}_1(x)\). Furthermore, it is assumed that the equilibrium velocity is zero, such that \(u_0 = 0\) and that the fluid is incompressible (\(\nabla \cdot \mathbf{u} = 0\)). Because quantities are able to vary in the x-direction, let \(D = \partial / \partial x\) for ease of notation and the set of equations can now be written as

\[
\begin{align*}
-i \omega \tilde{p}_1 + \tilde{u}_{1x} D \rho_0 &= 0 , \\
-i \omega \rho_0 \tilde{u}_{1x} &= \tilde{p}_1 a - D \tilde{p}_1 , \\
\omega \rho_0 \tilde{u}_{1y} &= k_y \tilde{p}_1 , \\
\omega \rho_0 \tilde{u}_{1z} &= k_z \tilde{p}_1 , \\
D \tilde{u}_{1x} + i k_y \tilde{u}_{1y} + i k_z \tilde{u}_{1z} &= 0 ,
\end{align*}
\]

(3.136-3.140)

where the unknowns in these five equations are \(\tilde{p}_1, \tilde{\rho}_1, \tilde{u}_{1x}, \tilde{u}_{1y},\) and \(\tilde{u}_{1z}\). The next step is to combine these equations into a single differential equation. First, substitute 3.138 and 3.139 into 3.140 and use the fact that \(k^2 = k_y^2 + k_z^2\) to obtain

\[
\frac{i \omega \rho_0}{k^2} D \tilde{u}_{1x} = \tilde{p}_1 ,
\]

(3.141)

and then combine 3.136 and 3.137 to get

\[
i \omega \rho_0 \tilde{u}_{1x} - \frac{i \tilde{u}_{1x}}{\omega} D \rho_0 a = D \tilde{p}_1 .
\]

(3.142)

Next, the derivative of 3.141 is taken and substituted into 3.142, resulting in

\[
k^2 \rho_0 \tilde{u}_{1x} - \frac{k^2}{\omega^2} a D \rho_0 \tilde{u}_{1x} = D \left( \rho_0 D \tilde{u}_{1x} \right) ,
\]

(3.143)

(3.144)

it is then trivial to write the differential equation in the following form

\[
\left( D^2 - k^2 + \frac{D \rho_0}{\rho_0} \left( \frac{ak^2}{\omega^2} + D \right) \right) \tilde{u}_{1x} = 0 .
\]

(3.145)

In the classic scenario, as illustrated in Figure 3-6, there is not a continuous density profile, but a discontinuous jump at the interface defined at \(x = 0\). To obtain the jump conditions,
Equation 3.143 is integrated across the interface from \(-\epsilon\) to \(+\epsilon\),
\[
\int_{-\epsilon}^{+\epsilon} \left( k^2 \rho_0 \ddot{u}_{1x} - \frac{k^2}{\omega^2} a D \rho_0 \ddot{u}_{1x} \right) \, dx = \int_{-\epsilon}^{+\epsilon} \left( D (\rho_0 D \ddot{u}_{1x}) \right) \, dx ,
\]
and it is noted that \(\ddot{u}_{1x}\) and \(D \ddot{u}_{1x}\) must be continuous at the interface,
\[
k^2 (\rho_l + \rho_h) \epsilon \ddot{u}_{1x} - \frac{k^2}{\omega^2} a [\rho_0] \ddot{u}_{1x} = [\rho_0 D \ddot{u}_{1x}] ,
\]
where \([Q]\) notation indicates evaluation at \(\pm \epsilon\) as \((Q|_{+\epsilon} - Q|_{-\epsilon})\). The limit is taken as \(\epsilon \to 0\) and the final jump condition can be written
\[
-\frac{k^2}{\omega^2} a \left( \rho_l|_{+\epsilon} - \rho_h|_{-\epsilon} \right) \ddot{u}_{1x} = \left( \rho_0 D \ddot{u}_{1x}|_{+\epsilon} - \rho_0 D \ddot{u}_{1x}|_{-\epsilon} \right) .
\]
In order to evaluate the jump condition, Equation 3.145 must be solved for \(\ddot{u}_{1x}\) after first eliminating the \(D \rho_0\) terms because the discontinuity is accounted for in the jump condition,
\[
(D^2 - k^2) \ddot{u}_{1x} = 0 .
\]
The solutions to this equation are simple exponentials,
\[
\ddot{u}_{1x} = Ae^{kx} + Be^{-kx} ,
\]
and using the boundary conditions that as \(x \to \pm \infty\), \(\ddot{u}_{1x} \to 0\) and that \(\ddot{u}_{1x}\) is continuous, the piecewise solution becomes
\[
\ddot{u}_{1x}(x) = \begin{cases} 
Ae^{kx} & x \leq 0 \\
Ae^{-kx} & x \geq 0
\end{cases} .
\]
Now that the solution to the perturbed velocity has been calculated, the jump condition 3.148 can be fully evaluated,
\[
-\frac{k^2}{\omega^2} a (\rho_l - \rho_h) \ddot{u}_{1x} = (-\rho_l k - \rho_h k) \ddot{u}_{1x} ,
\]
which is easily solved for the dispersion relation
\[
\omega = \sqrt{\frac{\rho_l - \rho_h}{\rho_l + \rho_h} ka} .
\]
This is the same expression given in Equation 3.129 and for completeness, let \(\omega = \omega_R + i\gamma\), such that the unstable growth rate \(\gamma\) may be expressed simply as
\[
\gamma = \sqrt{A_\gamma ka} ,
\]
where \(A_\gamma\) is the Attwood number as previously defined. As expected, if the density values were swapped, the fluid would be stable to RT growth under the defined acceleration field. The classic growth rate derivation illustrates the fundamental mechanism by which a fluid becomes unstable when the density gradient opposes the acceleration direction. The next
section discusses how this instability arises in an ablatively driven target that is directly
relevant to laser-produced plasmas and ICF implosions.

3.6.2 Ablative Rayleigh-Taylor

When a high-intensity laser irradiates a solid target, matter at the surface is ionized and
expands away from the target, as illustrated in Figure 3-7. The sharp increase in pressure
generated by the laser pulse creates a shock wave that propagates through and compresses
the solid target. Energy is deposited into plasma electrons up to the critical density as was
briefly discussed in Section 3.5.1. Thermal electrons deposit their energy at the ablation
front, heating the ablation region. More material is ablated away and the target is driven.
The ablation front is RT-unstable because the large density gradient opposes the accelera-
tion field in the ablation front reference frame. Unlike the discrete jump in density seen in
the stratified fluid problem from the previous section, the gradient here is continuous.

![Ablation Schematic](image)

Figure 3-7: A schematic drawing of an ablatively driven target. Laser energy coming in
from the right is absorbed in the underdense plasma up to the critical surface. Thermal
energy from the underdense region is transported to the ablation front through the overdense
plasma by electron thermal conduction.

**Continuous Density Profile**

Instead of two stratified fluids, \( \rho_l \) and \( \rho_h \), separated by a discontinuous boundary, let there
be a transition region between them, such that the initial density profile may be expressed

\[
\rho_0(x) = \begin{cases} 
\rho_h - \frac{\Delta \rho}{2} e^{2x/L} & x \leq 0 \\
\rho_l + \frac{\Delta \rho}{2} e^{-2x/L} & x \geq 0
\end{cases}
\]  

(3.155)

where \( \Delta \rho = \rho_h - \rho_l \) is the total density difference far from the interface and \( L \) is the scale
size of the continuous density transition region. To obtain a general condition for the linear
growth rate, Equation 3.143 is multiplied by \( \tilde{u}_{1x} \) to obtain

\[
k^2 \rho_0 \tilde{u}_{1x}^2 - \frac{k^2}{\omega^2} D \rho_0 \tilde{u}_{1x}^2 = D (\rho_0 \tilde{u}_{1x} D \tilde{u}_{1x}) - \rho_0 (D \tilde{u}_{1x}^2)^2.
\]  

(3.156)
This equation is then integrated over \( x \), where it is again recognized that as \( x \to \pm \infty \), \( \tilde{u}_{1x} \to 0 \), so that the fluid is stationary far away from the interface. The general solution for \( \omega \) may be written as

\[
\omega^2 = k^2 \int_{-\infty}^{+\infty} aD\rho_0 \tilde{u}_{1x}^2 dx \left/ \int_{-\infty}^{+\infty} \left[ \rho_0(D\tilde{u}_{1x})^2 + k^2\rho_0 \tilde{u}_{1x}^2 \right] dx + a\Delta \rho A^2 \right. 
\] (3.157)

Next, it is assumed that the eigenfunctions for the perturbed velocity are well approximated by those discussed previously in Equation 3.151. Inserting the piecewise functions for \( \tilde{u}_{1x} \) and \( \rho_0 \) into Equation 3.157 and evaluating the integrals results in

\[
\omega^2 = k^2 \frac{-a\Delta \rho A^2}{1+kL} \left[ \left( \frac{\rho_h}{k} - \frac{L\Delta \rho}{2(1+kL)} \right) + \left( \frac{\rho_l}{k} + \frac{L\Delta \rho}{2(1+kL)} \right) \right], 
\] (3.158)

where \( A \) here is the constant from the perturbed velocity eigenfunction. Reducing this equation yields the typical result for linear RT growth in a continuous density profile,

\[
\gamma_{RT, cont} = \sqrt{A_{t}ak} \left( \frac{1}{1+kL} \right), 
\] (3.159)

where the notation \( \omega = \omega_R + i\gamma \) has again been adopted, and the \( \rho \) subscript has been added to \( L \) to designate it as the density scale length. In the limit of long perturbation wavelengths, or equivalently short density scale lengths, such that \( kL \rho << 1 \), reproduces the classical stratified fluid result \( \gamma = \sqrt{A_{t}ka} \). In the opposite limit, for short wavelengths, or long scale lengths, \( kL \rho >> 1 \) that produces \( \gamma = \sqrt{A_{t}a/L} \rho \) indicating that growth rates of short perturbations are independent of wavelength and only dependent on the density scale length and acceleration at the transition region. In laser-irradiated targets the density at the ablation front \( (\rho_h) \) is much higher than the blow-off material \( (\rho_l) \) such that \( A_{t} \sim 1 \). Using this assumption, the first term in Equation 3.130 is reproduced and the second term of that equation, for ablative stabilization, is discussed in the next section.

**Ablative Stabilization of RT Growth**

Another modification to the RT growth rate in laser-irradiated targets is caused by the ejection (ablation) of material from the surface. Addressing this issue analytically has been investigated by others \( 42–44,50,51 \) and is beyond the scope of the current work. For a detailed theoretical treatment of the problem the reader is encouraged to see the referenced material. However, because ablative stabilization is an extremely important effect in laser-driven targets, a qualitative, heuristic description is provided.

As material is ablated away, the absolute location of the ablation front moves deeper into the solid material. The convection of the ablation front may be heuristically considered by moving into this reference frame. Recall that the solution to the perturbed velocity was

\[
u_{1x} = Ae^{ikr-\omega t}e^{kx}. 
\] (3.160)
Next, replace $\omega$ with $i\gamma$ and transform into the ablation front reference frame that moves into the material at the ablation velocity $v_a$, such that $x \rightarrow x - v_a t$, resulting in

$$u_{1x} = Ae^{ikx}e^{(\gamma - kv_a)t}e^{kx}. \quad (3.161)$$

The coefficient of $t$ in the exponent, $(\gamma - kv_a)$, is the effective growth rate as seen from the ablation front and the stabilizing effect of the ablation process is clearly demonstrated. In this heuristic derivation the stabilizing term, $kv_a$, has a coefficient of 1, where in directly-driven targets the coefficient is typically $\sim 3$. In the analytic derivation, other terms involving the ablation velocity arise, but are typically small and the ablative RT growth rate is

$$\gamma_{RT} = \sqrt{\frac{ka}{1 + kL_\rho} - \beta_{RT}kv_a}, \quad (3.162)$$

where $\beta_{RT}$ is $\sim 3$ for direct drive, and is the same expression given in Equation 3.130.

To illustrate the stabilizing effect of the ablation process a sample perturbation amplitude spectrum is linearly evolved in time and shown in Figure 3-8 for two different ablation velocities. The parameters used in these calculations are given in the caption and were equal, except for the ablation velocity, in both cases. The initial spectrum was uniform in amplitude at 5 nm. Spectra are shown in time steps of 0.5 ns and the factor of 2 difference in ablation velocity is shown to have a dramatic effect at smaller wavelengths. Moreover, the minimum wavelength that growth occurs at is proportional to $v_a^2$ and this is also demonstrated in Figure 3-8. The linear saturation line as a function of wavelength is also shown.

Figure 3-8: Perturbation growth due to ablative-RT for a sample uniform amplitude spectrum at 5 nm. The standard single-mode, linear-saturation line is shown where amplitudes reach $\lambda/10$. The saturation level using the multi-mode model developed by Haan for 2-D, ridge-like perturbations is also shown using a window size of $L_{det} = 400 \mu m$. Amplitude spectra are shown at several times with the following plasma parameters: $a = 130 \mu m/\text{ns}^2$, $L_\rho = 0.4 \mu m$, $\beta = 3$, and (a) $v_a = 4 \mu m/\text{ns}$ or (b) $v_a = 2 \mu m/\text{ns}$. 
at which point the perturbation amplitude is $\lambda/10$. This is the typical definition for linear saturation of a single mode perturbation, but is altered by the presence of multiple modes as described$^{52}$ by Haan. Linear saturation occurs at a lower perturbation height when multiple modes are present and is shown in Figure 3-8 for comparison.

Capsules in ICF implosions are susceptible to Rayleigh-Taylor growth during acceleration and deceleration of the shell. Ablative stabilization is essential for capsule integrity during spherical convergence and was recognized by Nuckolls et al.$^{29}$ at the birth of the inertial fusion energy program. Qualitatively, the stabilization phenomenon can be described$^{29}$ as the peaks of surface perturbations being ‘closer’ to the heat source, the critical surface. Thus, ablation occurs more quickly and higher local pressures are generated that reduce the perturbation amplitude. Other sources of stabilization have been predicted$^{53}$ to occur due to the ablation process such as vortex shedding, compressibility and thermal conduction smoothing. A detailed discussion of these effects is not necessary to grasp the main point here: stabilization occurs in ablatively driven targets and from a stability standpoint, higher ablation velocities are better and this is extremely good for the success of ICF.

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Chapter 4

Proton Radiography

Using protons to radiograph subjects instead of x rays provides a new window and insight into laser-matter interactions. X-ray radiography has been a standard diagnostic in many fields for imaging density distributions in objects. Protons are charged particles, and therefore interact with matter in a very different manner than x rays. Unlike x rays, charged particles are sensitive to both electromagnetic fields and matter, where the former is typically the quantity of interest in proton radiography experiments. Proton energy loss in matter is not typically used as an absolute measure of density distributions, but can provide a relative measurement. The real diagnostic strength of proton radiography lies in the ability to reveal electromagnetic field information from radiographic fluence images. This process inevitably involves some assumptions during the analysis that depend on the specific experiment. However, proton deflections caused by the Lorentz force provide an observable directly related to path-integrated electric and magnetic field strength in high energy density (HED) plasmas where other diagnostics, many times, prove impractical.

This chapter covers the experimental execution for typical proton radiography experiments. The exponential-spectrum backlighter and monoenergetic proton backlighter are contrasted. Use of a high-intensity, short-pulse laser for proton generation is covered in Section 4.1 and the monoenergetic, fusion backlighter is discussed in Section 4.2. All data concerned in this thesis was done using the monoenergetic backlighter and this diagnostic system is thoroughly discussed. Experimental results of backlighter isotropy are presented in Section 4.2.2. The use of CR-39 as a detecting medium for protons is covered in Section 4.2.3 with specific details on the characterization process using the Linear Electrostatic Ion Accelerator (LEIA). Experiments on the LEIA were conducted to investigate vacuum effects on the proton response in CR-39 and are discussed in detail in Section 4.3.3. Additionally, a benchmarked simulation tool has been developed to model the entire proton radiography system using the Geant4 toolkit and is thoroughly discussed in Section 4.4.
4.1 Sheath Accelerated Protons for Radiography

Radiography with protons was first done using an intense ($\sim 10^{19}$ W/cm$^2$) short-pulse laser incident onto a thin-foil target$^{2-5}$ to generate an exponential proton energy spectrum. When incident on a foil, the laser produces fast electrons at the front (irradiated) surface that pass through the material. Protons on the back surface see a large electric potential generated by charge separation and accelerate in a process known as target-normal sheath acceleration (TNSA),$^{3,6}$ see Figure 4-1. Protons emitted in this fashion have a continuous exponential spectrum with an endpoint-energy dependent on the incident laser intensity, foil material, and thickness, but can reach energies $\gtrsim 50$ MeV.

The short-pulse proton source provides high spatial ($\sim 10$ µm) and temporal ($\sim 10$ ps) resolution.$^7$ Images are typically recorded on a filtered stack of radiochromic (RC) film, where each film has a dominant energy window to which it is sensitive.$^7$ However, there can be a degeneracy in energy between the source continuum and energy loss in dense plasmas; depending on the configuration and field structure under observation. This was the first approach to proton radiography and was the only method until the High Energy Density Physics Division of the Plasma Science and Fusion Center at MIT developed a technique for producing a monoenergetic proton source for radiography (the reader is encouraged to other references$^{2-5,7}$ for further information regarding short-pulse proton radiography).
4.2 Fusion Protons for Radiography

Monoenergetic proton radiography has been used to infer path-integrated electric and magnetic field strengths in many HED physics experiments. This unique diagnostic technique provides a method to experimentally probe plasmas for electric and magnetic fields in regimes where other methods (Langmuir probes, B-dot probes, Faraday rotation, etc.) do not work or are impractical. Protons are deflected by electromagnetic fields in the plasma through the Lorentz force, but do not otherwise perturb the overall plasma evolution.

Fusion protons are generated through irradiation of an exploding-pusher capsule as illustrated in Figure 4-2. This unique backlighting source emits monoenergetic protons quasi-isotropically providing the ability to perform multiple experiments on a single shot. Furthermore, because the backlighter source is monoenergetic in nature, there is a one-to-one mapping of deflection angle to path-integrated field strength. The amount of deflection incurred by a charged particle due to E or B fields is proportional to the path-integrated field strength

\[
\theta_B = \frac{q}{\sqrt{2m_pE_p}} \int B_{\perp} dl, \quad (4.1)
\]

\[
\theta_E = \frac{q}{2E_p} \int E_{\perp} dl, \quad (4.2)
\]

where \( q \) is the particle charge, \( m_p \) the particle mass, and \( E_p \) the particle energy. \( B_{\perp} \) and \( E_{\perp} \) are the magnetic and electric field magnitudes perpendicular to the particle trajectory. In this way, path-integrated field strength information becomes encoded within modulations observed in proton fluence images.

Radiographs are recorded on CR-39 nuclear track detectors where absolute location and track characteristics are stored for every proton track. Typically, two sheets of CR-39 are fielded, each one filtered to register either DD or D³He protons whereby individ-


Figure 4-3: (a) Sample fusion proton spectra from an exploding-pusher backlighter capsule taken from OMEGA shot 51237. DD protons were measured at 3.6 MeV with FWHM of 320 keV and D\textsuperscript{3}He protons were measured at 15.3 MeV with FWHM of 670 keV. (b) Sample emission profile for D\textsuperscript{3}He protons over laid on the 1 ns square pulse used to drive the capsule on OMEGA shot 51237. Bang time was measured at 470 ps after laser onset with FWHM of 150 ps.

ual images of absolute proton fluence are easily generated for each species. To accurately extract quantitative information from proton radiographs, a thorough understanding of the proton source characteristics is necessary. This is the primary diagnostic technique used in work discussed in this thesis and a detailed description of the diagnostic methodology is provided in the following sections.

4.2.1 Backlighter Characteristics

The backlighter source used in monoenergetic proton radiography emits particles, not exponentially, but very close to Gaussian with a deviation from the mean of only a few percent;\textsuperscript{8,17} recall the discussion on exploding-pushers in Section 2.2.3 and specifically Equation 2.49. These targets consist of a thin-glass, spherical shell filled with equimolar D\textsubscript{2} (∼6 atm) and 3\textsuperscript{He} (∼12 atm) for a total pressure of ∼18 atm. Typically, capsules are ∼420 μm in diameter with a shell thickness of ∼2 μm SiO\textsubscript{2} (glass). This small capsule is compressed in the direct-drive fashion at the Omega laser facility where flexibility in the beam configuration allows for the implosion of the backlighter capsule as well as creation of the plasma to be studied. The fuel is shock-compressed to high temperatures (∼10 keV) and densities (∼10\textsuperscript{23} cm\textsuperscript{−3}) so that fusion can occur.

Exploding pushers have been used to generate fusion-protons for backlighting in many experiments\textsuperscript{8–13} at the Omega laser facility. Protons are produced through the reactions

\[
\begin{align*}
D + ^3\text{He} & \Rightarrow \alpha(3.6\text{ MeV}) + p(14.7\text{ MeV}) , \\
D + D & \Rightarrow T(1.01\text{ MeV}) + p(3.02\text{ MeV}) .
\end{align*}
\]

Characteristic spectra of DD and D\textsuperscript{3}He protons emitted from backlighter capsules are shown in Figure 4-3a. In typical configurations, backlighter capsules are irradiated with 20 OMEGA beams\textsuperscript{1} without smoothing by spectral dispersion (SSD) or distributed phase

\textsuperscript{1}Depending on the particular experimental configuration, more beams may be used to provide more laser energy on target.
plates (DPPs) for a total of \( \sim 9 \) kJ on target in a 1 ns square pulse as shown in Figure 4-3b. Fusion proton spectra are broadened (\( \sim 9\% \) and \( \sim 4\% \) FWHM for DD and \( \text{D}^3\text{He} \), respectively) by thermal effects (Equation 2.49). When nuclear production occurs during the laser pulse, as indicated in Figure 4-3b for these implosions, time-varying E fields around the implosion can also broaden the spectrum.\(^{18}\) The E fields are caused by a net positive charge on the capsule during laser irradiation and this charging effect produces an energy upshift of \( \sim 300-600 \) keV in fusion protons. In exploding-pushers of the specified dimensions, nuclear production always takes place during the 1 ns drive, and thus backlighter protons are always slightly higher in energy than the fusion birth spectrum would suggest.

**Spatial Resolution**

Proton radiography is subject to three main sources of image blurring: finite source size, scattering in the subject, and scattering in the detector. To analyze the effect on proton images, each source is characterized. However such a description does not account for any electromagnetic fields that might be present near the source, or in the subject. To first order, all three mechanisms are estimated to convolute the image with a Gaussian with a characteristic \( r_{1/e} \) (one-over-e-radius). The fusion source can be well approximated by a Gaussian emission profile in space with a \( 1/e \)-radius of \( r_{\text{src}} \sim 30 \) \( \mu \)m.\(^{17}\) Scattering is Gaussian in nature in the small angle approximation, as discussed in Section 3.2.1, and for energies above \( \sim 1 \) MeV, this is a sufficient estimation. Proton scattering in the target plasma is characterized by a \( 1/e \) scattering angle, \( \theta_{\text{targ}} \). Gaussian broadening in the detector is characterized by a \( 1/e \)-radius of \( r_{\text{det}} \). The blurring radii just mentioned are projected onto the detector and then demagnified to the subject plane (since experimental data is really the projection of the subject onto the detector plane).

Figure 4-2 shows a schematic of the generic radiography setup, with proper definitions of important parameters. The demagnified projections of each of these mechanisms can then be written as:

\[
R_{\text{src}} = \frac{M - 1}{M} r_{\text{src}} \quad \text{(4.5)}
\]

\[
R_{\text{targ}} = \frac{L_{\text{det}} - L_{\text{targ}}}{M} \theta_{\text{targ}} \quad \text{(4.6)}
\]

\[
R_{\text{det}} = \frac{1}{M} r_{\text{det}} \quad \text{(4.7)}
\]

The three blurring methods act together multiplicatively so that the total blurring of the image is the convolution with a Gaussian of the form:

\[
C(r) \sim e^{-\frac{r^2}{R_{\text{tot}}^2}} \quad \text{(4.8)}
\]

\[
R_{\text{tot}} \approx \sqrt{R_{\text{src}}^2 + R_{\text{targ}}^2 + R_{\text{det}}^2} \quad \text{(4.9)}
\]

The magnification of the system is typically \( \sim 30 \) so that \( R_{\text{src}} \approx r_{\text{src}} \). The broadening in the detector is dependent on the filtering chosen for the system, but can typically be estimated as \( \sim 15 \) \( \mu \)m–45 \( \mu \)m without demagnification.\(^{19}\) Therefore, in the subject plane, the detector broadening contribution is \( \lesssim 1 \) \( \mu \)m and can be ignored. The last mechanism affecting image broadening is that due to scattering in the subject, which is entirely dependent on the experiment. Such scattering can completely blur out the image, or have little to no
effect at all. Hence, for an experiment with little scattering in the subject, the resolution limit is defined by the size of the source. Otherwise, the square root of the quadrature sum of source size blurring and scattering in the subject sets the resolution limit for a given experiment.

**Temporal Resolution**

The proton temporal diagnostic (PTD)\textsuperscript{20} was used to measure peak fusion production (bang time) for D\textsuperscript{3}He protons. Previous experiments using 17, 20 or 30 beams on the backlighter, but still filled with 18 atm of D\textsuperscript{3}He, were examined. It was found that measured bang times fit a normal distribution well with a mean of 486±5 ps after laser onset and a standard error of 35±4 ps. When the on-target energy was increased by a factor of ∼2, no systematic change in bang time was observed. This result indicates that increasing on-target energy above ∼7700 J does not appreciably increase the shock transit time in these exploding-pusher capsules.

Timing of the proton source with respect to other laser beams is essential for radiography experiments. Without dedicating extra experiments to tuning timing fiducials, PTD has an absolute uncertainty of ±50 ps, dominating the timing error. However, based on many experiments, 95% of proton backlighters will have a bang time of 486±70 ps; though it should still be measured by PTD for each shot when diagnostic space is available. Also, the typical burn duration for these types of capsules was found to have a FWHM of ∼150 ps, that sets the temporal resolution of the radiography system.

### 4.2.2 Backlighter Isotropy

Contrary to short-pulse proton radiography, the exploding-pusher backlighter emits protons in a quasi-isotropic fashion. This unique feature allows for multiple experiments to be performed on a single shot; effectively doubling the amount of produceable data for viable experimental configurations. However, the temporal resolution is slightly longer and the spatial resolution slightly larger than the short-pulse system. With that said though, the monoenergetic nature of the proton source makes image interpretation somewhat simpler than that of the exponential source. Furthermore, the nature of the exploding-pusher proton source provides smoother fluence uniformity in general than from TNSA-generated protons.

Experiments were performed to examine backlighter isotropy for the monoenergetic proton source by fielding multiple yield diagnostics during a radiography campaign.

Diagnostics were fielded as indicated in Figure 4-4 to measure large and small scale proton-fluence uniformity. The charged-particle spectrometer (CPS)\textsuperscript{21,22} momentum analyzed charged particles passing through an aperture and energy spectra were recorded on CR-39 detectors. Multiple ten-inch manipulators (TIMs) fielded a variety of diagnostics. A properly filtered stack of 10 cm × 10 cm sheets of CR-39 was fielded in TIM2 to image the backlighter’s DD and D\textsuperscript{3}He protons. TIM3 held a single 7 cm round sheet of CR-39 that was filtered for DD protons only. To measure proton bang time and D\textsuperscript{3}He yield, PTD was fielded in TIM5. Lastly, an aluminum wedge range filter (WRF)\textsuperscript{15} in the KO1 diagnostic port\textsuperscript{ii} measured the time-integrated D\textsuperscript{3}He-proton energy spectra; these particular diagnostics are discussed further in Section 4.3.2. Radiographs of the backlighter in TIM2 and TIM3 provided short scale-length information on single sheets of CR-39, whereas long scale-length

\textsuperscript{ii}The so-called knock-on (KO) ports have since been replaced by nuclear diagnostic inserters (NDIs) on OMEGA.
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Figure 4-4: (a) An Aitoff projection of the OMEGA target chamber. The 60 beams are split into three legs. In typical proton radiography experiments, Legs 1 and 3 (blue) are used to drive a target and Leg 2 (red) drives the proton backlighter source. The diagnostics used to measure proton fluence are also shown (green) and labeled. Other diagnostic ports not used are labeled for reference. (b) A schematic of where isotropy diagnostics were located relative to the backlighter source held by a TPS in TIM4.

fluctuations were measured using the port-to-port variation in the inferred yield.

These diagnostics provided four measurements of D³He protons and three measurements of DD protons at different port locations. The results of multiple experiments to investigate backlighter isotropy are shown in Figure 4-5. Measurements of DD and D³He yields into 4π are shown for multiple port angles (lines of sight) in Figure 4-5a-b. Each yield measurement is represented by a different symbol and the statistical mean is denoted by ×. Error bars in these plots are calculated as the standard deviation (variance) of the yield measurements and are plotted against the average DD and D³He yields in Figure 4-5c-d. A slight increase in variance with average yield may be inferred from these data, but is inconclusive within the scatter. The global variance Σ for DD and D³He protons can be accurately characterized by a simple mean and deviation as Σ_{DD}≈16±7 % and Σ_{D³He}≈26±10 %, respectively. Local variation σ in fluence was assessed through ‘blank’ radiographs, shown in Figure 4-5g-h.

CR-39 sheets were fielded in TIM2 and TIM3 to image the backlighter without a subject in the field of view (FoV). These images provided proton fluence distributions over different solid angles. The 10 cm square CR-39 fielded in TIM2 was placed 27.18 cm from the backlighter covering ∼0.13 sr and 7 cm round CR-39 fielded in TIM3 were 21.88 cm away covering ∼0.08 sr. Because detectors are fielded at different distances, and we are interested in fluence fluctuations due to the backlighter itself, numerical statistics are removed from overall local fluence variation σ by:

\[ σ = \sqrt{σ^2_{Γ - meas} - σ^2_{Γ - stat}} \langle Γ_{meas} \rangle, \]  

where σ_{Γ - meas} is the measured statistical deviation of protons per steradian, \( ⟨Γ_{meas}⟩ \) is the statistical mean proton fluence used to normalize the variation across different experiments, and σ_{Γ - stat} is the statistical variation per steradian \( ∼\sqrt{⟨Γ_{meas}⟩} \). Figure 4-5e-f show the results of statistically corrected local variance measurements as a function of mean fluence.
Figure 4-5: Summary of backlighter proton emission isotropy data. (a) and (b) DD and D³He-proton yield measurements from different ports; error bars are the calculated standard deviations. (c) and (d) Global (long-scale) variance of yield measurements as a function of mean yield. Average variance Σ is shown by the solid line and the dashed lines are ± one standard deviation. (e) and (f) Local proton variance σ measured from radiographs in TIM2 (10 cm squares) and TIM3 (7 cm circles) as a function of mean proton fluence Γ. (g) and (h) Average power density spectra plotted as a function of angular frequency for two points in each (e) and (f) (outlined in black) with corresponding radiographs, where darker pixels indicate higher fluence. Frequencies ≥50 rad⁻¹ are shown to have amplitudes of ≤3% relative to the mean proton fluence level (normalized to 1). These data indicate that most of the local variance stems from long-scale perturbations.

over multiple experiments. The variance in DD proton fluence was observed to increase slightly with the mean proton fluence as shown in Figure 4-5e despite the two outliers; this trend is not pronounced within the scatter of the D³He measurements. Observance of this trend in DD, but not D³He, fluence measurements is indicative of deflections near the capsule due to electromagnetic fields, since the ≈3 MeV protons from the DD reaction will be deflected more than ≈15 MeV D³He protons. As in the case for global variance, the local variance of D³He protons (σ₃He) is measured to be slightly higher than that of the DD protons (σ₃He). This is most likely due to poorer statistics for D³He protons than DD protons for these small capsules. Measured proton fluence fluctuations were characterized using a discrete Fourier transform (DFT) technique.

Sample proton radiographs and corresponding average mode spectra are shown in Figure 4-5g-h. Lineouts of proton fluence were taken at multiple angles and processed using a one dimensional DFT technique with a Hann windowing function to reduce power leakage. The absolute sinusoidal amplitude αₐbs corresponding to a power density P_f at a given nonzero frequency is αₐbs ≈ √2P_f, where the proportionality constant is dependent on the normalization of the power spectra. However, the important metric here is the perturbation amplitude relative to the average (zero frequency) fluence α₀ ≈ √P₀. The normalized amplitude is defined as α = √2P_f/P₀. Furthermore, because α is the ampli-
tude of a sinusoid, the normalized RMS amplitude at a given nonzero frequency is simply 
\[ \alpha_{\text{rms}} = \sqrt{P_f/P_0}. \] Because spherical symmetry is assumed, DFTs over all angles are averaged to obtain an overall sense of mode structure in proton fluence.

