Transverse Modes in Large-Mode-Area Fibers

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To my wife and son
Curriculum Vitae

The author was born in Shenyang, Liaoning Province, China on February 17th, 1978. He attended Tsinghua University, Beijing, China from 1996 to 2003, and graduated with a Bachelor of Science degree in 2000 and a Master of Science degree in 2003. He came to the University of Rochester in the fall of 2003 and began graduate studies in the Department of Physics and Astronomy. He pursued his research in fiber optics under the direction of Professor John. R. Marciante and received the Master of Science degree from the University of Rochester in 2005.
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Abstract

This thesis is devoted to the study of transverse modes in large-mode-area (LMA) fibers. It highlights the importance of transverse spatial-hole burning (TSHB) effect in LMA multimode fibers through experiments and simulations. The measured beam-quality factor decreases until the gain becomes saturated in an amplified spontaneous emission (ASE) source based on an Yb-doped LMA multimode fiber. At saturation, the beam-quality factor reaches a minimum, beyond which it increases again. Numerical simulation trends based on a model using spatially resolved gain and transverse-mode decomposition of the optical field agree with the experimental results. A simplified model without TSHB is shown not fit to predict the observed behavior of beam quality in LMA fibers, especially at high powers. A comparison of both models shows that TSHB is also critical for properly modeling beam quality in LMA fiber amplifiers.

New precise modal decomposition methods for field distribution with phase information and intensity distribution without phase information respectively, are presented and extended to single-mode fiber characterization. In these new methods, different mode sets are calculated using varied sets of fiber parameters, and the modal power weights of each set are calculated using the experimentally extracted field or intensity distributions of the beam. The remaining residue is then minimized amongst the mode sets to extract the corresponding modal power weighting, phase differences, and even experimental fiber parameters. Experiments are carried out for both single-
mode fiber characterization and modal decomposition in few-mode fibers. The experimental results of the parameters obtained by the modal decomposition method agree well with the nominal or measured values.

The thesis also investigates helical-core fibers. An improved semi-analytic bend-loss model is derived that allows for the propagation of radiated fields outside the plane of the fiber bend. This allows for the modeling of small-bend radii for which the waveguide condition for total internal reflection is violated in a large angular spread of incident angles at the interface of the fiber core. This improved model is applied to LMA helical-core fibers (which require small-bend radii) for use as high-power fiber lasers and amplifiers, which enable bending loss to be utilized for mode selection without the deleterious effects of coiling straight-core fibers. A comparison of the fundamental-mode propagation loss between theory and experiments shows that this improved model is well matched using parameters of the fabricated helical-core fiber, but that more accurate measurements are needed for all parameters.
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Chapter One

Introduction

Since their inception in the 1970s, low-loss optical fibers have found applications in optical communications, optical sources, sensors and interferometers.

In recent years, fiber lasers and amplifiers have been widely used in high power applications such as material processing, industrial manufacturing and the military. Their main advantages are heat-dissipation capability, broad gain bandwidth, compactness, robustness and high efficiency.

The primary limitation in their power scaling is the onset of nonlinear effects. Optical fibers have relatively weak nonlinearities compared with other media, since silica is intrinsically not a highly nonlinear material [1]. However, the nonlinear effects are still important in optical fibers due to the small mode area and long propagation lengths.

The most important nonlinear effects in optical fibers are self-phase modulation (SPM), cross-phase modulation (XPM), four-wave mixing (FWM), stimulated Brillouin scattering (SBS), and stimulated Raman scattering (SRS). All of these effects are the third-order nonlinear effects, and have been studied extensively. For high-power narrow–band continuous-wave (CW) applications, the primary limitation in optical fibers is SBS due to its low threshold power.
Increasing the effective mode area can reduce the intensity of the optical field, and thus increase the nonlinear power thresholds. Therefore, large-mode-area (LMA) fibers are more suitable for high-power applications than standard single-mode fibers. Also, LMA fibers have higher damage threshold. LMA fibers have been the subject of intense research in recent years.

1.1 Large-Mode-Area Fibers

The simplest LMA fiber is conventional multimode step-index fiber. Since multimode step-index fibers support multiple transverse guided modes, the beam quality of the output beam is worse than that from standard single-mode fibers (it is worth noticing that the beam quality of output beams from multimode fibers is still better than that of most bulk solid-state lasers). To obtain better beam quality, many LMA fibers designs have been developed.

One design on the refractive index profile is low numerical aperture (NA) fibers. The NA is reduced as the mode area is increased to keep the normalized frequency (V number) always satisfying the single-mode condition [2]. However, the reduction of NA is limited by the controllability of the fabrication process. Also, for very low NA, the fiber modes become extremely susceptible to environmental perturbations. These factors impose a practical lower limit on the NA of 0.06 [3].

One design on the gain-doping profile is gain filtered fibers, where the dopants are distributed partially in the core with smaller radius or with non-rectangular shape [4]. Single-mode operation can be achieved by denying the gain
toward the edge of the core where most of the higher-order modes (HOMs) power is contained.

Ring-shape refractive index fiber combines refractive index design and gain profile design. It consists of a low-NA center core region and an outer ring with a raised index [5]. The outer ring gives an increased spot size for the HOMs and increases their susceptibility to bend loss. Only the low-NA center region is doped to achieve gain filtering for the HOMs.

Another design that combines refractive index design and gain profile design is gain-guided, index-antiguided (GG+AIG) fiber [6]. GG+AIG fibers have a very large gain coefficient in the large-diameter core and a negative index step from cladding to core so only the fundamental mode experiences more gain than loss.

One design utilizing long-period fiber gratings (LPG) is HOM fibers [7]. The signal is injected into the small single-mode core, and then converted to a desired HOM with zero azimuthal mode number in the large inner cladding by a LPG. Unlike the fundamental mode, these HOMS tend not to couple to other modes during propagation in the fiber. Another LPG at the output end converts the signal back to the fundamental mode.

One design utilizing different modal bending loss is coiled multimode step-index fibers [8]. The HOMs have higher bending loss than the fundamental mode. The coiling radius is chosen to provide high loss for the HOMs and negligible loss for the fundamental mode, such that HOMs will be filtered out and the fundamental mode will be guided.
Another design utilizing different modal bending loss is helical-core fibers [9, 10]. Helical-core fibers integrate coiling directly into the fabrication of the fiber itself without the mechanical stresses induced from tight bending which could lead to degradation and damage. The configuration of helical-core fibers is shown in Fig 1.1.

One design combining bend loss and modal coupling is chirally-coupled-core (CCC) fiber [11]. A CCC fiber contains a straight central core and at least one high-loss helical satellite core wrapped around the central core. By selecting suitable side-helix parameters, it is possible to obtain strong coupling between the HOMs of the central core and the very leaky satellite cores through quasi-phase matching, while leaving the fundamental mode in the central core.

Photonic crystal fibers (PCF), also called microstructured fibers, are a subset of LMA fibers with various designs. The common feature of PCF is the periodic structure in the cladding and an imperfection in the center as the core, which can be solid (silica), liquid, or air. PCF can be further divided into several subcategories.

Some PCF essentially operates by following the total internal reflection (TIR) in conventional fibers. The light in the fiber is guided in the high index core
surrounded by cladding, which are embedded with low index air holes. This arrangement gives better control of the effective cladding index, resulting in NA as low as 0.02.

Photonic band-gap (PBG) fibers are PCF utilizing the photonic band-gap effect [14]. The light in the fiber can be guided in the low index core (air), surrounded and confined by periodic high-index structures which act as two-dimensional Bragg mirrors.

Leakage channel fibers (LCF) are similar to PCF in that the solid glass core is surrounded by a few large holes, but they are not PCF strictly speaking, since they lack any radial periodic structure [15]. All modes in LCF are leaky in nature, due to the destruction of TIR induced by the holes. However, the arrangement and area of the holes can be designed for the fundamental mode to propagate with a negligible loss and all higher-order modes with high loss.

Multicore fiber (MCF) is another subset of LMA fibers. Only one mode is guided in each core of a MCF. When the cores are arrange close enough, they become coupled strongly through evanescent waves [16] and super-modes are formed among the array of cores. The total number of super-modes is equal to the total number of cores, but only the in-phase (fundamental) super-mode is desired. Different designs have been approached to selectively excite in-phase super-mode, such as using a Talbot cavity or Gaussian beam seed [17].

Multifilament-core (MFC) fiber is MCF with many very small cores (filaments) [18]. The field in the cores is less confined in MFC fibers than
conventional MCF and therefore leads to stronger coupling. The filaments of MFC fiber can be viewed as a large core with slightly higher refractive index than the cladding due to the small area of each filament, and therefore single mode cutoff condition is satisfied. Multicore photonic crystal fiber (MCPCF) is a combination of MCF and PCF [19].

Selective modal excitation is an important technique when using LMA fibers [17, 20]. When the intensity distribution of the seed beam best matches that of the fundamental mode, the output beam has the largest fraction of power in the fundamental mode.

Tapering can be used to achieve selective modal excitation. A double-clad (DC) multimode step-index fiber can be tapered down to a few microns of core radius where the pump fibers were side-spliced, providing a single-mode spatial filter [21]. In this way, the lasing of most higher-order modes can be suppressed.

The difference in loss due to Fresnel diffraction between the fundamental mode and the HOMs can be utilized to achieve selective modal excitation. In a laser cavity, the small gap distance between the active LMA fiber and the output coupling mirror can be adjusted and optimized for good beam quality [22]. In this way, the HOMs are filtered out due to higher Fresnel diffraction loss.

1.2 Transverse Spatial-Hole Burning

When a multimode optical beam is propagating in an active fiber, the gain of each mode is different, governed by the intensity distribution of each mode. The gain
of the fundamental mode is usually larger than those of the HOMs, since the field of the fundamental mode is confined in the doped core region more than those of the HOMs. Therefore, the power of fundamental mode increases faster than those of the HOMs during propagation in active fibers, and the overall beam quality is improved provided the gain profile is not altered as the signal is amplified.

At high power levels, the gain becomes saturated. Since the transverse intensity distribution is usually not uniform, the gain is more saturated where the transverse optical intensity is higher. This is the transverse spatial-hole burning (TSHB) effect. The TSHB effect in vertical-cavity surface emitting lasers (VCSELs) have been investigated intensively [23-25], but not in optical fibers.