Spatial frequencies in the detector plane were converted to angular frequencies \( f_\theta = 1/\theta \) for comparison of mode structure measurements at different distances from the backlighter; results are shown in Figure 4-5g-h. These amplitude spectra are not corrected for statistics, and it is clear that more proton fluence reduces the relative amplitude of high mode perturbations, as expected. The sample spectra shown clearly indicate that low mode perturbations dominate the local variance observed in Figure 4-5e-f. Amplitudes calculated for angular frequencies \( \geq 50 \, \text{rad}^{-1} \) are less than a few percent relative to the average proton fluence. These data indicate that when taking lineouts through proton fluence radiographs, local nonuniformities due to the backlighter are quite small for angles less than \( \sim 0.02 \) radians. However, long scale-length variation across a single CR-39 sample may be expected and must be considered when quantitatively analyzing proton fluence over large solid angles and when comparing to synthetic data.

### 4.2.3 CR-39 Nuclear Track Detectors

CR-39 is a clear plastic nuclear track detector\(^{1,15,16}\) utilized in all radiography data described in this thesis. CR-39 sheets were 1.5 mm thick and in the case of monoenergetic proton radiography, two sheets were stacked and individually filtered to be sensitive to one of the fusion products. Using detailed track information, proton fluence images may be generated, and because of the known relationship between particle energy and track diameter,\(^{1,16}\) an image of relative proton energy may also be produced. All CR-39-based nuclear diagnostics discussed herein rely on the predictable\(^{1,15,16}\) response of the plastic to charged particles.

As a charged particle travels through CR-39, it deposits energy in the plastic through Coulomb collisions with electrons, leaving a trail of destroyed polymer chains.\(^{24}\) Tracks of broken molecular chains and free radicals are made apparent through a chemical etching process utilizing 6N NaOH at 80°C. This exposes tracks because the etch rate of the track \( (v_t \sim 3.5-5.6 \, \mu\text{m/hr}) \) is faster than that of the bulk plastic \( (v_b \sim 3.3 \, \mu\text{m/hr}) \).\(^{25,26}\) The sensitivity of CR-39 to a specific particle species at a given incident energy is dependent on its restricted energy loss (REL) and defined by the ratio of track and bulk etch rates \( (V = v_t/v_b) \).\(^{26}\) After etching, CR-39 samples are scanned and individual track information is recorded for later analysis. The track diameter is used as a measure of the sensitivity and is a function of the etch-rate ratio \( (V \sim 1.7-1.05 \) for 1-5 MeV protons).

Figure 4-6a illustrates how the REL of a proton changes as it travels through CR-39 for three different incident energies; the typical depth for a 6-hour etch is also indicated \( (\sim 20 \, \mu\text{m}) \). Protons of these three energies leave very different tracks because of the distinct energy deposition profiles along the damage trail. For low-energy particles \( (i.e. \leq 0.8 \, \text{MeV}) \) where the etch depth has exceeded the range of the particle, a large circular crater is formed. This crater appears high in contrast relative to the background because light is mostly reflected. As this track is etched further, the crater wall becomes shallower, allowing more light to pass through, such that the track appears lower in contrast. Medium-energy protons \( (i.e. \sim 3.0 \, \text{MeV}) \) have a range larger than the etch depth and deposit enough energy along their path to create a deep conical pit. The pit wall internally reflects most light creating a very high contrast track. As the proton energy increases \( (i.e. \sim 7.1 \, \text{MeV}) \), the amount of energy deposited up to the etch depth diminishes and shallower conical pits are formed. These shallow pits do not reflect as much light and appear lower in contrast.
Figure 4-6: REL as a function of depth in CR-39 is shown for three incident proton energies in (a) with the typical depth for a 6-hour etch. Energy deposition along the damage trail controls the shape of the pit and therefore, the track appearance in the optical microscope system. A schematic of the pit shapes (b) and corresponding track images (c) are shown.

A schematic of the pit shapes and corresponding track images are shown in Figure 4-6b-c.

The specific manufacturing process of CR-39 has a large impact on the charged-particle sensitivity and response. For experiments discussed in this thesis, TasTrak® 1.5 mm thick CR-39 was used and etched in 6N NaOH at 80°C. Any changes in the plastic or etchant will alter the response and must be regularly characterized. Also, quantities such as the bulk etch rate \(v_b\) may change over time due to different manufacturing techniques used by a single company and must be assessed regularly. Because the predictable response of CR-39 is required for many nuclear diagnostics, samples are regularly exposed under controlled conditions on the MIT Linear Electrostatic Ion Accelerator (LEIA).
4.3 Characterizing the Response of CR-39

CR-39 is fielded in a number of nuclear diagnostics used in inertial confinement fusion (ICF) and HED physics experiments. The versatility of this detecting medium permits use under a wide variety of conditions. However, when utilizing this detector in new environmental conditions, any alteration of the response to charged particles must be characterized. In some experiments, extremely high, or low, fluences may occur and new processing techniques must be tested and verified to optimize the signal levels. Also, due to practical constraints at large ICF facilities, like the NIF or OMEGA, CR-39 samples may be exposed for prolonged periods to high vacuum environments which could affect the response. Furthermore, changes in manufacturing techniques or preprocessing of the plastic can change the response and necessitates experiments to assess the level of alteration. Characterization of CR-39 under varying environmental scenarios and development of new processing techniques are essential to sustaining and expanding the use of CR-39 in nuclear-diagnostics. Many of these characterization studies are performed at MIT using the LEIA.

This section will cover two studies that were performed to characterize CR-39 under different scenarios. The first experiments that are discussed are not only for CR-39 characterization, but also for calibration of diagnostics before being sent to the National Ignition Facility (NIF) or OMEGA. The second set of experiments presented here is the characterization of the effect that prolonged exposure to high vacuum can have on the proton response of CR-39. The LEIA system is both a research tool where different diagnostic studies are done, as well as a pedagogical machine for graduate students to acquire hands-on experience in a laboratory setting.

4.3.1 The Linear Electrostatic Ion Accelerator at MIT

Recent upgrades on the LEIA have improved beam performance and modularity of the system, see Figure 4-7. The new source, from National Electrostatics Corporation (NEC), generates a plasma discharge using a capacitively-coupled 300 watt radio frequency (RF) oscillator operating at 100 MHz. A bias is applied to ‘push’ positive ions out of the bottle where a set of permanent magnets aid in compressing the plasma beam at the exit canal. Next, an electrostatic lens may be biased to focus or defocus the beam before entering the acceleration tube. The machine uses a Cockcroft-Walton generator to step 120V AC up to ~150 kV DC voltage. The high voltage is applied to an array (‘the stack’) of electrostatic Einzel lenses where each step is separated by ~10 kV. The beam is adiabatically focused as it accelerates through each step of the high voltage stack and incident at target chamber center downstream where an erbium-deuteride target sits on a water-cooled copper finger. In order to provide both DD and D$_3$He fusion products, a $^3$He beam is run at low energy to dope the target with $^3$He ions. Under normal operating conditions, a ~140 kV deuteron beam is used to produce DD and D$_3$He fusion reactions in the target and these products are used for diagnostic experiments. For more details, the reader is encouraged to see the article by Sinenian et al. that discusses this machine’s capabilities in detail.
Figure 4-7: Multiple views of the Linear Electrostatic Ion Accelerator (LEIA): schematic top and side views, and a photograph of the LEIA system. Important diagnostics and components are labeled in the schematic views for reference.
4.3. CHARACTERIZING THE RESPONSE OF CR-39

4.3.2 Wedge Range Filter Spectrometers

A simple aluminum wedge has proven\(^1\)\(^5\) to be a very robust and versatile diagnostic. This diagnostic is the so-called WRF spectrometer that provides proton spectra in the energy range \(\sim 4-20\) MeV. These detectors have been fielded for many years at Omega, and recently over the past few years at the NIF. These spectrometers are commonly used to diagnose ICF experiments through measurements of primary and secondary fusion yields,\(^1\)^5 shell \(\rho R\) from downshifted charged fusion products,\(^2\)\(^9\) and fuel \(\rho R\) from scattered fuel ions (“knock-ons”).\(^3\)\(^0\) Calibrations of all wedges are performed using the LEIA.

\(^4\)\(^3\)He protons are used in conjunction with a ranging filter to provide a two-point calibration for each WRF as illustrated in Figure 4-8a. In this configuration, the two incident energies were measured at the correct location with a surface barrier detector (SBD) and determined to be 8.40 and 14.57 MeV. An approximately uniform fluence of \(^4\)\(^3\)He protons is generated by a deuteron beam incident onto the \(^3\)He-doped target. After going through the filter and wedge materials, the now continuous spectrum of protons results in a spec-
trum of diameters as a function of position on the CR-39, as shown by the contour plot in Figure 4-8b. Though any diameter range may serve, the calibrations are done using the 9-17 μm range and the resultant average fluence image is shown in Figure 4-8c. To improve statistics, lineouts averaged in the y-direction and a calibration is applied to map x-position to proton energy. The corresponding energy spectrum from this shot, using the nominal calibration, is plotted in Figure 4-8d. The position-to-energy map is adjusted through a software calibration using the measured and known incident energies. A 2-D calibration is used to account for wedges with large nonuniformities in the y-direction. Calibration of these detectors is extremely important for many diagnostics fielded at Omega and the NIF.

Because these simple detectors are used for many different diagnostic purposes, it is important to have a good understanding of the response of these detectors and the CR-39. It is clear that the diameter-to-energy mapping is very important in these detectors as indicated by the diameter cuts illustrated in Figure 4-8b. Furthermore, mapping of incident proton energy to resultant track diameter (at a specified etch time) is how the response of CR-39 is characterized. This can be done under various environmental, as well as processing (etching), conditions. One specific study, discussed in the next section, is the effect of prolonged vacuum exposure on the response of CR-39 to protons at various incident energies.

4.3.3 Vacuum Effects on Proton Response in CR-39

At large-scale ICF facilities CR-39 samples are left exposed to high vacuum (<10^{-3} Torr) for variable amounts of time (~1-3 hours at OMEGA, ~5-120 hours at the NIF) before and after irradiation by charged particles and neutrons. This necessitates characterization of the effects on CR-39 response to charged particles due to vacuum exposure.\(^1\)

The effect of vacuum exposure on track registration sensitivity for CR-39 from various manufacturers has previously been studied.\(^{31-34}\) It was shown that during the initial outgassing period, there is a drop in sensitivity (etch rate ratio) due to the changing oxygen profile in the plastic. Csige et al.\(^{32}\) observed a saturation point in the reduction of CR-39 sensitivity to 6.1 MeV alpha particles after 3 hours of vacuum exposure. It was also shown that if the plastic was immediately exposed to air post-irradiation, during the latent track-formation period (~minutes after irradiation), that the sensitivity could be partially recovered. These studies primarily used high-energy alpha particles, or other high-Z ions, and did not consider vacuum pressures below ~10^{-3} Torr. Golovchenko et al.\(^{35}\) investigated the sensitivity of multiple types of CR-39 to alpha particles in better vacuum conditions (P~4-20×10^{-5} Torr) for up to 10 hours of vacuum exposure. They observed varying amounts of sensitivity reduction for different CR-39 manufacturers and a sharper reduction in sensitivity for lower pressures. Typical pressures for vacuum conditions at OMEGA and the NIF are ~10^{-5} Torr and the primary particle of interest in CR-39-based diagnostics is the proton (and in some cases deuterons, tritons, or alphas). There is no previous study that has examined CR-39 sensitivity to MeV protons at vacuum pressures of ~10^{-5} Torr or lower.
4.3. CHARACTERIZING THE RESPONSE OF CR-39

Experimental Setup

Vacuum Chamber
~50 µA Deuteron Beam
~17° ~15 cm

Target

CR-39
SBD

Fusion Protons

Figure 4-9: A schematic of the experimental layout in the vacuum chamber is shown above. A deuteron beam is incident on a $^3$He-doped ErD$_2$ target. DD and D$^3$He fusion protons are produced and irradiate CR-39 samples exposed to various vacuum conditions. The number of particles incident on the CR-39 is controlled through in-situ counting using a surface barrier detector (SBD).

Configuration

Experiments were performed using the LEIA at MIT. Acceleration of a 140 kV deuteron beam onto a $^3$He-doped erbium-deuteride target produces the following fusion reactions:

\[
\begin{align*}
D + D & \Rightarrow ^3\text{He}(0.8 \text{ MeV}) + n(2.45 \text{ MeV}) , \\
D + D & \Rightarrow T(1.01 \text{ MeV}) + p(3.02 \text{ MeV}) , \quad (4.12) \\
D + ^3\text{He} & \Rightarrow \alpha(3.6 \text{ MeV}) + p(14.7 \text{ MeV}) . \quad (4.13)
\end{align*}
\]

In these experiments, individual CR-39 samples were placed in the vacuum chamber $\sim$15 cm from the target (Figure 4-9). A SBD situated 17° (or 34° for shutter experiments) from the CR-39 is used to count protons in-situ. The SBD provides an accurate measure of expected proton fluence at the CR-39 surface, thereby ensuring good statistics without saturating the sample. Because the CR-39 response is energy dependent, aluminum step-filters are used to range down DD- and D$^3$He-protons to provide various incident energies at the CR-39 surface. The SBD is used to accurately calibrate each filter pack before being fielded. There is a systematic energy uncertainty in the SBD calibration at the time was $\pm$75 keV and is transferred to the associated mean filter pack energy.

In order to run the high voltage ion beam, the entire system must be at high vacuum (pressures of $\sim$10$^{-5}$ Torr) to avoid arcing. The system achieves high vacuum through a combination of roughing and turbo pumps. First, a roughing pump is used to bring the chamber down to $\sim$5×10$^{-2}$ Torr which takes $\sim$10 minutes. After reaching rough vacuum, the turbo-pump is “valved-in” and brings the chamber pressure down to less than 10$^{-5}$ Torr.
in ~30-45 minutes. Before fusion products irradiate the CR-39 sample, it is exposed to continuously decreasing pressure in the chamber. The pump-down process described emulates the basic procedure for fielding CR-39-based nuclear diagnostics on OMEGA and the NIF. This level of vacuum exposure serves as the baseline for comparison to different vacuum exposure conditions.

Two different experiments were performed to examine the effect on the response of 1.5 mm thick CR-39 due to vacuum exposure. (1) CR-39 samples were irradiated with fusion protons at the baseline vacuum exposure and kept in high vacuum for different amounts of time after irradiation. (2) CR-39 samples were brought to the baseline and kept in vacuum for extended periods of time before proton irradiation. During these prolonged periods in high vacuum, the pressure continues to drop and saturates at \( \sim 10^{-7} \) Torr after \( \sim 16 \) hours.

**Processing and Analysis**

After an experiment is finished, the vacuum chamber is vented with dry nitrogen. Once the \( \text{N}_2 \) pressure of the system reaches ambient atmosphere (after \( \sim 5 \) minutes), CR-39 samples are removed from the chamber. During irradiation and venting time, latent track formation may occur without reintroducing oxygen to the system. CR-39 samples typically sit at room temperature and pressure for a day (or more) before processing begins. A 6N sodium-hydroxide (NaOH) solution is used at 80°C to etch each sample. All samples in this study were etched for 6 hours. After etching, the samples are scanned using an automated, optical microscope system whereby the diameter, eccentricity, and contrast of each pit are recorded for analysis. The spatial resolution is set by the optical parameters of the microscope system and was \( \sim 0.3 \) µm in these data.

The proton birth spectrum is narrow, but broadened when passing through the filter pack, this in turn produces a spectrum of diameters on the CR-39. Gaussian fits are used to measure the peaks of the energy and diameter distributions (see Appendix B for further details). The resulting random uncertainties in mean diameter and energy are calculated from the 95% confidence bounds in the fits and found to be \( \leq 0.05 \) µm and \( \leq 10 \) keV, respectively. Total uncertainties in energy and diameter measurements are smaller than the symbols used.

**Results of Vacuum Exposure After Proton Irradiation**

Four individual samples of CR-39 were irradiated with DD-protons and then left at high vacuum. Figure 4-10 shows the resulting diameter versus energy (D vs E) curves for four different vacuum exposure times. The response of CR-39 to 1-3 MeV protons is observed to be stable to vacuum exposure after proton irradiation for up to 67 hours. The slight decrease in mean diameter observed at longer vacuum times may be due to vacuum exposure, but these deviations are easily within typical piece-to-piece variation.

The observed stability in CR-39 sensitivity when exposed to vacuum after irradiation is easily understood through the process by which tracks are formed. Latent track formation in CR-39 is known to take place shortly after irradiation. The etch rate ratio is affected by the oxygen profile during the track formation process. Tracks have already formed in the first few minutes after irradiation and are therefore insensitive to an extended period in a high vacuum environment.
4.3. CHARACTERIZING THE RESPONSE OF CR-39

Figure 4-10: The stability of CR-39 response to 1-3 MeV protons is illustrated when exposed to high vacuum after irradiation. Mean diameter vs. energy curves are shown for various vacuum exposure times (given as time left in vacuum after irradiation). The 3-hour curve is obscured by the 24-hour curve.

Results of Vacuum Exposure Before Proton Irradiation

Six individual CR-39 samples were exposed for various amounts of time in high vacuum before proton irradiation. Both DD- and D³He-protons were used to probe the response of CR-39 to protons in the energy range of 1-9 MeV. Figure 4-11 displays the resulting D vs. E curves for six different vacuum exposure times. Up to 16 hours of vacuum exposure shows only small changes (∼15-30%) in D vs. E. These small deviations, however, oscillate about the baseline curve and are consistent with typical piece-to-piece variation. At 68 hours of vacuum exposure, a large decrease in mean diameter is observed for most proton energies. Protons at the two highest energies, ∼7.1 MeV and ∼8.6 MeV, have become undetectable at the longest vacuum exposure time (see Appendix B for further details).

In order to address piece-to-piece variation in D vs. E observed in Figure 4-11, a vacuum shutter system was utilized. The shutter allowed for irradiation across small discrete areas with identical filtering schemes. To investigate the diameter (energy) resolution on a single sample, a ∼6 μm Al filter was used to expose CR-39 at six different positions to ∼2.9 MeV protons at the baseline vacuum exposure. The time between ∼3 min exposures was ∼2 min so the whole experiment lasted ∼30 min, whereby no measurable vacuum effects are expected. The experiment was performed on two CR-39 samples and the results are shown in Figure 4-12. Measurements indicate a deviation from the mean of ∼4% and maximum difference of ∼6%. For comparing diameters (energies) at different positions on a single sample, these data indicate a systematic error of ∼3%/cm. Because these protons are products of beam fusion, there is a kinematic spread in incident energy of ±50 keV from one side to the other, but this is not sufficient to explain the observed deviations in mean diameter. Also, both samples were fielded identically, but illustrate opposite trends in mean diameter with respect to position indicating that kinematic energy shift could not be responsible for the observed deviations. Small inhomogeneities in the polymer could explain this level of discrepancy and provide a lower limit on energy resolution across a single CR-39 sample.
Figure 4-11: Six D vs. E curves are shown for CR-39 exposed to high vacuum before proton irradiation. Exposure times given correspond to vacuum exposure after the baseline was achieved. Small oscillations in D vs. E are observed for up to 16 hours in vacuum. However, at 68 hours the average diameter has decreased greatly for all proton energies. The highest energy protons at $\sim 7.0$ and $\sim 8.6$ MeV are no longer detectable.

To investigate the effects of vacuum exposure before proton irradiation on a single sample the shutter system was utilized. Two samples were fielded with aluminum step filters, one irradiated with DD-protons and the other with D$^3$He-protons. The shutter allowed for proton irradiation at different vacuum times without breaking vacuum and reintroducing oxygen to the sample. Figure 4-13 shows the mean diameter as a function of vacuum exposure time before irradiation for eight incident proton energies on two CR-39 samples. Over the 12 hour vacuum exposure, deviations up to $\sim 10\%$ are observed. However, this magnitude of deviation may be expected when comparing diameters from areas $\sim 3$ cm apart (as indicated in Figure 4-12). Therefore, modest vacuum exposure times ($\leq 12$ hr) before proton irradiation does not alter the response from the baseline exposure any more than expected from typical piece-to-piece variation.

In order to accurately probe the effects of an extremely long vacuum exposure before proton irradiation, a single CR-39 sample was used with a single $\sim 6$ $\mu$m aluminum filter on the shutter. The experiment was performed on two separate samples with two similar, but separate, pump-down sequences. CR-39 samples were irradiated with DD protons at multiple levels of vacuum exposure up to $\sim 5$ days, a NIF-relevant vacuum exposure time. Figure 4-14 shows the resulting mean diameter of $\sim 2.9$ MeV protons. After analyzing data from Sample 1, a second experiment was performed to confirm the trend with an extra sample time at $\sim 54$ hours. For both samples, a continuous drop in sensitivity is observed up to the $\sim 34$-hour exposure time in Sample 1 and the $\sim 54$-hour mark in Sample 2. The observed reduction in track diameter is explained by a constantly declining oxygen profile due to extended time in high vacuum. Mean track diameters are slightly higher at the $\sim 100$-hour exposure than the preceding time in both samples. However, sensitivity significantly recovers at the $\sim 125$-hour mark. At this time, no explanation is given for the resurgence of CR-39 sensitivity at vacuum times $>100$ hours.
4.3. CHARACTERIZING THE RESPONSE OF CR-39

Figure 4-12: (a) The number density (tracks/cm²) image for Sample 2 is shown where darker indicates more tracks. Each sample was irradiated by ∼2.9 MeV protons using a shutter system at the baseline vacuum exposure time. (b) Mean diameters are shown to differ as a function of position by ∼3%/cm. The opposite trends observed between Sample 1 and 2 indicate that kinematic energy spread is not responsible for observed diameter deviations.

In summary, CR-39 exposed to high vacuum before proton irradiation shows a dependency on the level of vacuum exposure. This effect is attributed to the continually changing oxygen profile in the CR-39 sample as a function of time with some component due to spatial inhomogeneities in the polymer. For vacuum exposure times less than 16 hours, the deviations observed in D vs. E are of comparable magnitude and shape as typical piece-to-piece variations. As vacuum exposure time is increased to ∼70 hours, overall sensitivity of the plastic decreases to the point that higher energy protons (smaller diameter tracks) become completely undetectable. The results also indicate that at extremely long vacuum exposure times (>100 hours) before irradiation, the sensitivity significantly recovers due to an unknown source (see Appendix B for detailed analysis information).

Conclusions

CR-39 is a plastic nuclear track detector used in many nuclear diagnostics for the ICF program and as a detecting medium for proton radiography in various HEDP experiments. When fielded on large-scale facilities, CR-39 may be exposed for hours (at OMEGA), or days (at the NIF), to high vacuum before and after irradiation by charged particles. It has been previously shown that exposure to rough vacuum has a dramatic effect on CR-39 sensitivity. However, in any high vacuum system, there is a transitory period from atmospheric pressure, through rough vacuum, to high vacuum. During this process, CR-39 inevitably loses some sensitivity because of out gassing and the declining oxygen profile. The question of the predictability of CR-39 sensitivity after the transitory period is important to the successful implementation for quantitative applications at OMEGA and the NIF.

It was shown that prolonged exposure to high vacuum after irradiation at the baseline
had no effect on CR-39 sensitivity (Figure 4-10). This is sensible because latent track formation is dependent not only on ionization characteristics during irradiation, but also the complex physico-chemical processes that take place only minutes afterwards.\textsuperscript{32} When left in high vacuum for modest exposure times (<16 hours) before proton irradiation, CR-39 response was not strongly affected. Shutter experiments performed on single CR-39 samples indicated that vacuum times \( \leq 12 \) hours were consistent with expected diameter deviations (\( \sim 3\% / \text{cm} \)) across a single piece (Figure 4-13). The exact nature of the vacuum effect was indistinguishable from piece-to-piece variations for up to 16 hours of vacuum exposure, but strongly reduced sensitivity by the 68-hour mark (Figure 4-11).

Long exposure times (>20 hours) before proton irradiation exhibited a strong affect on CR-39 response and must be treated carefully. This study suggests that the exact oxygen profile and small-scale polymer inhomogeneities in CR-39 are responsible for some observed piece-to-piece variations in the sensitivity to protons. These intrinsic factors are a function of the manufacturing process and environmental conditions prior to particle irradiation. Piece-to-piece variation of these intrinsic factors affect CR-39 sensitivity as much, or more than, vacuum exposure up to 16 hours. This inconsistency is relatively small and accounted for when calibrating CR-39. However, for extended vacuum exposure times, the oxygen profile changes drastically and its effect is clearly visible in measured track diameters. A method to calibrate CR-39 sensitivity in these long vacuum exposures is underway.
4.3. CHARACTERIZING THE RESPONSE OF CR-39

Figure 4-14: The mean diameter of ∼2.9 MeV protons as a function of vacuum exposure before irradiation. Two similar experiments were performed on two separate CR-39 samples. Similar trends are observed for both protons on each sample for vacuum times less than 54 hours. At vacuum exposure levels >100 hours an unexplained recovery in sensitivity is observed. This anomalous behavior may be due to a longer track formation period when the oxygen profile has been so greatly depleted. This explanation would imply that reintroduction of oxygen into the chamber after the typical ∼5 min waiting period was still impacting trace formation.
4.4 Geant4 - A Monte Carlo Code for Particle Tracking

To model proton radiography experiments, a Monte Carlo code was written using the Geant4 toolkit. The geometry and tracking code is an open source library of functions written in C++. The experimental system was constructed within the Geant4 framework (version 4.9.4.p01) through proper geometry, material, and physics package implementation. To this end, a simulation has been developed employing a finite Gaussian proton source for DD and D³He protons of finite spectral width to create synthetic proton radiographs of various subject types using actual experimental configurations.

Accurate modeling is necessary for quantitative interpretation of proton fluence modulations in some radiography experiments. Tracking of protons through electromagnetic fields is performed in Geant4 using a standard 4th-order Runge-Kutta algorithm. Because Geant4 is open source, the user may define an electromagnetic field of arbitrary complexity and choose from a number of different solvers for the equations of motion. Currently, the simulation implements simple fields due to spherical shells of charge, cylindrical shells of charge or current, or sinusoidal E or B fields of varying spatial dimension. Modeling of the Lorentz force is relatively straightforward; its effect does not change whether particles are traversing a plasma of spatially varying parameters, or standard cold matter. However, the interaction of imaging protons with matter is a collisional process and dependent on the local properties of the material, as was discussed in Sections 3.2.1 and 3.2.2.

4.4.1 The Cold Matter Approximation

The binary Coulomb interaction of high energy ∼MeV ions with a background medium was thoroughly discussed in Section 3.2. Differences between cold-matter (CM) and plasma media were specifically discussed for energy loss and scattering of incident ions. Many freely available numerical codes only implement ion interactions with cold matter. Therefore, it is essential to calculate the effect of using the cold matter approximation to model ions through various plasma environments.

The accuracy of the cold matter approximation was assessed in a sample plasma environment relevant to experiments discussed in Chapters 6 and 7. The experiments are explained in detail in these chapters, but some laser parameters are given here for context. In these experiments a ∼20 µm thick CH foil was illuminated with 0.351 µm laser light, as illustrated in Figure 4-15a, at an intensity of ∼4×10¹⁴ W/cm² in a 2 ns square pulse. The plasma evolution was simulated using the 2-D radiation-hydrodynamic code DRACO (see Section 6.2) and plasma profiles relevant to scattering and energy loss are shown in Figure 4-15b at a sample time of 1.3 ns after onset of the laser drive. In this plot, the laser is incident from the right and the foil is driven to the left. At the time shown, the cold, dense foil is observed at a position of ∼20 µm and the ablated plasma expands to the right. A sample trajectory is also indicated for imaging protons in these experiments and it is clear that these protons experience a wide variety of plasma conditions.

The coupling parameter Γ was calculated using Equation 3.4 and plotted with other plasma parameters. Figure 4-15b illustrates the dynamic range of conditions in a typical laser-produced plasma and shows that in most regions, the plasma is weakly coupled. The only exception being inside the foil where it appears to be moderately coupled. However, according to DRACO predictions this ‘moderately coupled’ region is scarcely ionized with an average ionization state <Z> less than one. In this domain, a transition treatment between

iiithThe QGSP BERT physics package was used.
4.4. GEANT4 - A MONTE CARLO CODE FOR PARTICLE TRACKING

For a CH(1:1.38) plasma

\( \rho = 1.04 \text{ g/cc} \) (cold matter)

Path-integrated 15.1-MeV:
- Plasma Energy Loss = -0.0764 MeV
- CM Energy Loss = -0.0703 MeV
- Plasma/CM = 109%
- Plasma Scattering = 0.00213 radians
- CM Scattering = 0.00203 radians
  - Plasma/CM = 105%

Path-integrated 3.3-MeV:
- Plasma Energy Loss = -0.227 MeV
- CM Energy Loss = -0.247 MeV
- Plasma/CM = 92%
- Plasma Scattering = 0.00975 radians
- CM Scattering = 0.00930 radians
  - Plasma/CM = 105%

Figure 4-15: (a) Sample laser-ablation plasma parameters as simulated by DRACO. Relevant plasma parameters \( n_e, \rho, T, <Z>, \) and \( \Gamma \) are shown as a function of position with a sample proton trajectory illustrated. Contours in T-\( \rho \) space are illustrated for DD (dotted) and D\(^3\)He (dashed) protons for the ratio between plasma and CM rms deflection angle in (b), and for the ratio between plasma and CM energy loss in (c). The solid line in c) and d) corresponds to the path in T-\( \rho \) space experience by the proton along the trajectory indicated in b) from left to right.

Calculations were performed over many orders of magnitude in plasma density and temperature. Though it is not always true in the ablated plasma, thermal equilibrium between electrons and ions was assumed in these calculations for estimation purposes. Proton radiography discussed in this thesis focuses on DD and D\(^3\)He fusion products, so comparative results are shown for typical proton energies of 3.3 MeV (dotted) and 15.1 MeV (dashed) in Figure 4-15b-c. The trajectory through the sample laser-ablated plasma is indicated in both plots for reference. It is noted that the majority of energy loss and scattering takes place in the low-density, low-temperature region. However, imaging protons sample the entire plasma and a path integral is necessary in both the CM and plasma limits to assess the total energy loss and scattering.
Table 4.1: Results for path integration of plasma and CM scattering angle and stopping power. Integration was performed along the trajectory shown in Figure 4-15 for the sample plasma environment.

<table>
<thead>
<tr>
<th></th>
<th>3.3 MeV (DD-p)</th>
<th>15.1 MeV (D³He-p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scattering Angle: Plasma/CM</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>Stopping Power: Plasma/CM</td>
<td>0.92</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Contours of the plasma-to-CM scattering ratio are shown in Figure 4-15c for 3 and 15 MeV protons in T-ρ space. In the proton scattering formulation discussed in Section 3.2.1, the only change relative to the CM approximation enters in the logarithm and is not dependent on incident energy. For this reason, there is no noticeable difference in the plasma-to-CM scattering angle ratio for 3.3 and 15.1 MeV protons as shown in Figure 4-15b. It is clear that at low temperatures and high densities, there is little difference between scattering in plasma and scattering in CM as indicated by the ‘1.0’ contour. To estimate the accuracy of the CM approximation, the sample trajectory shown in Figure 4-15c was path-integrated for both plasma and CM scattering cases through T-ρ space,

\[ \theta \approx \sqrt{\int d<\theta^2>} \tag{4.14} \]

This calculation indicated that the effective CM scattering would be \(~5\%\) lower than that of plasma for both 3 and 15 MeV protons along this trajectory. Because the ratio of scattering angles results in a ratio of Coulomb logarithms, the difference between 3 and 15 MeV protons is negligible as illustrated by the overlaid dotted and dashed curves, but this is not the case when looking at stopping power.

Contours of the plasma-to-CM stopping power ratio are shown in Figure 4-15d for 3 and 15 MeV protons in T-ρ space. At low temperature and low density, plasma stopping power strongly deviates from the CM value for these high energy protons. As temperature increases at a constant density, a temperature threshold is reached when the electron speed in the plasma is near the test proton speed, and the plasma stopping becomes weaker than that in cold matter of the same density. This threshold is obviously reached at a lower temperature for 3 MeV protons than 15 MeV protons. Plasma stopping is also weaker than CM at low temperatures and high densities because InΛ decreases with increasing density. Again, to estimate the accuracy of the CM approximation, the sample trajectory is path-integrated on the surface in T-ρ space,

\[ \Delta E \approx \int \left( \frac{dE}{dx} \right) dx \tag{4.15} \]

It was found that 15 MeV protons would have \(~9\%\) higher stopping power in the plasma, whereas 3 MeV protons would have a lower stopping power by \(~8\%\). In these calculations, the absolute energy loss was assumed negligible\(^{iv}\) and is sufficient for estimation purposes here. The results of both calculations are shown in Table 4.1 and indicate that in this plasma

\(^{iv}\)Energy lost from protons was \(~80\) keV and \(~250\) keV for 15.1 and 3.3 MeV protons respectively.
environment, the CM approximation is $\lesssim 5\%$ lower than expected scattering in a plasma and that CM stopping power is within $\sim 10\%$ of the Li-Petrasso value. These deviations are within the accuracy of the analytic modeling discussed for these phenomena.

The cold matter approximation has been shown to be accurate to $\lesssim 5\%$ for proton scattering and to $\lesssim 10\%$ for energy loss in typical proton radiography experiments of laser-foil interactions. Every experiment is different, and in some cases the CM approximation is insufficient and a more complex plasma model will need to be used. Furthermore, the plasma stopping power model implemented here assumes a fully ionized plasma and so this the ratio represents an upper bound on the error of the CM approximation. Nevertheless, for experiments discussed here and in many laser-foil experiments, the CM approximation is adequate and within the uncertainty of the presented analytical models. Therefore the collision physics currently implemented in Geant4 can be used to model trajectories in proton radiography experiments.

### 4.4.2 Benchmark Experiments

Proton radiographs of non-irradiated targets were used to benchmark Geant4 simulations and validate the collision-physics package implementation. These ‘cold’ targets were characterized and modeled in the Geant4 framework. Coulomb collisions between imaging protons and target material causes energy loss and trajectory deflection. A 24 $\mu$m-thick CH shell was radiographed using 3 and 15 MeV protons as illustrated in Figure 4-16a. The capsule target provides a useful benchmark for proton scattering through variable path lengths of material. Figure 4-16b and c show the comparison between synthetic and experimental radiographs for 3 and 15 MeV protons, respectively. Radial lineouts produced from synthetic Geant4 radiographs (dashed) agree very well with experimental data (solid). Experimental radiographs are aligned with synthetic radiographs for both energies and reproduce the observed data accurately.

![Figure 4-16](image)

*Figure 4-16: (a) Experimental setup for capsule radiographs. Synthetic and experimental data are shown for 3 MeV (b) and 15 MeV (c) proton radiographs of an unimploded CH capsule from OMEGA shot 46531. The top half of each radiograph is from experimental data and the bottom half is simulated. Corresponding radial lineouts are shown by the solid line (experimental) and the dashed line (simulated).*
Proton fluence amplitude modulation was benchmarked using a variable frequency nickel mesh and CH foils of different thicknesses. The 35 μm-thick Ni mesh was electroformed with hole spacings of λ~230, 150, 90 μm; in this geometry, these spacings correspond to \( f_\theta \sim 110, 170, 290 \) rad\(^{-1}\) respectively. The mesh splits the quasi-isotropic proton flux into ‘beamlets’ at variable spatial frequencies. The rms amplitude modulation (\( \alpha_{rms} \)), with no additional CH, is shown to increase with hole spacing in Figure 4-17b. A separate experiment had beamlets incident onto 25, 50, 75, and 100 μm-thick CH foils. Lineouts taken of the resultant proton radiograph are shown in Figure 4-17c for the λ~230 μm wavelength. Proton fluence was normalized for comparison of different CH thicknesses and amplitude modulation is shown to decrease with increased CH foil thickness as expected. The rms amplitude modulation was calculated and plotted as a function of CH thickness for \( \lambda \sim 230 \) μm (○) in Figure 4-17d. Proton beamlets of λ~150 μm were only observed through the 25 μm-thick foil, whereas \( \lambda \sim 230 \) μm was measured through CH thicknesses up to 100 μm. However, beamlets of λ~90 μm were not resolvable through any CH thicknesses due to blurring caused by Coulomb scattering in the foil. Simulations of these experiments were found to track measured data reasonably well, thereby verifying the modeling capabilities for proton radiography in the Geant4 framework when the cold-matter approximation is sufficient to describe the Coulomb interactions.
4.5. SUMMARY

Geant4 provides the user with the ability to simultaneously model particle interactions in a target and the subsequent detector physics in arbitrary geometries. This capability can be extremely useful for detectors with complex response functions, such as magnet-based spectrometers, detector systems which rely on nuclear reactions, or stacks of film. This capability has been exploited in Geant4 to determine the response function for magnet-based diagnostics, specifically the magnetic-recoil-spectrometer (MRS). Furthermore, the complex detector response of an exponential proton spectrum incident onto a filtered stack of radiochromic film, as in short-pulse proton radiography, may be self-consistently modeled in the Geant4 framework to deduce quantitative information from the images. In the work described here, a realistic backlighter source of both 3 and 15 MeV protons was modeled through a target and then the detector stack for a comprehensive simulation of the experiment, from source to detector.

4.5 Summary

The exploding-pusher proton source discussed here has been recently used in many experiments and is in strong contrast to the TNSA-generated proton source used previously by others. A summary of characteristics for the two proton sources is given in Table 4.2 using typical parameters from OMEGA-EP, though other facilities will have slightly different source characteristics. TNSA-generated protons typically have a smaller source size, shorter pulse duration, and higher peak energy than the exploding-pusher-generated protons. However, TNSA protons are produced in an exponential spectrum, where energy-loss in the target can create a degeneracy not present when using the monoenergetic source. The exploding-pusher source generates protons isotropically and can be utilized for multiple ex-

<table>
<thead>
<tr>
<th>Source Size [µm]</th>
<th>Exploding-pusher</th>
<th>TNSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration (FWHM) [ps]</td>
<td>~150</td>
<td>~1-10</td>
</tr>
<tr>
<td>Peak Energy [MeV]</td>
<td>~15</td>
<td>~60</td>
</tr>
<tr>
<td>Energy Spectrum</td>
<td>Monoenergetic</td>
<td>Exponential</td>
</tr>
<tr>
<td>Drive Requirements</td>
<td>multi-beam (implosion)</td>
<td>high-intensity (&gt;10^{19} W/cm²)</td>
</tr>
<tr>
<td>Detecting Medium</td>
<td>CR-39</td>
<td>RC Film</td>
</tr>
<tr>
<td>Detection Process</td>
<td>proton counting (track detection)</td>
<td>dosimetric (optical depth)</td>
</tr>
</tbody>
</table>
experiments in a single shot. Contrarily, the TNSA source protons are forwardly directed, but can provide radiographs of the target at multiple times in a single shot due to the difference in proton time-of-flight.

Monoenergetic proton radiography has been used in many experiments to measure path-integrated electromagnetic fields in HED plasmas where other methods prove ineffective. Proton emittance isotropy of these exploding-pusher backlighter capsules has been characterized on a global and local scale. Multiple yield diagnostics were fielded to quantify the global deviation of both proton species and were measured to be $\Sigma_{DD} \approx 16 \pm 7\%$ and $\Sigma_{D^3He} \approx 26 \pm 10\%$. Local variation was measured on single sheets of CR-39 using ‘blank’ radiographs of the backlighter source. It was shown that local fluence variation was dominated by low angular frequency modes $f_\theta \lesssim 50\text{ rad}^{-1}$ and that variations of a few percent should be expected on shorter spatial scales.

Typical media for proton detection also differs between these two proton sources. The ability to count individual tracks and directly measure relative proton fluence using CR-39 removes the necessary deconvolution when using RC film. Using a mesh in an experiment, may remove the necessity of knowing the relative fluence to make a quantitative measurement at a cost of further energy degeneracy (in the case of TNSA) and at a cost to spatial resolution. These two complimentary diagnostic tools differ substantially in source characteristics and both come with a unique set of challenges. However, regardless of the proton source, Coulomb collisions in the target will cause energy loss and scattering and these effects have been modeled and benchmarked for the monoenergetic source.

A new simulation tool has been developed to model monoenergetic proton radiography experiments using the Geant4 open-source framework. Realistic spectral source profiles, exact detector geometries, arbitrary electromagnetic field maps, and generic target mass distributions have been implemented. The physics packages currently implemented address Coulomb interactions in the cold-matter approximation and do not account for plasma effects. Due to the minimal amount of energy-loss and scattering experienced by MeV protons under plasma conditions discussed herein, the cold-matter approximation was shown to accurately approximate the collisional behavior to $\lesssim 10\%$ which is within the uncertainty of the analytic formulations used. Geant4 modeling was benchmarked against multiple experimental radiographs of non-irradiated targets. This simulation tool is used to generate synthetic radiographs for quantitative comparisons with experimental data as well as to aid in experimental design.

**4.5.1 Future Work**

The development of a plasma physics module for Geant4 would significantly improve the modeling capabilities of the code. Geant4 is open-source and those who maintain the code encourage users to develop their own physics modules that can be integrated into future releases, however, is not a trivial undertaking. It would require a flag within the material type, or a new material type, that recognizes the need to use a separate plasma physics module. Within this module, algorithms could be written to apply the desired physics model for charged particle stopping power and Coulomb scattering. Additionally, models for photo absorption and other relevant plasma processes could be implemented for general use. Any plasma volume would have the plasma parameters defined as material properties that would be used as arguments to plasma-physics functions. Moreover, experiments for testing plasma-physics models for stopping power would complement this new physics implementation in Geant4 with results for benchmarking.
References


Chapter 5

Measurements of Return Currents in Inertial Fusion Targets

In current inertial confinement fusion (ICF) experiments, high-intensity lasers are incident onto a target that is held in place by a mechanical support. When targets are irradiated, laser energy preferentially heats electrons in the expanding plasma. The electrons are accelerated away from system, resulting in a net positive charge on the target and surrounding mechanical structures. These support structures (stalks) illustrate a fundamental difference between the idealized target physics of interest and what the actual experimental configuration must entail. The stalk provides a direct electrical connection between the irradiated target and the rest of the target chamber. Large potentials generated near laser-irradiated targets due to high-energy electrons leaving the system drive return currents through the target’s stalk. The interactions between the target and the chamber, via the stalk, should be considered when investigating the target physics of the experiment.

Previous work has been done to characterize charge separation effects and return currents. Pearlman et al. directly measured the target potential after a 50 ps laser pulse was incident onto a target. They observed a potential decay time over a few ns consistent with electron depletion in the plasma followed by slower ions. Also, it was found that the peak voltage reached and the decay time observed were functions of the vacuum chamber size (ranging from 12.7-25.4 cm) consistent with a capacitance model of the system. Hicks et al. measured fast protons with energies of \( \gtrsim 1 \) MeV from spherical implosions. These fast ions are pulled by the escaping electron population and provide another metric for the target potential. Recently, fast-ion measurements from spherical implosions on OMEGA were used to infer the target capacitance as a function of intensity for ambient and cryogenic targets. Benjamin et al. directly observed the return current of laser-irradiated targets using optical photographs. They demonstrated that ohmic heating caused by the return currents was dominant over other plausible heating sources: thermal conduction, plasma radiation, and hot-electron propagation. However, detailed measurements of where and how this current flows were not made.

Plasmas generated by rapid ohmic heating of a thin wire are found in other areas of high energy density (HED) physics research as well, namely wire-array Z-pinches. There has been significant effort made to measure spatial distributions of current and magnetic fields in Z-pinch plasmas. For the fast voltage pulses in experiments discussed here with rise times of order \( \sim 200 \) ps, the skin depth \( (\delta) \) of the return current is of order \( \sim 50 \) \( \mu \)m suggesting that initial ionization occurs on the outer edges of the stalk. The current flowing
through this plasma generates azimuthal B fields. Conventional field measurement tech-
niques, specifically Faraday rotation and B-dot probes, have limitations as wire Z-pinch
plasmas have very high density in the core and significant density and temperature gradi-
ents in the coronal plasma. Proton deflectometry can provide information about current
strength as well as magnetic field topology in these plasmas. Monoenergetic proton radiog-
raphy has been used to investigate current profiles in Z-pinch-like plasmas generated around
the stalks of inertial fusion targets.

In this chapter, the first spatially resolved measurements of return currents and charge
separation in laser-driven ICF targets is discussed. The experimental configuration is pre-
sented in Section 5.1. Modeling of proton trajectories using Geant4 and the associated
forward fitting techniques used when comparing synthetic and experimental radiographs
are covered in Section 5.2. Experimental results for picket-pulse experiments are shown in
Section 5.3. Finally, this chapter concludes with a discussion in Section 5.4 on these results
and the implications in relevant HED research.
5.1. STALK EXPERIMENTS

In these experiments, one of the primary physics goals was to obtain charged particle radiographs of spherical targets illuminated by 40 OMEGA beams and the surrounding field structure. Laser beams on these targets used smoothing by spectral dispersion (SSD) and SG4 distributed phase plates (DPPs). This work was concerned with radiographing and quantifying field structures in the corona of the irradiated target. In addition to these data, proton radiographs were taken of the stalks holding these targets to quantify charging effects and return currents generated by laser illumination.

An Aitoff projection of the OMEGA chamber configuration used in these experiments is shown in Figure 5-1a and indicates laser beams used for driving the backlighter as well as the spherical target at target chamber center (TCC). Figure 5-1b illustrates how the quasi-isotropic character of this backlighter can be used to simultaneously radiograph the spherical target in TIM6 and the stalk in TIM1. The stalk is not perpendicular to the backlighter-CR39 optical axis, but at an angle $\theta \approx 39.4^\circ$ such that the optical characteristics along the stalk differ. Because this angle is $<90^\circ$, net proton deflections can result due to return-current-generated azimuthal magnetic fields around the stalk; if the stalk were perpendicular, then no observable net deflections due to azimuthal magnetic fields would occur as discussed in Section 5.2. Radiographs were taken with DD protons and recorded on 1.5 mm-thick sheets of CR-39.

After proton exposure, CR-39 samples were etched in a 6N NaOH solution for 1-6 hours, depending on fluence level, to reveal tracks created by charged particles. Etched CR-39 samples were scanned using an automated optical microscope system, whereby track locations, diameters, eccentricities, and contrast levels (relative to the background) were recorded and stored for later analysis. In this way, proton fluence images were generated, and because of the relationship between particle energy and track diameter, an image of relative proton energy may also be produced. Proton radiographs of stalks holding spherical targets at TCC were recorded on 7 cm round CR-39 nuclear track detectors.

Figure 5-1: (a) Aitoff projection of the OMEGA target chamber. Legs 1 and 3 were used to irradiate the target at TCC and Leg 2 was used to drive the backlighter. Proton radiography diagnostics were fielded in TIM1 and TIM6. (b) A schematic of the experimental geometry. The backlighter is a thin-glass capsule filled with $D^3He$ placed 9 mm from TCC. Spherical capsules at TCC had a diameter of $\sim 860 \mu m$ with a 35 $\mu m$ thick CH shell and were filled with 1 atm $H_2$. The relative directions of TIM1 and TIM6 are shown for reference.
The stalk assemblies that secured targets at TCC consisted of a series of boron (B) and silicon-carbide (SiC) fibers. These materials had initial densities of $\rho_B \approx 2.38 \text{ g/cm}^3$ and $\rho_{\text{SiC}} \approx 3.1 \text{ g/cm}^3$, a schematic diagram of the assembly is shown in Figure 5-2c. A tripod of B fibers extends from the connector pin that attaches to the target positioner. Two of these fibers are used to support a third $\sim 35 \text{ mm}$ away from the connector pin. The third B fiber continues further by $\sim 8 \text{ mm}$ where a single SiC adjoins to the end. The stalk assembly terminates at the target with a single thin SiC fiber $\sim 1 \text{ mm}$ long; a summary of stalk components is given in Table 5.1. Proton radiographs taken in TIM1 image the stalks with a $\sim 4.3 \text{ mm}$ diameter field of view as shown in Figure 5-2c. Electric and magnetic fields generated around target stalks were radiographed using monoenergetic DD-fusion protons.

Table 5.1: Characteristic lengths and diameters of stalk components. TIM1 has a $\sim 4.3 \text{ mm}$ diameter field of view near the S2-S3 junction.

<table>
<thead>
<tr>
<th>Label</th>
<th>Material</th>
<th>Length [mm]</th>
<th>OD [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: B Fiber Tripod</td>
<td>Boron</td>
<td>$\sim 35$</td>
<td>$\sim 145$</td>
</tr>
<tr>
<td>S2: B Fiber</td>
<td>Boron</td>
<td>$\sim 8$</td>
<td>$\sim 145$</td>
</tr>
<tr>
<td>S3: SiC Fiber</td>
<td>Silicon Carbide</td>
<td>$\sim 13$</td>
<td>$\sim 82$</td>
</tr>
<tr>
<td>S4: SiC Fiber</td>
<td>Silicon Carbide</td>
<td>$\sim 1$</td>
<td>$\sim 17$</td>
</tr>
</tbody>
</table>
5.2 Electromagnetic Fields Around Target Stalks

During target illumination, laser energy is preferentially deposited to plasma electrons through collisional absorption, resonance heating, and a number of instability channels.\textsuperscript{16,17} A two-temperature electron distribution is often observed,\textsuperscript{18,19} characterized by a ‘hot’ ($T_h$) and a ‘cold’ ($T_c$) temperature. The hot distribution is typically generated through resonance absorption near the critical surface and through laser-plasma instabilities such as the two-plasmon decay (TPD) near the quarter critical surface. The so-called cold electrons are dominantly heated through collisional absorption and thermalize with the local ion population. However, those electrons in the high-energy tail of the Maxwellian distribution may also be considered ‘hot’ in the sense that they have energies comparable to those in the hot population. These hot electrons are approximately collisionless and eventually escape the local system, leaving a net positive charge on the target. The supporting stalk provides a path for return currents to neutralize this charge separation, as indicated in Figure 5-2a. Monoenergetic proton radiographs were taken to quantify the strength and location of return currents and residual charge induced by the escaping electron population.

Proton radiographs of target stalks were simulated using the Geant4 modeling tool discussed in Section 4.4. The simplicity of the experimental geometry shown in Figure 5-2a allowed for a relatively straightforward implementation. To capture the essential deflection physics, axisymmetric E and B fields were assumed. A constant current $I$ and linear charge density $\lambda$ were distributed uniformly within separate cylindrical annuli around the cold stalk providing an axisymmetric field structure. This model approximates a physical situation where the current preferentially flows in the expanding hot plasma due to its lower resistivity, and the positive potential manifests as a charge imbalance.

5.2.1 The Concentric Cylinder Model

A cross section of the stalk with E-field-related and B-field-related annuli is illustrated in Figure 5-3a. A total of six parameters characterize simulated B and E fields: the current $I$ and charge density $\lambda$; the mean radii of each annulus, $R_B$ and $R_E$; and the corresponding widths, $W_B$ and $W_E$. Because this geometry is assumed symmetric in azimuth as well as along the stalk, fields vary only as a function of radius. E fields are directed radially outward and B fields are azimuthal around the stalk. The field equations presently implemented in the Geant4 model are:

$$E_r(r) = \begin{cases} 
\frac{\lambda}{2\pi\epsilon_0 R_E, o} \frac{r}{R_E, o} \frac{1 - \left(\frac{R_E, i}{R_E, o}\right)^2}{1 - \left(\frac{R_E, i}{R_E, o}\right)^2} & 0 < r < R_{E,i} \\
\frac{\lambda}{2\pi\epsilon_0} & R_{E,i} < r < R_{E,o} \\
\frac{\lambda}{2\pi\epsilon_0} & R_{E,o} < r < \infty 
\end{cases} \quad (5.1)$$

$$B_\theta(r) = \begin{cases} 
\frac{\mu_0 I}{2\pi R_B, o} \frac{r}{R_B, o} \frac{1 - \left(\frac{R_B, i}{R_B, o}\right)^2}{1 - \left(\frac{R_B, i}{R_B, o}\right)^2} & 0 < r < R_{B,i} \\
\frac{\mu_0 I}{2\pi} & R_{B,i} < r < R_{B,o} \\
\frac{\mu_0 I}{2\pi} & R_{B,o} < r < \infty 
\end{cases} \quad (5.2)$$

where the inner ($R_i = R - W/2$) and outer ($R_o = R + W/2$) radii have been used instead of the mean ($R$) and width ($W$) for notational convenience. These six independently defined parameters determine the appearance of resultant proton radiographs in a specified geometry. Two sample simulations are illustrated in Figure 5-3b.
The first case is a simple 140 \( \mu \text{m} \) diameter boron stalk, component S2 in the stalk assembly, with no fields. The stalk stops and scatters DD protons as demonstrated by the white ‘shadow’ in the mean fluence \(<N>\) image, but the resultant increase in fluence to the right and left of the stalk is too diffuse to be observed. However, in the mean energy \(<E>\) image, intermittent dark pixels observed on either side of the stalk indicate points of lower average energy due to the protons scattering through the edges of the stalk; areas with no incident particles are shown white in the stalk shadow. Furthermore, the slight variation of optical magnification, caused by the angle of the stalk, is only faintly observed in the fluence image. The second Geant4 simulation example presented in Figure 5-3b has implemented axisymmetric E and B fields. Differing electromagnetic optical characteristics produce features that vary substantially along the stalk in the fluence image.

In Figure 5-3c, two fluence lineouts of the second case in Figure 5-3b are shown from different positions along the stalk axis (indicated by arrows). In this simulation \( I=7 \text{ kA} \) and \( \lambda=1.3 \text{ } \mu\text{C/m} \) and spatial parameters were set as follows: \( R_B=675 \text{ } \mu\text{m} \), \( R_E=250 \text{ } \mu\text{m} \), \( W_B=350 \text{ } \mu\text{m} \), and \( W_E=100 \text{ } \mu\text{m} \). A fluence asymmetry is clearly observed due to the differing electromagnetic optics experienced by imaging protons at different locations along the stalk. However, in many cases the qualitative features labeled in Figure 5-3c may be intuitively explained by the parameters defined in the simulation.
Figure 5-4: Geant4 simulation tests for variations in modeling of proton radiographs of stalk fields. Experimental and synthetic radiographs are shown on the left with the lineout regions indicated by the boxes; these lineouts are reproduced in each of the plots in a), b), and c) for comparison. (a) Illustrates the effects of slightly modifying the energy of the protons or the magnification of the system. (b) Demonstrates that reversing the current direction drastically alters radiographic characteristics and that leaving the ratio of current to charge density constant doesn’t keep radiographs consistent. (c) Indicates the effects of altering the stalk orientation relative to the optical axis.

With the current directed away from the target, resultant B fields act to focus protons towards the stalk generating the halo, whereas the positive potential generates electric fields which deflect protons away from the stalk and produce the valley. The intensity (height) of the halo is dominantly due to the strength of the current. The location and breadth of the halo are determined primarily by the mean radius \( R_B \) and width \( W_B \) of the current annulus, respectively. Whereas, the depth of the valley is directly related to the strength of the linear charge density \( \lambda \), and its location and width determined by \( R_E \) and \( W_E \) as one would expect. The shadow is obviously caused by scattering and stopping of protons in the cold stalk material as previously discussed. In this particular simulation these effects may seem to be fairly decoupled, however, this is not the case. The precise development of the halo and valley is a result of the opposing electric and magnetic forces. The relative positions and magnitudes of these forces, as defined by the six simulation parameters, determine the characteristic features in proton radiographs in a truly coupled manner.

This model follows a reasonable physical interpretation and was chosen for its relative simplicity and ability to match the observed data. Currents and charges limited to the stalk material alone were not sufficient to explain the data. However, the uniform current
and charge distributions implemented provide an absolute measure of current and charge accumulation while capturing the important effects on proton deflections. Current and charge measurements are inferred by iterating on the simulation until reasonable agreement with the observed data (forward fitting) is reached.

The impact of modeling parameter variation is demonstrated in Figure 5-4. A sample proton radiograph from OMEGA shot 51247 is shown with the corresponding ‘best-fit’ synthetic Geant4 radiograph. Nominally, the mean proton energy is 3.3 MeV and the geometric magnification is M=16.1 at the center of the radiograph. Lineouts are indicated in the radiographs and reproduced in all of the plots. In Figure 5-4a, an increase of 300 keV in mean proton energy is shown to have a minimal impact of the resultant lineout. However, increasing the magnification by \( \sim 10\% \) broadens the halo and shifts the peak without strongly affecting the halo height. Figure 5-4b demonstrates that reversing the current (directing it toward the target) erases the halo completely by defocusing, instead of focusing, incoming protons. Furthermore, keeping the ratio of current to charge density constant, does not result in the same radiographic characteristics; the absolute values of these individual parameters are important. Finally, variations in stalk orientation are illustrated in Figure 5-4c. If the stalk were flipped 180\( ^\circ \), the magnetic optics would be similar to the reverse current scenario, but the geometric magnification now varies in the opposite direction. This results in larger magnification at the top of the image, not the bottom, and the features now flare out at the top instead of the bottom. Additionally, if the stalk were perpendicular to the optical axis, magnetic deflections cancel out and the resultant features in the radiograph are due only to electric fields.

Uncertainties in measurements are estimated based on the sensitivity of synthetic data to variations in input parameters and the variation of experimental data due to different analysis parameters (i.e. lineout width and location). Synthetic radiographs were shown in Figure 5-4 where extreme variations of these parameters were implemented, and the effects discussed. Non-axisymmetric behavior observed in experimental data also contributes some uncertainty to the inferred measurement, but those areas are not used in comparisons with synthetic radiographs. Taking these sources of error into consideration, constant error bars are conservatively estimated to be \( \pm 0.5 \) kA and \( \pm 0.3 \) \( \mu \)C/m for the current and charge density respectively. Uncertainties in spatial parameters \( R_{B/E} \) and \( W_{B/E} \) are not shown in plots, but are estimated to be \( \pm 50 \) \( \mu \)m.

5.2.2 Electrical Circuit Properties

The stalk is one component of the comprehensive electrical circuit created by the target-chamber system. It can be represented as a resistor and inductor in series as discussed in Section 5.3.3. The inductance may be calculated by setting the inductive energy equal to the energy stored in the magnetic field,

\[
\frac{1}{2} L_S I^2 = \oint \frac{B^2}{2\mu_0} dV .
\]

Solving this equation for the stalk inductance \( L_S \), one obtains

\[
\frac{L_S}{\ell} = \frac{1}{\mu_0} \oint \left( \frac{B}{T} \right)^2 r dr d\phi ,
\]
5.2. ELECTROMAGNETIC FIELDS AROUND TARGET STALKS

where $L_S/\ell$ is the stalk inductance per unit length, $I$ is the current, $B$ is the magnetic field, and $\mu_0$ is the permeability of free space. As previously discussed, the current annulus is characterized by $R_B$ and $W_B$, or equally the inner ($R_{B,i}$) and outer ($R_{B,o}$) radii. This geometry results in two contribution regions for the stalk inductance: within the annulus ($R_{B,i} < r < R_{B,o}$) and without ($r > R_{B,o}$). The magnetic field generated by this current was given in Equation 5.2 and can now be used in Equation 5.4 to calculate the inductance per unit length,

$$\frac{L_S}{\ell} = \frac{\mu_0}{2\pi} \left[ \ln \frac{R_\infty}{R_{B,o}} + \frac{1}{(1 - R_n^2)^2} \left( \frac{1}{4} - R_n^2 + \frac{1}{4} R_n^4 \right) \right],$$  \hspace{1cm} (5.5)

where $R_n \equiv R_{B,i}/R_{B,o}$ is the ratio of inner to outer radii and $R_\infty$ is the upper bound on the radial integral. Using $R_n$ as a small expansion parameter, the inductance per unit length up to $O(R_n^2)$ can be written

$$\frac{L_S}{\ell} = \frac{\mu_0}{2\pi} \left[ \ln \frac{R_\infty}{R_{B,o}} + \frac{1}{4} \left( 1 + 2 R_n^2 \right) \right].$$  \hspace{1cm} (5.6)

The log and $1/4$ terms originate from the magnetic field outside of the cylinder and are the dominant contributors to the inductance. The other terms are from the integral within the annulus and are typically small. From Equation 5.6, it is clear that the inductance per unit length diverges as $R_\infty \to \infty$ because of the axisymmetric assumption, so a realistic limit must be used. The exact value chosen is not critical, however, because it enters as the argument in the natural logarithm; the field of view size is used such that $R_\infty \approx 5$ mm. Thus, for typical parameter values, the inductance will be of order $\sim \mu$H/m.

The resistivity of the boron fibers were measured at room temperature to be $\sim 10^8$ $\Omega$/m, putting this material in the semiconductor category. To estimate the plasma resistivity ($\eta$), the Spitzer form ($\eta \propto T_e^{-3/2}$) was assumed for a coronal Boron plasma. Again, the meaningful quantity here is the resistance per unit length ($R_S/\ell$) and it is simply related to the resistivity by the current carrying geometry

$$\frac{R_S}{\ell} = \frac{\eta}{\pi R_{B,o}^2 \left( 1 - R_n^2 \right)}. $$  \hspace{1cm} (5.7)

Both the resistivity and the electron temperature of the coronal plasma will be dynamically evolving. Unlike the inductance, the resistance per unit length will drastically change throughout the plasma evolution. However, it can be useful to calculate these circuit parameters to gain insight to the complex dynamics of the system. For example, from Equations 5.6 and 5.7, a current decay time constant can be simply estimated as $\tau_I \approx L_S/R_S$. Circuit parameters were calculated for data taken in these experiments and discussed in the following sections.
5.3 Picketed-pulse Experiments

A 35 µm-thick plastic (CH) shell was irradiated by 40 OMEGA beams with a picketed laser pulse. Figure 5-5a illustrates the laser power as a function of time for the RD1501p pulse used in these experiments. An initial picket prior to the main drive has been shown to improve target stability\textsuperscript{20} and the use of multiple pickets has been demonstrated to increase\textsuperscript{21} fuel compression. The drive segment of this picketed pulse consisted of two intensity plateaus, the first at $I \approx 1 \times 10^{14}$ W/cm$^2$ followed by a stronger drive of $I \approx 4 \times 10^{14}$ W/cm$^2$.

Capsule stalks were imaged at four different times relative to the onset of the laser drive as indicated ($\bullet$) in Figure 5-5a to investigate return currents from these targets. Both fluence $<N>$ and mean energy $<E>$ images were generated from $\sim 3.3$ MeV proton radiographs and are shown for each sampled time in Figure 5-5b. In fluence images, darker pixels indicate higher fluence and in energy images, darker pixels indicate lower energy. Because electrons are leaving the target, a residual positive charge accumulates on the capsule and stalk with a return current directed away from the capsule as illustrated in Figure 5-2a. At the beginning of the main drive, the net positive charge increases and the position and strength of the return current evolves. Recall that the field of view is near the junction of the S2-S3 segments. In some images, the S3 (SiC) segment of the stalk is not visible, so the discussion is limited to the S2 (B) segment at the top (chamber-side) of the images. Similar features are observed at the bottom (target-side) of the stalk, though the diameter, material, and optical characteristics are different.

Proton fluence images reveal dynamic electric and magnetic field structure around capsule stalks over the duration of the laser pulse. The first image in Figure 5-5b at 1.1 ns shows no sign of the stalk shadow, but a fluence enhancement is observed in its place. This is caused by the skin current flowing near the stalk surface, that causes fields to focus protons to where the shadow would have been. By 1.9 ns, the valley and the halo have become well formed. However, the stalk shadow is still not fully visible and may be due to some residual current flowing near the stalk, though most current flows in the outer expanding plasma, generating the halo. At 2.4 ns, the high intensity plateau has been reached and the stalk shadow is visible. At this time, all of the current is flowing in the outer region and a well formed valley is visible. After the laser pulse has turned off, at 3.4 ns a strong positive...
charge and return current are still prevalent. In the last two radiographs, some instabilities are observed jetting out from the stalk which reveal themselves in both the fluence and energy images; these stochastic features are not modeled.

5.3.1 Comparison of Synthetic and Experimental Radiographs

To infer measurements of the six modeled parameters, simulations are iterated upon and compared with experimental radiographs until reasonable agreement is achieved. Figure 5-6 illustrates two examples from the picketed laser-pulse data set. The amplitude of features observed in fluence images are directly related to the magnitudes of the current \( I \) and charge density \( \lambda \), though the location and shape are strongly coupled to all parameters. Therefore, a unique solution (within error bars) may be found by comparing experimental and synthetic radiographs. The level of agreement demonstrated in Figure 5-6 was achieved for all stalk images individually after \( \sim 15-20 \) iterations.

In Figure 5-6, the solid lines indicate experimental data and dashed lines represent the ‘solution’ simulation for that experiment. Synthetic and experimental lineouts are normalized for comparison and shown to agree reasonably well. Misalignment between synthetic lineouts and experimental data is accounted for in the conservative measurement uncertainties previously discussed. However, in these models the actual outer diameter of the stalk at each time is unknown. Therefore, discrepancies in the width of the stalk shadow are not considered when iterating in search of a field solution; the important comparison points are the location and depth of the valley, and the location, height, and width of the halo. Using

\[
\delta \approx 50 \, \mu m
\]

Figure 5-6: Radiographic comparison using lineouts from experimental (solid) and synthetic (dashed) data. Comparisons are shown for (a) \( t = 1.1 \) ns and (b) \( t = 2.4 \) ns. The magnitudes of \( I \) and \( \lambda \) used for the simulated data shown are labeled above their respective annuli. The current-carrying annulus near the stalk in a) has a width consistent with the approximate skin depth \( \delta \) and the plasma expands and broadens later in time.
this iterative procedure and the comparison points of interest just mentioned, reasonable agreement with experimental data from each shot was achieved and measurements of the six input parameters inferred.

5.3.2 Measurements of Field Characteristics

Measurements of current dynamics are illustrated in Figure 5-7a. In this plot, bars represent the width of the annuli where current and charge were uniformly distributed, and the mean radius is illustrated by the connecting lines. These measurements demonstrate that the charge imbalance annuli stay relatively stationary throughout the sampled times, whereas the current flow is quite dynamic. Current begins near the surface of the stalk in a thin annulus, and migrates outwards in time. Though, the inner radii of the current flow remain approximately stationary from 1.9 ns and it is only the outer radii that move outwards. This is consistent with the hot expanding plasma model previously discussed. A simple calculation reveals that the outer radius initially expands at a speed of \( \sim 800 \, \mu \text{m/ns} \) and then between 1.9 and 3.4 ns, continues at \( \sim 200-250 \, \mu \text{m/ns} \). The mean radius initially moves outwards at \( \sim 650 \, \mu \text{m/ns} \) and then at \( \sim 130 \, \mu \text{m/ns} \) thereafter within the sampled times. The mean velocity estimates would be consistent with an initial supersonic expansion of the diffuse, coronal Boron plasma with a temperature of \( T_i \gtrsim 500 \, \text{eV} \). However, temperatures of this magnitude are unlikely assuming ohmic dissipation on these timescales (\( \lesssim 1 \, \text{ns} \)).