Due to the TSHB effect, the gain of the fundamental mode will decrease faster than those of the HOMs, since the field of the fundamental mode saturates the gain according to its profile, leaving the rest of the gain for the HOMs to utilize. Therefore, the power of the fundamental mode increases slower than those of the HOMs during propagation in saturated active fibers, and the overall beam quality is degraded. Therefore, the TSHB effect is important in LMA fibers and requires investigation.

### 1.3 Modal Decomposition

If an optical beam is composed of multiple modes, the modal weights can be obtained by decomposing the total power of the beam into the mode components.

Modal decomposition in multimode fibers and the other LMA fibers is gaining more importance in many applications, such as laser beam characterization,
higher-order-mode-dispersion compensation, mode-beating generation, pumping scheme optimization, and multi-path interference [26-28].

\[ M^2 \], the beam-quality factor, is defined as the ratio of a beam’s divergence to that of a Gaussian beam with the same beam waist width [29]. For a beam composed with multiple modes, its beam-quality factor is affected by the phase differences between the modes and can be very low even with a significant amount of power contained in the HOMs [30, 31]. The phase differences between the modes can change due to environmental fluctuations, which lead to fluctuation of the beam quality. Therefore, a near-diffraction-limited \( M^2 \) does not necessarily mean single-mode operation, nor does it mean that beam will always have good beam quality. In contrast to using \( M^2 \), performing a modal decomposition yields the power fraction in each mode (modal power weights), which are usually stable, and possibly the phase differences between the modes. Therefore, modal decomposition is a very useful technique for measuring the beam quality in a fiber system.

Modal decomposition can be carried out given the field distribution with intensity and phase information, which is usually extracted from interferometric measurements, wavefront curvature measurements, and phase retrieval with transport-of-intensity equation [27, 32-35].

Various modal decomposition methods without direct access to phase information have also been developed. A least-squares-fitting method [36] is based on a single intensity measurement in the Fourier plane with modal weights and phases as fitting parameters. A phase-space tomography method [27] utilizes the beam’s mutual
intensity profile which can be retrieved from Wigner or ambiguity functions obtained by phase-space tomography. A phase-retrieval method [37] is based on near-field and far-field measurements and indirectly accesses the phase information using Gerchberg-Saxton phase-retrieval algorithm. A hologram method [38] utilizes computer-generated holographic filters to generate diffraction patterns of cross correlation functions. The S² imaging method resolves the modal interference patterns spatially and spectrally [39].

The intensity distribution of each mode must be known for most of these methods, which are determined by fiber and imaging parameters. The S² imaging method does not require intensity distributions of modes, but is limited to broadband sources. To obtain accurate results, the fiber and imaging parameters must be known precisely. Therefore, a precise modal decomposition method is needed, which has not been investigated yet.

### 1.4 Helical-Core Fibers

As discussed in Section 1.1, coiling is a common technique to suppress the HOMs in LMA fibers by introducing a bending loss that becomes increasingly larger with the mode order [40]. According to industry standards, the bend radius of coiled fibers with 125-μm cladding diameter is 2.54 cm for long-term reliability without a pull load and increases linearly with cladding diameter [41]. For conventional (step-index) fiber-waveguide geometries, the induced stresses from tight bending which is necessary for single-mode operation are close to or over this limit, and can lead to
degradation and damage of the fiber, especially while under constant exposure to high optical powers.

Helical-core fibers integrate coiling directly into the fabrication of the fibers without the penalties introduced by coiling. In the geometry as shown in Fig. 1.1, the helical core provides effective bending loss for the HOMs in the core and promotes pump absorption if cladding-pumped, and the cylindrical cladding confines pump light in the fiber. The fabrication of a helical-core fiber is simple as shown in Fig. 1.2: a preform is created with a core that is offset from the center of the perform, and then the preform is rotated at a constant speed as the fiber is drawn. The ratio of the pitch of the helix to the offset of the helix is usually very large and torsional stress is absent; therefore, the torsion effect in conventional coiled fibers can be neglected in helical-core fibers. These stress-free helical-core fibers can meet the requirements of robustness, repeatability, and stability for commercial systems much better than coiled multimode fibers because of their built-in bending loss.

Helical-core fibers were originally designed and fabricated for passive transmission systems to provide circular birefringence [42]. Helical-core LMA fibers were then proposed to provide single-mode operation [43], and fabricated and investigated in high power fiber lasers [9]. Furthermore, helical-core microstructured fibers, shown in Fig. 1.3, have been recently demonstrated with single-mode
operation [44]. Other LMA fiber designs can be incorporated into helical-core fibers and generate more advanced concepts. Therefore, helical-core fibers have a very promising future and are worth investigation.

![Section of a helical-core microstructured fiber](image)

Fig. 1.3. (a) Cross section and (b) side view of a helical-core microstructured fiber.

### 1.5 Outline

This thesis is focused on the transverse modes in LMA fibers. The remainder of the thesis is arranged into four chapters.

In Chapter 2, the transverse spatial-hole burning effect and modal competition in LMA fiber is investigated numerically and experimentally. In Chapter 3, precise modal decomposition methods are developed for field distributions with phase information and intensity distributions without phase information, respectively. The methods are extended to single-mode fiber characterization. In Chapter 4, an improved bend loss model is developed, and the properties of helical-core fibers are
investigated through modeling and measurements. In Chapter 5, this thesis is summarized and some further research is suggested.
References


Chapter Two

Transverse Spatial-Hole Burning in LMA Fibers

In this chapter, the transverse spatial-hole burning (TSHB) effect in LMA fibers is investigated in an amplified spontaneous emission (ASE) source based on an ytterbium-doped (YD) double-clad (DC) LMA fiber.

The experimental setup is presented in Section 2.1. The measured beam-quality factor and output power as functions of input pump power are shown in Section 2.2. In Section 2.3, a localized multimode model is introduced with spatially resolved gain and the equations to calculate beam-quality factor are presented. The results of numerical simulations based on this model are compared to experimental results and extrapolated to higher power in Section 2.4. In Section 2.5, the validity of a simplified model that does not include the TSHB effect is discussed. The theoretical models are further applied to fiber amplifiers in Section 2.6.

2.1 Experimental Configuration

Rare-earth-doped fiber ASE sources have been used in various applications as incoherent broadband optical sources, such as fiber sensors [1], fiber optics gyroscope (FOG) [2, 3], and optical communication systems [4], optical coherence tomography (OCT) [5]. The interests on utilizing multimode fibers in these applications have grown rapidly in recent years [5-8]. The LMA fiber ASE source also provides a good
example to investigate modal competition and the TSHB effect in fibers due to its incoherent nature which leads to simple inter-modal phase relationships.

The Yb-doped gain medium have many advantages with diode pumping, including larger absorption bandwidth, longer upper-state lifetime, and three to four times lower thermal loading per unit pump power.

The double-clad structure is more readily able to scale to high power because its geometry permits cladding pumping by low brightness diodes as shown in Fig. 2.1.

![Fig. 2.1. Depiction of a double-clad fiber.](image)

The experimental arrangement used for this work is shown schematically in Fig. 2.2. A YD DC fiber was utilized with a length of 6.5 m, a core diameter of 30 µm, an inner cladding diameter of 260 µm, and a core NA of 0.06. The fiber was coiled at a radius of 9.5 cm for packaging purpose only. The back end of the fiber is angle cleaved to prevent reflection of the forward ASE. A fiber-coupled laser diode provided forward Continuous-Wave (CW) pump at 976 nm. A dichroic mirror was set between an aspheric lens and a compound lens to extract the backward ASE output.
The ASE output power was measured by a powermeter and the beam widths (defined as the second moment) were measured by a charge-coupled-device (CCD) camera placed at different distances. $M^2$, the beam-quality factor, in the x and y directions was calculated by fitting the beam widths and distances to a polynomial using a least-squares-fitting method [9, 10],

$$W^2(z) = W_0^2 + M^4 \left( \frac{\lambda}{\pi W_0} \right)^2 (z - z_0)^2,$$

where $W(z)$ is the beam width at distances $z$, $W_0$ and $z_0$ are the respective beam width and distance at the beam waist where the beam width is minimal, and $\lambda$ is the wavelength. Although $M^2$ can not directly provide modal power weights, it is a good indicator of how the modal power weights change.

### 2.2 Experimental Results
The ASE output power is plotted as a function of input pump power in Fig. 2.3. The figure shows that the output power increases exponentially and quasi-linearly when the input pump power is below and above ~6 W respectively, which means the gain becomes saturated at 6-W input pump power. The entire gain is not saturated along the whole fiber, but the gain with high signal (i.e. near the output end of the fiber) experiences saturation. This “soft” threshold is called as the pump threshold for saturation in this work.

![Output power from the ASE source as a function of input pump power](image)

Fig. 2.3 The output power from the ASE source as a function of input pump power. The dashed vertical line is the pump threshold for saturation.

The ASE spectrum at 7-W input pump power is plotted in Fig. 2.4, and the beam-quality factor is plotted as a function of the input pump power in Fig. 2.5.
Fig. 2.4 The spectrum of ASE source at 7-W input pump power.

Fig. 2.5 The beam-quality factor of the ASE source as a function of input pump power in the (a) x and (b) y directions. The dashed vertical line is the pump threshold for saturation.

Compared with Fig. 2.3, Fig. 2.5 shows that the beam quality improves with input pump power below the pump threshold for saturation and optimized as the gain medium becomes saturated.
To fully understand the physics behind this beam-quality behavior, numerical simulations based on theoretical modeling must be developed.

### 2.3 Localized Model

In order to investigate the TSHB effect in LMA fiber sources, a spatially-resolved localized model is extended to multiple transverse-spatial modes of ASE [11]. In this model, the total optical power is quantized into the transverse modes while the population inversion retains spatial dependence. In this way, the TSHB effect can be accounted for while retaining the simplicity and transparency of a mode-based picture. Similar treatment has also been developed for modeling VCSELs [12].