The initial stalk plasma expansion is consistent with a Coulombic explosion of singly ionized Boron ions. Figure 5-7b shows the temporal evolution of the stalk current (\( I \)) and charge density (\( \lambda \)) as inferred from proton radiographs. The return current was found to increase from \( \sim 2 \, \text{kA} \) at 1.1 ns to \( \sim 7 \, \text{kA} \) at 2.4 ns and slightly decays a few hundred ps after the laser turns off. When the fast (\( \sim \text{kA} \)) skin current flows, it is confined to a thin layer (or order \( \sim \delta \)) at the edge of the stalk as shown in Figure 5-6a. Some ionization of stalk ions is required to be consistent with observations, suggesting temperatures of \( T_i \gtrsim 10 \, \text{eV} \) due to initial ohmic heating. Furthermore, the current will preferentially flow within the expanding coronal plasma due to the reduced resistivity. Charge accumulation was shown to initially increase in time and quickly plateau near \( \sim 1.4 \, \mu \text{C/m} \) at 1.9 ns resulting in an approximate potential of \( \sim 60 \, \text{kV} \). The asymptotic velocity of Boron ions can be estimated by letting the electric potential energy equal the kinetic energy,

\[
\langle Z \rangle \phi_{\text{stalk}} \approx \frac{1}{2} m_i v_{\text{exp}}^2 ,
\]

where \( \langle Z \rangle \) is the average charge state, \( \phi_{\text{stalk}} \) is the positive potential at the stalk, \( m_i \) is the ion mass, and \( v_{\text{exp}} \) is the expansion velocity. Assuming singly ionized Boron at a potential of \( \sim 60 \, \text{kV} \), the expansion velocity is \( \sim 1000 \, \mu \text{m/ns} \). This is consistent with the initial outer plasma expansion of \( \sim 800 \, \mu \text{m/ns} \) given the model estimates and measurement uncertainties. After the initial expansion, plasma ions see a shielded potential and expand at a slower rate. The initial size of the current-carrying annulus is set by a Coulombic explosion of stalk ions due to the local potential and scales as \( \sqrt{\delta} \).

The current annuli, characterized by \( R_B \) and \( W_B \), were used with Equations 5.6 and 5.7 to calculate the inductance and resistance per unit length respectively. The results of these calculations are shown in Figure 5-7b as a function of time. To estimate resistivity, the Spitzer form (\( \eta \propto T_e^{-3/2} \)) was assumed for a singly ionized Boron plasma with \( T_e \sim 10 \, \text{eV} \). It is important to note the low effective resistivity per unit length of the plasma \( \sim 10 \, \Omega/\text{m} \), as compared to the room temperature measurements of \( \sim 10^8 \, \Omega/\text{m} \) for the Boron fiber. From
these calculations, an estimated time constant ($\tau \sim L/R$) was found to increase from $\sim 0.5$ ns to $\sim 30$ ns.\(^1\) The variability in this time is dominated by the changing resistance of the current-carrying plasma since inductance is relatively constant after the initial Coulombic expansion. The true decay time however, is related to the complex circuit system briefly discussed in Section 5.3.3.

Electromagnetic field strengths were calculated from inferred current and charge density distributions and are shown in Figure 5-7d. Peak magnetic field magnitudes were found to decrease sharply from $\sim 4$ T to $\sim 1$ T and remained approximately constant throughout the sampled times. The observed decrease is clearly attributed to the expansion of the current location despite the increase in current magnitude. Whereas peak $E$ fields increase sharply early in time and were found to be approximately constant at $\sim 80$ MV/m. These experimental data demonstrate the first measurements\(^{23}\) of dynamic current location in a ‘single-wire z-pinch’ geometry under the specified experimental conditions. Furthermore, these are the first observations of $\sim$ kA return currents from capsule implosions in the OMEGA target chamber.

\(^{1}\)An incorrect temperature estimate of 500 eV was used in the publication\(^{23}\) of this work to calculate plasma resistance and $L/R$ time constants because at the time of publication it was believed that the expansion was thermally driven. Also, a larger $R_{\infty}$ was used for inductance estimates, but previous values are still within a factor of 2 of those shown here and are consistent within the infinite cylinder assumption already made.
5.3.3 The OMEGA Target Chamber Circuit

The OMEGA target chamber and irradiated target may be portrayed by the circuit shown in Figure 5-8 as discussed by Sinenian et al. The model consists of a capacitive target at an initial peak voltage, driven by electrons leaving the system. Discharging of the capacitor occurs through two main branches: positive ablator ions streaming off the target, and return currents through the stalk support structure. Of specific interest to radiographs discussed in this chapter, is the current path along the stalk as outlined in Figure 5-8b. Solving this circuit problem is nontrivial and will not be discussed here, for detailed information regarding the circuit solution, the reader is encouraged to see the article by Sinenian et al.

The goal of this section is to briefly discuss the first measurements of return currents from laser-irradiated spherical targets in the context of the OMEGA circuit model.

Measurements shown in Figure 5-7 demonstrated the dynamic nature of the return current in the coronal stalk plasma as well as the corresponding variability in the inductance and resistance per unit length. Inductance measurements were not very sensitive to uncertainties in the location of the current since it is dominated by fields outside the current carrying region and the outer radius goes into the natural logarithm in Equation 5.6. This means that under the specified experimental conditions and model assumptions, the inductance per unit length is fairly well understood. The resistance, on the other hand, relies on an estimate of the coronal temperature and may be off by factors of a few. However, the effective stalk length changes in time and is harder to accurately define. In this model, it is the capsule voltage that is ‘fixed’ by the laser-target interaction, and this can be estimated from the stalk measurements.

A schematic of the stalk assembly is shown in Figure 5-9a to illustrate that the potential on the stalk decreases away from the target. The target potential \( \phi_{\text{target}} \) can be estimated as

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\( ^{ii} \) Direct measurements of the peak voltage using ablator-ion endpoint energy was not possible in these experiments due to contaminant ablator ions from the proton backlighter capsule.
5.3. PICKETED-PULSE EXPERIMENTS

Figure 5-9: (a) A schematic diagram of the stalk assembly with specific locations of interest called out with approximate lengths. The potential falls from the target potential at the capsule to zero at the sheath edge. (b) The target potential ($\phi_{\text{target}}$) was estimated as the sum total of the stalk potential ($\phi_{\text{stalk}}$), the drop due to inductance ($\Delta \phi_L$), and the drop due to resistance ($\Delta \phi_R$) as a function of time. Using the measured inductances and resistances from Figure 5-7b for the stalk, the predicted target potential (dashed) from the full model (courtesy of N. Sinenian) is also shown. The laser drive is plotted for reference.

The measured potential at the stalk $\phi_{\text{stalk}}$ plus the voltage drop due to the stalk inductance $\Delta \phi_L$ and resistance $\Delta \phi_R$ between the measurement point on the stalk and the target,

$$\phi_{\text{target}} \approx \phi_{\text{stalk}} + \Delta \phi_L + \Delta \phi_R,$$

where the voltage drop due to the stalk resistance is approximated by

$$\Delta \phi_R \approx I_S \left( \frac{R_S}{\ell} L_1 \right),$$

and the drop due to self-inductance of the expanding coronal plasma is estimated by

$$\Delta \phi_L \approx L_S' \frac{dI_S}{dt},$$

with the stalk inductance for a finite cylinder\(^9\) ($L_S'$) given by

$$L_S' = \frac{\mu_0}{2\pi} \ell \left[ \ln \left( \frac{\ell}{R_{B,o}} + \sqrt{\left( \frac{\ell}{R_{B,o}} \right)^2 + 1} \right) - \frac{R_{B,o}}{\ell} \left( \sqrt{\left( \frac{\ell}{R_{B,o}} \right)^2 + 1} + 1 \right) \right].$$

The infinite cylinder approximation was not used because it tends to break down as the annulus size becomes comparable to the length and plays a larger role. \(^{iii}\) Finally, the stalk

\(^9\)Equations 5.10 and 5.11 were also used in the development of the model by Sinenian et al.\(^9\) and are used here for comparison with that model.
potential at the radiograph location may be approximated by expressing the E field as $-\nabla \phi$ and using Equation 5.1 to obtain

$$\phi_{stalk} \approx \frac{\lambda}{2\pi \epsilon_0} \ln \left( \frac{R_\infty}{R_{E,o}} \right),$$

(5.13)

where $R_\infty \approx 5$ mm and $R_{E,o}$ is the outer edge of the charged cylinder and is approximately constant ($\sim 350$ μm) as shown in Figure 5-7a. Furthermore, it has been assumed that the inductance and resistance are constant from the target to the radiograph location. This assumption is not so important for the resistance, as discussed in the next paragraph, but the inductance per unit length is simply a function of the geometry of the current carrying cylinder and this will vary along the stalk at any given point in time. However, this changing cylindrical radius is in the argument of the natural logarithm reducing the affect of this assumption. It is therefore a good first approximation for the purposes of demonstrating the viability of the circuit model under these experimental conditions.

Each of the components used to estimate the target potential are shown in Figure 5-9b. The first important observation is that the voltage drop due to the stalk resistance $\Delta \phi_R$ is negligible in comparison with other quantities, thus the large uncertainty in the resistance measurements are unimportant in this context. The estimated target potential is dominated by the sum of the measured stalk potential $\phi_{stalk}$ and the voltage drop due to the stalk inductance $\Delta \phi_L$. It is noted that the last data point indicates a ‘negative’ $\Delta \phi_L$, this is due to the inductor opposing the voltage decay. This in turn reduces the current thereby changing the sign of $dI_s/dt$ and indicating that the target potential is actually lower than the stalk measurement. The target potential is shown to peak at $\sim 100$ kV in rough agreement with the scaling given by Sinenian et al.\(^5\) for target potential as a function of laser intensity. The potential is approximately constant (within errors) up to $t \sim 2.4$ ns at which time the current begins to decay as was shown in Figure 5-7b. The circuit model used for comparison was developed for 60-beam implosions, whereas in these experiments 40 beams were used. Furthermore, in these experiments the backlighter capsule was also in the chamber and was irradiated at a higher intensity than the primary plastic capsule targets. These differences in the effective circuit may account for the lingering potential observed in these data, whereas the typical rise time assumed the for 60-beam-implosion model is of order $\sim 200$ ps.\(^9\) Nonetheless, using the timing of the peak current as the starting point for the voltage decay, this model (dashed line) predicts\(^iv\) that the target potential decays shortly after the drive, in rough agreement with the estimated measurements given the differences in experimental conditions.

\(^{iv}\)The exact time that the decay begins is unknown due to the low number of data points, but must be $\gtrsim 2.4$ ns since potential estimates put to this point is roughly constant, so that time is used in this rough comparison.
5.4 Summary

The first measurements of ∼kA return currents in capsule implosions at OMEGA were made using monoenergetic proton radiography. Using the benchmarked Geant4 simulation discussed in Section 4.4, the current and potential near the stalk were inferred from ∼3.3 MeV-proton radiographs. A simple axisymmetric, uniform distribution of current and charge was used to interpret proton radiographs. This analysis inherently neglects the variance in stalk properties and any observed asymmetric behavior, such as jet-like structures at later times. Because these structures appear in both fluence and energy images, they must be due to either mass ejection or strong local electric fields. Nonetheless, the inferred current and potential profiles using this model were shown to resemble the data well for these experiments and provided good estimates of the circuit properties of the stalk. It was also shown that the current begins near the stalk surface and is consistent with an ionized layer near the predicted skin depth of the system. The current-carrying plasma was shown to expand outward due to the induced Coulomb explosion of stalk ions. These measurements allowed for the first quantification of the circuit properties of the stalk plasma during and after target irradiation.

A detailed circuit model has been developed by Sinenian et al. to predict the voltage decay on laser-irradiated targets in the OMEGA target chamber. Using the inferred measurements of circuit properties from the stalk, an estimate of the potential at the target as a function of time was calculated. The circuit model was qualitatively discussed and it was shown that it agrees reasonably well with the potential measurements discussed herein given the differences in experimental conditions. In addition to ICF-related topics, the current-carrying stalk system also resembles an exploding wire Z-pinch configuration with a low current (typical currents in wire-array Z-pinch are of order ∼1 MA).

These radiographs also demonstrated the feasibility of using proton deflectometry to map electric and magnetic field evolution in a ‘single wire’-style Z-pinch configuration. The skewed angle of the stalk allowed protons to be sensitive to both self-generated electric and magnetic fields. For currents of a few kA, ∼3.3 MeV protons provided a reasonable amount of deflection without leaving the field of view. If larger currents were present, higher energy protons would be needed to properly map the field evolution. In these experiments, return currents were measured and found to increase from ∼2 kA to ∼7 kA during a picketed laser pulse. Observations made herein motivate further investigation of dynamic current flow measurements in larger machines, such as the Z-accelerator. Furthermore, short-pulse proton radiography with Z-beamlet could provide high temporal and spatial resolution of field structure in advanced pinch configurations at Z.

5.4.1 Future Work

Results presented herein provided measurements of the dynamic location of current flow in laser-generated Z-pinch plasmas. The model discussed provided an absolute measure of current and charge accumulation while capturing the dominant effects on proton deflections. Perturbations to this distribution are expected to alter the details of the observed features, but the effect on inferred magnitudes and mean locations will be minimal due to the dependence of optical characteristics on the absolute field magnitudes. Accurate modeling of the Z-pinch plasma evolution would determine deviations from the uniform distributions currently implemented. Stalk radiographs with a larger field of view, or at multiple places along the stalk, would demonstrate the variation of plasma parameters along the stalk.
To further investigate the stalk effect on the circuit model of the system, data taken at earlier and later times relative to the laser pulse could be performed. Measurements at early times would provide more information on how the target charges up and at later times would verify the voltage decay behavior. A separate study could be done to experimentally investigate the inductance behavior as a function of laser intensity, or equivalently peak voltage. It was shown that the inductance should scale with the peak voltage as $\sqrt{\phi}$, but experimental verification would be ideal. The current data set only provides information on the inductance behavior for a peak target potential of $\sim 100$ kV, whereas the typical peak voltage in experiments with on-target intensities of $\sim 1 \times 10^{15} \text{ W/cm}^2$ are much higher $\sim 800-1200$ kV.

References


Chapter 6

RT-induced Electromagnetic Fields During the Linear Growth Phase

The Rayleigh-Taylor (RT)\textsuperscript{1,2} instability is a concern for capsule integrity in inertial confinement fusion (ICF).\textsuperscript{3} In the classic stratified fluid problem, the RT instability occurs when a high-density fluid is supported by a lower-density fluid. For small amplitude perturbations at a single wavelength ($\lambda = 2\pi/k$), the growth rate along the interface of these fluids is

$$\gamma_{RT} = \sqrt{A_t a k},$$

where $A_t$ is the Atwood number and $a$ is the acceleration.\textsuperscript{1,2} In laser-matter interactions, as seen in inertial confinement fusion, a continuous density profile is created whereby the ablating mass accelerates into the lighter, expanding plasma, forming an RT-unstable region at the ablation front.

During linear growth, perturbations on the ablation surface grow approximately exponentially ($h(t) \approx h_0 e^{\gamma_{RT} t}$) until reaching the saturation point when $h \approx \lambda/10$, thereafter growing at a slower rate.\textsuperscript{4} The ablative nature of the RT-instability in laser-produced plasmas has been predicted,\textsuperscript{5–7} and verified,\textsuperscript{8–10} to have a stabilizing effect on the linear growth rate. For an ablatively driven target $A_t \approx 1$ and the linear growth rate is:\textsuperscript{5}

$$\gamma_{RT} = \sqrt{\frac{ka}{1 + kL_\rho}} - \beta_{RT} k v_a,$$  \hspace{1cm} (6.1)

where $L_\rho$ is the density scale length, $\beta_{RT}$ is the ablative stabilization coefficient ($\beta_{RT} \approx 3$ for direct-drive)\textsuperscript{11} and $v_a$ is the ablation velocity. The ablative, linear growth rate illustrates the stabilization provided by this process, such that perturbations smaller than $\lambda \approx 2\pi \beta_{RT} V_a^2/a$ ($\sim 10$ \textmu m for typical parameters) are linearly stable. However, the fluids involved with an ablatively driven target are not charge-neutral, but are plasmas consisting of separate populations of ions and electrons.

During the ablation process, dynamic charge separation and subsequent current generation will create magnetic fields within the plasma.\textsuperscript{12,13} By comparing magnetic energy density with energy in fluid vorticity, Evans’\textsuperscript{14} formulation demonstrated that for an RT-unstable CH plasma, growth rates of wavelengths less than $\sim 5$ \textmu m would be affected by self-generated magnetic fields. In laser-ablation systems, the growth rate given in Equation 6.1 shows that small wavelengths, which may be affected by magnetic fields, are ablately stabilized. Even though magnetic fields may not play an important role during linear growth of relevant wavelengths, they may potentially affect energy transport from the under-dense plasma to the ablation surface.
Figure 6-1: Schematic drawing of laser-foil interactions with important regions labeled. The ablation front is unstable to RT growth because of the acceleration in the opposite direction of the density gradient.

To drive a target through the ablation process, energy must be efficiently deposited to the ablation surface. Energy provided by thermal electrons is conducted through the overdense region to the ablation surface as illustrated in Figure 6-1. Acceleration of ablated material into the overdense plasma generates an RT-unstable region because of the large, acceleration-opposing density gradient. Surface perturbations on the target will grow because of this instability and induce magnetic fields. Electron thermal conduction across a magnetic field ($\kappa_\perp$) is reduced from the classical value ($\kappa_\parallel$),

$$\frac{\kappa_\perp}{\kappa_\parallel} = \frac{\gamma'_1 \chi^2 + \gamma'_0}{\gamma_0 (\chi^4 + \delta_1 \chi^2 + \delta_0)} ,$$

where the coefficients $\gamma'_1, \gamma'_0, \gamma_0, \delta_1, \delta_0$ are given by Braginskii$^{15}$ and were given in Table 3.1 as a function of Z. The Hall parameter $\chi$ is a quantity describing the characteristic number of collisions undergone by a thermal electron while gyrating about a magnetic-field line and can be expressed as $\chi = \omega_{ce} \tau_{ei}$, where $\omega_{ce}$ and $\tau_{ei}$ are the electron cyclotron frequency and characteristic collision time, respectively. The Hall parameter can be expressed in relevant units by,

$$\chi \approx 20 \frac{B T_e^{3/2}}{Z n_e \ln \Lambda} ,$$

where $B$ is in Tesla, the electron temperature $T_e$ is in keV, density $n_e$ is in $10^{20}$ cm$^{-3}$, $Z$ is the average ionization state, and $\ln \Lambda$ is the Coulomb logarithm. The Hall parameter characterizes the reduction in thermal conduction due to magnetic fields. A Hall parameter value as small as $\sim 0.3$ will reduce thermal conduction to $\sim 40$ % of the classical value in a CH(1:1.38) plasma.

RT-unstable plasma configurations occur in many systems: in laser-matter interactions,$^{10}$ during the acceleration and deceleration phases in inertial confinement fusion,$^{16}$ during core-collapse of supernovae,$^{17}$ in stellar coronae,$^{18}$ and in other astrophysical phenomena.$^{19}$ The so-called Biermann battery$^{20}$ is the dominant source of self-generated magnetic fields in plasmas. This source term has been predicted to cause field generation due
to the RT instability in astrophysical contexts\textsuperscript{21} as well as in laser-plasma interactions.\textsuperscript{22} Mima \textit{et al.}\textsuperscript{23} and Nishiguchi \textit{et al.}\textsuperscript{24} investigated different models and environments for magnetic field generation, but both predicted peak field strengths on the order of $\sim$10-100 T. Fields of this magnitude near the critical surface in directly-driven ICF capsules can drastically affect electron thermal conduction and inhibit effective ablative drive.

This chapter covers results obtained from the experimental investigation into the linear regime of laser-driven RT. Foils with $\lambda \approx$120 $\mu$m wavelength surface perturbations were used in both x-ray and proton radiography experimental configurations discussed in Section 6.1. Hydrodynamic plasma evolution using the DRACO code is demonstrated in Section 6.2 and post-processed simulations discussed in Section 6.3 illustrate the predicted RT-induced field structures. Measurements and analysis of early-time, linear growth are presented in Section 6.5 and implications discussed in Section 6.6. Finally, the chapter concludes in Section 6.7 with a summary of the results of this work and recommendations for future directions of research in related topics.

Table 6.1: List of Braginskii coefficients used in Equation 6.2 for inhibition of electron thermal condition. Approximate values were inferred from Table 3.1 for a fully ionized CH(1:1.38) plasma with a mean ionization state of $Z \approx 3.1$.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>6.15</td>
</tr>
<tr>
<td>$\gamma'$</td>
<td>3.46</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.564</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>9.56</td>
</tr>
</tbody>
</table>
6.1 Experimental Investigation of RT-induced EM Fields

Both proton and x-ray radiography experiments were performed on the OMEGA laser using the setups shown in Figure 6-2a-c and Figure 6-2d-f, respectively. Imaging protons are sensitive to both areal density and electromagnetic fields such that fluence modulations in proton radiographs are due to a combination of these effects, as illustrated in Figure 6-2b. The primary goal of these experiments was to relate proton fluence modulations to path-integrated field strengths. Independent measurements of areal density modulations were made using well established x-ray radiographic techniques.

X-ray radiographs provided measurements of density-modulation growth in the target. Face-on images were obtained using ~1.3 keV x-rays from a uranium backlighter and a streak camera having a temporal resolution of ~80 ps and a spatial resolution of ~10 μm. Streaked images were recorded on Kodak T-Max 3200 film and digitized using a Perkins-Elmer PDS microdensitometer. The optical depth (OD) measured by scanning the film may be converted to an areal density by:

$$\rho L = \frac{OD}{\mu_U},$$

where the conversion factor for uranium and the equipment used has been calculated as $\mu_U \approx 0.95 \text{ cm}^2/\text{mg}$, such that $\rho L$ is in mg/cm$^2$. These measurements provide empirical data on the growth rate of areal density modulations in these laser-irradiated foils.
Monoenergetic proton radiography\textsuperscript{28,29} was used to probe RT-induced field structures in these experiments. A \( \sim 2 \) \( \mu \)m-thin-glass, explosive-pusher filled with 18 atm of equimolar \( \text{D}_3\text{He} \) gas was imploded by up to 20 OMEGA laser beams. This backlighting technique provides a quasi-isotropic, monoenergetic (\( \sim 15 \) MeV) proton source with an approximately Gaussian emission profile with a FWHM of \( \sim 45 \) \( \mu \)m and burn duration of \( \sim 150 \) ps, as demonstrated in many experiments.\textsuperscript{30–33} Imaging protons were incident on 10 cm \( \times \) 10 cm CR-39 detectors that were filter-matched to range \( \sim 15 \) MeV protons down to \( \sim 4 \) MeV, where CR-39 has 100\% detection efficiency.\textsuperscript{34} After exposure, the CR-39 was processed in \( 80^\circ \text{C} \) 6N NaOH solution to reveal tracks left by the protons. Each piece of CR-39 was scanned using a digital optical-microscope system and individual track locations and characteristics were retained by the system for analysis.

Foil surfaces were either flat or seeded with ridge-like 2-D sinusoidal modulations. The exact laser configuration was not constant across all experiments, however drive characteristics were nominally equal. The laser drive in all cases was a 2 ns square pulse with a total of \( \sim 3300 \) J of energy on-target. All drive beams implemented SG4 distributed phase plates (DPPs)\textsuperscript{35} to provide a \( \sim 750 \) \( \mu \)m diameter spot and a drive intensity of \( I \lesssim 4 \times 10^{14} \) W/cm\textsuperscript{2}. The beams used smoothing by spectral dispersion (SSD)\textsuperscript{36} and distributed polarization rotators (DPRs)\textsuperscript{37} to reduce the speckle in laser intensity. In proton radiography experiments, CH foils and CR-39 detectors were located \( \sim 1 \) cm and \( \sim 30 \) cm from the backlighter, respectively, providing a magnification of M\( \sim 30 \). The strength of path-integrated mass and fields, as illustrated in Figure 6-2c, in conjunction with the optical geometry determines the amount of proton deflection.

The quasi-uniform\textsuperscript{38} flux of \( \sim 15 \) MeV protons provided by the backlighter is perturbed through inhomogeneous mass distributions and electromagnetic fields in the plasma. Modulations in proton flux are caused by deflections perpendicular to proton trajectories. The amount of deflection undergone by a particle caused by B or E fields is proportional to the path-integrated field strength, \( \theta_B \propto B_\perp L_B \) and \( \theta_E \propto E_\perp L_E \), where \( B_\perp \) and \( E_\perp \) are the magnetic and electric field magnitudes perpendicular to the particle trajectory, respectively, with corresponding length scales \( L_B \) and \( L_E \). Information regarding path-integrated field strength is encoded within proton fluence modulations. RT-induced modulations cause local broadening of the proton fluence due to Coulomb scattering and the Lorentz force, as illustrated in Figure 6-2b. The total fluence modulation is due to a combination of perturbing effects from both field deflections and Coulomb scattering. The effects of scattering are accounted for through Monte Carlo modeling of Coulomb interactions using measured areal density modulations from x ray data.
6.2 Modeling Plasma Evolution

The radiation-hydrodynamic code DRACO\textsuperscript{39,40} was used to model laser-foil interactions in these experiments. Calculations were done to self-consistently evolve the foil hydrodynamics, though no electric or magnetic fields were included. The no-field approximation is typically sufficient for predicting the hydrodynamics in these types of plasmas\textsuperscript{10,41} due to the high ratio of plasma pressure to magnetic pressure ($\beta = 2\mu_0 p / B^2 \gtrsim 10^4$ under typical conditions).

6.2.1 Physics Implemented in DRACO

DRACO was run using a 2-D cylindrical geometry, assuming azimuthal symmetry and open boundary conditions. These calculations were done after the experiments and implemented the incident angles and energies for individual beams in a super-gaussian beam spot for the fielded SG4 DPPs. Beams were incident onto a 21-μm-thick CH foil with sinusoidal perturbations of wavelength 120 μm and initial amplitude of 0.27 μm. Beams were azimuthally symmetric and they irradiated the CH foil on axis. To simulate the flux-limited heat flow in these calculations, the harmonic mean of the classical and flux-limited heat conduction was used with a constant flux limiter of $f = 0.06$.

The hydrodynamic calculations were performed using a two-fluid model, consisting of the electron fluid and a single population of ions. For multi-species materials, such as CH, the single ion fluid is composed of particles with the mean mass and charge based on the atomic composition of the material. An equation-of-state (EOS) is needed to relate the two state variables ($T$ and $\rho$) to a third, typically the internal energy or fluid pressure. The SESAME\textsuperscript{42–45} tables are used in DRACO to provide the EOS for both electrons an ions. Thermal electronic contributions are tabulated using finite temperature Thomas-Fermi-Dirac (TFD) theory. At solid densities, the ionic contribution is treated with Debye theory and analytic fits of available data, with a smooth transition to an ideal gas at high densities.

![SESAME Pressure Contours](image)

Figure 6-3: SESAME tables for the equation-of-state in CH for (a) ionic pressure, (b) electronic pressure, and (c) total pressure. A sample proton trajectory (thick solid) as it passes through the RT foil is shown for reference. Pressure contours are shown from 10 kbar to 1 Gbar in temperature-density space. The solid lines are the values from the table and the dotted lines are the equivalent ideal EOS $p \propto \rho T$. Deviations from the ideal case become more prominent at lower temperatures and higher densities.
temperatures. At high densities, an interpolation is made to the TFD theory. The interested reader is encouraged to visit the SESAME website for more detailed information. These tabulated values are used to quickly reference an EOS for a specific fluid in DRACO without necessitating more calculations during the hydrodynamic computation.

Pressure contours are plotted in $T$-$\rho$ space for the CH EOS from the SESAME tables in Figure 6-3. The ion and electron contributions are shown in Figure 6-3a and b, respectively, with the total pressure shown in Figure 6-3c. Contours from tabulated data (solid) are plotted for pressures between 10 kbar and 1 Gbar with the dotted lines representing the ideal gas law, $p \propto \rho T$. A proton trajectory\(^1\) (thick solid) through the simulated CH plasma at $t = 1.5$ ns is shown for reference. It is clear that the ion fluid is well approximated by an ideal gas along the trajectory through the sample plasma. The electron fluid however, deviates from the ideal EOS at lower temperatures. DRACO simulations using the aforementioned physics implementations have previously been shown\(^10\) to reproduce drive conditions well at intensities below $\sim 5 \times 10^{14}$ W/cm\(^2\) for CH foils.

Table 6.2: This table provides a top-level overview of the physics implemented in the DRACO simulations used in this work.

<table>
<thead>
<tr>
<th>Physics</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation</td>
<td>emission and absorption</td>
</tr>
<tr>
<td>Hydrodynamics</td>
<td>two fluids, two temperatures:</td>
</tr>
<tr>
<td></td>
<td>one thermalized ion fluid</td>
</tr>
<tr>
<td></td>
<td>one thermalized electron fluid</td>
</tr>
<tr>
<td>Equation-of-State</td>
<td>ions: SESAME tables</td>
</tr>
<tr>
<td></td>
<td>electrons: SESAME tables</td>
</tr>
<tr>
<td>Heat flow</td>
<td>harmonic mean of classical and limited</td>
</tr>
<tr>
<td></td>
<td>$(f = 0.06)$</td>
</tr>
<tr>
<td>Electromagnetic Fields</td>
<td>NO implementation</td>
</tr>
</tbody>
</table>

6.2.2 Simulated Hydrodynamic Results

Predicted hydrodynamic results are shown in Figure 6-4 for three sample times during the 2 ns laser pulse. One-dimensional quantities were obtained by averaging over 120 $\mu$m (a single wavelength) in radius and plotted as a function of distance on-axis and illustrated in the left column of Figure 6-4. In these plots, the lasers were incident from the right and the ablation (Abl.), critical (Crit.), and quarter critical (Quart. Crit.) surfaces are labeled for reference. The bulk of the foil is clearly shown by the density-peak on the left side of each plot and is observed to move towards the left. The maximum density was calculated to be $\sim 2.5$ g/cm\(^3\) indicating a $\sim 2.5$ compression factor. Additionally, an approximately constant mass ablation rate was calculated as $\dot{m}_a \approx 4 \times 10^5$ g/cm\(^2\)/s, corresponding to an ablation velocity of $v_a \sim 1.5$ $\mu$m/ns. In the reference frame of the ablation front, the acceleration is

\(^1\)The same trajectory as discussed in Section 4.4.1
directed toward the right and the density gradient is towards the left, generating an RT unstable region.

Two-dimensional contours of electron density (solid) and temperature (long dash) are plotted in the column on the right of Figure 6-4 corresponding to the three sample times. Peak number density contours were set to $2.5 \times 10^{23}$ cm$^{-3}$ ($\sim 0.8$ g/cm$^3$) and are highlighted by thicker solid (orange) lines in each plot. Number density contours decrease by increments of $8 \times 10^{22}$ cm$^{-3}$, such that the short-dashed line on the far right within each plot is at $10^{22}$ cm$^{-3}$ (approximately the critical density). Electron temperature contours are labeled in increments of 200 eV and shown to sharply increase from the ablation front to the critical surface. Additionally, Rayleigh-Taylor growth of the sinusoidal surface perturbation is clearly illustrated by the growing amplitude of the peak density contours.
Figure 6-4: Predictions from the DRACO simulation for three sample times. In the left column, fluid density (short dash), electron number density (solid), and electron temperature (long dash) were averaged in ‘radius’ to provide 1-D profiles along the axis (lasers come in from the right). In the right column, corresponding 2-D number density contours (solid) are shown with electron temperature contours (long dash) up to the critical surfaces at each time: (a) 1.1 ns, (b) 1.3 ns, (c) 1.5 ns after drive laser onset.
6.3 RT-induced Electromagnetic Fields

Self-generated, electromagnetic fields have been observed\textsuperscript{12,13,31,46} in many laser-produced plasmas. Electric-field generation primarily occurs in response to gradients in the electron pressure, whereas the dominant source of self-generated magnetic fields is related to perpendicular gradients in the electron temperature and density. Figure 6-4a-c demonstrates that such non-collinear temperature and density gradients occur in laser-ablated targets due to Rayleigh-Taylor growth. Section 3.4 covered the general mechanisms involved in self-generated electromagnetic fields in plasmas. In this section, a brief review of the magnetohydrodynamic (MHD) equations is given and concludes with post-processed DRACO simulations demonstrating the predicted RT-induced field structures.