In the model, the gain medium is treated as a simplified two-level system [11, 13, 14]. The rate equations for YD DC fiber amplifiers are given by

\[
\frac{dn_2(r,\phi,z)}{dt} = \sum_k \frac{P_k(z)i_k(r,\phi)}{\hbar \nu_k} \sigma_{ak} n_1(r,\phi,z) - \sum_k \frac{P_k(z)i_k(r,\phi)}{\hbar \nu_k} \sigma_{ek} n_2(r,\phi,z) - \frac{n_2(r,\phi,z)}{\tau}, \tag{2.2}
\]

\[
n_t(r,\phi,z) = n_1(r,\phi,z) + n_2(r,\phi,z), \tag{2.3}
\]

where the mode order \( k \) denotes any combination of beam propagation direction (+, −), wavelength (\( \lambda \)), transverse-mode order (\( \nu, m \)), and orientation (even, odd); \( n_1, n_2 \), and \( n_t \) are ground-level, upper-level, and total ytterbium ion density, respectively, as a
function of time and spatial coordinates; \( \sigma_a \) and \( \sigma_e \) are the absorption and emission cross sections of ytterbium ions, respectively; and \( \tau \) is the upper-state lifetime.

\( P_k(z) \), the power of the \( k^{th} \) mode at position \( z \) in the fiber, is the integration of the light-intensity distribution \( I_k(r,\phi,z) \) over the radial and azimuthal coordinates:

\[
P_k(z) = \int_0^{2\pi} \int_0^\infty I_k(r,\phi,z) r \, dr \, d\phi.
\] (2.4)

The normalized modal-intensity distribution \( i_k(r,\phi) \) is defined as

\[
i_k(r,\phi) = \frac{I_k(r,\phi,z)}{P_k(z)},
\] (2.5)

and is determined by the spatial shape of the mode and therefore independent of \( z \).

The terms at the right side of Equation 2.2 describe the effects of absorption, stimulated emission, and spontaneous emission, respectively. Note that the interference terms are neglected in this model. This assumption is correct for transverse modes of ASE because they do not interfere with each other due to their incoherence nature. For a coherent multimode beam, this model could be modified by adding the interference terms.

In the steady-state case, the time derivative in Equation 2.2 is set to zero and the inversion is solved as
The numerator accounts for small signal gain and the summation in the denominator accounts for the TSHB effect.

The propagation equations are given by

\[
\frac{dP_k(z)}{dz} = u_k \sigma_{ek} \left[ P_k(z) + m h \frac{c^2}{\lambda_k^3} \Delta\lambda_k \right] \int_{0}^{2\pi} \int_{0}^{\pi} i_k(r,\phi) n_z(r,\phi,z) r dr d\phi
\]

\[
- u_k \sigma_{ak} P_k(z) \int_{0}^{2\pi} \int_{0}^{\pi} i_k(r,\phi) m_1(r,\phi,z) r dr d\phi - u_k \alpha P_k(z),
\]

where \( u_k = 1 \) for the modes traveling in the forward direction or \( u_k = -1 \) in the backward direction, \( m \) is the number of polarizations of each mode, \( \Delta\lambda_k \) is the bandwidth, and \( \alpha \) is the fiber-loss term. The terms at the right side of Equation 2.7 describe the effects of stimulated emission, spontaneous emission, absorption, and scattering loss, respectively. This model can be readily adapted to account for mode coupling by inserting the term \( + \sum_{j} (\kappa_{jk} P_j - \kappa_{kj} P_k) \) into Equation 2.7, where \( \kappa_{jk} \) is the coupling coefficient for power transfer from the \( j^{th} \) to the \( k^{th} \) mode. Specific mode coupling coefficients are difficult to obtain experimentally and are neglected in this analysis in order to clearly model the impact of the TSHB effect in LMA fibers.
The ASE and pump have different optical properties. The modes of ASE propagate in the fiber in both directions, but the pump propagates only in the forward direction. The bandwidth of ASE is relatively narrow, as shown in Fig. 2.4, so the ASE is simplified as a single spectral mode. The pump light is considered to be a single spectral mode with $\Delta \lambda_k = 0$ (no spontaneous emission at the pump wavelength).

Under the weakly guided approximation, the transverse modes of ASE can be represented by linearly polarized (LP) modes [15]. For the LP$_{vm}$ mode, the normalized optical intensity $i_{vm}(r, \phi)$ and the normalized electric field distribution $E_{vm}(r, \phi)$ can be written as

$$i_{vm}(r, \phi) = E_{vm}^2(r, \phi),$$  \hspace{1cm} (2.8)

$$E_{vm}(r, \phi) = \begin{cases} 
b J_v(\kappa_{vm}r) f_v(\phi) & r < a_{core} \\
b J_v(\kappa_{vm}a_{core}) K_v(y_{vm}r) f_v(\phi) \frac{K_v(y_{vm}a_{core})}{K_v(\kappa_{vm}a_{core})} & r \geq a_{core} 
\end{cases},$$  \hspace{1cm} (2.9)

where $\nu$ and $m$ are the azimuthal and radial mode numbers, respectively, $J_v$ and $K_v$ are Bessel function of the first kind and modified Bessel function of the second kind, respectively, $a_{core}$ is the radius of core, and $b$ is the normalization coefficient of the electric field.
The transverse attenuation coefficient of the mode in the inner cladding $\gamma_{vm}$ and the transverse wave vector $\kappa_{vm}$ are solutions of following system of equations [16]:

$$\kappa_{vm} \frac{J_{v-1}(\kappa_{vm}a_{\text{core}})}{J_{v}(\kappa_{vm}a_{\text{core}})} = -\gamma_{vm} \frac{K_{v-1}(\gamma_{vm}a_{\text{core}})}{K_{v}(\gamma_{vm}a_{\text{core}})},$$  \hspace{1cm} (2.10)

$$\kappa_{vm}^2 + \gamma_{vm}^2 = V^2/a_{\text{core}}^2.$$ \hspace{1cm} (2.11)

The $V$ number and NA are defined as

$$V = \frac{2\pi}{\lambda} a_{\text{core}} \text{NA},$$ \hspace{1cm} (2.12)

$$\text{NA} = \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2},$$ \hspace{1cm} (2.13)

where $\lambda$ is the wavelength, and $n_{\text{core}}$ and $n_{\text{clad}}$ are the refractive indexes in the core and cladding, respectively.

The azimuthal component $f_v(\phi)$ is equal to 1 for those transverse modes with zero azimuthal mode number, and given by

$$f_v(\phi) = \begin{cases} 
\cos(v\phi) & \text{even} \\
\sin(v\phi) & \text{odd} 
\end{cases},$$ \hspace{1cm} (2.14)

for the other transverse modes with even or odd orientation.
Since the area of the inner cladding is much larger than the core, the highly multimode pump light can be simplified as one transverse mode effectively being uniformly distributed across the inner cladding and the core, which means the intensity distribution of pump $I_{pump}$ and normalized intensity distribution of pump $i_{pump}$ can be considered independent of radial and azimuthal coordinates. The normalized intensity distribution of the pump in the inner cladding and the core is then obtained from Equations 2.4 and 2.5 by

$$i_{pump} = \frac{1}{\pi a_{clad}^2}, \quad (2.15)$$

where $a_{clad}$ is the radius of inner cladding.

The output power is the sum of the backward output power contained in each mode and given by

$$P_{output} = \sum_{\nu, m} \mathcal{P}_{\nu m}^{-}(0). \quad (2.16)$$

The output modal power weight $\alpha_{\nu m}$ of the $LP_{\nu m}$ mode is defined as

$$\alpha_{\nu m} = \frac{P_{\nu m}^{-}(0)}{P_{output}}. \quad (2.17)$$

The transverse modes with the same mode numbers but different orientations will have the same modal power weight for ASE due to spectral symmetry.
The beam-quality factor of optical beam can be calculated given the optical field distribution [17]. Since the electric field of ASE can be treated as symmetric and real, many terms in the equations to calculate the beam-quality factors vanish. The equations can then be simplified as

\[
M_x^2 = 2 \left[ \iint \alpha_{vm} \left( \frac{\partial E_{vm}(r,\phi)}{\partial x} \right)^2 rdrd\phi \right] \left[ \iint x^2 \sum \alpha_{vm} \left| E_{vm}(r,\phi) \right|^2 rdrd\phi \right], \tag{2.18}
\]

\[
M_y^2 = 2 \left[ \iint \alpha_{vm} \left( \frac{\partial E_{vm}(r,\phi)}{\partial y} \right)^2 rdrd\phi \right] \left[ \iint y^2 \sum \alpha_{vm} \left| E_{vm}(r,\phi) \right|^2 rdrd\phi \right]. \tag{2.19}
\]

2.4 Numerical Simulations

Initial boundary conditions are needed to solve the propagation Equation 2.7 and are specified at \( z = 0 \) and \( z = L \) as

\[
P_{\text{pump}}^+(0) = P_0,
\]

\[
P_{\text{pump}}^-(L) = 0,
\]

\[
P_{vm}^+(0) = 0,
\]

\[
P_{vm}^-(L) = 0,
\]

(2.20)
where $P_0$ is the forward pump power injected into the fiber and $L$ is the length of the fiber. For ASE sources, the input signal is zero.