The basic assumptions involved in this model are as follows: electron inertia is unimportant on the hydrodynamic time scales of interest (\(m_e \rightarrow 0\)) and it is recognized that viscosity is dominated by ion motion and is ignored in the electron equations. This results in the basic formulation presented by Braginskii\textsuperscript{15} for the electric field,

\[
E \approx -\nabla p_e e_0 n_e - V_e \times B + R_e e_0 n_e ,
\] (6.5)

where \(p_e, n_e,\) and \(e_0\) are the electron pressure, number density, and charge respectively. Collisional effects are contained within \(R_e\) and the bulk electron fluid velocity is \(V_e\). Next, using Equation 6.5 and Faraday’s Law, the equation governing magnetic field evolution may be written

\[
\frac{\partial B}{\partial t} \approx \nabla \times \left( \frac{\nabla p_e}{e_0 n_e} \right) + \nabla \times (V_i \times B) + \frac{\eta}{\mu_0} \nabla^2 B ,
\] (6.6)

where the electron fluid velocity has been replaced by the ion fluid velocity \(V_i\) and current density \(j = e_0 n_e (V_i - V_e)\). It is noted that the collisionless Hall term is second order in \(B\) and has therefore been neglected. Next, for these calculations, the plasma is assumed collisionless forming the Hall MHD equations for field generation,

\[
E \approx -\nabla p_e e_0 n_e - V_e \times B ,
\] (6.7)

\[
B \approx -\frac{m_i}{e_0 (Z + 1)} \xi ,
\] (6.8)

where \(\xi = \nabla \times V_i\) is the fluid vorticity, \(m_i\) is the ion mass, \(Z\) is the effective charge state, and electrons and ions were assumed to be in thermal equilibrium. It is clear from Equation 6.7 that electric fields are mainly generated in response to the electron pressure gradient\textsuperscript{12,13,15} with an additional component due to the collisionless Hall effect. Whereas, Equation 6.8 illustrates that magnetic fields are simply proportional to the fluid vorticity in the Hall MHD limit. However, the neglection of collisional terms may not be warranted in all cases and will be discussed in Section 6.6 as it pertains to these experiments.

Predicted hydrodynamic results from DRACO were post-processed using Equations 6.7 and 6.8 to calculate magnetic and electric field structure under these experimental conditions. Figure 6-5a-c illustrates the Hall MHD electromagnetic field structures generated by the plasma for the same sample times as shown in Figure 6-4. During linear growth, sinusoidal surface perturbations lead to sinusoidal fields, as expected. Furthermore, these...
Figure 6-5: B and E field contour plots calculated from hydrodynamic DRACO simulations. Contour levels are identified at the top of each plot, where negative contours are dotted, positive contours are long-dashed, and the zero contour is a solid line. Peak electron density contours (thick solid) from Figure 6-4 are also shown with the dashed line on the right indicating the critical surface at each time: (a) 1.1 ns, (b) 1.3 ns, and (c) 1.5 ns.

calculations indicate that fields are generated in a narrow space near the ablation surface, then grow and expand toward the critical surface in time. This work demonstrates a technique to measure the sinusoidal fields caused by the Biermann-battery source generated during linear RT-growth.
Figure 6-6: (a) Lineout of a synthetic image. The modulation wave vector was angled at \( \theta = 120^\circ \) as indicated in the radiograph. (b) Measured rms amplitude modulation at the fundamental frequency as a function of lineout angle for both Raw (dashed) and Filtered (solid) data. (c) Power spectra of the lineout taken at \( \theta = 120^\circ \) for both Raw (dashed) and Filtered (solid) data. The Wiener filter (dotted) used is shown for reference.

### 6.4 Fourier Analysis Technique

The features of interest in this work are linear perturbations to the proton fluence on a scale length near the seeded perturbation wavelength on the foil. A Fourier treatment is therefore a natural method to analyze data. Proton radiographs are produced from digital scans of the CR-39. In this form, each pixel of the image has a value corresponding to the number of protons incident per unit area, i.e., proton fluence. X-ray radiographs are made from digital scans of the exposed film where each pixel value corresponds to the optical depth measured. Lineouts are taken to quantitatively analyze amplitude modulations in proton fluence and optical depth. The analysis of a synthetic image of 120 \( \mu \)m wavelength modulations is illustrated in Figure 6-6.

The synthetic image in Figure 6-6a was generated with a sinusoidal amplitude of 0.05 with a mean of 1. White noise with an amplitude of \( \pm 0.5 \) was added to illustrate an image with a 0.1 signal-to-background ratio. The lineout along the wave vector, corresponding to an angle of \( \theta = 120^\circ \), is shown. Amplitude modulation measurements \( (\alpha_{\text{rms}}) \) were made from image lineouts at 10° increments from 0° to 180°, as illustrated in Figure 6-6b. Modulations are shown to flatten as the lineout orientation becomes perpendicular to the wave vector, as expected since relevant features are averaged out. For this reason, a clear peak in amplitude modulation was observed at 120° in Figure 6-6b.

A discrete Fourier transform (DFT) of each lineout provides the power density spectrum. A sample spectrum from the lineout illustrated in Figure 6-6a is shown in Figure 6-6c. The frequency of interest is the fundamental frequency \( (1/\lambda) \) as derived from the known perturbation wavelength. The amplitude modulation \( (\alpha_f) \) at a spatial frequency \( f \) is proportional to the square root of the power density \( (P_f) \) at that frequency, \[ \alpha_f \propto \sqrt{P_f}. \] To optimize the accuracy of the spectral power, a Hann-windowing function is utilized in the DFT to avoid power leakage and the Nyquist frequency is set such that the fundamental frequency is centered on a DFT bin, as illustrated in Figure 6-6c. To compare different radiographs, the normalized root mean square (rms) amplitude modulation \( \alpha_{\text{rms}} \) is defined relative to the background at zero-frequency \( P_0 \) (DC offset) as \[ \alpha_{\text{rms}} \equiv \sqrt{P_f/P_0}. \] This metric quantifies...
the rms of a sinusoid at frequency $f$ relative to the mean and is plotted in Figure 6-6b for the synthetic image.

A range of angles near perpendicular to the wavevector is deduced from the ‘Raw’ amplitude modulation ($\times$) measurements in Figure 6-6b and used to calculate an average noise spectrum. In this example, lineouts at angles from $0^\circ$ to $30^\circ$ were averaged to generate a Weiner-filter, or estimated noise spectrum, and is shown (dotted) in Figure 6-6c. The sample spectrum shown for $\theta=120^\circ$ clearly demonstrates that the power at the fundamental frequency is well above the noise, even for a 0.1 signal-to-noise ratio.ii This filter was applied\(^{47}\) to power spectra at all angles and the corresponding ‘Filtered’ amplitude modulation was calculated and is shown (o) in Figure 6-6b. The implemented rms modulation for this synthetic image was 0.035 and the filtered measurements indicate an rms amplitude of $\alpha_{\text{rms}}=0.035\pm0.006$ at the correct wave vector angle of $120^\circ$. Errors in amplitude modulation measurements are primarily due to statistical variation in the image. When calculating a lineout, as seen in Figure 6-6a, pixels perpendicular to the lineout direction are averaged. The standard deviation of the mean pixel value is the uncertainty at each point along the lineout. These uncertainties are propagated through the DFT in the manner described by Fornis-Marquina et al.,\(^{48}\) resulting in an uncertainty $\Delta$ in the $\alpha_{\text{rms}}$ measurement due to statistical variation. If the lineout is wide, this error can be quite small and does not capture the true uncertainty in the $\alpha_{\text{rms}}$ analysis.

Amplitude modulation measurements are calculated from a number ($S$) of thinner, sections within the overall lineout envelope. A single $\alpha_{\text{rms},i}$ is calculated for each section with an uncertainty $\Delta_i$ and the total $\Delta \alpha_{\text{rms}}$ value is obtained as the weighted average

$$\alpha_{\text{rms}} = \frac{\sum_i S \alpha_{\text{rms},i}}{\sum_i S \frac{1}{\Delta_i}},$$

(6.9)

with a weighted statistical uncertainty $\Delta_N$ given by

$$\Delta_N = \frac{S}{\sum_i S \frac{1}{\Delta_i}}.\quad (6.10)$$

The standard deviation of the mean is calculated from the lineout sections and characterizes the variation in the DFT across the lineout, $\Delta_{\text{DFT}}$. Both uncertainties are added in quadrature and represent the total error, $\Delta \alpha_{\text{rms}} = \sqrt{\Delta_{\text{DFT}}^2 + \Delta_N^2}$, in the measurement of a single lineout. This procedure is used on every lineout and the uncertainties in the sample case are illustrated by the error bars in Figure 6-6b at each angle.

\(^{ii}\)raw powers below the noise are set to zero and not shown on the log scale.
Figure 6-7: (a) Sample x-ray radiographs at three times relative to the 2 ns laser drive; scale size is given in the target plane and the lineout direction is indicated. (b) Inferred areal density lineouts from the radiographs shown in a). The nominal value for 0 ns is shown for comparison. (c) Measured rms areal densities (●) from x-ray radiographs and predicted values (solid) from DRACO. (d) Sample proton fluence radiographs at similar times as x ray images in a); scale size is given in the target plane and lineout direction is indicated. Flat-foil radiographs are shown for comparison. (e) Corresponding lineouts for modulated-foil radiographs in d) are normalized for comparison across different shots. (f) Measured rms fluence variations (▲) in proton radiographs. Expected rms variation due to mass only (●) was calculated using density distributions from x-ray data.

6.5 Experimental Results

A summary of x-ray and proton radiographic results are shown in Figure 6-7. Sample x-ray radiographs are shown in Figure 6-7a at 1.2, 1.3, and 1.5 ns after the onset of the 2 ns laser drive. X-ray radiographs provided areal density modulations and were used with the mass ablation rate of $\dot{m}_a \approx 4 \times 10^5$ g/cm$^2$/s to generate the lineouts shown in Figure 6-7b that illustrate mass ablation and perturbation growth. The rms areal densities ($\langle \rho L \rangle_{\text{rms}}$) were calculated from these data and plotted in Figure 6-7c. Predicted areal density modulations from DRACO were benchmarked with x ray radiographs as shown by the solid line in Figure 6-7c. An exponential fit to $\langle \rho L \rangle_{\text{rms}}$ measurements, independent of simulations, indicate linear growth up to $t \sim 1.5$ ns with a growth rate of $\gamma \approx 2.2$ ns$^{-1}$.

Proton radiographs of modulated foils were taken over the course of three different shot days, providing data at multiple times during plasma evolution. Figure 6-7d shows sample proton fluence radiographs corresponding to similar times as sample x-ray radiographs shown in Figure 6-7a. The visible ring structure in all images illustrates the edge of the laser spot. Within the laser spot irradiation is uniform and steady RT growth is expected; it is

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iiiX ray radiographs were from OMEGA shot 50870.
ivSample proton radiographs were from OMEGA shots 49109 (1.2 ns), 61721 (1.3 ns), 49111 (1.5 ns) for modulated foils and 50610 (1.2 ns), 50610 (1.4 ns) for flat foils.
6.5. EXPERIMENTAL RESULTS

AmpMod Models

0.01
0.02
0.04
0.06
0.08
0.10
α

≈ 1.4 × 10^{-4} e^{3.19t}

0.02
0.04
0.06
0.08
0.10
α

B/E

≈ 7.4 <θ>_{rms}

Time (t) [ns]

(a)

0.00
1 1.2 1.4 1.6

<θ>_{rms} [deg]

(b)

Figure 6-8: (a) Amplitude modulation due to x-ray-measured areal-density modulations as a function of time (•) with an exponential fit. (b) Modeled amplitude modulation caused by sinusoidal deflection angles at the target (•) with a linear fit.

this inner region that is used for analysis. During the times sampled, coherent linear features were observed in modulated-foil proton radiographs. However, radiographs of flat foils at similar times reveal stochastic characteristics in comparison. Lineouts from modulated-foil radiographs were normalized for comparison across different experiments and shown in Figure 6-7e.

All radiographs were analyzed using the DFT technique discussed in Section 6.4. Resultant amplitude modulation in proton fluence was found to grow during the 2 ns drive and is shown (▲) in Figure 6-7f. It is expected that proton fluence modulations at the fundamental frequency will grow in time due simply to the increase in Coulomb scattering experienced by protons traversing RT spikes. This contribution to \( \alpha_{\text{rms}} \) was calculated by implementing areal density distributions derived from x-ray radiographs into the Geant4 simulation discussed in Section 4.4. Expected proton fluence amplitude modulation due to mass alone was shown (•) in Figure 6-7f to be 3-5 times less than measured values.

The total amplitude modulation observed in proton fluence is due to a combination of perturbing effects from both field deflections (\( \alpha_{B/E} \)) and Coulomb scattering (\( \alpha_{\text{mass}} \)),

\[
\alpha_{\text{rms}}^2 = \alpha_{B/E}^2 + \alpha_{\text{mass}}^2.
\]

The Coulomb scattering component was assessed from the x-ray inferred areal density modulations and shown to be small in comparison to the measured values. An exponential fit to calculated \( \alpha_{\text{mass}} \) values is shown in Figure 6-8a. Proton deflections due to B or E fields can be expressed by

\[
\theta_B = \frac{q}{\sqrt{2 m_p E_p}} \int B \perp dl, \quad (6.11)
\]

\[
\theta_E = \frac{q}{2 E_p} \int E \perp dl, \quad (6.12)
\]

where \( q \) is the proton charge, \( m_p \) the mass, and \( E_p \) the energy.

The small contribution to \( \alpha_{\text{rms}} \) due to scattering was removed, and the residual was attributed to deflections due to RT-induced magnetic and/or electric fields,

\[
\alpha_{B/E} = \sqrt{\alpha_{\text{rms}}^2 - \alpha_{\text{mass}}^2}. \quad (6.13)
\]
Figure 6-9: Inferred path-integrated quantities (▲) are calculated from measured \( \alpha_{\text{rms}} \) values if deflections are caused by (a) B-fields and mass, or (b) E-fields and mass. Simulated B fields indicate an approximate upper estimate and are a factor of \( \sim 2 \) higher than inferred values, whereas simulated E fields are a factor of \( \sim 100 \) too low to account for measured proton fluence modulations. (c) Estimated B field amplitudes (▲) inferred from path-integrated measurements. The field-structure scale-length is the perturbation height as determined by the experimentally determined growth rate and initial foil conditions. B-field amplitudes predicted by the ideal MHD model are shown (solid) for comparison. The predicted plasma \( \beta \) is also shown using the ablation parameters \( n_e \sim 9 \times 10^{22} \text{ cm}^{-3} \) and \( T_e \sim 300 \text{ eV} \).

During linear growth, a sinusoidally varying field structure develops, as discussed in Section 6.3 and demonstrated in Figure 6-5, which causes sinusoidal deflections. Expected \( \alpha_{B/E} \) values were found to be linearly proportional to the rms deflection angle \( \langle \alpha \rangle_{\text{rms}} \propto \langle \theta \rangle_{\text{rms}} \) as shown in Figure 6-8b. Protons were modeled in the experimental geometry with sinusoidal deflections occurring at the target,

\[
\theta = \langle \theta \rangle_{\text{rms}} \sqrt{2} \sin(k_y y),
\]

where \( \langle \theta \rangle_{\text{rms}} \) is directly proportional to the rms path-integrated field strength for monoenergetic protons. In this way, rms magnetic fields (\( \langle BL \rangle_{\text{rms}} \)) or electric fields (\( \langle EL \rangle_{\text{rms}} \)) were inferred from proton fluence modulation measurements. Residual amplitude modulations were attributed to only magnetic fields (Figure 6-9a) or only electric fields (Figure 6-9b).

Hall MHD calculations were performed to determine whether B or E fields dominated proton deflections at the target. Hydrodynamic calculations from DRACO were post-processed to compute B and E fields as shown in Figure 6-5. Fields were path-integrated from the peak density to the critical surface and the rms value calculated at each time step. The results of these calculations were compared with the experimentally determined values as shown by the solid lines in Figure 6-9a-b. Even in this ideal limit, E fields are \( \sim 100 \) times too small to account for the observed proton deflections. B fields, on the other hand, were predicted to be a factor of \( \sim 2 \) too high to explain the observations. Therefore, it was magnetic, not electric, fields that were responsible for the large fluence modulations observed in proton radiographs.
6.6 Discussion

Measurements deduced from proton radiographs are inherently path-integrated quantities. Because of the complexity of the environments they are traversing, inversion techniques are difficult, if not impossible, to apply. Therefore, in order to estimate magnetic field strengths from path-integrated measurements shown in Figure 6-9a, some knowledge of the scale size of these field structures is needed.

The natural scale size of RT-induced fields is the perturbation height. This claim is verified by the contour plots shown in Figure 6-5a-c. From these calculations, it is clearly observed that the spatial extent of the fields grow in time along with the perturbation due to RT growth. The highest contours (field strengths) are found near the ablation surface and are comparable in size to the peak-to-valley (P-V) perturbation height. Using the initial P-V height ($h_0 \approx 0.54 \mu m$) and the experimentally determined growth rate ($\gamma_{RT} \approx 2.2 \text{ ns}^{-1}$), the perturbation height as a function of time can be estimated as $h(t) \approx h_0 e^{\gamma_{RT}t}$. Subsequently, the magnetic field amplitude may be estimated from the path-integrated measurements by the following relation, $B_{max} \approx \sqrt{2\langle BL\rangle_{rms}}/h$.

Resultant B-field amplitude estimates are illustrated (▲) in Figure 6-9c along with predictions (solid line) from the Hall MHD model. B field contour locations plotted in Figure 6-5 indicate that, at these times, B fields occupy the dense ($n_e \sim 9 \times 10^{22} \text{ cm}^{-3}$), cold ($T_e \sim 300 \text{ eV}$) region near the ablation front. Using these plasma conditions and the B-field estimates from Figure 6-9c in Equation 6.2, it was found that thermal conduction will be ‘reduced’ to $\sim 99.7\%$ of its free-streaming value; a negligible effect. Moreover, under these conditions, the plasma $\beta$ is shown in Figure 6-9 to be $\gtrsim 10^4$ during the observation times, validating the no-field assumption implemented in the hydrodynamic simulations. If the scale-size assumed was too large, and the B field amplitudes were actually higher (within factors of a few) than estimated, the effect on thermal conduction in these plasma conditions is still minimal due to the high collision frequency.

These measurements indicate a negligible effect on electron thermal conduction due to B fields during linear growth under the specified target and laser conditions. However, it is clear from the ideal calculations illustrated in Figure 6-5 that the B field structure grows in time and begins to extend toward the critical surface. Furthermore, as RT growth continues into the non-linear regime, spikes will ‘fall’ closer to the critical surface generating fields further away from the ablation front. Plasma conditions near the critical surface ($n_{cr} \sim 10^{22} \text{ cm}^{-3}$ and $T_e \sim 800 \text{ eV}$) are different than those at the ablation front and in this environment a B field of $\sim 10 \text{ T}$ can reduce thermal conduction to $\sim 80\%$ of its free-streaming value. It must be noted that B-field calculations presented herein neglected the Nernst and diffusion effects due to computational constraints and may alter the field dynamics.

The Nernst velocity ($V_{\text{Nernst}}$) is a collisional effect due to temperature gradients in the plasma which cause the fluid to convect with the heat flow ($\mathbf{q}$), as $\mathbf{V}_{\text{Nernst}} \propto \mathbf{q} \propto -\nabla T$. In the overdense region, Nernst advection flows away from the critical surface and towards the ablation front. Therefore, any magnetic fields that are generated closer to the critical surface will feel an additional convective force towards the ablation front. Furthermore, as magnetic fields are convected toward the denser, colder regions of the ablation front, the local resistivity increases and magnetic diffusion may play a bigger role in the dynamics. Additionally, the field amplification typically associated with Nernst convection in this region will no longer hold near the ablation front. With higher resistivity, the frozen-in

\footnote{Recent results from nonlinear RT experiments done by Gao et al.\cite{51} show $\sim \text{MG}$ magnetic fields due to highly nonlinear RT growth under different laser conditions used in experiments discussed in this thesis.}
mechanism that causes the amplification is no longer present and the fields can diffuse in the cold plasma. The field diffusion time $\tau_{diff}$ can be approximated by

$$\tau_{diff} \approx \frac{\mu_0}{(\eta k^2)},$$  \hfill (6.15)

where $\eta$ is the resistivity and $k$ is the wavenumber. At $\sim 300$ eV temperatures in this plasma, $\tau_{diff} \sim 1$ ns which is of the same order as the RT-growth time scale. Measurements shown in Figure 6-9 were consistently lower than the ideal calculations, suggesting that diffusion of B fields into the colder, denser plasma may be occurring.

Collisional effects causing magnetic field diffusion are contained in the $R$ terms in Equation 3.115. Inclusion of the frictional force reveals the diffusion mechanism in the B field evolution equation,

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \times \left( \frac{\nabla p_e}{\epsilon_0 n_e} \right) + \nabla \times (\mathbf{V}_i \times \mathbf{B}) - \nabla \times (D_m \nabla \times \mathbf{B}),$$  \hfill (6.16)

where $D_m$ is the magnetic diffusion coefficient and is related to the plasma resistivity by $D_m = \eta/\mu_0$. To estimate the reduction in field strength due to diffusive effects, a simple correction factor$^{24}$ may be used in Equation 6.8,

$$
B \approx \frac{1}{1 + \frac{k^2 D_m}{\gamma_{RT}} \frac{-m_i}{\epsilon_0 (Z + 1)} \xi},
$$  \hfill (6.17)

where $k$ is the wavenumber of the perturbations under investigation and $\gamma_{RT}$ is the growth rate. Using this formalism with the experimentally determined RT growth rate and assuming the plasma temperature near the ablation front is $\sim 300$ eV, this calculation results in a correction factor of $\sim 0.4$ implying that the ideal calculation overestimates the field magnitude by $\sim 2.5$ times. A correction of this magnitude would account for the discrepancy observed between the experimental data and the ideal predictions illustrated in Figure 6-9a.

The magnetic Reynolds number ($Re_m$) is a fundamental parameter used to determine the importance of diffusion in a conducting fluid. It is defined as $Re_m = V L/D_m$, where $V$ is the flow velocity and $L$ is the scale length. Assuming that the flow is relatively constant (to within factors of a few) at $\sim 150$ $\mu$m/ns, and that the scale length is determined by the temperature gradients as shown in Figure 6-4, the magnetic Reynolds numbers at various locations were estimated: near the ablation front, $Re_m \sim 0.1$; at the critical surface, $Re_m \sim 50$; and at the quarter critical surface (in the corona), $Re_m \sim 2000$. The Reynolds number discussion will be analyzed in more detail in Chapter 7. In general, as the plasma becomes hotter and more homogenous (longer scale lengths), diffusion becomes less important and the B fields are ‘frozen-in’ to the plasma. However, RT-generated fields occur near the ablation surface where $Re_m \lesssim 1$ and diffusion will play a bigger role.
6.7 Summary

Path-integrated measurements of RT-induced magnetic fields during the linear growth phase were made using a combination of x-ray and proton radiographic techniques. Experiments were performed using planar targets with initial surface perturbations at a wavelength of $\lambda \sim 120 \, \mu m$. Field-strength information was encoded within proton fluence modulations due to the relationship between the deflection of a monoenergetic proton beam and the path-integrated field strength. Radiographs were analyzed using a discrete-Fourier-transform technique to recover information at the known wavelength of interest. X-ray measurements provided experimental areal-density modulations at the target and a growth rate of $\sim 2 \, \text{ns}^{-1}$ was inferred. From these measurements, areal density modulations were shown to contribute little to the overall amplitude modulation observed in proton fluence radiographs. Density-corrected proton measurements were then used to infer path-integrated field strengths.

Amplitude modulation in proton radiographs were shown to be dominated by magnetic deflections. Path-integrated measurements exhibited an increase from $\sim 10 \, \text{T-} \mu \text{m}$ to $\sim 100 \, \text{T-} \mu \text{m}$ during the linear growth phase. Radiation-hydrodynamic simulations performed with DRACO were post-processed to calculate magnetic field structures in the Hall MHD limit for comparison with data. Under these conditions, B fields were shown to be proportional to fluid vorticity and due to the high ($\gtrsim 10^4$) plasma beta, do not greatly affect the bulk hydrodynamics. Path-integrated B-field measurements were found to be a factor of $\sim 2$ lower than predictions because of the neglected collisional effects, Nernst and diffusion. B-fields were shown to be generated near the ablation front, where plasma conditions generate higher resistivity. In this environment, diffusion may play a larger role in the dynamics, thereby reducing the B-field strength in the experiments relative to the ideal calculations.

Under the plasma and laser conditions explored in these experiments, RT-induced B fields due to 120-µm wavelength perturbations were shown to have a negligible effect on electron thermal conduction. Furthermore, B fields generated near the ablation front will, in general, have a minimal effect on thermal conduction due to the high collisionality in that region. Additionally, Nernst advection will act to push fields into the colder, denser plasma where resistivity is higher and diffusion may more readily occur. Of greater concern are B fields generated by non-linear RT evolution near the critical surface where inhibition of thermal conduction may occur at lower B-field magnitudes. Proton images were taken at later times to investigate RT-induced fields near linear saturation. These images exhibited coherent cellular structure that were initially thought to be caused by early-onset of nonlinear RT growth, these data are discussed in the next chapter.

6.7.1 Future Work

These were the first measurements of RT-induced magnetic fields and will be useful for benchmarking codes that implement a more complete description of magnetic field generation. The results presented here did not include all of the terms in the magnetic field evolution equation, specifically the diffusion and Nernst terms as they may play an important role for fields generated near the cold, dense, ablation front. Some simulation codes used in HED science already have this capability, but due to practical constraints were not accessible for comparison in this work. However, the results discussed herein provide a clear path to perform these types of benchmarks that were not previously possible.

New experiments could be performed to investigate RT-induced B-field generation under different conditions. All experiments discussed here used a 2 ns square pulse, whereas
ignition style pulses use pickets and a ramped main drive. However, to avoid compromising the RT-induced field measurements, images must be taken early enough to avoid the cellular field structures and may necessitate higher initial perturbation amplitudes. Instead of using a directly-driven foil, indirect-drive experiments could be performed where the seeded foil is accelerated by the x-ray drive generated inside of a hohlraum. These experiments could be done using the monoenergetic-proton backlighter, so long as the seeded wavelengths are $\gtrsim 100$ µm. However, shorter wavelengths could be investigated with finer spatial and temporal resolution obtained using a short-pulse-laser to generate backlighter protons as discussed in Section 4.1. Though some development would be necessary to quantitatively analyze radiographs generated in this way.

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Chapter 7

Coherent Electromagnetic Fields in Coronal Plasmas

Unexpected results were obtained from radiography experiments performed to investigate B fields generated by Rayleigh-Taylor (RT) near linear saturation, when the perturbation amplitude approaches \( \sim 10\% \) of the wavelength. Proton data indicated strong nonlinear behavior earlier in time than would be predicted from simple growth estimates. X-ray radiographs indicated a growth rate of \( \gamma_{120} \sim 2 \text{ ns}^{-1} \) for plastic (CH) foils with \( \lambda = 120 \mu\text{m} \) perturbations. These foils had an initial amplitude of \( \sim 0.27 \mu\text{m} \) and reach linear saturation near \( \sim 1.9 \text{ ns} \). The exact initial perturbation height is unknown because of shock propagation through the foil before acceleration begins, but linear saturation should begin around \( \sim 1.9 \text{ ns} \). Moreover, the preimposed mode would saturate and transition into the nonlinear regime. Interestingly, coherent cellular features appeared in proton radiographs at times \( \geq 1.6 \text{ ns} \) regardless of the preimposed surface conditions and the characteristic scale size of these cells was found to be temporally invariant. These data were found to provide valuable insight to previously observed filamentary fields in the underdense corona.

An overview of previously observed fields around laser-irradiated spherical targets is given in Section 7.1 to provide background information and context of these findings. Planar experiments that provide axial views of these filaments are similar to those discussed in Chapter 6, but a brief review is given on the configuration and target details in Section 7.2. Analysis techniques used to quantify these cellular structures is described in Section 7.3. Experimental results and analyses are given in Section 7.4 and it is shown that cellular fields are not caused by RT growth of surface perturbations. A review of other field-generation mechanisms relevant to these experiments is provided in Section 7.5 and it is demonstrated that the likely cause of coherent field structures is the magnetothermal instability (MTI). This chapter concludes with a summary of these unexpected findings in Section 7.7 and future research directions opened by these results.
7.1 Coronal Filamentary Fields

Monoenergetic-proton radiography\(^1,2\) has been used to image coronal field structures in laser-irradiated CH spheres.\(^3\) Figure 7-1a shows the experimental setup used in these experiments. A solid CH sphere with a diameter of \(\sim860\ \mu\text{m}\) was irradiated by 0.351 \(\mu\text{m}\) laser light with an intensity of \(\sim2\times10^{14}\ \text{W/cm}^2\) in a 1 ns laser pulse and implemented full beam smoothing\(^4,5\) and SG4 distributed phase plates.\(^6\) The resultant blow-off was probed using 15 MeV fusion-protons to image electromagnetic fields in the underdense coronal plasma.

Self-generated magnetic fields originate from the Biermann battery\(^7\) source caused by perpendicular temperature and density gradients as discussed in Section 3.4. For the purposes of this discussion, the magnetic field evolution equation (Equation 3.115) is written

\[
\frac{\partial \mathbf{B}}{\partial t} \approx \frac{\nabla T_e \times \nabla n_e}{\epsilon_0 n_e} + \nabla \times (V_{\text{adv}} \times \mathbf{B}) - \nabla \times (D_m \nabla \times \mathbf{B}),
\]

where \(T_e\) is the electron temperature, \(n_e\) is the electron density, \(\epsilon_0\) is the unit charge, and \(D_m\) (\(= \eta/\mu_0\)) is the diffusion coefficient. In this formalism electron inertia and second order terms in \(\mathbf{B}\) have been neglected, but other collisional terms were retained, namely the Nernst and diffusion effects. The advection velocity \(V_{\text{adv}}\) is the vector sum of the fluid velocity \(V_i\) and the so-called Nernst velocity\(^8\) \(V_{\text{Nernst}}\). Advection by the Nernst effect arises because the magnetic field can move with the heat-conducting electron population and is
7.1. CORONAL FILAMENTARY FIELDS

thus proportional to the temperature gradient,

\[ \mathbf{V}_{\text{Nernst}} \approx -\frac{\beta''_0}{\delta_0} \tau_{ei} m_e \nabla T_e, \]  

(7.2)

where \( m_e \) is the electron mass, \( \tau_{ei} \) is the electron-ion collision time, and \( \beta''_0 \) and \( \delta_0 \) are Braginskii coefficients.\(^9\) The weakly magnetized approximation (Hall parameter \( \chi \ll 1 \)) has been implemented and for the CH plasmas discussed herein \( \beta''_0/\delta_0 \approx 2.5 \). The Nernst effect contributes to the total convection of the magnetic field along with field diffusion described by the third term in Equation 7.1. The first term in Equation 7.1, the Biermann battery or thermoelectric\(^{10} \) source, is the dominant generation mechanism of self-generated magnetic fields in plasmas.

Figure 7-1b illustrates four proton-fluence radiographs taken before, during, and after the 1 ns laser pulse. The field of view of the detector in the target plane is 3 mm × 3 mm in each image. Protons are stopped in the solid sphere which results in the white ‘shadow’ seen in each image. No coronal field structures were observed until the end of the laser drive, \( \sim 1 \) ns under these conditions, though fields around the stalk, like those discussed in Chapter 5, were observed in the image at \( \sim 0.6 \) ns. Because these are solid spheres there is bulk acceleration indicating that RT, as discussed in Chapter 6, is not the cause of these field structures.\(^{1} \) Séguin \textit{et al.} demonstrated that the observed filamentary structures are quickly (\( \sim 200 \) ps) generated between \( \sim 0.6 \) ns and \( \sim 0.8 \) ns in these experiments and that the filaments expand radially outwards with the blow-off plasma.

The magnetic field evolution described by Equation 7.1 shows that fields will advect in the direction of \( \mathbf{V}_{\text{adv}} \). The advection velocity is dependent on both the fluid and Nernst velocities, and will change as a function of time and position. Plasma evolution in these experiments was modeled using the 1-D radiation-hydrodynamic code LILAC\(^{11} \) and the resultant velocity profiles at 1 ns are shown in Figure 7-1c as a function of distance from the ablation surface. The Nernst velocity was calculated using Equation 7.2 and the predicted temperature profiles and changes directions at the peak temperature as indicated in the plot. It is clear that the advection velocity will change directions at some point in the plasma, and under these specific plasma conditions this occurs \( \sim 100 \) \( \mu m \) from the ablation front. B fields generated inside this transition region will advect inwards to the ablation surface, otherwise they expand out with the plasma.

Filamentary structures are observed throughout the coronal plasma, thus fields must be generated in a region where they will convect out with the expanding plasma. In these images the filaments appear to extend to the ablation surface of the sphere, however the inherent 3-D nature of the spherical geometry precludes any definitive assessment here. Though filaments clearly expand with the coronal plasma and are even present after the laser drive has ended. Quantitative analysis of the filament size is difficult in this geometry, but through Monte Carlo simulations was predicted\(^3 \) to be of order \( \sim 150 \) \( \mu m \) at the quarter-critical surface. Face-on imaging of laser-irradiated planar foils can be used to further probe these filamentary fields on-axis.