The parameters used in numerical simulation are listed in Table 2.1. The emission and absorption cross sections and $\Delta \lambda_{\text{ASE}}$, the bandwidth of ASE, are calculated based on the spectrum in Fig. 2.4 [18].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_t$</td>
<td>$1.02 \times 10^{26}$ m$^{-3}$</td>
</tr>
<tr>
<td>$\lambda_{\text{ASE}}$</td>
<td>1041 nm</td>
</tr>
<tr>
<td>$\lambda_{\text{pump}}$</td>
<td>976 nm</td>
</tr>
<tr>
<td>$\sigma_{a\text{ ASE}}$</td>
<td>$3.32 \times 10^{-26}$ m$^2$</td>
</tr>
<tr>
<td>$\sigma_{e\text{ ASE}}$</td>
<td>$4.98 \times 10^{-25}$ m$^2$</td>
</tr>
<tr>
<td>$\sigma_{a\text{ pump}}$</td>
<td>$2.48 \times 10^{-24}$ m$^2$</td>
</tr>
<tr>
<td>$\sigma_{e\text{ pump}}$</td>
<td>$2.48 \times 10^{-24}$ m$^2$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.84 ms</td>
</tr>
<tr>
<td>$\Delta \lambda_{\text{ASE}}$</td>
<td>10 nm</td>
</tr>
<tr>
<td>$m$</td>
<td>2</td>
</tr>
<tr>
<td>$L$</td>
<td>6.5 m</td>
</tr>
<tr>
<td>$a_{\text{core}}$</td>
<td>15 µm</td>
</tr>
<tr>
<td>$a_{\text{clad}}$</td>
<td>130 µm</td>
</tr>
<tr>
<td>NA</td>
<td>0.06</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.003 m$^{-1}$</td>
</tr>
</tbody>
</table>
Given the initial boundary conditions, the propagation Equation 2.7 is resolved by standard numerical integration techniques. The ASE output power and the output modal power weights are obtained by Equations 2.16 and 2.17, and then the beam-quality factor is calculated by Equations 2.18 and 2.19. The ASE output power, the output modal power weights, and the beam-quality factor as functions of input pump power up to 25 W are calculated and plotted in Figs. 2.6-2.8 respectively.

![Graph](image)

Fig. 2.6 The ASE output power as a function of input pump power from the localized model (solid) and experiments (dotted). The dashed vertical line is the pump threshold for saturation.

Fig. 2.6 shows that the output power and the pump threshold for saturation from simulation are close to those from the experiment. Fig. 2.7 shows that all the transverse modes have nearly the same output modal power weights at very low input pump power. The output modal power weights of the lower-order modes (LP$_{01}$ and LP$_{11}$ in this case) increase with pump power while those of the higher-order modes
(LP_{01}, LP_{11}, and LP_{31} in this case) decrease. Most importantly, the lower-order modes maximize near the pump threshold for saturation, while the higher-order modes minimize.

Fig. 2.7 The output modal power weights as a function of input pump power from the localized model. The dashed vertical line is the pump threshold for saturation.

From Equations 2.18 and 2.19, it is obvious that the output modal power weights determine the beam-quality factor, since the beam-quality factor of each LP transverse mode is fixed. Generally speaking, the lower-order modes have smaller beam-quality factor, while the higher-order modes have larger beam-quality factor [17]. So the beam-quality factor decreases, minimizes, and increases when the output
modal power weights of the lower-order modes increase, maximize, and decrease, respectively.

This behavior is manifest in Fig. 2.8, which shows that the beam-quality factor minimizes near the pump threshold for saturation, which agrees with the experimental results. The behavior of beam-quality factor follows directly from the behavior of output modal power weights as shown in Fig. 2.8. The output modal power weights, and thus the beam-quality factor, are determined by the gain of each transverse mode, which depends on the overlap of mode field distribution and population inversion distribution.

![Fig. 2.8 The beam-quality factor as a function of input pump power from the localized model. The dashed vertical line is the pump threshold for saturation.](image)

The upper-level dopant distribution across the injection fiber end for various pump power is plotted in Fig. 2.9.
When the pump power is 5W and below the pump threshold for saturation, the population inversion is nearly uniform across the core, so the modal gain is nearly proportional to the fraction of the mode in the core. The intensity distribution of the fundamental (LP_{01}) mode and a higher-order mode (LP_{31}) mode in the fiber core are plotted in Fig. 2.10, which shows the fields of the lower-order modes are more confined in the core. Therefore, the lower-order modes have larger gain than the higher-order modes, as shown in Table 2.2. In this small-signal regime, the powers in the modes with larger gain increase faster than those in the modes with smaller gain. Therefore, the output modal power weights of the lower-order modes increase and the beam quality improves.

Above the pump threshold for saturation, the TSHB effect is shown in the upper-level dopant distribution with 7- and 15-W input pump power in Fig. 2.9,
where the gain profile is much more saturated in the center of the core than on the edge. Since the lower-order modes are more concentrated in the center of the core, the gain of the lower-order modes decreases relative to the gain of higher-order modes. In the saturation region, the faster the gain in the modes decreases, the slower the power in the modes increases. So under the impact of the TSHB effect the output-fraction factors of lower-order modes decrease and the beam quality degrades.

Fig. 2.10 The intensity distribution of (a) LP_{01} mode and (b) LP_{31} mode in the fiber core.

Table 2.2 The Ratio of the Gain of the HOMs to That of the Fundamental Mode with 5-W Pump Power.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\frac{g_{LP_{vm}}}{g_{LP_{01}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP_{11}</td>
<td>95.5%</td>
</tr>
<tr>
<td>LP_{21}</td>
<td>89.4%</td>
</tr>
<tr>
<td>LP_{02}</td>
<td>84.8%</td>
</tr>
<tr>
<td>LP_{31}</td>
<td>76.9%</td>
</tr>
</tbody>
</table>
2.5 The Validity of Simplified Model

The rate and propagation equations are often simplified by replacing transverse space integrals with overlap integrals, especially in single-mode fibers [14, 19]. The validity of such simplification in multimode fibers is discussed below.

The rate equations of such a simplified model are given by

\[
\frac{d\phi(z)}{dt} = \sum_k \frac{P_k(z)\Gamma_k\sigma_a}{\hbar v_k A} n_1(z) - \sum_k \frac{P_k(z)\Gamma_k\sigma_{ek}}{\hbar v_k A} n_2(z) - \frac{n_2(z)}{\tau}, \tag{2.21}
\]

\[
n_t(z) = n_1(z) + n_2(z), \tag{2.22}
\]

where \(n_1\) and \(n_2\) represent average ground-level and upper-level ytterbium ion density across the fiber cross section, respectively, \(A\) is the area of the core cross section, and \(\Gamma_k\) is the overlap integral between the mode and dopants.

The overlap integral of ASE modes is given by

\[
\Gamma_{vm} = \int_0^{2\pi} \int_0^{2\pi} i_{vm}(r,\phi)n_t(r,\phi,z)\,rdrd\phi \int_0^{\infty} n_t(r,\phi,z)\,rdrd\phi. \tag{2.23}
\]

If the dopant is distributing uniformly in the fiber core, \(\Gamma_{vm}\) depends only on the mode field and can be simplified as

\[
\Gamma_{vm} = \int_0^{2\pi} \int_0^{\infty} i_{vm}(r,\phi)drd\phi. \tag{2.24}
\]
The overlap integral of pump is given by

\[ \Gamma_{pump} = \frac{a^2_{\text{core}}}{a^2_{\text{clad}}}. \]  

(2.25)

In the steady-state case, \( n_2 \) is solved as

\[ n_2(z) = n_t \frac{\sum_k P_k(z) \Gamma_k \sigma_{ak}}{1 + \sum_k P_k(z) \Gamma_k (\sigma_{ak} + \sigma_{ek})}. \]  

(2.26)

Since the upper-level dopant distribution depends only on the longitudinal coordinate \( z \) and is independent of radial and azimuthal coordinates, TSHB is not included in the simplified model. The saturation effect is included as an averaged level across the core.

The simplified propagation equation is given by

\[ \frac{dP_k(z)}{dz} = u_k \sigma_{ek} \left[ P_k(z) + m h c^2 \Delta \lambda_k \right] \Gamma_k n_2(z) - u_k \sigma_{ak} P_k(z) \Gamma_k n_1(z) - u_k \alpha P_k(z). \]  

(2.27)

Given the same initial boundary conditions as Equation 2.20, the propagation Equation 2.27 is resolved. The output power as a function of input pump power is the same in the simplified model as the localized model. However, the modal properties are significantly different. The output modal power weights and beam-quality factor
as functions of input pump power up to 25 W in the simplified model compared to the localized model are shown in Figs. 2.11 and 2.12.

Fig. 2.11 The output modal power weights as a function of input pump power from the localized (solid) and simplified (dashed) models. The dashed vertical line is the pump threshold for saturation.

Fig. 2.12 The beam-quality factor as a function of input pump power from the localized (solid) and simplified (dashed) models. The dashed vertical line is the pump threshold for saturation.
Fig. 2.11 shows that the output modal power weights in the simplified model are the same as those in the localized model when the input pump power is below the pump threshold for saturation. However, the output modal power weights of the lower-order modes in the simplified model keep increasing slower beyond the pump threshold for saturation. Similarly, Fig. 2.12 shows that the beam-quality factor in the simplified model is the same as that in the localized model at the input pump power below the pump threshold for saturation. However, above the pump threshold for saturation, the beam-quality factor in the simplified model keeps decreasing slower.

The behaviors of output modal power weights and the beam-quality factor are consistent and can be explained as follows: in the simplified model, the gain of each transverse mode is always proportional to the fraction of the mode in the core, so the simplified model gives the same simulation results as the localized model below the pump threshold of saturation. Above the pump threshold for saturation, the gain of each mode decreases at the same rate, so the power in each mode increases nearly at the same rate. Therefore, the output modal power weight of each mode approaches constant, as does the beam-quality factor, at very high input pump power.

The failure to show the minimum of beam-quality factor near the pump threshold for saturation proves that the simplified model is not valid and the TSHB effect is required to model LMA multimode fibers when dealing with beam quality.
2.6 Fiber Amplifiers

Fiber amplifiers are of more importance than fiber ASE sources in high-power applications. As mentioned in Section 2.3, the localized model assumed no interference, which is true for an optical beam from incoherent sources like ASE, but not for coherent sources like fiber amplifiers. The calculations that follow include these interference terms.

For the purpose of simplicity, we assume the input beam is linear polarized, only the LP_{01} and LP_{11} modes are coupled into the fiber amplifier, and the power contained in the LP_{11} mode is evenly distributed in the two orientations.