\(^{1}\)The ablation front still ‘accelerates’ because as material is ablated away, the front moves and has some acceleration associated with it. However, this acceleration is much much smaller than the bulk acceleration experienced by a freely moving target and is not considered in most cases.
7.2 Planar Radiography Experiments

Monoenergetic-proton and x-ray radiography experiments were performed on the OMEGA laser using the configuration shown in Figure 7-2. Protons are sensitive to both mass and field modulations through Coulomb scattering and the Lorentz force, respectively. X rays are sensitive only to density modulations in the target. The complementarity of these two diagnostic techniques provides information to address density and field distributions during plasma evolution. Unlike the solid sphere experiments, these foils will be accelerated and RT growth of density perturbations is expected to occur. Areal density distributions and growth of perturbations are characterized by x-ray radiographs, whereas protons also sample the path-integrated field structures.

The proton backlighter capsules, filled with 18 atm D\textsuperscript{3}He gas, were imploded using 20 OMEGA beams to produce fusion protons. Each proton radiography experiment gives a single ‘snapshot’ in time of the laser-foil interaction and multiple experiments with different laser timings provide a series of radiographs illustrating the plasma evolution. This backlighting technique provides a temporal resolution of \(\sim 150\) ps and D\textsuperscript{3}He-fusion protons (\(E_p \sim 15\) MeV) are produced by an approximately Gaussian source with a FWHM of \(\sim 45\) \(\mu\)m. 15-MeV proton radiographs were recorded on filtered CR-39 plastic nuclear track detectors. After exposure, CR-39 samples were etched to reveal tracks left by the incident protons. Through the etching process, signal tracks in the plastic were revealed and pieces scanned using an optical microscope system.\textsuperscript{13} From these digital scans, radiographs were processed and proton fluence images were created.

X-ray radiographs were taken using a laser-irradiated Uranium foil and filtered to provide \(\sim 1.3\) keV x-rays for optimum contrast through \(\sim 20\) \(\mu\)m CH. Images were recorded on film using a framing camera\textsuperscript{14,15} with a temporal resolution of \(\sim 80\) ps and a spatial resolution of...
The apertured camera provides multiple images of a single foil during its evolution, yielding multiple radiographs from a single experiment. The measured optical depth image may be directly converted to an areal density map of the target for comparison with proton radiographs which are sensitive to both density modulations and field deflections at the foil.

Four different types of CH foils of varying thicknesses and surface perturbations, as described in Table 7.1, were used over multiple shot days. Foil surfaces were either flat, seeded with ridge-like 2-D sinusoidal modulations, or 3-D eggcrate-like sinusoidal modulations. The laser spot was shaped by SG4 distributed phase plates (DPPs), smoothed by spectral dispersion (SSD) and distributed polarization rotators (DPRs) were implemented to minimize the broadband imprint from the laser spot. Twelve beams were overlapped to provide a drive intensity of $I \lesssim 4 \times 10^{14}$ W/cm$^2$ within a $\sim 750 \mu$m diameter spot using a 2 ns square pulse delivering a total of $\sim 3300$ J of energy on target. Proton and x-ray radiographs were taken at times between 1.2 ns and 2.4 ns after laser onset to provide data on the plasma during and after the drive.

Table 7.1: Characteristic metrology of the four CH foil types used in these experiments. Initial ambient foil density was $\rho = 1.03$ g/cm$^3$. Variations in thickness ($l_0$), wavelength ($\lambda_0$), and sinusoidal amplitude ($a_0$) are all $\lesssim 10 \%$. The value given for the eggcrate foil (3-D) is the diagonal peak-to-peak wavelength.

<table>
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<tr>
<th>Label</th>
<th>$l_0 [\mu m]$</th>
<th>$\lambda_0 [\mu m]$</th>
<th>$a_0 [\mu m]$</th>
</tr>
</thead>
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<tr>
<td>Flat Foil</td>
<td>21</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>120 $\mu m$ (2-D)</td>
<td>21</td>
<td>120</td>
<td>0.27</td>
</tr>
<tr>
<td>180 $\mu m$ (2-D)</td>
<td>23</td>
<td>180</td>
<td>0.55</td>
</tr>
<tr>
<td>115 $\mu m$ (3-D)</td>
<td>26</td>
<td>115</td>
<td>0.56</td>
</tr>
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</table>
7.3 Proton-Radiograph Analysis Techniques

Field evolution imaged by protons at times $\gtrsim 1.6$ ns revealed 3-D cellular features. The two observables of interest in proton radiographs are the characteristic isotropic spatial wavelength of the features ($\lambda_{AC}$), if one exists, and the normalized broadband rms amplitude ($\sigma_{\text{rms}}$) which is a measure of the strength of the features. The latter may be calculated from the distribution of fluence values after removing the statistical component,

$$\sigma_{\text{rms}} = \frac{\sqrt{\sigma_{\text{meas}}^2 - \sigma_{\text{stat}}^2}}{\mu_{\text{meas}}},$$

(7.3)

where $\sigma_{\text{meas}}$ is the measured statistical deviation of protons per pixel, $\mu_{\text{meas}}$ is the statistical mean proton fluence per pixel used to normalize the variation across different experiments, and $\sigma_{\text{stat}}$ is the numeric statistical variation per pixel $\sim \sqrt{\mu_{\text{meas}}}$. Deducing the characteristic spatial scale of the cellular features requires a more complex analysis.

An autocorrelation (AC) algorithm is used on each proton-fluence image to determine the isotropic scale-size of the observed features. This procedure begins by calculating the 2-D FFT of a square region that results in a 2-D array $C(k_x, k_y)$ of complex Fourier coefficients,

$$C(k_x, k_y) = \mathcal{F}\{I(x, y)\},$$

(7.4)

where $\mathcal{F}$ denotes the FFT algorithm and $I(x, y)$ is the array of pixel values in the area of interest. The autocorrelation coefficients $A(x, y)$ may then be calculated directly from the Fourier coefficients,

$$A(x, y) = \mathcal{F}^{-1}\{C^* C\},$$

(7.5)

where $C^*$ is the complex conjugate of the Fourier coefficient array and $\mathcal{F}^{-1}$ is the inverse FFT algorithm. The autocorrelation coefficients, as defined above, represent how well the image correlates with itself. The AC coefficients are azimuthally averaged in space, thus eliminating one of the spatial dimensions and producing a 1-D (isotropic) representation of the AC coefficients. If present, an isotropic scale-size will be revealed by this 1-D representation and may be extracted from each proton radiograph.

Figure 7-3 illustrates two examples of the AC analysis with synthetic data and two examples from experimental radiographs. Two synthetic images were generated with 2-D sinusoidal functions (eggcrates) with a wavelength ($\lambda_0$) of 150 $\mu$m in both directions, resulting in a diagonal peak-to-peak wavelength $\lambda_{0,D} = (\sqrt{2}/2)\lambda_0$ of 106 $\mu$m. The two images in Figure 7-3a illustrate the differences between amplitudes of 10% (top) and 30% (bottom) of the mean. The corresponding normalized 1-D AC spectra are shown by the solid and dashed lines, respectively and show that higher amplitudes in the image result in higher amplitudes in the AC coefficients, as expected. The synthetic images used have a single spatial frequency over many wavelengths, which results in the decaying oscillations observed at harmonics of the fundamental wavelength (first peak) in the AC spectra. The dominant scale size was measured to be $113\pm8$ $\mu$m in both cases as illustrated by the dotted line. This measurement is consistent with the original diagonal peak-to-peak wavelength; AC coefficients near the lateral wavelength of 150 $\mu$m averaged out during the process. This analysis procedure accurately measures the dominant scale size in these images to within the uncertainty of the measurement.
7.3. PROTON-RADIOGRAPH ANALYSIS TECHNIQUES

Figure 7-3: (a) Two synthetic images with eggcrate-like perturbations with a diagonal peak-to-peak wavelength of $\sim 106 \mu m$ and sinusoidal amplitudes of 10% (top) and 30% (bottom) of the mean. The corresponding AC spectra are shown for both images with the measured dominant scale size of the features as $113 \pm 8 \mu m$. (b) Experimental radiographs from 1.4 ns (top) and 1.6 ns (bottom) with corresponding AC spectra. The earlier image does not show a peak in the AC spectrum, suggesting that there is no isotropic scale size in the image. Whereas, the radiograph at 1.6 ns clearly shows a peak at $185 \pm 10 \mu m$ indicating a dominant feature in the image.

The dominant scale size of isotropic features in an image was calculated from 1-D AC spectra by fitting a curve to the first observed peak at a length greater than zero. A 2nd-degree polynomial was found to fit most data better than a Gaussian, or other peaked functions. The primary goal of the fit is to algorithmically obtain an accurate measurement of the peak. Furthermore, using the fitted curve, an uncertainty in the measured peak position may be estimated. The width of the peak represents the uncertainty in the dominant scale size of the observed features, though the typical FWHM metric is not a well-defined quantity in most cases. Therefore, the width of the parabola is taken at the point where it has reached 95% of the value at the peak. This width is the uncertainty in the measured scale size of cellular features.

Figure 7-3b illustrates two sample proton radiographs of laser-irradiated CH foils with $\sim 120 \mu m$ ridge-like perturbations. The first image at $t \sim 1.4$ ns shows the expected linear behavior in the image and the corresponding AC coefficient spectrum (solid) does not indicate any peak after the initial fall off, suggesting that there is no dominant, isotropic scale-size. The bottom image in Figure 7-3b occurs later in time, at $t \sim 1.6$ ns, and the corresponding AC coefficient spectrum (dashed) is shown. This spectrum shows a faster fall off, indicating higher amplitude modulations than the previous image, and a dominant scale size peak at $185 \pm 10 \mu m$. This systematic analysis technique provides an accurate measurement of the dominant scale size, when present, of isotropic features observed in radiographic images.
CHAPTER 7. COHERENT ELECTROMAGNETIC FIELD STRUCTURE

7.4 Experimental Results

7.4.1 Proton Radiographs

A comprehensive summary of proton-fluence radiographs is shown in Figure 7-4. Radiographs were taken after the first nanosecond of the 2 ns laser drive for each of the four different foil types. Proton radiographs of all foil types taken before \(\sim 1.5\) ns indicate minor variance across the analysis region, though coherent linear features were observed in 2-D modulated foils at times \(\lesssim 1.5\) ns. These features were discussed in Chapter 6 and shown to be caused by Rayleigh-Taylor-induced magnetic fields.\(^{17,18}\) A rapid transition \((\lesssim 200\) ps\) occurs near \(t\sim 1.5\) ns whereby fluence radiographs of all foil types show a drastic change in appearance. Some underlying linear features most likely due to RT-induced fields are still observable, especially in 180-\(\mu m\)-foil images at times \(\gtrsim 1.6\) ns, but the 3-D cellular structure is still prevalent and dominates proton radiographs at these times. These features are consistent with an axial view of the previously observed filamentary fields.

Cellular structure was shown to begin during the laser drive and continued well after the end of the pulse. The dominant scale size of these features \(\lambda_{AC}\) and the rms amplitude modulation \(\sigma_{\text{rms}}\) were calculated to characterize the scale and strength of the filamentary field structures. The results of the autocorrelation analysis (as described in Section 7.3) are plotted in Figure 7-5a. It is clearly demonstrated that these features have an approximately constant scale size that can be characterized by \(\lambda_{AC} \approx 210\ \mu m\) with a standard deviation of \(\pm 30\ \mu m\). Radiographs taken before cellular-onset were analyzed, but did not reveal a dominant scale size and are thus not shown in Figure 7-5a. The separation scale of filaments is shown to be constant in size immediately after initial onset and that it does not change in time or depend on the initial foil surface conditions. This suggests that that filamentary fields are generated away from regions affected by the shape of the ablation surface, i.e., in the expanding, underdense corona.
7.4. EXPERIMENTAL RESULTS

7.4.1 Nonlinear Proton Analysis

Figure 7-5: (a) Dominant scale size of cellular features observed in proton radiographs as a function of time. For times $\lesssim 1.5$ ns, no dominant scale size was measured. (b) Normalized broadband rms amplitude as a function of time. The rms amplitude is shown to grow in time similarly for all foil types and continue after the laser drive has ended.

Figure 7-5b illustrates the normalized rms amplitude as a function of time for all four foil types. Radiographs at earlier times ($\lesssim 1.5$ ns) are shown to have normalized rms amplitudes of $\lesssim 20\%$, though no dominant scale size was observed. The normalized broadband rms characterizes the amplitude of proton deflections and thus path-integrated field strength. It is clearly shown that proton deflections grow at the same rate for all foil types during the laser drive. Filaments causing the cellular features are created during the laser pulse, but do not lose appreciable path-integrated strength after the drive ends.

7.4.2 X-ray Radiographs

All four foil types were radiographed with x rays to characterize density distributions in these laser-foil interactions. Modulations in areal density arise due to RT-growth of surface perturbations$^{15}$ and laser-imprinted$^{19}$ intensity modulations. X-ray radiographs shown in Figure 7-6a illustrate the evolution of areal density modulations for all target types. In these images, lighter pixels indicate higher areal density in the target (more x-ray absorption). Flat-foil radiographs show no significant features until late in time ($t \sim 2.2$ ns), at which point small-scale ($\sim 30\ \mu\text{m}$) structure due to laser imprint$^{20}$ becomes apparent. However, x-ray radiographs of modulated foils clearly show dramatic features at the seeded wavelengths
due to RT-growth. The amplitude of the 120-µm-wavelength foil increases in time with a rate of $\gamma_{120} \sim 2$ ns$^{-1}$ and these data agree well with DRACO$^{21}$ radiation-hydrodynamic simulations as shown in Figure 7-6b. Some small-scale structure, also at $\sim 30$ µm, is visible at late times in modulated foils, but these features are much lower in amplitude than the dominant preimposed modulations.

X-ray radiographs do not show similar features as those observed in late-time proton fluence images. Cellular structure in areal density has been observed$^{19}$ under different experimental conditions due to laser imprint, but in experiments discussed herein the 3-D structure has not had enough time to strongly develop. A Fourier analysis was performed on the x-ray images at $\sim 2.2$ ns to compare the relative amplitudes of the observed features. The resultant spectra are shown in Figure 7-7 and demonstrate that the $\sim 30$ µm features in both cases are approximately equal in amplitude and are consistent with RT-growth of laser-imprinted perturbations. In the modulated foil case, the amplitude of 120 µm perturbations is $\sim 5$ times higher than $\sim 30$ µm features.
Proton deflections due to RT-induced\textsuperscript{17} B fields are dominated by fields at the seeded wavelength of 120 µm in the modulated-foil case at times \( \lesssim 1.5 \) ns. B fields created by RT growth occur near the ablation surface and, neglecting diffusion, are proportional to the fluid vorticity. The peak field scales\textsuperscript{22} with perturbation parameters as \( |B(t)| \propto \frac{h(t)}{\lambda \gamma} \), where \( h \) is the perturbation amplitude, \( \lambda \) is the wavelength, and \( \gamma \) is the growth rate. Proton deflections, though, are proportional to the path-integrated field strength \( \langle BL_B \rangle \) where the field scale-length \( L_B \sim h \).\textsuperscript{18} Measured RT growth rates\textsuperscript{15} for both wavelengths, \( \gamma_{30} \sim 4.5 \) ns\textsuperscript{-1} and \( \gamma_{120} \sim 2 \) ns\textsuperscript{-1}, were used with the fact that \( h \propto \langle \rho L \rangle_{\mathrm{rms}} \) to estimate the relative magnitudes of \( \langle BL_B \rangle \) between the two perturbation wavelengths. If proton deflections late in time were due only to RT-induced B fields, simple estimates show that deflections due to \( \lambda=120 \) µm are \( \sim 15 \) times higher than those for \( \lambda=30 \) µm. Moreover, this is a lower limit because field diffusion effects, not included here, affect shorter wavelengths more than longer ones. Also, this analysis was for images at \( \sim 2.2 \) ns, whereas proton images illustrate strong cellular features by \( \gtrsim 1.6 \) ns. If RT-induced B fields were the dominant field structure in modulated-foil experiments, proton images would exhibit strong features throughout the pulse corresponding to the preimposed surface perturbations. Instead, coherent cellular structures likely caused by filamentary fields in the underdense corona are observed in flat and modulated-foil images at late times.

Cellular fields were shown to dominate proton fluence images of all foil types at sample times \( \gtrsim 1.6 \) ns. Recall from Figure 6-9 that the path-integrated field strength of RT-induced B fields in the 120-µm foil case was \( \sim 100 \) T·µm at \( t \sim 1.5 \) ns. Using the collisionless field evolution model discussed in Section 6.3, the predicted upper estimate of \( \langle BL_B \rangle \) due to RT increases to \( \sim 500 \) T·µm by the end of the pulse. Thus, at the time of cellular-field onset, \( \langle BL_B \rangle \) from these 3-D structures must be \( \gtrsim 100 \) T·µm and increase in time to \( \gtrsim 500 \) T·µm in order to continue dominating proton fluence images. Since these fields are independent of foil-surface conditions, and are likely occurring in the underdense corona, the path-length can be of order \( \sim 1 \) mm indicating that B-field magnitudes can be as low as \( \sim 0.1 \) T, but must be relatively coherent throughout the corona.

Figure 7-7: X-ray images taken at 2.2 ns of both flat and modulated foils are shown on the left. Lineout directions are indicated by arrows in the images, where spectra from multiple lineouts were averaged for the flat foil case and lineouts parallel (Mod) and perpendicular (no Mod) to the perturbation wave vector are shown for the 120 µm case.
Figure 7-8: (a) Predicted DRACO profiles of electron temperature and density taken at \( \sim 1.5 \) ns. Primary field-generating instabilities are listed and the locations where they occur in the plasma are indicated. The ablation, critical, and \( \frac{1}{4} \)-critical surfaces are labeled for reference. (b) Profiles of the fluid, Nernst, and advection velocities calculated from DRACO simulations. Distances are given relative to the ablation front and positive velocities are pointed outward. These calculations predict that the advection velocity changes direction near the quarter-critical surface.

### 7.5 Overview of Instabilities in Laser-produced Plasmas

Magnetic fields may be generated by a range of instabilities and sources as described in detail by Haines.\(^{10}\) The primary sources of concern here are outlined in Figure 7-8a and where they occur in the sample plasma at 1.5 ns. In addition to these instabilities, laser-plasma interactions (LPI) can locally generate B fields through generation of hot-electron currents in the coronal plasma. Hard x-ray detectors\(^{23}\) and scattered-light streak cameras\(^{24}\) were fielded to observe LPI-related plasma behavior in the corona of these laser-irradiated planar foils. Hard x-ray signals were not observed, eliminating two-plasmon decay (TPD) as the cause. Additionally, scattered light due to stimulated Brillouin or Raman scattering was also not observed, eliminating these instabilities as a source of hot-electron currents. Filamentation of laser ‘hot-spots’ could not be responsible because the average intensity was too low\(^{25}\) and all beams were spectrally smoothed.\(^{4}\) LPI-generated hot-electron currents were determined not to be the source of the observed coherent field structures.

The so-called Weibel\(^{26}\) instability is generated by electron-temperature anisotropy in the plasma. The typical collisionless Weibel (CLW) instability is only relevant at the very edge of the corona\(^{10}\) when \( \lambda_{mfp}/(c/\omega_{pe}) \gtrsim 10^3 \), where \( \lambda_{mfp} \) is the mean-free-path and \( c/\omega_{pe} \) is the collisionless skin depth. In these calculations, this parameter is \( \lesssim 500 \) and thus CLW is a very unlikely field source. The collisional Weibel (CW) instability has been predicted to generate fields under ablatively driven conditions.\(^{27}\) These fields tend to grow fastest near the over dense region (\( n_e \gtrsim n_{crit} \)), but when the Nernst effect and field diffusion were included, the instability was shown to be stabilized. Nonetheless, if these fields exist under the specific experimental conditions here, they would be confined to the overdense region.

The electrothermal instability (ETI), first described by Haines,\(^{28}\) occurs between the ablation and critical surfaces in ablatively driven systems. This instability originates from a spatial perturbation to the background electron temperature that is perpendicular to the
Table 7.2: The list of possible mechanisms that were investigated as the cause of observed cellular features. Through a detailed process of elimination, the likely cause was shown to be the magnetothermal instability.

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<tr>
<th>Generation Mechanism</th>
<th>Cause?</th>
<th>Reasoning</th>
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<tbody>
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<td>Rayleigh-Taylor</td>
<td>very unlikely</td>
<td>Empirically demonstrated not to be the cause</td>
</tr>
<tr>
<td>Electrothermal Instability</td>
<td>very unlikely</td>
<td>Restricted to the over dense region</td>
</tr>
<tr>
<td>Collisional Weibel</td>
<td>very unlikely</td>
<td>Growth is reduced in the corona and confined by the Nernst effect</td>
</tr>
<tr>
<td>Laser-Plasma Instabilities</td>
<td>unlikely</td>
<td>No signals from multiple diagnostics</td>
</tr>
<tr>
<td>Collisionless Weibel</td>
<td>very unlikely</td>
<td>Occurs at the very edge of the corona</td>
</tr>
<tr>
<td>Magnetothermal Instability</td>
<td>likely</td>
<td>Grows fastest just outside the peak temperature</td>
</tr>
</tbody>
</table>

heat flux. The temperature perturbation leads to nonuniform ohmic heating due to the change in plasma resistivity, that positively feeds back on the perturbation. The fastest growing ETI mode in these plasmas, \( \lambda_{ETI} \sim 60 \, \mu\text{m} \), is much smaller than the observed \( \sim 210 \, \mu\text{m} \) scale size, therefore this instability is unlikely to be the source of the observed cellular fields. Furthermore, fields generated by the electrothermal instability are also confined to the high density region.

The Nernst effect in these experiments is calculated to be strongest just outside the critical surface where \( \mathbf{V}_{\text{Nernst}} \sim 700 \, \mu\text{m/ns} \) and is much larger than the fluid velocity. Fields generated here are quickly advected towards the ablation front. This can result in amplification of the B field due to the increasing electron density, though this amplification is reduced by diffusion effects. DRACO calculations presented herein implemented the local thermal equilibrium (LTE) approximation\(^{ii}\) and a constant flux limiter of \( f = 0.06 \) which has been shown\(^{15}\) to reproduce drive conditions well at intensities \( \lesssim 5 \times 10^{14} \, \text{W/cm}^2 \). These calculation indicate that the advection velocity changes directions near the \( 1/4 \)-critical surface. Coherent fields observed in planar and spherical experiments must be generated where outward advection can occur.

The MTI\(^{30}\) is sourced by a seed B field generated by the Biermann battery. The difference in thermal conduction\(^9\) due to the presence of this B field acts to enhance the perturbation. This instability can occur when the temperature and density gradients in the direction of heat flow are aligned. Coronal plasmas outside of the peak temperature satisfy this condition. In this region the Nernst velocity is aligned with the fluid flow as shown in Figure 7-8b and B fields generated in this region will advect out with the coronal plasma. Furthermore, using the analysis from Tidman and Shanny\(^{30}\) together with predicted temperature and density profiles, the fastest growing MTI mode in these experiments is found to be \( \lambda_{MTI} \sim 200-300 \, \mu\text{m} \) at these times. This wavelength is near the dominant cellular scale size and B fields generated by MTI will advect outwards, consistent with observations.

\(^{ii}\)Recent work by Willingale et al.\(^{29}\) demonstrated magnetic field advection due to the Nernst effect in the corona at speeds \( \sim 8000 \, \mu\text{m/ns} \). Though, these experiments had \( I \sim 10^{15} \, \text{W/cm}^2 \) with \( \lambda = 1.053 \, \mu\text{m} \), whereby hot-electron generation is more prominent and nonlocal thermal conduction plays a significant role.
7.6 Discussion

Laser-solid interactions create plasmas conditions that vary greatly as a function of position from the solid material as demonstrated in Figure 7-8a. Plasma conditions go from cold \( T_e \sim 300 \text{ eV} \) and dense \( n_e \sim 10^{23} \text{ cm}^{-3} \) near the ablation front to hot \( T_e \sim 2 \text{ keV} \) and sparse \( n_e \sim 10^{21} \text{ cm}^{-3} \) in the underdense corona. The plasma fluid velocity also varies and, at the time shown in Figure 7-8b, changes direction from inward flow to outward expansion just outside the critical surface. These spatial profiles also evolve in time due to the plasma expansion as shown in Figure 7-9. These varied conditions give rise to differences in relevant physics mechanisms.

7.6.1 Plasma Conditions

Coherent magnetic field structures were observed in these experiments. Predicted plasma conditions must then confirm that the flow is laminar such that coherence is possible and that field diffusion is minimal. The magnetic Reynolds number \( (Re_m) \) relates B-field advection to diffusion in the plasma. This can be expressed as \( Re_m = V_{adv} L / D_m \), where \( V_{adv} \) is the field advection velocity, \( L \) is the scale length of the plasma, and \( D_m \) is the magnetic diffusion coefficient. Figure 7-9 shows how \( Re_m \) varies as a function of position in the plasma at three times during the evolution and demonstrates that in most of the plasma, field ad-
vection dominates over diffusion. This is due to the low resistivity away from the ablation front and is consistent with B fields being frozen-in to the flow. The sharp dip in $\text{Re}_m$ near the $\frac{1}{4}$-critical surface at each time corresponds to the location where the advection velocity goes to zero. After the drive ends, $\text{Re}_m$ decreases indicating that diffusion is becoming more important, as expected since the plasma is cooling down.

The Reynolds number ($\text{Re}$) characterizes whether the fluid flow is laminar ($\text{Re}\lesssim 2300$) or turbulent ($\text{Re}\gtrsim 4000$).\(^{31}\) This dimensionless number compares inertial to viscous forces and can be written $\text{Re} = \frac{V_iL}{\nu}$, where $V_i$ is the fluid velocity, $L$ is the scale length of the plasma, and $\nu$ is the kinematic viscosity of plasma ions. Figure 7-9 illustrates how $\text{Re}$ profiles change as a function of time, though calculations indicate that the flow is laminar everywhere during the entirety of the evolution. The Reynolds number increases near the ablation front during the drive due to the decrease in viscosity. X-ray radiographs though, demonstrated coherent RT-growth at the ablation front in modulated foils during the entirety of the laser drive. These calculations demonstrate that plasma flow is dominantly laminar in nature allowing for coherent features.

Magnetic fields generated outside the $\frac{1}{4}$-critical surface will be strongly advected outwards in these experiments. Velocity profiles calculated from DRACO simulations are shown in the third column of Figure 7-9 and show how the B-field advection velocity evolves in time. DRACO predicts peak coronal temperatures just outside the $\frac{1}{4}$-critical surface and therefore the Nernst velocity changes directions at this point from inward to outward. The corresponding field advection velocity is shown to change directions near the $\frac{1}{4}$-critical surface even as the plasma expands. After the drive ends, all velocity profiles tend to flatten out, as expected. The diffusion coefficient profiles are also shown in these plots and illustrate that during the drive, field diffusion only plays a role near the ablation front. When the drive ends, the plasma cools and diffusion becomes more important throughout the plasma. However, even after the drive is off, the diffusion coefficient far out in the corona is $\sim 40$ $\mu$m$^2$/ns, so the diffusion time for a $\sim 200$ $\mu$m B-field is $\sim 25$ ns. Magnetic fields generated in the hot corona during the drive will not dissipate until nanoseconds later.

### 7.6.2 Basics of the Magnetothermal Instability

The classic work on the magnetothermal instability (MTI) was done by Tidman and Shanny\(^{30}\) in 1974. This original work, that neglected plasma flow, was followed up by Ogasawara et al.\(^{32}\) and Hirao et al.\(^{33}\) who considered hydrodynamic and Nernst effects, respectively. Haines\(^{10}\) discussed this instability briefly among other field-generating instabilities and recently Bissell, Kingham, and Ridgers\(^{34}\) extended this theory to include the presence of an externally applied B field. All discussion given here\(^{iii}\) and calculations made are based on the classic work by Tidman and Shanny, where the fastest growing mode in a CH plasma has a growth rate ($\gamma_{\text{MTI}}$) and wavelength ($\lambda_{\text{MTI}}$) given by

\begin{align}
\gamma_{\text{MTI}} &\approx 1.65 \times 10^8 \frac{T_e^{5/2}}{n_e Z L_p L_T \ln \Lambda} \left[ 1 \text{ ns} \right], \\
\lambda_{\text{MTI}} &\approx 2 \times 10^{-4} \sqrt{\frac{L_p L_T Z \ln \Lambda}{n_e \lambda_D^3 T_e^{1/2}}} \left[ \mu\text{m} \right].
\end{align}

\(^{iii}\)It is noted that in these laser-produced plasmas, the fluid flow and Nernst advection should be included in more advanced analyses. For estimation purposes presented here however, the classic theory is sufficient to demonstrate the viability of the MTI as the source of the observed field structures.
In the preceding equations $Z$ is the average charge state, $\ln \Lambda$ is the Coulomb logarithm, the electron temperature $T_e$ is in keV, density $n_e$ in $10^{20}$ cm$^{-3}$, the Debye length $\lambda_D$ is in $\mu$m, and the scale lengths for temperature ($L_T$) and density ($L_\rho$) are also in $\mu$m. The positive feedback process competes with field diffusion due to the finite plasma conductivity that affects shorter wavelengths more readily. Figure 7-10 shows the basic idea and feedback mechanism of this instability.

Parallel temperature and density gradients, as required for the classic MTI, occur outside the peak temperature in laser-produced plasmas, as schematically represented in Figure 7-10. In the MTI process, a seed magnetic field may be created by a temperature perturbation $T_1$ perpendicular to the original gradient. The perturbed B field $B_1$ alters the heat conduction as was discussed in Section 3.3.2 and provides positive feedback on the perturbed temperature in this geometry, thus increasing the temperature to $T'_1$ and enhancing the perturbed B-field strength to $B'_1$. As schematically shown in Figure 7-10, it is not necessary to have a zeroth-order B field, the seed field is generated by the temperature perturbation and the background density profile. However, in order for instability to occur, the perturbed B field must be strong enough to alter the heat conduction. From this requirement, an order of magnitude can be placed on the field by setting a minimum Hall parameter ($\chi$). At his level of detail, the exact value is not critical, but let $\chi_{\text{min}} \approx 0.1$ in Equation 6.2 such that cross-field transport ($\kappa_\perp$) is $\sim 14\%$ less than parallel transport ($\kappa_\parallel$). Then, using Equation 6.3, with underdense plasma conditions of $T_e \sim 1.8$ keV, $n_e \sim 20 \times 10^{20}$ cm$^{-3}$, $Z = 3.1$, and $\ln \Lambda \sim 7$, it is clear that a B-field strength of only $\sim 1$ Tesla can alter the heat conduction in this way. Even though this qualitative analysis neglects fluid flow and field advection, it describes the general driving mechanism of this instability and provides some plausible field magnitudes. For a detailed explanation of the this theory, the interested reader is encouraged to see the references mentioned at the beginning of this section.
Figure 7-11: Profiles of the growth rate $\gamma_{MTI}$ (short dash) and wavelength $\lambda_{MTI}$ for the fastest growing mode are shown for (a) the spherical case at 0.8 ns and (b) the planar case at 1.5 ns. Simulated plasma profiles are used with Equations 7.6 and 7.7 to generate the curves shown here and plotted as a function of (a) radial distance and (b) axial distance. An average wavelength $<\lambda_{MTI}>$ was calculated at each time by averaging $\lambda_{MTI}$ over this space and using the growth rate as a weighting factor. (c) The resulting average wavelength is plotted as a function of time for the planar (solid) and spherical (dash-dot) cases with the respective drives (dotted) shown at the bottom in arbitrary units. The inferred characteristic size of cellular structures determined from planar proton radiographs is also shown at $<\lambda_{MTI}>=210 \ \mu m$.

### 7.6.3 The MTI in Experiments

Sections 7.4 and 7.5 described in detail that the likely cause of field structure in proton radiographs of planar foils and solid spheres is the magnetothermal instability. Besides the inherent difference in geometry, 1-D (planar) versus 3-D (spherical), these experiments were also performed at different intensities, $\sim 4 \times 10^{14}$ (planar) versus $\sim 2 \times 10^{14}$ (spherical) W/cm$^2$. One notable difference between the spherical and planar experiments is the apparent onset time of the instability based on observation of the fields. In the planar case, cellular structure appears at $\sim 1.5$ ns and in the spherical case discussed here, filaments were shown to develop at $\sim 0.8$ ns. Other spherical experiments by Séguin et al.$^3$ were done at an intensity of $\sim 6 \times 10^{14}$ W/cm$^2$ and demonstrated a slightly earlier onset of $\sim 0.6$ ns (though not shown or discussed here). The planar experiments that provided a more accurate measure of the filament separation of $\sim 210 \ \mu m$ were done at an intensity between the two spherical data sets, but had an onset much later in time. The exact cause of the apparent onset is not well understood at this time, but is an effect due to both the measurement technique (i.e. the sensitivity of radiographing proton deflections and limited temporal resolution) and physical plasma conditions.

The MTI grows from a seed magnetic field in the coronal plasma that could come from
a number of different perturbative sources, laser nonuniformities, plasma waves, etc. Thus, the initial mode distribution is not well understood or characterized and varies spatially and temporally which makes quantitative analysis of this instability very difficult. However, some insight may be gained by simply looking at the fastest growing modes as described by Equations 7.6 and 7.7. Figure 7-11a and b illustrate the spatial variance of $\lambda_{MTI}$ (solid) and $\gamma_{MTI}$ (short dash) at single instances as calculated from LILAC and DRACO profiles, respectively. The fastest growing wavelength is very large near the peak temperature due to the long temperature scale lengths, but levels off quickly and slowly increases farther out due to decreasing densities in both the spherical and planar cases. The growth rate peaks at $\sim5$ ns$^{-1}$ in the planar case and decreases thereafter, whereas $\gamma_{MTI}$ levels off at $\sim10$ ns$^{-1}$ in the spherical case before diverging at larger radii due to the rapidly decreasing density. These profiles change as a function of time, but the snap-shots shown here provide an idea of the differences between the planar and spherical cases.

The sample profiles shown in Figure 7-11a and b were taken near the apparent onset time, at $\sim0.8$ ns for the spherical case and at $\sim1.5$ ns for the planar case. These plots demonstrate that near the onset time, the predicted growth rates of the fastest growing modes are $\sim2$ times faster in the spherical case than in the planar case, consistent with the observed difference in onset time. A given plasma profile provides a spectrum of ‘fastest growing modes’ throughout the plasma, though in both the spherical and planar cases these modes are predicted to be between $\sim200-300$ $\mu$m, again consistent with observations. To assess some aspect of temporal variance, an average wavelength $<\lambda_{MTI}>$ is defined by

$$<\lambda_{MTI}> = \frac{\sum_i \lambda_{MTI,i} \times \gamma_{MTI,i}}{\sum_i \gamma_{MTI,i}},$$

where each $\lambda_{MTI,i}$ is weighted by the associated growth rate $\gamma_{MTI,i}$ and is summed over the MTI-unstable region in the underdense corona at each time step. The results of these calculations for both spherical and planar simulations are shown in Figure 7-11c. The laser drives for both configurations are also shown (dotted) for reference and it is clear that $<\lambda_{MTI}>$ rapidly increases after the drive turns off due to temperature profiles flattening out as the plasma cools. These calculations suggest that MTI occurs early in the drive at smaller wavelengths, though this is not observed in experiments. Rather, a rapid transition ($\sim200$ ps) was demonstrated in both the spherical and planar experiments. Interestingly though, the measured characteristic size of cellular structures, $\sim210$ $\mu$m, crosses the spherical and planar curves in Figure 7-11c near the observed onset times for each case!

The MTI is the likely cause of the observed cellular features in planar experiments and filamentary field structures in spherical experiments. Many pieces of evidence have been shown that support this hypothesis, though the observed rapid onset of these field structures is not well understood. There has been no mention here of how the strength of MTI-generated B-fields varies as a function of wavelength and this is not discussed in the literature. If the proton radiographic technique implemented here is only sensitive to fields of this scale, then the smaller fields that exist at earlier times would not be detected. However, this is unlikely because the difference between the side-on (spherical) and face-on (planar) experiments provide different paths through these complex field structures and the spatial resolution in these experiments was $\sim50$ $\mu$m well below the predicted scale of fields earlier in time. A more likely cause is related to the complex evolution of these fields in a moving fluid and how fields generated in this plasma region can be quickly advected away from the source location. As illustrated in Figure 7-9 for the planar case, the advection
velocity magnitude is higher and the gradient steeper at earlier times which could stabilize the MTI at earlier times. These results have suggested the likely cause for the observed field structures and provided a direction for further theoretical work to address the complex spatial and temporal evolution of fields in these dynamic plasmas.
7.7 Summary

Proton and x-ray radiography experiments have been used to further investigate filamentary fields previously observed in laser-irradiated spherical targets. Planar experiments provided an axial view of the filamentary fields and showed coherent cellular field structures independent of initial surface conditions, suggesting field generation in the corona. Furthermore, filamentary fields observed in spherical experiments existed far from the ablation surface so must be generated in a region where they can be advected outwards. Through numerical calculations, the field advection velocity was shown to switch direction from inward to outward in the underdense corona because of the combination of fluid flow and the Nernst effect. These coherent fields structures were shown to dominate proton radiographs of planar foils at late times over other sources of electromagnetic fields.

Through a detailed discussion of various field-generating instabilities, it was shown that the likely cause for these fields is the magnetothermal instability. Numerical calculations based on DRACO simulations demonstrated that plasma conditions were laminar and dominated by field advection over diffusion in the corona, consistent with observations of coherent field features. The basic physics of the MTI was described and calculations of the fastest growing modes demonstrated that field structures at the observed scale size may be expected from this instability. Moreover, MTI-generated fields are created near the peak coronal temperature where they are advected outwards with the plasma. This instability occurs because of altered heat conduction in the corona and could therefore affect heating uniformity in directly-driven targets. These results have identified the likely source of coronal fields previously observed in directly-driven spherical targets. Initial calculations indicate consistency with experiments, and thus motivate further numerical and experimental exploration into field-generation by this instability.

7.7.1 Future Work

Flat-foil radiographs discussed here were not intended to investigate filamentary field structures, though proved to provide valuable insight into the source of these fields. Planar experiments were ideal for accurately measuring the size of filamentary fields under a single laser configuration. New experiments using different laser intensities could provide further modeling constraints based on differences in the observed scale size or onset time. Also, changing the foil material to Beryllium or silicon-doped CH could be empirically interesting to the inertial confinement fusion (ICF) program as these materials may be used in future ignition targets. However, proton radiography alone is insufficient for further investigation of these fields and coronal plasma conditions should be experimentally determined using, for example, Thomson scattering diagnostics. Coronal measurements would determine the plasma conditions where these fields occur and could be used to verify predictions.

Further theoretical work in field generation by MTI would be very useful to extend the discussion presented here. The analytic works by Ogasawara et al., Hirao et al., and Bissell et al. provide good starting points to a more complete model for MTI in these laser-produced plasmas. Field calculations that implement all relevant mechanisms, including plasma flow, the Nernst effect, and 3-D resistivity models, would provide more accurate spatial and temporal predictions of field-mode spectra. The calculations presented in Section 7.6 were done using the simpler theory from Tidman and Shanny and provided valuable information that further supports the hypothesis that the MTI is the cause for these field structures. However, these ‘simple’ calculations also demonstrated the complexity of
field generation by the MTI in these plasmas and motivate further numeric investigation with more advanced theory. The existing data sets provide various plasma conditions in different geometries that could be used to compare with new numerical calculations.

References


Chapter 8

Conclusion

Many years of work in the high energy density (HED) Physics Division at MIT have culminated in multiple publications of original research that was accomplished in collaboration with colleagues at several scientific institutions. Performing cutting-edge research in HED science requires access to the most sophisticated laser facilities in the world. Experiments discussed in this thesis utilized a novel monoenergetic backlighter system that, to-date, may only be used on the OMEGA laser system. This imaging system was conceived and developed by the HED Physics Division at MIT, but would not have been successful without the ability to test and characterize the CR-39 plastic nuclear track detectors on the MIT Linear Electrostatic Ion Accelerator (LEIA) system.

An experimental program to investigate the effects on proton-sensitivity in CR-39 due to prolonged exposure to high-vacuum environments was executed using the LEIA, as discussed in Section 4.3.3. It was found that vacuum exposure post-irradiation had no effect on the registration sensitivity of proton tracks. However, when left in vacuum for times $\gtrsim 16$ hours before irradiation, a strong reduction in proton sensitivity was observed due to the decreasing oxygen profile in the plastic. Furthermore, an as yet unexplained resurgence in proton sensitivity was shown to exist for extremely long vacuum exposure times greater than $\sim 100$ hours. However, CR-39 used in monoenergetic proton radiography experiments discussed here were typically exposed to high-vacuum for less than 1 hour before irradiation, well within the limits revealed by this study.

The exploding-pusher backlighter capsules used in experiments at OMEGA were filled with 18 atm of D$_3$He gas and imploded with up to 20 beams. The mainly shock-driven implosion compresses and heats the fuel to produce monoenergetic fusion-protons from the DD ($\sim 3.3$ MeV) and D$_3$He ($\sim 15.1$ MeV) reactions. Due to the nature of this type of backlighter, protons are emitted in an isotropic fashion. The uniformity of proton emittance from these capsules was investigated using multiple lines-of-sight to diagnose the yield as a function of angular position (global variance) and through ‘blank’ radiographs of the source itself (local variance), as discussed in Section 4.2.2. It was found that global variance for DD and D$_3$He protons was $\Sigma_{DD} \approx 16 \pm 7 \%$ and $\Sigma_{D_3He} \approx 26 \pm 10 \%$, respectively. Of direct importance to radiography experiments discussed herein, local variance for both DD and D$_3$He protons was shown to be less than a few percent of the mean for angular frequencies $\gtrsim 50 \text{ rad}^{-1}$. The accurate characterization of the backlighter source paved the way for benchmarking simulations.

\textsuperscript{1}Typical energies listed here are upshifted from the birth energy due to fusion production occurring during the laser drive whereby a net potential exists on the capsule.
An experimental modeling tool was developed\(^2\) using the Geant4 framework to simulate complete radiography experiments, as discussed in Section 4.4. Geant4 is an open-source library of functions written in C++ and provides the functionality necessary to define complex geometries and include different physics implementations. The code written to simulate radiography experiments was benchmarked against non-irradiated targets since the cold-matter (CM) approximation is the only form of Coulomb scattering physics currently implemented in the library. It was shown that for typical laser-irradiated CH foil plasma parameters, the CM approximation is accurate to \(\lesssim 5\%\) for proton scattering and to \(\lesssim 10\%\) for energy loss. In addition to modeling Coulomb interactions in matter, the Geant4 code also implements charged-particle tracking through arbitrarily defined electric and magnetic fields, which is necessary to interpret some proton radiography experiments.

Irradiated targets are always physically connected to a stalk structure held in place by a mechanical device that positions the target-stalk assembly. Monoenergetic proton radiographs were taken of stalks connected to spherical targets that were irradiated by 40 beams on OMEGA. When a target is irradiated, electrons are preferentially heated and some fraction of them escape the target-stalk system resulting in a net positive charge on the target assembly. This in-turn drives a current through the stalk to neutralize the charge imbalance. For the first time, return currents were measured\(^3\) from an irradiated capsule target at OMEGA. Using the Geant4 simulation to interpret proton radiographs, it was shown that the current increased from \(\sim 2\) to \(\sim 7\) kA during a picketed laser-pulse, as discussed in Chapter 5. Furthermore, it was observed that the current begins near the stalk surface and moves out due to the induced Coulombic explosion. These measurements provided important information regarding the stalk’s role in the overall OMEGA target-chamber circuit model.

The Rayleigh-Taylor (RT) instability arises in many physical systems, of specific interest here is when this occurs in a laser-produced plasma environment where separate electron and ion populations allow for magnetic field generation. The Biermann battery field-generation mechanism was investigated using laser-irradiated CH foils with pre-imposed 2-D ridge-like perturbations, as discussed in Chapter 6. A combination of monoenergetic-proton and x-ray radiography provided the necessary information to decouple the Coulomb-scattering and Lorentz-force contributions to observed proton deflections. Furthermore, x-ray radiographs were used to benchmark radiation-hydrodynamic simulations performed using the DRACO code. The first measurements\(^4\) of RT-induced magnetic fields due to the Biermann-battery indicate an increase from \(\sim 3\) to \(\sim 9\) T during linear growth. Moreover, it was suggested that diffusion effects play a non-negligible role in the magnetic field evolution\(^5\) through comparisons with post-processed DRACO simulations. Finally, it was conjectured that RT-induced magnetic fields during linear growth will not significantly affect electron thermal conduction in directly-driven targets due to the high collisionality near the ablation surface, where the fields are generated. However, later in time during nonlinear growth, RT-induced fields may be generated closer to the critical surface and some inhibition of thermal transport (from the classical value) may be expected.

When investigating RT-induced fields later in time, an early onset of 3-D field structure was observed. These fields were found to be analogous to previously observed filamentary structures around directly-driven spherical targets. Proton radiographs of planar foils showed the onset of cellular fields at \(\sim 1.5\) ns into the 2 ns drive pulse for all foils, irrespective of the initial surface perturbations, as discussed in Chapter 7. Corresponding x-ray radiographs demonstrated the expected RT-growth in foils with preimposed surface modulations, and did not indicate strong nonlinear features at times relevant to proton ob-
servations. This suggested that these cellular fields were generated away from the ablation front where initial surface perturbations have an effect on plasma conditions. Using both the planar and spherical data, a detailed discussion of various field-generating instabilities revealed that the magnetothermal instability (MTI) is the likely cause of these fields. The measured scale size of the cellular features, $210\pm30 \, \mu m$, was the same for all foils and did not measurably change in time, consistent with predictions from the fastest growing modes of the MTI. These results have identified the likely source of coronal fields previously observed in directly-driven spherical targets and thus motivate further investigation into field-generation by this instability.

Present day laser facilities have opened the doors for experimental investigation of a broad range of topics in HED science. High intensity and high power lasers of sub-micron-wavelength light allow access to extreme physical environments that previously only existed in the context of various astrophysical phenomena and stellar, or planetary, interiors. Creating new physical environments in the lab often necessitates the development of new diagnostic techniques. Monoenergetic proton radiography has been shown to be a useful system for diagnosing magnetic fields in a variety of HED-plasma environments and will be used in many other experiments in the future.

References


Appendix A

OMEGA Data Summary

Data discussed in this thesis were acquired on multiple shot days and experimental campaigns performed on the OMEGA laser. Each section in this appendix provides a summary table of OMEGA shots with important measurements listed for the specific campaign. Each of these data sets were discussed in previous chapters. Section A.1 covers the fluence uniformity of the monoenergetic-proton backlighter as discussed in Chapter 4. Section A.2 provides the shots used to investigate return currents in target stalks as presented in Chapter 5. Finally, Section A.3 shows a complete list of all foil radiographs generated for investigations into RT-induced fields as discussed in Chapter 6 and MTI-generated fields in Chapter 7.
## A.1 Proton-Fluence Uniformity

Table A.1: D³He-proton fluence uniformity summary. Yields inferred from each diagnostic, the average, and the global variance are in units of 10⁷. Local variance is in [%/sr].

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<th>CPS2 (H1)</th>
<th>WRF (KO1)</th>
<th>10×10 (TIM2) Average</th>
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<th>Local Var.</th>
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</table>

Table A.2: DD-proton fluence uniformity summary. Yields inferred from each diagnostic, the average, and the global variance are given in units of 10⁸. Local variance is in [%/sr].

<table>
<thead>
<tr>
<th>OMEGA Shot</th>
<th>CPS2 (H1)</th>
<th>7-cm (TIM3)</th>
<th>10×10 (TIM2) 10×10 Average</th>
<th>Global Var.</th>
<th>Local Var. (TIM2)</th>
<th>Local Var. (TIM3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51237</td>
<td>2.5</td>
<td>3.9</td>
<td>3.1</td>
<td>3.2</td>
<td>0.7</td>
<td>17.2</td>
</tr>
<tr>
<td>51238</td>
<td>1.9</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>0.2</td>
<td>17.2</td>
</tr>
<tr>
<td>51239</td>
<td>2.0</td>
<td>1.4</td>
<td>1.9</td>
<td>1.8</td>
<td>0.3</td>
<td>13.4</td>
</tr>
<tr>
<td>51240</td>
<td>1.4</td>
<td>1.6</td>
<td>1.6</td>
<td>1.5</td>
<td>0.1</td>
<td>22.5</td>
</tr>
<tr>
<td>51241</td>
<td>2.6</td>
<td>2.1</td>
<td>1.4</td>
<td>2.0</td>
<td>0.6</td>
<td>22.9</td>
</tr>
<tr>
<td>51242</td>
<td>1.8</td>
<td>2.4</td>
<td>1.9</td>
<td>2.0</td>
<td>0.4</td>
<td>65.0</td>
</tr>
<tr>
<td>51243</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>51244</td>
<td>1.4</td>
<td>2.0</td>
<td>1.7</td>
<td>1.7</td>
<td>0.3</td>
<td>10.1</td>
</tr>
<tr>
<td>51246</td>
<td>8.7</td>
<td>1.1</td>
<td>0.9</td>
<td>0.9</td>
<td>0.1</td>
<td>18.1</td>
</tr>
<tr>
<td>51247</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.4</td>
<td>0.1</td>
<td>12.6</td>
</tr>
<tr>
<td>51250</td>
<td>1.2</td>
<td>1.7</td>
<td>1.4</td>
<td>1.4</td>
<td>0.2</td>
<td>20.2</td>
</tr>
</tbody>
</table>
A.2 Stalk-Field Characteristics

Table A.3: The following list provides a summary of all proton radiographs taken of stalks holding targets that were irradiated by a picketed (RD1501p) laser pulse.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>51244</td>
<td>1.1</td>
<td>2</td>
<td>90</td>
<td>40</td>
<td>1.15</td>
<td>0.3</td>
<td>450</td>
<td>70</td>
<td>14.3</td>
</tr>
<tr>
<td>51246</td>
<td>1.9</td>
<td>5</td>
<td>610</td>
<td>280</td>
<td>0.76</td>
<td>1.4</td>
<td>320</td>
<td>100</td>
<td>0.3</td>
</tr>
<tr>
<td>51247</td>
<td>2.4</td>
<td>7</td>
<td>675</td>
<td>350</td>
<td>0.74</td>
<td>1.3</td>
<td>250</td>
<td>100</td>
<td>0.2</td>
</tr>
<tr>
<td>51250</td>
<td>3.4</td>
<td>6</td>
<td>800</td>
<td>600</td>
<td>0.70</td>
<td>1.5</td>
<td>310</td>
<td>100</td>
<td>0.1</td>
</tr>
</tbody>
</table>
### A.3 Planar-Foil Experiments

Table A.4: The following list provides a summary of all proton radiographs of CH foils.

<table>
<thead>
<tr>
<th>Foil Type</th>
<th>OMEGA Shot-Port</th>
<th>Timing [ns]</th>
<th>rms [%]</th>
<th>Single Mode ( \alpha_{\text{rms}} )</th>
<th>Dominant Scale Size [( \mu \text{m} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>Flat</td>
<td>50610-T4</td>
<td>1.23</td>
<td>0.191</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50610-T6</td>
<td>1.37</td>
<td>0.206</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Flat</td>
<td>50607-T4</td>
<td>1.77</td>
<td>0.246</td>
<td>252±17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50611-T4</td>
<td>1.83</td>
<td>0.351</td>
<td>200±05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50611-T6</td>
<td>2.37</td>
<td>0.389</td>
<td>259±14</td>
</tr>
</tbody>
</table>

120 \( \mu \text{m} \) (2-D)

| Flat      | Flat            | 50606-T4    | 1.20    | 0.127                         | 1.5±0.4                          |
|           |                 | 49109-T3    | 1.24    | 0.102                         | 2.4±0.1                          |
|           | Flat            | 61728-T6    | 1.27    | 0.121                         | 1.5±0.6                          |
|           |                 | 61721-T6    | 1.33    | 0.100                         | 1.9±0.2                          |
|           | Flat            | 61721-T4    | 1.34    | 0.100                         | 2.9±0.3                          |
|           |                 | 50606-T6    | 1.35    | 0.146                         | 4.1±1.3                          |
|           |                 | 49111-T3    | 1.51    | 0.282                         | 7.6±1.7                          |
|           |                 | 61726-T4    | 1.68    | 0.292                         | 185±10                           |
|           |                 | 50607-T6    | 1.82    | 0.287                         | 225±15                           |
|           |                 | 49112-T3    | 1.91    | 0.352                         | 185±17                           |
|           | Flat            | 61727-T4    | 1.96    | 0.355                         | 204±09                           |
|           |                 | 50608-T6    | 2.37    | 0.460                         | 213±23                           |

180 \( \mu \text{m} \) (2-D)

| Flat      | Flat            | 61728-T4    | 1.43    | 0.160                         | 5.8±0.9                          |
|           |                 | 61724-T4    | 1.52    | 0.228                         | 227±12                           |
|           | Flat            | 61724-T6    | 1.54    | 0.212                         | 222±07                           |
|           |                 | 61723-T4    | 1.75    | 0.243                         | 216±01                           |
|           | Flat            | 61723-T6    | 1.77    | 0.234                         | 213±03                           |
|           |                 | 61726-T6    | 2.04    | 0.385                         | 235±20                           |
|           | Flat            | 61727-T6    | 2.33    | 0.425                         | 260±30                           |

115 \( \mu \text{m} \) (3-D)

| Flat      | Flat            | 50612-T4    | 1.22    | 0.191                         | -                                |
|           |                 | 50612-T6    | 1.36    | 0.206                         | -                                |
|           | Flat            | 50608-T4    | 1.83    | 0.312                         | 173±08                           |
|           |                 | 50613-T4    | 1.85    | 0.344                         | 269±19                           |
|           | Flat            | 50613-T6    | 2.40    | 0.365                         | 171±06                           |
Appendix B

Analysis Details for CR-39 Vacuum Experiments

Experiments were performed to investigate the effects prolonged vacuum exposure may have on the proton response in CR-39. This appendix describes the analysis procedure for some of this data (see Table B.1 for reference shot numbers). Proton signal tracks were separated by properly defined contrast and diameter limits. The relative size and location of different energy and vacuum exposure windows are obviously known from the experimental setup and filtering schemes used. Each window is analyzed separately and contrast/diameter limits set individually. After signal tracks are pulled from intrinsic noise, the diameters are binned and fit to Gaussians for an accurate measure of the average diameter.

Table B.1: LEIA shot numbers for data used in the study to characterize the effects of vacuum exposure on CR-39 proton response.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Shot Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 4-10,4-11 - 0hr</td>
<td>2009102201</td>
</tr>
<tr>
<td>Figure 4-10 - 3/24/67 hr</td>
<td>200908 0601/0602/0701</td>
</tr>
<tr>
<td>Figure 4-11 - 1 hr</td>
<td>2009102201</td>
</tr>
<tr>
<td>Figure 4-11 - 3 hr</td>
<td>2009120201</td>
</tr>
<tr>
<td>Figure 4-11 - 6 hr</td>
<td>2009120301</td>
</tr>
<tr>
<td>Figure 4-11 - 16 hr</td>
<td>2009120302</td>
</tr>
<tr>
<td>Figure 4-11 - 68 hr</td>
<td>2009111601</td>
</tr>
<tr>
<td>Figure 4-12 - Sample 1/2</td>
<td>20110420 07/01</td>
</tr>
<tr>
<td>Figure 4-13 - E\leq3 MeV/E\geq5 MeV</td>
<td>201004 1501/0201</td>
</tr>
<tr>
<td>Figure 4-14 - Sample 1/2</td>
<td>2010 092801/100701</td>
</tr>
</tbody>
</table>
Figure B-1: Contrast vs. diameter contour plots are shown for different incident proton energies used for the ‘0 hrs’ data set from Figure 4-11. Protons are ranged down through Aluminum filters to the energies labeled a)-h). The data have been background subtracted, however intrinsic noise tracks still dominate the low contrast, low diameter areas. The proton track peak moves from $\sim 20 \mu m$ in a) at $\sim 0.5$ MeV to $\sim 3 \mu m$ in h) at $\sim 8.8$ MeV. (A DD-triton peak is also observed in e) because the filter is thin enough to allow tritons through). (i) Gaussian fits (lines) are shown along with the data used for the fit (black points). For the baseline vacuum exposure, Gaussian fits provide good representations of proton track distributions.

Figure B-1 illustrates contour plots in contrast-diameter space for proton tracks behind different filter windows at the baseline (‘0 hr’) vacuum exposure shown in Figure 4-11. Here, proton tracks have high contrast and noise dominates lower contrast. However, proton tracks on both sides of the probed energy range get smaller in diameter and lower in contrast, and in extreme cases are not separable from the intrinsic noise. It is clearly seen in Figure B-1a-h that the mean track diameter changes as a function of mean incident energy, and the shape of the diameter distribution is evolving. Corresponding proton track diameter distributions are shown in Figure B-1i with Gaussian fits.
Figure B-2: Contrast vs. diameter contour plots are shown for the ‘68 hrs’ data set from Figure 4-11. Lower energy windows show a large reduction in measured proton track diameters. Tracks in the high-energy windows are not measurable as shown in (g) and (h). (i) For energy windows ≤2.3 MeV, track diameter distributions are still well approximated by Gaussians. At higher incident energies (≥3 MeV), diameter distributions become peaked towards smaller diameters. For the two highest energies, tracks were not measurable.

Similar contour-diameter plots and corresponding diameter distributions are shown for the ‘68 hrs’ data set from Figure 4-11 in Figure B-2. It is easily seen that prolonged exposure to high vacuum before irradiation has affected the CR-39 response to incident protons. Low energy protons (≤2.3 MeV) are observed to have a large reduction in mean diameter with higher energies (≥3 MeV) beginning to blend in with intrinsic noise tracks. As the incident energy is raised, it is increasingly difficult to distinguish data from noise and eventually tracks are no longer detected, as seen in Figure B-2g-h.
Figure B-3 shows sample contrast versus diameter contour plots from data shown in Figure 4-14 for Sample 1. In Figure B-3a the \( \sim 2.9 \) MeV proton peak is clearly seen at \( \sim 14 \) \( \mu m \), the broader \( \sim 0.4 \) MeV triton peak at \( \sim 20 \) \( \mu m \), and intrinsic noise tracks are dispersed in the lower left at low contrast. Proton tracks are reduced in size as the vacuum exposure increases, as seen in Figure B-3b-f.

Proton and triton tracks follow similar evolutions with increased vacuum exposure time. However, the triton peak becomes much lower in contrast and begins to blend in with intrinsic noise tracks, while the proton tracks continue to stay well separated at high contrast. It is easily seen in Figure B-3f that at \( \sim 125 \) hr of vacuum exposure before irradiation, proton tracks have become larger than the previous \( \sim 100 \) hr exposure time and the tritons have begun coming out of the intrinsic noise level. This recovery in CR-39 sensitivity is not well understood, but is absolutely unambiguous in the data for both CR-39 samples for both triton and proton tracks.

For simple analysis, proton tracks are isolated from intrinsic noise using practical diameter and contrast limits. The resultant diameter distributions are well represented by Gaussians and fit accordingly. These means are reported in the figures of this paper. Sample Gaussian fits to proton data from are illustrated in Figure B-3g. Black points correspond to track distributions measured at each individual vacuum exposure time. Gaussian fits are shown for all six exposure-times from Figure B-3. It is easily seen that the diameter distributions are Gaussian and the uncertainty in the fits, as calculated by the 95% confidence bounds, are quite small (\( \leq 0.05 \) \( \mu m \)).
Appendix C

Geant4 Framework Overview

The Geant4 toolkit is an open-source Monte Carlo framework written in C++ and is used for simulating particle interactions through arbitrary geometries defined by the user. The code can be readily downloaded from the Geant4 website which provides the code and all needed libraries, as well as a suite of examples that address various ‘typical’ problems. In the context of work discussed in this thesis, a simulation tool was written to emulate proton radiographs of different types of targets where proton trajectories are tracked through electromagnetic fields and scattered in matter (using the cold-matter (CM) approximation, see Section 4.4.1). This tool was discussed thoroughly in Section 4.4 that covered physics implementation and benchmark experiments. This appendix is meant to serve as a rough user’s guide to the proton radiography code and provide some qualitative insight to the inner workings of the Geant4 framework.
C.1 Geant4 Generic Workflow

Geant4 is a complex object-oriented code and many of the intricacies will not be discussed here, but details on many standard classes can be found in the Geant4 manual and reference guide. Every Geant4 program is different and is specifically written to address a particular problem. The programmer can choose what physics packages to implement, how the geometry is defined, what types of particles to track, how to track them, and how the output is formatted. With so much freedom in defining the problem of interest, some basic features are required for every simulation.

The top level object that includes the main() function must instantiate a few basic classes. First, the RunManager() class handles all things related to tracking a virtual particle through different steppers, physics implementations, and essentially manages everything related to a run. The RunManager() has a default class file within the Geant4 library. A user-defined ‘DetectorConstruction’ class, that inherits from G4VUserDetectorConstruction, must be instantiated in the main() function. This class file contains all things related to defining the problem geometry, including materials and objects within the problem domain. The last instance that must be defined for a simulation to compile is the ‘PhysicsList’. This class can be user-defined and contain as many (or as few) physics packages as needed. However, there are a number of default physics lists which contain many of the ‘typical’ physics packages, and these are recommended for most users. These three classes are necessary for all simulations, but at this point, no rules for creating or tracking particles have been made.

Particles are generated and tracked through class objects typically involving the word ‘Action’ in their names. The user-defined object that creates particles typically inherits from the G4VUserPrimaryGeneratorAction class. Standard particles are defined in the G4ParticleTable and are created with a chosen energy and vectorized momentum using the G4ParticleGun class. The particle gun can ‘shoot’ multiple particles of a single, or varying, type, energy, and momentum. After adding all the desired particles to the particle gun for the current event, the GeneratePrimaryVertex() function must be called from the particle gun object within the GeneratePrimaries() function of the ‘PrimaryGeneratorAction’ class. At this point, it is important to define what ‘event’ means in the Geant4 framework.

An ‘event’ in Geant4 refers to an entire execution of the simulation. This could include one particle, or many particles. If there is a desired distribution of particles from the source, the entire distribution of particles could be defined in a single event, or multiple events could be run to simulate one particle that samples the desired distribution. These are the two extremes, of the continuum of possible source definitions when sampling distribution functions. The primary accessibility difference between these two implementations is that exact particle and track information is lost after each event. There is also a computation-time tradeoff between the two methods, in that the more particles and tracks that must be kept for a given event takes up memory and at some point can slow down the simulation. Though, on the other hand, there is a finite amount of overhead that must be accounted for on each event, so a single particle per event may not be optimal. Using typical radiography geometries on my personal machine, the optimum number of protons was found to be 5000-10000 per event. More particles were run in a single simulation by executing multiple events and track information was recorded at the end of each event.

The user defined EventAction class object inherits from the G4UserEventAction class. This object has two primary user-defined functions, BeginOfEventAction() and EndOfEventAction(). These functions are run before, or after, the ‘event’ and can be used to access and record specific track information stored in the so-called sensitive detector.
Particle tracking occurs throughout a particle’s entire trajectory, but information is only accessible when passing through a volume that has been identified as a sensitive detector. This is done in the ‘DetectorConstruction’ class by using the `SetSensitiveDetector()` function of a `G4LogicalVolume` object. This function takes an argument of a separate user-defined ‘Tracker’ class object that inherits from `G4VSensitiveDetector`. The `ProcessHits()` function of this object is called during every step taken in a sensitive volume. This function has access to the user-defined ‘Hit’ class, that inherits from `G4VHit`, and contains information about the current state of the particle: position, momentum, energy, etc. This hit is stored in a ‘HitCollection’ vector that is also defined in the ‘Hit’ class. This vector of hits can be accessed in the `EndOfEventAction()` function in the `EventAction` object, particle information can be retrieved, and can be subsequently stored for use in post-processing.

This section has outlined some of the main classes necessary for a useful Geant4 simulation. However, the simplest way to write a new simulation is to start with an existing example code and adapt accordingly. Many steps have been left out of this brief overview, but should provide the reader with some useful things to look for when going through a sample code before adapting it. The ‘N02’ example was used as a starting point for the proton radiography simulation, but this is unrecognizable in its current state.
C.2 Running the Proton Radiography Simulation

The proton radiography simulation written using the Geant4 framework implements realistic source and detector geometries. It has gone under heavy modification over the past 6 years, and will continue to need it as experimental objectives evolve, thus the object-oriented framework is ideal. The code has been written in a modular form so that new radiography subjects, or proton sources can be easily implemented by writing new modules. However, the code does not have to be recompiled if running simulations using existing subjects and sources because all input parameters are read from text files that the user can change between runs.

This simulation is done in ‘batch’ mode, there is not an interface command line as with many Geant4 simulations. The code searches for input files in the PRSim_X/Param_Files/CurrSetup/ folder at the beginning of the simulation. There are 5 basic components that are defined in this folder in multiple text files: the run file (RDT), source geometry, subject geometry, detector pack, and output file (CPSA) information. Each of these will be discussed in the following paragraphs. There are a few setup folders containing text files of default problems for reference.

The RDT file allows the user to define basic information like the simulation name, the type of source, and the number of subjects to be created. Here the ‘source type’ is either ‘Capsule’ or ‘Beam’. In this file the user defines the ‘number of objects’ and this must correlate with the following list of ‘object types’. This list must be composed of the file names of the subjects with the order in which they are created. For example, the ‘Default_25CH-Mesh_CurrSetup’ folder lists 3 objects: SingleFoil_1, Mesh_2, and SingleFoil_3. This will simulate a foil, a mesh, and another foil based on input parameters listed in those files. If an electromagnetic field is also to be implemented, it must be called out in this file and the object with which it is associated. The number of events is also defined here that indicates the number of times particles will be created based on the ‘SourceParticles’ file.

The ‘Capsule’ source type is useful for simulating the monoenergetic backlighter used in many proton radiography experiments. In this case, two files must be defined. The ‘CapsuleSource’ file is used to define the shell and gas in the source. This implementation would be useful when trying to simulate interactions with the source capsule, however this feature has not been used in most cases and the ‘srcShellMater’ is typically set to ‘Vacuum’ to avoid any energy loss of source particles.