The initial boundary conditions are changed to

\[
P^+_{\text{pump}}(0) = P_0,
\]

\[
P^-_{\text{pump}}(L) = 0,
\]

\[
P^+_{01}(0) = P_s \chi,
\]

\[
P^+_{11}(0) = P_s (1 - \chi),
\]

\[
P^+_{\nu m}(0) = 0, \text{ otherwise}
\]

\[
P^-_{\nu m}(L) = 0,
\]

where \( P_s \) is the total signal power and \( \chi \) is input modal power weight of LP_{01} mode.
The normalized electric field distribution of output beam can be written as

\[ E(r, \phi) = \sum_{\nu, m} \sqrt{\alpha_{vm}} E_{vm}(r, \phi) e^{-i\beta_{vm}L}, \quad (2.29) \]

where the propagation coefficient \( \beta_{vm} \) is given by [16]

\[ \beta_{vm}^2 = \left( \frac{2\pi}{\lambda} \right)^2 n_{\text{core}}^2 - \kappa_{vm}^2. \quad (2.30) \]

In this form, modal dispersion is included. No initial phase difference is considered between the modes since the two modes are assumed to be excited by a single-mode input beam (for example, by misalignment). The beam-quality factors are calculated directly from Ref. 18 without simplification used to obtain Equations 2.18 and 2.19.

The new parameters used in the simulation and those that differ from Table 2.1 are listed in Table 2.3: \( \lambda_s \) is the wavelength of the signal, \( \sigma_{as} \) and \( \sigma_{es} \) are the absorption and emission cross sections of the signal, \( \Delta\lambda_{\text{ASE}} \) is set to be the same as the bandwidth of the signal, and \( P_0 \) is set far above the pump threshold for saturation, which is true in most high-power applications.

The calculations show that the output power is nearly the same in both models and does not change as the input-fraction factor. For both models, the output modal power weight of the fundamental mode and the beam-quality factor are calculated as a function of the input modal power weight of the fundamental mode from 0.4 to 1 (shown in Figs. 2.13 and 2.14).
Table 2.3. New and changed parameters used in simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_s$</td>
<td>1053 nm</td>
</tr>
<tr>
<td>$\sigma_{as}$</td>
<td>$2.07 \times 10^{-26}$ m$^2$</td>
</tr>
<tr>
<td>$\sigma_{es}$</td>
<td>$3.63 \times 10^{-25}$ m$^2$</td>
</tr>
<tr>
<td>$P_s$</td>
<td>10 W</td>
</tr>
<tr>
<td>$n_{core}$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\Delta\lambda_{ASE}$</td>
<td>0.1 nm</td>
</tr>
<tr>
<td>$P_0$</td>
<td>1.5 kW</td>
</tr>
</tbody>
</table>

Fig. 2.13 The output modal power weight as a function of input modal power weight of the fundamental mode from the localized (solid) and simplified (dashed) models.

Fig. 2.13 shows that the output modal power weight of the fundamental mode in the localized model is smaller than the corresponding input modal power weight, while the output modal power weight of the fundamental mode in the simplified model is larger. These behaviors can be explained as follows. The input pump power
used in the simulations is well above the pump threshold for saturation. In the localized model, the gain of the fundamental mode is less than that of the LP$_{11}$ mode due to the TSHB effect. Therefore, the fundamental mode is amplified less than the LP$_{11}$ mode, leading to a smaller output modal power weight of the fundamental mode. However, in the simplified model, the TSHB effect is ignored and the gain of the fundamental mode is always larger than that of the LP$_{11}$ mode. In this case, the fundamental mode is always amplified more than the LP$_{11}$ mode, leading to larger output modal power weight of the fundamental mode.

Fig. 2.14 shows that the beam-quality factor from the simplified model is significantly underestimated compared to that from the localized model, due to the underestimation of the output modal power weight of the fundamental mode. This difference underscores the importance of the TSHB effect on beam quality in LMA fiber amplifiers for high-power applications.

Fig. 2.14 The beam-quality factor as a function of input modal power weight of the fundamental mode from the localized (solid) and the simplified (dashed) models.
2.7 Conclusions

In conclusion, the importance of the TSHB effect in LMA multimode fibers was revealed through experiments and simulations. The measured beam-quality factor decreases until the gain becomes saturated in an ASE source based on an Yb-doped, LMA multimode fiber. At saturation, the beam-quality factor reaches a minimum, beyond which it increases again. Numerical simulation trends based on a model using spatially resolved gain and transverse-mode decomposition of the optical field agree with the experimental results. A simplified model without TSHB is shown not fit to predict the observed behavior of beam quality in LMA fibers, especially at high powers. A comparison of both models shows that the TSHB effect is also critical for properly modeling beam quality in LMA fiber amplifiers.
References


Chapter Three

Modal Decomposition and Fiber Characterization

Modal decomposition in multimode and LMA fibers is gaining more importance in many applications. To obtain accurate results, the fiber and imaging parameters must be known precisely. In this chapter, new methods for precise modal decomposition are developed by finding the minimum of the residue with variable parameters.

In Section 3.1, a precise modal decomposition method is developed for a given field distribution with intensity and phase information, and extended to single-mode fiber characterization. In Section 3.2, a precise modal decomposition method following similar principle is developed without direct phase information and extended to single-mode fiber characterization too. In Sections 3.3 and 3.4, the experiments of single-mode fiber characterization and modal decomposition in few-mode fibers, respectively, are carried out using the method described in Section 3.2.

3.1 Modal Decomposition with Phase Information

The linearly polarized (LP) and normalized electric field $E$ of a coherent beam from a step-index fiber can be written as a superposition of orthogonally normalized LP modes $e_i$, 

.. _modal-decomposition-with-phase-information:
\[ E(x, y) = \sum_i c_i e_i(x, y) = \sum_i a_i e^{i\phi} e_i(x, y), \]

\[ a_i = |c_i|, \]

\[ \phi_i = \text{arg}(c_i), \]

\[ \langle e_i | e_j \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e_i(x, y)e_j^*(x, y)dx\,dy = \delta_{ij}, \]

where \( c_i \) is the modal coefficient, \( a_i \) is the amplitude of the modal coefficient, and \( \Phi_i \) is the modal phase.

From Equation 3.1, if the spatial field distributions of the modes and the beam are known, the modal coefficient is determined by the spatial overlap of the mode field with that of the beam field,

\[ c_i = \langle E | e_i \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y)e_i^*(x, y)dx\,dy, \]

The modal power weight \( \alpha_i \) can be obtained by

\[ \alpha_i = a_i^2 = |c_i|^2 = \langle E | e_i \rangle^2, \]

and the modal phase can be obtained by Equation 3.1.

The field distribution of the beam can be extracted from interferometric measurements, wavefront curvature measurements, and phase retrieval with transport-of-intensity equation as discussed in Section 1.3. However, the field distribution of modes can not be extracted individually and must be calculated.
The field distribution of modes inside the fiber can be calculated through Equations 2.9-2.14 and are dependent on fiber specifications. The most important fiber parameters are the NA and the core radius. Imaging and alignment parameters such as the magnification ratio and the origin of the coordinate system are also important. An example of the latter one is as shown in Fig. 3.1.

Fig. 3.1 Mixed fundamental mode and LP_{11} mode.

The origin of the coordinate system in Fig. 3.1 is the geometry center of the image which could be taken from an intensity measurement to a beam of mixed fundamental (LP_{01}) mode and LP_{11} mode, instead of where the intensity is highest, which is where the origin would be for a beam of fundamental mode. Misplacing the origin of the coordinate system with \( \Delta x \) and \( \Delta y \) will lead to a mistaken field distribution \( e(x - \Delta x, y - \Delta y) \), and therefore lead to incorrect modal power weights.

Using Equations 3.1-3.3 requires precise knowledge of both the modes and the measured beam to obtain accurate results. However, the specifications and
parameters required to define the modes and measure the beam are often given with large error or are hard to obtain in practice. A method for precise modal decomposition is developed in this work that eliminates the obstacles.

### 3.1.1 Correct Modes and Residue

Suppose \( \epsilon_i \) is a set of orthogonally normalized modes calculated from inaccurate parameters. The correct mode \( e_i \) and the total field \( E \) be expressed as a linear superposition of \( \epsilon_i \) with a remainder term \( \Delta_i \) and \( \Delta \),

\[
e_i = \sum_j \langle e_i | \epsilon_j' \rangle \epsilon_j' + \Delta_i,
\]

\[
\langle \Delta_i | \epsilon_j' \rangle = 0,
\]

\[
E = \sum_i c_i e_i = \sum_i c_i \left( \sum_j \langle e_i | \epsilon_j' \rangle \epsilon_j' + \Delta_i \right) = \sum_i c_i \epsilon'_i + \Delta,
\]

\[
c'_i = \langle E | \epsilon'_i \rangle = \sum_i \langle e_i | \epsilon'_i \rangle c_i,
\]

\[
\Delta = \sum_i c_i \Delta_i.
\]

Consider the sum of modal power weights of the a set of accurate and inaccurate modes,
\[ Q = \sum_i \alpha_i = \sum_i |c_i|^2 = \langle E|E \rangle = 1, \]

(3.5)

\[ Q' = \sum_i \alpha'_i = \sum_i |c'_i|^2 = \langle E|E \rangle - \langle \Delta|\Delta \rangle = 1 - \langle \Delta|\Delta \rangle \leq 1. \]

Only when \( \Delta = 0 \), \( Q' = 1 \). From Equation 3.4, for a limited number of \( c_i \), this condition will not be met in practice.

Therefore, the set of modes calculated from correct parameters will give the largest sum of power weights among all the calculated mode sets. This relationship can be utilized to find the accurate modes and the correct parameters. At the same time, the residue \( R \), which is defined as the difference between the unity and the sum of the power weights of a set of modes, will be the minimum for the correct modes, as shown in Equation 3.6. This relationship can be utilized to find the accurate modes and the correct fiber parameters too and will be used in this work.

\[ R = 1 - Q = 0, \]

(3.6)

\[ R' = 1 - Q' = \langle \Delta|\Delta \rangle \geq 0 = R. \]

The procedure for precise modal decomposition with phase information can be summarized as follows,

1. Calculate different sets of modes from varied parameters.
2. Calculate the modal coefficients for these sets of modes against the measured field.
3. Calculate the residues for these sets of modes.

4. Find the minimum of the residues and the corresponding modal power weights, phases, and parameters.

### 3.1.2 Application to Single-Mode Fibers

Although this precise modal decomposition method is developed for multimode fibers, it can be applied to single-mode fibers for fiber characterization purposes.