The ‘CapsuleSourceParticles’ file defines actual source particle characteristics. Particles are emitted from a spatially Gaussian source with a thermalized velocity distribution with a defined temperature. In this file the number of particle types (n) must be called out. If n is greater than one, each of the subsequent parameters must have n defined values. Three particle types are currently supported: D³He protons and alphas, and DD protons. The difference in implementation occurs in the definition of the doppler width based on the fusion product of interest and defined ion temperature. The source size is also defined here by the so-called $1/e$ radius. Finally, the $4\pi$ capsule yield is given here and multiples of this are made through multiple events as defined in the RDT. The program does not simulate particles in all directions, it uses the subject and detector geometry to determine a cone angle and simulates the correct fraction of the isotropic yield in the proper direction.

There are 5 subjects available to radiograph: Capsule, Cylinder, Mesh, RTFoil, and SingleFoil. To go along with these, there are two types of fields implemented: ChargeShell and Wire. Multiple objects can be implemented in a single simulation, as demonstrated by the ‘Default_25CH-Mesh_CurrSetup’ problem, however care must be taken not to allow vol-
C.2. RUNNING THE PROTON RADIOGRAPHY SIMULATION

...umes to overlap. Each type of object has a specific list of parameters that must be defined. The ‘Default Copies for Backup’ folder contains versions of all object and field types that have been implemented. Needed files should be copied and put into the ‘CurrSetup’ folder for the current simulation and the parameters adapted for the problem to be addressed. The position of an object is obviously very important and, unless otherwise stated, always refers to the location of the centroid of the object relative to the origin (particle source) of the simulation. All object and field parameter definitions are given a short clear description (including units) above the line where they are to be defined. Geant4 can implement materials of arbitrary atomic composition, but this simulation contains a MaterialsConstruction class object that has all the defined available materials. If a new material is needed, this file can be opened and changed accordingly.

The detector implementation is based on the standard CR-39 proton radiography pack. This consists of two CR-39 pieces, one for DD-protons (Bert) and one for D\(^3\)He-protons (Ernie), with associated filtering for each piece. The ‘DetectorPack’ parameter file contains information for defining these four objects. Both pieces of CR-39 are defined as sensitive detectors in the ‘GeometryConstruction’ class. At the end of each event, particles with energies less than 10 MeV are recorded and an artificial scan file is created.

The ‘CPSA_Info’ text file contains all the needed information to create the artificial scan file for both the Bert and Ernie pieces. In most instances, this file will not need to be altered, unless the goal is to emulate exact parameters of a specific scan system. The current file uses parameters taken from one of the scan systems used in the past. In order to be somewhat realistic, a standard diameter-versus-energy curve can be used to convert proton energies to approximate diameters. This assumes a 6 hour etch time, and is meant only to give rough numbers for diameter distributions and should not be directly compared to actual diameter distributions; this is beyond the scope here. The primary goal with this output is to be able to use the standard analysis program to post-process simulated proton radiographs. In this file, there is also a flag that can be set to output a basic text file with all the particle information and this could be post-processed using another program.

Files are saved to a folder with the simulations name into the PRSim_X/Saves/ folder for later use. A copy of the input parameters used for the problem setup will be saved with the artificial scan files. Another useful file that is created in the main PRSim_X/ folder after the simulation is finished is the ‘G4Data0.heprep’. This file is meant to be used with the Wired program that must be installed separately. The script ‘RunWired.sh’ will need to be adapted to accommodate where the Wired program is located, but can be used to check the problem geometry in 3-D using a nice user interface. This is not necessary to run the code, but is a useful debugging tool to verify the geometry.

The main objective of this section was to qualitatively describe how one could use the current implementation of the PRSim_X code to simulate monoenergetic proton radiography experiments. However, it must be clearly noted that this simulation is not ‘user friendly’ and should not be used without understanding the code! The best way to do this is to go through it, and see how the actual implementation is done. I have outlined the basics of running the program without changing the source, but have left out many details that will really only be understood after following the code flow. If a new material or object is needed to accommodate new experimental goals, the source will need to be changed and recompiled.
C.2.1 Adding a New Radiography Object

The PRSim\_X code was written with modularity in mind, specifically with respect to radiography subjects. The source and detector pack can stay essentially unchanged, but new experiments can have various objects of interest. The ‘DetectorConstruction’, as discussed in Section C.1, in this Geant4 implementation is done with the PRSimGeometryConstruction class. This object is quite simple, it simply calls ‘Construction’ objects for the particle source, the detector pack, the electromagnetic fields, and for objects. All of the standard code seen in the Geant4 examples within the ‘DetectorConstruction’ class are modularly contained in each of the different ‘Construction’ objects. The ‘Construction’ objects are all similar in framework, but this section will focus on the ObjectConstruction class and the interested reader can view other ‘Construction’ objects as needed.

All radiography object classes inherit from ObjectConstruction and have two primary functions, ReadInputFile() and Construct(). The ReadInputFile() function is essentially a parser for the text file associated with this type of object. This is different for every object, but many of the algorithms already written can be copied from already written object classes for reading in doubles, integers, strings, etc. The Construct() function contains all the geometry definitions needed for the radiography object.

Any volume defined in the Geant4 framework must have three different class instances associated with it. The so-called ‘solid’ class inherits from G4CSGSolid and is the basic class that defines the geometric parameters of the volume. Geant4 has many predefined solid classes and can be referenced from the Geometry chapter in the developers guide. Next, the ‘logical volume’ is defined by instantiating a G4LogicalVolume using the solid object as an argument. Lastly, the so-called ‘physical volume’ is created by the G4VPhysicalVolume class using the logical volume as an argument. The physical volume defines the location of the logical volume centroid relative to the ‘parent’ logical volume. The volume may also be rotated relative to the ‘parent’ volume when instantiating the physical volume.

Many of the subtle details of volume instantiation order is already accounted for in the PRSim\_X code. For example, the ‘parent’ logical volume for all objects in a single simulation is defined in the ObjectConstruction class. This logical objectBox must be defined before the child objects that reside within. This is all accounted for in the ObjectConstruction class and algorithms have been defined for each type of object to make sure that the object box is large enough to accommodate the objects going into it. When a new radiography object is implemented, a corresponding case for that object must be added to the ObjectConstruction::ConstructObject() function.

This section has provided a brief overview of changes to the source code that must be made to add a new object class to the PRSim\_X program. The simplest way to begin this process, is to copy an existing class file and adjust accordingly. If new sources or field structures are needed, a similar procedure for adjusting the SourceConstruction and EMFieldConstruction classes would be required. For a different detector geometry, however, a new implementation of the DetPackConstruction class would be needed. After any changes are made to the source code, it must be recompiled before running the program.
Appendix D

Useful Dimensionless Parameters

Plasmas exist under varying environmental conditions and dimensionless numbers help to compare important physics processes in different regimes. Figure D-1 illustrates how some important dimensionless parameters vary in a sample laser-produced plasma. The magnetic Reynolds ($Re_m$), Péclet ($Pe$), Reynolds ($Re$), Lundquist ($S$), and Schmidt ($Sc$) numbers are shown for a ‘typical’ plasma environment taken from DRACO calculations discussed in Chapter 6. Table D.1 provides a summary of these parameters and typical values are given for the sample plasma environment.

![Figure D-1: (a) Simulated DRACO profiles discussed in Chapter 6 used to demonstrate how some dimensionless numbers vary in these types of plasmas. (b) Sample profiles of dimensionless numbers in a realistic laser-produced plasma environment. Values for the Lundquist number were calculated for a 1 T field strength.](image-url)
Table D.1: Some useful dimensionless numbers in laser-produced plasmas with the following parameters: $V$ is the fluid velocity, $L$ is the relevant scale length, $\nu$ is the viscosity, $\alpha$ is the thermal diffusivity, $D$ is the mass diffusion coefficient, $V_{\text{adv}}$ is the magnetic field advection velocity, $D_m$ is the magnetic field diffusion coefficient, and $V_A$ is the Alfvén speed.

<table>
<thead>
<tr>
<th>Number</th>
<th>Equation</th>
<th>Description</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds</td>
<td>$Re = \frac{VL}{\nu}$</td>
<td>Ratio of inertial to viscous forces and determines flow conditions: laminar for $Re \lesssim 2300$, turbulent for $Re \gtrsim 4000$</td>
<td>$10^1 - 10^3$</td>
</tr>
<tr>
<td>Schmidt</td>
<td>$Sc = \frac{\nu}{D}$</td>
<td>Ratio of momentum to mass diffusivity and determines the effective mass transfer mechanism: diffusion for small Sc, convection for large Sc</td>
<td>$10^{-1} - 10^2$</td>
</tr>
<tr>
<td>Péclet</td>
<td>$Pe = \frac{VL}{D}$</td>
<td>Ratio of advection to diffusion of mass and determines dominant mechanism: advection for large Pe, diffusion for small Pe</td>
<td>$10^2 - 10^4$</td>
</tr>
<tr>
<td>Magnetic Reynolds</td>
<td>$Re_m = \frac{V_{\text{adv}}L}{D_m}$</td>
<td>Ratio of magnetic advection to diffusion and determines dominant mechanism: advection for large $Re_m$, diffusion for small $Re_m$</td>
<td>$10^2 - 10^4$</td>
</tr>
<tr>
<td>Lundquist</td>
<td>$S = \frac{V_A L}{D_m}$</td>
<td>Ratio of Alfvén wave propagation velocity to magnetic field diffusion speed and determines if waves propagate before fields diffuse out</td>
<td>$10^{-2} - 10^1$</td>
</tr>
</tbody>
</table>
Appendix E

Basics of Plasma Bubbles

When high-intensity lasers irradiate a solid target, the surface is rapidly ionized and the hot plasma expands outwards. In the case of an irradiated flat foil, as shown in the schematic on the left of Figure E-1, an expanding plasma bubble is formed. Perpendicular temperature and density gradients near the bubble edge generate strong (~1 MG) azimuthal magnetic fields as indicated. Outwardly directed electric fields are also created near the bubble edge due to the electron pressure gradient. Monoenergetic-proton deflectometry has been previously used to directly detect and diagnose these field structures.

Of interest to work discussed in this thesis, are spontaneous electromagnetic fields generated by the plasma within the expanding bubble as was discussed in Chapters 6 and 7. To minimize the effects due to the bubble fields, proton probing of the plasma occurred

![Figure E-1](image_url)

Figure E-1: (a) A schematic of bulk electromagnetic fields generated by an expanding plasma bubble. Azimuthal B-fields (solid red) are concentrated at the outer edges of the bubble and there is a net E field (short dashed blue) pointing outwards. (b) Sample proton deflections (long dashed green) due to the plasma-bubble fields.
after the bubble had sufficiently expanded, such that the bubble edges were away from the region of interest. This necessary delay is dependent on the expansion velocity of the bubble, that is a function of the foil material and laser parameters. Additionally, directionality of protons relative to the field structures plays a role.

There are two orientations illustrated in Figure E-1 where the top (defocusing) configuration is preferable when studying fields within the plasma bubble. In this geometry protons deflected by the bubble fields are directed away from the central region, whereas if protons were incident from the opposite direction, the azimuthal B fields would deflect them inwards. When the ‘focusing’ geometry is used, observable structures within the bubble may be potentially contaminated, as demonstrated by Petrasso et al. and shown in Figure E-1. This problem can, to some extent, be alleviated by allowing sufficient time for bubble expansion, though this time is longer than that of the ‘defocusing’ configuration. In planar experiments discussed in this thesis the ‘defocusing’ configuration was used and all sample times were $\gtrsim 1.2$ ns giving the bubble sufficient time to expand.

References


Appendix F

A Shaped OMEGA Laser Spot

In many laser-matter interactions it is preferable to have a laser beam with a flat intensity profile. At OMEGA, beam profile shaping is achieved using distributed phase plates (DPPs). An empirical discussion is given here of the so-called super-Gaussian (SG) phase plates that are relevant to the work described in Chapters 6 and 7. In these experiments, 12 OMEGA beams were overlapped to provide an average intensity of $\lesssim 4 \times 10^{14}$ W/cm$^2$. These beams were incident onto a flat foil from different angles so that the beam at the target, or equivalent target-plane (ETP), is projected onto a flat surface. The multiple beam configuration was not the same across different shot days, resulting in small beam projection variations, though the nominal intensities were constant on all experiments and blow-off plasma tends to smooth out these variations. This appendix provides empirical information on SG4-shaped beam profiles and demonstrates some of the realistic deviations that are expected from the ideal configuration.

The ultraviolet (UV) ETP image of an OMEGA laser beam shaped by an SG4 DPP is shown at best focus in Figure F-1a. This data was taken using beam 46 from shot 61335 and delivered $\sim 427$ J of energy in a 1 ns pulse. Smoothing by spectral dispersion (SSD)$^1$ and distributed polarization rotator (DPR)$^2$ were also implemented, analogous to the configuration in all flat-foil experiments. The 4 designation in ‘SG4’ indicates the nominal power $n$ of the super-Gaussian function. To quantify the laser profile, an arbitrary 2-D super-Gaussian function was fit to the intensity image using

$$I(x, y) = I_0 \exp \left[ -\left( \frac{(x - x_0)^2}{2\sigma_x^2} + \frac{(y - y_0)^2}{2\sigma_y^2} \right)^{n/2} \right],$$

where $x_0$ and $y_0$ are the 2-D spatial offsets for the coordinates system, $\sigma_x$ and $\sigma_y$ are the 2-D standard deviations, and $I_0$ is the peak intensity. The best fit using this form gave $\sigma_x \approx 255$ $\mu$m and $\sigma_y \approx 247$ $\mu$m indicating good circularity to within $\sim 3\%$. The SG power was found to be $n \approx 4.3$ and this fit resulted in an R-squared value of 0.994 indicating very good agreement with the empirical data. A synthetic laser spot was generated using this functional form and is shown at the bottom of Figure F-1a on the same intensity scale as the data and demonstrates good agreement. Normalized radial intensity profiles that are shown in Figure F-1b illustrate that the asymmetry is minimal.

The $r_{1/e}$ (long dash) is shown in Figure F-1b for reference using an average radial profile and was found to be $\sim 350$ $\mu$m. It is important to note that the definition of the laser spot ‘edge’ is somewhat ambiguous. For the purposes of characterizing the uniformly
irradiated region, the one-over-e radius is used here. However, the exact value chosen for a single beam is not critical because when multiple beams are overlapped and are incident at different angles, the effective laser spot can be altered as shown in Figure F-1c. Moreover, the blow-off plasma decouples the drive beams from the ablation surface within the first \( \sim 50-100 \) ps of irradiation which further blurs and smooths the irradiation profile.\(^3\) Beams are obliquely incident onto the foil such that the distance to the projection plane (the foil) varies within the laser spot and is dependent on the incident angle.

In flat-foil experiments discussed in Chapters 6 and 7, the maximum angle of incidence for a single beam is 58.8°. Due to the ‘soccer ball’ configuration of beams on OMEGA, along a specified axis, beams may be split into three ‘cones’ characterized by a cone half-angle: Cone-1 is at 23.2°, Cone-2 is at 47.8°, and Cone-3 is at 58.8°. Neglecting converging and diverging effects near the location of best focus from the F/6.7 lenses, the effective increase in spot size can be (over) estimated due simply to the projection (1/cosine effect) of the spot. These calculations are summarized in Table F.1, but it is emphasized that

<table>
<thead>
<tr>
<th>Beams</th>
<th>Radial Enhancement</th>
<th>Areal Enhancement</th>
<th>Intensity Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone-1</td>
<td>( \sim 1.1 )</td>
<td>( \sim 1.2 )</td>
<td>( \sim 0.83 )</td>
</tr>
<tr>
<td>Cone-2</td>
<td>( \sim 1.5 )</td>
<td>( \sim 2.2 )</td>
<td>( \sim 0.44 )</td>
</tr>
<tr>
<td>Cone-3</td>
<td>( \sim 1.9 )</td>
<td>( \sim 3.7 )</td>
<td>( \sim 0.27 )</td>
</tr>
</tbody>
</table>
Figure F-2: (a) Four SG4 laser spot images at different focal positions. Here, a negative focal distance indicates a position that is past the location at best focus, such that the beam diverges with distance. The top row contains raw spot images and the second row shows the super-Gaussian fit to each laser spot. For each set of images the color scale is the same, but the scale changes between focal positions (columns). (b) Peak-normalized radial lineouts of the super-Gaussian fits for each of the images in a). Similar images (c) and radial plots (d) are shown for a beam with no phase plate. Best focus spots are not available due to diagnostic damage concerns. The 1-D fit profiles in d) may appear to have good flat-top quality, but this ignores the high-intensity speckle that is of greater concern in many experiments and is apparent by the color schemes in the images of c).

these are over estimates. Smoothing of the intensity profile caused by the plasma mitigates these seemingly extreme effects after the first ~100 ps. The beam configuration shown in Figure F-1c used 4 beams from each of the 3 cones, assuming only projectional effects would predict an unrealistic intensity reduction of ~50%! It is very difficult to accurately calculate the effective laser intensity under these conditions without numerical simulations. In this thesis, the nominal laser energy and pulse duration is used with a spot diameter of 700 µm and the intensity is typically quoted as $\lesssim 4 \times 10^{14}$ W/cm². In these planar experiments, it is assumed that the center region is uniformly accelerated. This is confirmed a posteriori through comparisons of x-ray radiographs with simulations, as the complex heat transfer mechanisms involved are not easily used to estimate the profile. Uniform acceleration, to first order, originates from the relatively flat intensity profiles provided by the overlapping, SG4-shaped laser beams.

The detailed profile shape, as well as the effective spot size, is a function of the focal position. Figure F-2 illustrates the defocusing effect (negative offset) on SG4-shaped beams.
Each data-and-fit image pair in Figure F-2a is normalized to the peak intensity in the data image. These images demonstrate that high-intensity speckles are prominent as the beam defocuses and lower-amplitude, longer-wavelength modes are also more apparent. Radial profiles of the best SG fits for each of the images is shown in Figure F-2b and clearly illustrates the defocusing effect on the overall spot profile. Similar images and plots are shown for a beam with no phase plate for comparison in Figure F-2c-d.

Changes in profile shape must be considered when performing experiments with defocused laser beams. This brief chapter illustrated some characteristics of typical beam profiles on the OMEGA laser using SG4 phase plates. At best focus, these provide a \(~700 \ \mu m\) diameter (1/e intensity contour) spot. The super-Gaussian profile was found to have a power of \(~4.3\) that provides a relatively flat intensity profile. Additionally, SSD was used in the laser spots shown here which minimizes the speckle observed at best focus. In general, DPPs serve to shape the overall beam envelope and SSD smooths out high-intensity speckle. For detailed information about these smoothing and shaping techniques, the reader is encouraged to see following references.

References


ablation front
A location in the overdense region defined as the point where the density drops to \(\sim 37 \% (1/e)\) of its peak value. Also termed the ablation surface, it moves with the accelerated material (peak density), and is unstable to the Rayleigh-Taylor (RT) instability.
[94, 95, 153]

ablator
The material that is ejected during laser-matter interactions. In inertial confinement fusion it is the outer capsule (typically plastic) that holds the fuel. Irradiation of the surface rapidly heats and ionizes the material that expands outwards.
[40]

adiabat
A measure of the relative degeneracy of a fluid. It is defined as the ratio of the fluid pressure to the Fermi pressure. A fluid may be more readily compressed when done at a low adiabat.
[40]

areal density
The integrated density along a specified path. For diagnostic purposes, this quantity refers to the integral of density along the particle’s trajectory.
[17, 43, 70]

bang time
This corresponds to the time that peak fusion production occurs. In ideal exploding-pusher implosions, there exists only a shock bang time occurring during the shock rebound after spherical convergence. In a compressive implosion, there is both a shock and compression bang time, associated with shock convergence and peak compression, respectively.
[106]

Biermann battery
This term comes from noncollinear gradients in electron temperature and density in plasma and is the primary source of self-generated magnetic fields.
[82, 154, 178, 200, 230]

burn fraction
The fraction of fuel that is burned in an implosion. In an ignited ICF implosion, burn
fractions of \( \sim 30\% \) are anticipated. [17, 47]

**burn parameter**
A parameter that quantifies the burn fraction for a specific reaction. The burn parameter has units of areal density, and in the limit that the areal density of the fuel is much larger than the burn parameter, the burn fraction approaches unity. [17, 47]

**collisional absorption**
This is a mechanism of absorbing electromagnetic (EM) waves in a plasma by which coherent motion of plasma electrons created by the wave is lost to random ion motion through Coulomb collisions. Also known as inverse bremsstrahlung, this is the preferred method of absorption in inertial confinement fusion (ICF) because of the resultant smooth energy deposition profile. [86, 229]

**compression-burn**
The period of most fusion production during an inertial confinement fusion implosion. After the first strong shock converges and rebounds. The shock reverberates between spherical convergence and the incoming dense fuel. The cold fuel stagnates at peak compression of the multiply shocked vapor, generating high temperatures and densities producing many fusion reactions. [40, 227]

**coupling parameter**
The ratio of the Coulombic potential energy to random thermal energy in a plasma. This parameter determines whether the collective-electrostatic effects \( (\Gamma << 1) \) or binary-collisional effects \( (\Gamma >> 1) \) will dominate the behavior. It is also inversely proportional to the number of electrons in a Debye sphere. [58]

**CR-39**
A plastic nuclear track detector used extensively in nuclear diagnostics developed by the high energy density (HED) physics division at MIT. [7, 199, 226]

**critical density**
The density at which the electron plasma frequency is equal to the incoming electromagnetic wave frequency. [17, 86]

**cross section**
A quantity that describes the likelihood of a specific interaction to take place between particles. It has units of area, but represents the probability for the specified reaction to occur. [36, 231]

**cyclotron frequency**
Also called the ‘gyro’ frequency, this quantity is the angular frequency of a charged
particle gyrating about a magnetic field line.
[18]

**D vs E**
The relationship between the measured diameter of tracks on CR-39 and the incident energy of particles. To interpret wedge range filter (WRF) spectrometer data, this curve is needed for incident protons.
[116]

**Debye length**
A quantity that characterizes the local electrical screening distance in a plasma.
[18, 57]

diamagnetic
This identifies phenomena that occur in a direction perpendicular to the magnetic field and the vector quantity driving the effect.
[62, 80]

direct-drive
A method of inertial confinement fusion whereby the capsule is imploded through direct laser illumination of the surface.
[34, 39]

**DRACO**
A multi-dimensional radiation-hydrodynamic code in use at the Laboratory for Laser Energetics (LLE).
[29, 86, 122, 158, 171, 186, 191, 196]

**electrothermal instability**
In laser-produced plasmas, this instability occurs in the highly resistive overdense region near the ablation surface. Temperature asymmetries here cause inhomogeneous ohmic heating due to the differences in resistivity that further enhances the temperature perturbations.
[21, 188]

**exploding-pusher**
An idealized type of implosion where the shell is thin enough so that there is no compression-burn, but only an initial shock-burn. The implosion physics for this type of implosion is much simpler than an ignition-style capsule and serves as an ideal diagnostic tool.
[48, 103, 104, 106, 128, 157, 199, 225, 232]

**flux-limited heat flow**
A phenomenological form representing heat flux in a plasma as a fraction of the free-streaming limit. A classical definition of conduction using a random walk approach usually overestimates heat flux, and the so-called ‘flux-limited’ heat flow is implemented in many cases. Possible mechanisms causing this behavior include large magnetic fields and ion acoustic turbulence.
[41, 158]
fusion
The nuclear process by which the two reacting nuclei bond together, fuse. The sum of masses of the product nuclei are less than that of the reactants and the mass difference is converted into energy through \( E = \Delta mc^2 \).
[33]

gain
In the context of inertial fusion energy, it is defined as the total fusion energy output of a single implosion divided by the total laser energy on target.
[18, 37]

Geant4
A simulation was written using this toolkit to aid in experimental design and data interpretation for proton radiography.
[29, 68, 101, 122, 128, 134, 211]

Hall MHD
Refers to taking the collisionless limit in the analytic magnetohydrodynamic (MHD) framework.
[81]

Hall parameter
This quantity characterizes the level of electron magnetization as the product of the electron cyclotron frequency and electron-ion collision time. If \( \chi > > 1 \), then the electrons are strongly magnetized and many gyrations occur before a characteristic collision. Conversely, if \( \chi < < 1 \), then the electrons are not magnetized and any organized motion that the magnetic field would impose is randomized through collisions.
[18, 62, 154, 179, 192]

high energy density
A physics regime where energy densities are greater than 1 Mbar. In most instances substances in this regime are in a fully or partially ionized state.
[7, 21, 27, 33, 59, 101, 133, 199, 226]

hot-spot ignition
An ideal fuel-mass configuration for achieving ignition in inertial confinement fusion. A cold, dense, shell of DT fuel is spherically compressed through multiple shocks and spherical convergence. DT vapor within is also compressed and heated by the shocks while the cold fuel spherically converges around it. The DT vapor becomes very hot (the 'hot-spot') producing fusion reactions. When ignited, DT-\( \alpha \) particles are stopped within the ‘hot-spot’, depositing their energy and initiating a self-propagating fusion-burn wave through the cold, dense fuel.
[37]

ignition threshold factor
A dimensionless parameter normalized to unity for the marginal ignition case. This quantity provides scaling laws for various implosion parameters describing the progress
towards ignition (see Section 2.2.2).

**implosion velocity**

The average speed of the incoming fuel in an ICF implosion. It is typically averaged over the entirety of the fuel and weighted by the local density.

**indirect-drive**

A method of inertial confinement fusion whereby the capsule is placed within a cylinder, called a hohlraum, made of a high-Z material, such as gold or uranium. Lasers are incident onto the walls of the hohlraum creating an x-ray oven with black body temperatures of order \( \sim 300 \text{ eV} \). The x rays ablate the capsule surface to indirectly drive the implosion.

**inertial confinement fusion**

One of the primary methods of achieving net energy production through nuclear fusion reactions, the other primary method being magnetic confinement. Inertially confined plasmas are generated through the implosion of spherical capsules using high intensity radiation. The plasma is held together by its own inertia, during which time fusion reactions occur, and then expands out into the vacuum.

**interstellar medium**

The matter that exists throughout a galaxy between star systems. It is an extremely dilute mixture of gas, dust, and cosmic rays.

**inverse bremsstrahlung**

see collisional absorption.

**ion-acoustic wave**

A plasma wave oscillation in both ion and electron densities that moves at the local sound speed. Also known as ‘acoustic’ waves or simply ‘sound’ waves.

**Kelvin-Helmholtz**

A hydrodynamic instability that occurs in systems where there is a net velocity shear parallel to the perturbation wave vector.

**Laboratory for Laser Energetics**

This is the lab that houses the Omega laser facility and is associated with the University of Rochester (see [http://www.lle.rochester.edu/](http://www.lle.rochester.edu/)).

**Landau damping**

A collisionless absorption process whereby resonant electrons, with velocities near the
wave phase velocity, absorb energy by ‘surfing’ the wave. Also known as ‘resonance
absorption’, this mechanism tends to generate hot electrons with energies a few times
higher than the typical thermal velocity.
[86]

Langmuir wave
A wave in the electron density that oscillates at the electron plasma frequency. It is
also known as an ‘electron plasma wave’, or simply a ‘plasmon’.
[86, 231, 232]

laser-plasma interactions
A field of research under the broad umbrella of plasma physics. This field is most often
related to inertial confinement fusion due to the use of high intensity lasers interacting
with matter. In many instances, the most relevant ‘interactions’ are energy absorption,
refraction (scattering), and laser-driven instabilities (see Section 3.5.1).
[22, 34, 85, 188]

Light Amplification by Stimulated Emission of Radiation
A coherent source of light by which amplification is achieved through resonant inter-
actions with materials compatible with a specific wavelength of radiation.
[21, 28]

LILAC
A one-dimensional radiation-hydrodynamic code in use at the LLE and often used to
predict capsule performance.
[179]

Linear Electrostatic Ion Accelerator
An ion accelerator at MIT used for nuclear diagnostic development. The machine can
achieve acceleration voltages up to $\sim 150$ kV and produces both DD and $D^3He$ fusion
reactions.
[7, 22, 29, 101, 110, 199]

magnetothermal instability
Based on the classic theory, this instability occurs in the underdense corona outside
the peak temperature where the gradients in temperature and density are parallel. A
temperature perturbation in this region will produce a magnetic field generated by
the Biermann battery. The B field will alter the heat conduction and enhance the
temperature perturbation, thereby enhancing the B field.
[19, 22, 177, 191, 201]

mean free path
The characteristic distance traveled by a particle between collisions.
[19, 70]

Monte Carlo
A mathematical method whereby pseudo-random numbers may be used to predict
behavior with a specified probability distribution.
[157, 179, 211, 228]
National Ignition Facility
A laser facility located at Lawrence Livermore National Laboratory (LLNL) in Livermore, CA. Currently, this is the most advanced laser facility in the world and has the capability to achieve ignition in ICF. This facility has 192 beams arranged in a symmetric, hemispherical pattern with primary experiments being performed in the indirect-drive configuration. [22, 28, 50, 111, 233]

Nernst effect
This collisional effect describes magnetic field advection with the heat-conducting electron population. In the local thermal equilibrium (LTE) limit, the advection velocity (like heat conduction) is proportional, but opposite in direction, to the electron temperature gradient. [20, 82, 83, 178, 189, 196]

OMEGA
This notation is shorthand for the OMEGA-60 laser system and is part of the Omega laser facility. [28, 30, 111, 114, 119, 133, 135, 146, 156, 199, 203, 221, 224, 232]

Omega laser facility
A laser facility located in Rochester, NY composed of two separate systems: OMEGA-60 and OMEGA-EP. The OMEGA-60 system is a spherically symmetric arrangement of 60 beams in a truncated icosahedron (‘soccer ball’) pattern capable of delivering ~30 kJ on target. The OMEGA-EP (extended performance) system is composed of 4 beams with 2 beams having the capability to run with pulses as short as ~1 ps. [7, 51, 104, 229, 231]

peak-to-valley
This term refers to the amplitude of a perturbation from the ‘peak’ (high point) to the ‘valley’ (low point). In a purely sinusoidal function of a single frequency, this is twice the sinusoidal amplitude and is a factor of $2\sqrt{2}$ times larger than the rms value. [22, 169]

plasma frequency
The natural oscillation frequency of an electrostatic wave in a plasma. [18, 57, 226, 229]

plasmon
A wave within a plasma, typically refers to a Langmuir wave. [41, 86, 229]

Q-value
The energy released in a fusion reaction as calculated by $E = \Delta mc^2$, where $\Delta m$ is the difference in mass between the reactant and product nuclei. [35]

Rayleigh-Taylor
A hydrodynamic instability that occurs in systems where the density gradient opposes
the acceleration field.

Richtmyer-Meshkov
A hydrodynamic instability that occurs in systems after a shock crosses a fluid interface, thereby imposing an impulse force at transit time.

Rutherford cross section
The angular differential cross section for Coulomb scattering of charged particles.

SESAME
A composition of equation-of-state (EOS) tables for various materials maintained by the Los Alamos National Laboratory (LANL).

shock-burn
A period of fusion production after the rebound of the first spherically converging shock. In a true exploding-pusher implosion, the only fusion production occurs as the initial shock rebounds from the center further heating and compressing the incoming gas.

stimulated Brillouin scattering
A laser-plasma instability whereby the incoming photon decays into an ion-acoustic wave and a scattered photon.

stimulated Raman scattering
A laser-plasma instability whereby the incoming photon decays into a Langmuir wave and a scattered photon.

stopping power
The differential energy loss per unit length of a charged particle through matter.

supernova
The final phase of stellar evolution for massive stars ($\gtrsim 10 \ M_\odot$). In core-collapse supernovae, this occurs when the fusion production in the star can no longer sustain the core under its own gravity and the collapse causes a violent explosion of the outer layers of stellar material resulting in a neutron star or black hole.

supernova remnant
The resultant nebulous object created by material ejected from a supernova explosion into the interstellar medium (ISM).
**target-normal sheath acceleration**

A mechanism that creates high-energy ions through charge separation. In laser-irradiated targets, high-energy electrons quickly leave the system resulting in a net positive potential. This charge separation drives positive ions off the material with the most flux coming off normal to the surface. It is a standard technique for producing a spectrum of high-energy protons used for radiography.

[22, 102]

**ten-inch manipulator**

A diagnostic port at OMEGA that can house user-developed diagnostics.

[22, 52, 106]

**two-plasmon decay**

A laser-plasma instability whereby the incoming photon decays into two separate Langmuir waves.

[22, 87, 137, 188]

**Weiner-filter**

An estimated noise spectrum used to filter discrete Fourier transform (DFT) data in the analysis of radiographs from RT experiments.

[165]

**Z-accelerator**

This is the pulsed power device, also called the Z machine, located at the Sandia National Laboratory (SNL). It uses large capacitor banks to release ∼MA currents through different types of wire-array geometries.

[149, 233]

**Z-beamlet**

A single National Ignition Facility (NIF)-like laser beam used to generate x rays or protons for radiography experiments on the Z-accelerator.

[149]