The set of modes $e_i$ or $e'_i$ is simplified as the fundamental mode $e$ or $e'$ in single mode fibers. Equations 3.4 are simplified as,

$$e = \langle e | e' \rangle e' + \Delta,$$

$$\langle \Delta | e' \rangle = 0,$$

$$E = e = c' e' + \Delta,$$

$$c' = \langle E | e' \rangle = \langle e | e' \rangle.$$

and Equations 3.5 are simplified as,

$$Q = |c|^2 = \langle E | E \rangle = 1,$$

$$Q' = |c'|^2 = \langle E | E \rangle - \langle \Delta | \Delta \rangle \leq 1.$$

Equations 3.6 remain as the same in the single-mode case.
The procedure for single-mode fiber characterization (assuming the availability of phase information) can be summarized as follows,

1. Calculate different fundamental modes from varied parameters.
2. Calculate the residues for these modes against the measured field.
3. Find the minimum of the residues and thus the corresponding fiber parameters.

3.2 Modal Decomposition without Phase Information

3.2.1 Linear Independence of Modal Self- and Cross-Products

Since the (complex) field distribution cannot be measured directly and must be extracted from intensity measurements, modal decomposition directly from intensity distribution is highly desirable.

From Equations 3.1, the normalized optical intensity $I$ of a linearly polarized coherent beam from a step-index fiber can be written as a superposition of self-products $e_i^2$ and cross-products $e_i e_j$ of the field distributions of orthogonally normalized LP modes $e_i$, 
\[ I(x, y) = E(x, y)E^*(x, y) \]
\[ = \left( \sum_{i} a_i e^{\phi_i} e_i(x, y) \right) \left( \sum_{i} a_i e^{\phi_i} e_i(x, y) \right)^* \]
\[ = \sum_{i} a_i^2 e_i^2(x, y) + \sum_{i<j} 2a_i a_j \cos(\Delta \Phi_{ij}) e_i(x, y) e_j(x, y), \] (3.9)

\[ \Delta \Phi_{ij} = \phi_i - \phi_j, \]

where \( \Delta \Phi_{ij} \) is the modal phase difference.

\( e_i^2 \), and \( e_i \) can be viewed as vectors in the \( x-y \) plane. If they are linearly independent, the expression of \( I \) of \( e_i^2 \) and \( e_i \) will be unique, and modal decomposition with intensity-only information of the beam can be achieved by obtaining \( a_i \) and \( \Delta \Phi_{ij} \) following a procedure similar to that for modal decomposition with the field distribution of the beam.

The condition for linear independence of \( e_i^2 \) and \( e_i \) is that \( b_k = 0 \) is the only solution (which is called a trivial solution) to the equation

\[ \sum b_k g_k(x, y) = 0, \] (3.10)

where the vector \( g_k \) is either \( e_i^2 \) or \( e_i \).

For a few-mode fiber containing only LP\(_{01}\) and LP\(_{11}\) modes (the simplest case of multimode fiber), Equation 3.10 can be expressed as

\[ b_1 e_{01}^2 + b_2 e_{11,even}^2 + b_3 e_{11,odd}^2 + b_4 e_{01} e_{11,even} + b_5 e_{01} e_{11,odd} + b_6 e_{01} e_{11,even} e_{11,odd} = 0, \] (3.11)
which can be physically separated into two regions: in the core and in the cladding. Only the core part is considered here, since if \( e_i^2 \) and \( e_ie_j \) are linearly independent in the core, they are linearly independent in the whole cross section.

From Equations 2.4-2.14, Equation 3.11 in the core can be rewritten as

\[
\begin{align*}
& b_1 J_0^2(\kappa_{01}r) + b_2 J_1^2(\kappa_{11}r) \cos^2(\phi) + b_3 J_1^2(\kappa_{11}r) \sin^2(\phi) \\
& + b_4 J_0(\kappa_{01}r) J_1(\kappa_{11}r) \cos(\phi) + b_5 J_0(\kappa_{01}r) J_1(\kappa_{11}r) \sin(\phi) \\
& + b_6 J_1^2(\kappa_{11}r) \cos(\phi) \sin(\phi) = 0, \\
\end{align*}
\]

(3.11)

where the normalization coefficient \( b_{vm} \) of the electric field in Equation 2.9 is omitted for simplicity. This does not affect the linear dependence properties of the vectors.

Expanding Equation 3.11 as Fourier series yields,

\[
\begin{align*}
& b_1 J_0^2(\kappa_{01}r) + b_2 J_1^2(\kappa_{11}r) \cos(2\phi) + \frac{1}{2} \\
& + b_3 J_1^2(\kappa_{11}r) \frac{1 - \cos(2\phi)}{2} + b_4 J_0(\kappa_{01}r) J_1(\kappa_{11}r) \cos(\phi) \\
& + b_5 J_0(\kappa_{01}r) J_1(\kappa_{11}r) \sin(\phi) + b_6 J_1^2(\kappa_{11}r) \frac{\sin(2\phi)}{2} = 0, \\
\end{align*}
\]

(3.12)

For Equation 3.12 to be valid for any \( \Phi \), the following conditions must be satisfied,
which results in,

\[ b_1 J_0^2(\kappa_0 r) + \frac{b_2 + b_3}{2} J_1^2(\kappa_1 r) = 0, \]

\[ b_2 J_0(\kappa_0 r) J_1(\kappa_1 r) = 0, \]

\[ b_3 J_0(\kappa_0 r) J_1(\kappa_1 r) = 0, \]

\[ \frac{b_2 - b_3}{2} J_1^2(\kappa_1 r) = 0, \]

\[ \frac{b_6}{2} J_1^2(\kappa_1 r) = 0, \]

\[ \text{(3.13)} \]

At \( r=0 \), \( J_1^2(0)=0 \), and Equations 3.14 gives

\[ b_2 - b_3 = 0, \]

\[ b_4 = b_5 = b_6 = 0. \]

\[ \text{(3.14)} \]

From Equations 3.14 and 3.16,
\[ b_2 = 0. \] (3.17)

From Equations 3.14 to 3.17, since all \( b_k \) must be 0 for Equation 3.10 to be valid, \( e_i^2 \) and \( e_ie_j \) are linearly independent.

For a multimode fiber with the higher order modes, the proof is more complicated. Equation 3.10 is can be expressed as

\[
\sum_{\nu_1, \nu_2} b^2_{\nu_1\nu_2} e_{\nu_1\nu_2}^2 + \sum_{\nu_1, \nu_2, \nu_3, \nu_4} b_{\nu_1\nu_2\nu_3\nu_4} e_{\nu_1\nu_2\nu_3\nu_4} = 0,
\] (3.18)

where \( o = 1 \) or 2 for even or odd orientation of modes and is omitted for the modes with zero azimuthal mode number since they only have one orientation which is even.

Again, considering only the core region here, Equation 3.18 is expanded as a Fourier series, like Equation 3.12,

\[
A_0(r) + \sum_{n=0} A_{2n}(r) \cos(2n\phi) + \sum_{n=0} A_{2n+1} \cos((2n+1)\phi)
+ \sum_{n=0} B_{2n}(r) \sin(2n\phi) + \sum_{n=0} B_{2n+1} \sin((2n+1)\phi) = 0,
\] (3.19)

where the Fourier cosine coefficient \( A_n \) and sine coefficient \( B_n \) are defined as...
$$A_0 (r) = 2 \sum_{m} b_{0m} J^2_{0m} (\kappa_{0m} r) + 2 \sum_{m < m_2} b_{0m,0m_2} J_{0m} (\kappa_{0m} r) J_{0m_2} (\kappa_{0m_2} r)$$  
\begin{align*}
+ \sum_{v>0,m} (b_{v0} m + b_{v1} m) J^2_{sm} (\kappa_{0m} r) + \sum_{v>0,m < m_2} (b_{v0m_1,0m_2} + b_{v0m_1,2m_2}) J_{v0m_1} (\kappa_{0m_1} r) J_{v0m_2} (\kappa_{0m_2} r) \\
+ \sum_{v=2n,m_1 < m_2} (b_{v0m_1,1m_2} - b_{v0m_1,2m_2}) J_{v0m_1} (\kappa_{0m_1} r) J_{v0m_2} (\kappa_{0m_2} r) \\
+ \sum_{v_1 + v_2 = 2n,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} - b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r)
\end{align*}

$$A_{2n} (r) = 2 \sum_{v=2n,m_1 < m_2} b_{0m_1 m_2} J_{0m} (\kappa_{0m_1} r) J_{m} (\kappa_{0m_2} r)$$  
\begin{align*}
+ \sum_{v_1 + v_2 = 2n,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} + b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r) \\
+ \sum_{v_1 + v_2 = 2n,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} - b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r)
\end{align*}

$$A_{2n+1} (r) = 2 \sum_{v=2n+1,m_1 < m_2} b_{0m_1 m_2} J_{0m} (\kappa_{0m_1} r) J_{m} (\kappa_{0m_2} r)$$  
\begin{align*}
+ \sum_{v_1 + v_2 = 2n+1,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} + b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r) \\
+ \sum_{v_1 + v_2 = 2n+1,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} - b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r)
\end{align*}

$$B_2 (r) = 2 \sum_{v=2n,m_1,m_2} b_{0m_1 m_2} J^2_{0m} (\kappa_{0m} r) + \sum_{v=n,m_1 < m_2} (b_{v0m_1 m_2} + b_{v0m_1 m_2}) J_{v0m_1} (\kappa_{0m_1} r) J_{v0m_2} (\kappa_{0m_2} r)$$  
\begin{align*}
+ \sum_{v_1 + v_2 = 2n,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} + b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r) \\
+ \sum_{v_1 + v_2 = 2n,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} - b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r)
\end{align*}

$$B_{2n+1} (r) = 2 \sum_{v=2n+1,m_1,m_2} b_{0m_1 m_2} J_{0m} (\kappa_{0m_1} r) J_{1m} (\kappa_{0m_2} r)$$  
\begin{align*}
+ \sum_{v_1 + v_2 = 2n+1,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} - b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r) \\
+ \sum_{v_1 + v_2 = 2n+1,v_1,v_2 > 0,m_1,m_2} (b_{v_1 v_2 m_1 m_2} + b_{v_1 v_2 m_1 m_2}) J_{v_1 m_1} (\kappa_{0m_1} r) J_{v_2 m_2} (\kappa_{0m_2} r)
\end{align*}

Again, the normalization coefficient $b_{1m}$ of the electric field in Equation 2.9 is omitted for simplicity, which does not affect the linear dependence properties of the vectors.
According to Equation 3.10, all Fourier coefficients are functions of \( J_{vm}(\kappa_{vm}r) \), which can be expanded as a power series [2],

\[
J_{vm}(\kappa_{vm}r) = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!(l + \nu)^\frac{2l + \nu}{2}} \left( \frac{\kappa_{vm}r}{2} \right)^{2l + \nu}. \tag{3.21}
\]

Inserting Equation 3.21 into Equation 3.20 and applying the condition that all Fourier coefficients must be zero yields,

\[
A_0(r) = 2\sum_{m} b_{0m} \left( \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left( \frac{\kappa_{0m}r}{2} \right)^{2l} \right)^2
+ 2 \sum_{m_1 < m_2} b_{0m_10m_2} \left( \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left( \frac{\kappa_{0m_1}r}{2} \right)^{2l} \right) \left( \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left( \frac{\kappa_{0m_2}r}{2} \right)^{2l} \right)
+ \sum_{\nu > 0, m_1 < m_2} \left( b_{\nu m_11m_2} + b_{\nu m_12m_2} \right) \left( \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left( \frac{\kappa_{\nu m_1}r}{2} \right)^{2l + \nu} \right) \left( \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left( \frac{\kappa_{\nu m_2}r}{2} \right)^{2l + \nu} \right)
= \sum_{s=0}^{\infty} \sum_{l=0}^{s} \sum_{m} b_{0m} \frac{(-1)^l}{l!(s - l)!!} 2^{s - 1} \kappa_{0m}^{-2s} r^{-2s}
+ \sum_{s=0}^{\infty} \sum_{l=0}^{s} \sum_{m_1 < m_2} b_{0m_10m_2} \frac{(-1)^s}{(s - l)!!} 2^{s - 1} \kappa_{0m_1}^{-2s} \kappa_{0m_2}^{-2s} r^{-2s}
+ \sum_{s=0}^{\infty} \sum_{l=0}^{s} \sum_{\nu > 0, m} \left( b_{\nu m_11m_2} + b_{\nu m_12m_2} \right) \frac{(-1)^s}{l!(s - l)!!(s - l + \nu)!!} 2^{s + \nu} \kappa_{0m}^{-2s - 2\nu} r^{-2s - 2\nu}
+ \sum_{s=0}^{\infty} \sum_{l=0}^{s} \sum_{\nu > 0, m_1 < m_2} \left( b_{\nu m_11m_2} + b_{\nu m_12m_2} \right) \frac{(-1)^s}{l!(s - l)!!(s - l + \nu)!!} 2^{s + \nu} \kappa_{0m_1}^{-2s - 2\nu} \kappa_{0m_2}^{-2s - 2\nu} r^{-2s - 2\nu} \tag{3.22}
\]
3.12 to be valid for any $r$, all of the power coefficients in Equation 3.22 must be zero,

\[
\begin{align*}
\sum_{s=0}^{\infty} \left( \frac{(-1)^s}{s! (s-l)!} \right)^2 \left( \sum_{m=0}^{s} b_{0m} k_{0m}^{2s} + \sum_{m\neq 0, m_{1}, m_{2} \leq m_{2}} b_{0m_{1}0m_{2}} k_{0m_{1}}^{2l-2s} k_{0m_{2}}^{2} \right) r^{2s} + \\
+ \sum_{s=0}^{\infty} \left( \frac{(-1)^s}{s! (s-l)!} \right)^2 \left( \sum_{m=0}^{s} \left( b_{0m_{1}} + b_{0m_{2}} \right) k_{0m}^{2s} + \sum_{m\neq 0, m_{1}, m_{2} \leq m_{2}} \left( b_{0m_{1}1m_{2}} + b_{0m_{2}1m_{2}} \right) k_{0m_{1}}^{2l+v} k_{0m_{2}}^{2l-2s+3v} \right) r^{2s+2v} \\
= 0 \\
\end{align*}
\]

(3.22 continued)

The other Fourier coefficients expansions are considered later. For Equation 3.12 to be valid for any $r$, all of the power coefficients in Equation 3.22 must be zero,

\[
\begin{align*}
\sum_{s=0}^{\infty} \left( \frac{(-1)^s}{s! (s-l)!} \right)^2 \left( \sum_{m=0}^{s} b_{0m} k_{0m}^{2s} + \sum_{m\neq 0, m_{1}, m_{2} \leq m_{2}} b_{0m_{1}0m_{2}} k_{0m_{1}}^{2l-2s} k_{0m_{2}}^{2} \right) r^{2s} + \\
+ \sum_{s=0}^{\infty} \left( \frac{(-1)^s}{s! (s-l)!} \right)^2 \left( \sum_{m=0}^{s} \left( b_{0m_{1}} + b_{0m_{2}} \right) k_{0m}^{2s} + \sum_{m\neq 0, m_{1}, m_{2} \leq m_{2}} \left( b_{0m_{1}1m_{2}} + b_{0m_{2}1m_{2}} \right) k_{0m_{1}}^{2l+v} k_{0m_{2}}^{2l-2s+3v} \right) r^{2s+2v} \\
= 0 \\
\end{align*}
\]

(3.23)

\[
\begin{align*}
&= 0, \\
&\text{ } \text{ } s = 0, \ 1, \ \ldots \ \infty. \\
\end{align*}
\]
Equation 3.23 is a system of linear equations with \( b_{0m}, (b_{m1} + b_{m2}), b_{0m0m2}, \) and \( b_{m1m1m1} + b_{m2m2m2} \) as variables. Suppose the total number of variables is \( N \), then the coefficient matrix of the system of linear equations is a \( \infty \times N \) matrix. With limited number of \( \kappa_{im} \), the rank of the coefficient matrix is \( N \). Therefore, the system of linear equations only has a trivial solution, which gives,

\[
\begin{align*}
    b_{0m} &= 0, \\
    b_{m1} + b_{m2} &= 0, \\
    b_{0m1m1} &= 0, \quad m_1 < m_2 \\
    b_{m1m1m1} + b_{m2m2m2} &= 0, \quad \nu > 0, m_1 < m_2.
\end{align*}
\] (3.24)

Expanding the other Fourier coefficients as power series following the same procedure gives,

\[
\begin{align*}
    b_{0m1m1m1} &= 0, \quad \nu = 2n > 0, m_1 < m_2 \\
    b_{m1} - b_{m2} &= 0, \quad \nu > 0 \\
    b_{m1m1m1} - b_{m2m2m2} &= 0, \quad \nu > 0, m_1 < m_2 \quad (3.25) \\
    b_{\nu_1m1 \nu_2 m2} + b_{\nu_1m2 \nu_2 m2} &= 0, \quad \nu_1 - \nu_2 = 2n, \nu_2 > 0 \\
    b_{\nu_1m1 \nu_2 m2} - b_{\nu_1m2 \nu_2 m2} &= 0, \quad \nu_1 + \nu_2 = 2n, \nu_1 > \nu_2 > 0
\end{align*}
\]
where \( n = 1,2, \ldots \)

From Equations 3.24-3.28, \( b_k = 0 \). Since all \( b_k \) must be 0 for Equation 3.10 to be valid, \( e_i^2 \) and \( e_ie_j \) are linearly independent in multimode fibers. The preceding proof of linear independence of \( e_i^2 \) and \( e_ie_j \) in few-mode fibers is a special case of that in multimode fibers.
3.2.2 Transformation of Coordinates

Since the vectors $e_i^2$ and $e_ie_j$ are linearly independent, they form a basis of the vector space $S = \sum b_k g_k$, where the vector $g_k$ is either $e_i^2$ or $e_ie_j$ as in Equation 3.10.

From Equation 3.9, $I$ obviously belongs to $S$, so $I$ can be uniquely represented as a linear combination of $e_i^2$ and $e_ie_j$ with $a_i^2$ and $2a_i a_j \cos(\Delta \phi_y)$ as coordinates [1]. However, these coordinates cannot be extracted directly as in Equation 3.2, since $g_k$ are not orthogonally normalized vectors like the modes $e_i$. Using the Gram–Schmidt process [3], $g_k$ can be transformed to orthogonally normalized vectors $h_k$ with

$$\langle h_i | h_j \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_i(x, y) h_j^*(x, y) dx dy = \delta_{ij}. \quad (3.29)$$

The normalized intensity distribution $I'$ can be expressed in the basis of $g_k$ and $h_k$ as,

$$\langle I' | I' \rangle = 1,$$

$$I' = \sum_k \xi_k^* g_k = g \xi,$$  \hspace{1cm} (3.30)

$$I' = \sum_k \eta_k h_k = h \eta,$$

where the row vectors $g$ and $h$, and the column vectors $\xi$ and $\eta$ are,
\[ g = (g_1, g_2, \ldots, g_K), \]

\[ h = (h_1, h_2, \ldots, h_K), \]  

(3.31)

\[
\xi = \begin{pmatrix}
\xi_1 \\
\xi_2 \\
\vdots \\
\xi_K
\end{pmatrix}, \quad \eta = \begin{pmatrix}
\eta_1 \\
\eta_2 \\
\vdots \\
\eta_K
\end{pmatrix}
\]

\( \eta \), the coordinates of \( I' \) in the basis \( h_k \), can be calculated as,

\[
\eta_k = \langle I'|h_k \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I'(x,y) h_k^*(x,y) dx \, dy. \tag{3.32}
\]

\( h \) can be transformed to \( g \) through a transformation matrix,

\[ g = hA, \]  

(3.33)

where \( A_{ij} \), the elements of the transformation matrix, can be calculated as,

\[
A_{ij} = \langle g_j | h_i \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_j(x,y) h_i^*(x,y) dx \, dy. \tag{3.34}
\]

\( \xi \), the coordinates of \( I' \) in the basis \( g_k \), can be calculated as,

\[ \xi = A^{-1} \eta, \]  

(3.35)

where \( A^{-1} \), the inverse of \( A \), always exists since \( g_k \) are linearly independent.

From Equation 3.30,
Comparing Equation 3.36 and Equation 3.9, \( a_i^2 \) is \( \langle I \rangle^{1/2} \xi_i \) for the self-product term \( e_i^2 \), from which the modal coefficient and modal power weights can be obtained, and \( \Delta \phi_j \) is \( \arccos \left( \frac{\langle I \rangle^{1/2} \xi_i}{2a_i a_j} \right) \) for the cross-product term \( e_i e_j \), from which the modal phase differences can be obtained.

### 3.2.3 Correct Modes and Residue

This above method requires precise field distribution of the modes and the beam to get accurate results. However, these specifications and parameters are often given with large error or hard to obtain in practice. A method for precise modal decomposition without phase information is developed in this work which is similar to that with phase information presented in Section 3.1.

Suppose \( g'_k \) is the self- and cross-products of the field distributions of modes calculated from inaccurate parameters, and \( h'_k \) is the orthogonally normalized basis transformed from \( g'_k \).
The correct orthogonally normalized basis $h_k$ and the normalized intensity distribution $I'$ can be expressed as a linear superposition of $h'_k$ with a remainder term $\Delta_k$ and $\Delta$,

$$h_k = \sum_j \langle h_k | h'_j \rangle h'_j + \Delta_k,$$

$$\langle \Delta_k | h'_j \rangle = 0,$$

$$I' = \sum_k \eta_k h_k = \sum_k \eta_k \left( \sum_j \langle h_k | h'_j \rangle h'_j + \Delta_k \right) = \sum_j \eta'_j h'_j + \Delta,$$

$$\eta'_j = \langle I'|h'_j \rangle = \sum_k \langle h_k | h'_j \rangle \eta_k,$$

$$\Delta = \sum_k \eta_k \Delta_j.$$

Consider the sum of the coordinates squared in the accurate and inaccurate basis,

$$Q = \sum_k |\eta_k|^2 = \langle I'|I' \rangle = 1,$$

$$Q' = \sum_j |\eta'_j|^2 = \langle I'|I' \rangle - \langle \Delta | \Delta \rangle = 1 - \langle \Delta | \Delta \rangle \leq 1. \quad (3.38)$$

Only when $\Delta=0$, $Q' = 1$. From Equation 3.37, for a limited number of $\eta_k$, this condition will not be met in practice.

The residue $R$, which is defined as the difference between the unity and the sum of the coordinates squared in a basis, will be the minimum for the correct basis,
as shown in Equation 3.39. This relationship can also be utilized to find the accurate modes and the correct fiber parameters and will be used in this work.

\[ R = 1 - Q = 0, \]

\[ R' = 1 - Q' = \langle \Delta | \Delta \rangle \geq 0 = R. \]

(3.39)

The procedure for precise modal decomposition without phase information can be summarized as follows,

1. Calculate different sets of modes from varied parameters.
2. Calculate the self-products \( e_i^2 \) and cross-products \( e_i e_j \) for these sets of modes.
3. Orthogonally normalize the basis \( g_k (e_i^2 \text{ and } e_i e_j) \) to the basis \( h_k \) using the Gram–Schmidt process and calculate the transformation matrix \( A \) for these sets of modes.
4. Normalize \( I \) to \( I' \).
5. Calculate \( \eta \), the coordinates of \( I' \) in the basis \( h_k \), with spatial overlap of \( I' \) and \( h_k \) for these sets of modes.
6. Calculate \( \xi \), the coordinates of \( I' \) in the basis \( g_k \), with \( \eta \) and \( A^{-1} \) for these sets of modes.
7. Calculate the residues for these sets of modes.
8. Find the minimum of the residues and the corresponding coordinates \( \eta \) and fiber parameters.
9. Calculate the modal coefficient and modal power weights from the self-product terms.

10. Calculate the modal phase differences from the cross-product terms with the modal coefficients.

### 3.2.4 Application to Single-Mode Fibers

This precise modal decomposition method can be applied to single-mode fibers for characterization purposes.

The set of modes $e_i$ is simplified as the fundamental mode $e$ in single-mode fibers. The basis $g_k$ are simplified as $e_i^2$. The basis $h_k$ are normalized $e_i^2$.

The procedure for single-mode fiber characterization can be summarized as follows,

1. Calculate different fundamental modes from varied parameters.
2. Calculate and normalize the self-product $e_i^2$ for these modes.
3. Normalize $I$ to $I'$.
4. Calculate the residues for these modes.
5. Find the minimum of the residues and the corresponding fiber parameters.

### 3.3 Experiment of Single-Mode Fiber Characterization
3.3.1 Experimental Method

The experimental setup for single-mode characterization is as shown in Fig. 3.2. A single-frequency narrow-linewidth distributed feedback (DFB) fiber laser is used as optical source to provide 1053-nm light and delivered with SM980 (passive) fiber, which is a single-mode fiber at 1053 nm. An objective lens is used to form a magnified image of the optical intensity distribution on the fiber facet to a CCD camera.

Since the lens pupil is much larger than the optical beam width, the convolution with the Fraunhofer diffraction pattern of the lens pupil can be ignored, and the image is a scaled and inverted version of the object [4].

The intensity distribution on the fiber facet \( I_o \) is given by,

\[
I_o \left( \frac{x-x_0}{M}, \frac{y-y_0}{M} \right) = |M|^2 I(x,y), \tag{3.40}
\]

where \( I \) is the intensity distribution of the image measured by the CCD camera, \( (x_0,y_0) \) is the origin of the coordinate system of fiber facet which is up to characterize.
The magnification $M$ is given by,

$$M = -\frac{z_2}{z_1}, \quad (3.41)$$

where $z_1$ and $z_2$ are the distances from the lens to the object and the image respectively. In practice, the distance from the lens to the object is hard to measure precisely. From the lens law,

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}, \quad (3.42)$$

$M$ can be obtained from $z_2$ and $f$, which is the focal length of the lens,

$$M = 1 - \frac{z_2}{f}. \quad (3.41)$$

The procedure in Section 3.2.4 is used for characterization. The varied parameters are $M$, $a_{\text{core}}$, $V$, $x_0$ and $y_0$. $z_2$ and $a_{\text{core}}$ are also measured to be compared with the calculation results.

### 3.3.2 Experimental Results
The image taken by the CCD camera is shown in Fig. 3.3. Before the fiber characterization procedure, the noise in the image induced by the camera needs to be removed. The fringes are removed by filtering out the higher-order terms in the Fourier transformation. The background noise level is calculated from the “dark” regions of the image and removed from the entire image. The central region of the image after processing is shown in Fig. 3.4.

The calculation results using the procedure defined in Section 3.2.4 are shown in Table 3.1. The recovered image from the extracted parameters is shown in Fig. 3.5.

The difference between the generated image and the image after processing, which is the intensity distribution of the image after processing minus that of the generated image, is shown in Fig. 3.6. The relative difference is less than 2.5% and can be induced by the aberration of the lens.

Figure 3.3 Measured image of intensity distribution from SM980 fiber
Figure 3.4 Image of intensity distribution from SM980 fiber after processing.

Figure 3.5 Recovered image from the extracted parameters of SM980 fiber.
Figure 3.6 Difference between the recovered image and the image after processing of SM980 fiber.

Table 3.1. Parameters Extracted in Fiber Characterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>M</td>
</tr>
<tr>
<td>$V$</td>
<td>1.7783</td>
</tr>
<tr>
<td>$x_0$</td>
<td>30.260</td>
</tr>
<tr>
<td>$y_0$</td>
<td>30.240</td>
</tr>
<tr>
<td>$R$</td>
<td>0.0011348</td>
</tr>
</tbody>
</table>

The core radius is measured as $2.4 \pm 0.1$ µm according to the microscope image of cleaved SM980 fiber, as shown in Fig. 3.7. $|M|$ is calculated as $186 \pm 3$ from Equation 3.41, with $z_2=3.37 \pm 0.05$ m and $f=18$ mm.
Given the extracted value of $V = 1.7783$, the NA = 0.1192–0.1296 using Equation 2.12. The NA of SM980 fiber is 0.13–0.15 in the specification sheet provided by the vendor. Therefore, the NA can be regarded as matching with the experimental results if the core radius is close to 2.3 µm. At this core radius, the extracted value of $|M| a_{core} = 430.05$ µm will give $|M| = 187$, which agrees with the experimental result.

![Fig. 3.7 Microscope image of cleaved SM980 fiber.](image)

### 3.4 Experiment of Modal Decomposition in Few-Mode Fibers

#### 3.4.1 Experimental Method

The experimental setup for modal decomposition in few-mode fibers is shown in Fig. 3.8. SM980 fiber in Fig 3.2 is replaced by SMF-28e fiber which is few-mode fiber at 1053 nm.
The procedure in Section 3.2.3 is used for modal decomposition. The varied parameters are $M|a_{core}$, $V$, $x_0$ and $y_0$. $a_{core}$ are measured to compare with the calculation results, and $z_2$ is unchanged.

### 3.4.2 Experimental Result

The image taken by the CCD camera is shown in Fig. 3.9. The central region of the image after processing as described in Section 3.3.2 is shown in Fig. 3.10.
The calculation results are listed in Table 3.2, where the mode order 1-3 represents the fundamental mode, the $\text{LP}_{11}$ even mode and the $\text{LP}_{11}$ odd mode respectively.

The total phase difference $\Delta = \Delta \Phi_{12} + \Delta \Phi_{23} - \Delta \Phi_{13} = 0.0381 \pi$, instead of the theoretical value of zero. This inconsistency comes from noise and is averaged into the phase differences as error resulting in the revised parameters listed in Table 3.3.

The recovered image from the extracted parameters is shown in Fig. 3.11. The difference between the recovered image and the image after processing is shown in Fig. 3.12, which has a similar pattern to Fig 3.6 in that the brightest part is reduced in the generated image and the outer circle is increased, which suggests a systematic error such as lens aberration.
Table 3.2 Parameters Extracted in Modal Decomposition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>M</td>
</tr>
<tr>
<td>$V$</td>
<td>2.7916</td>
</tr>
<tr>
<td>$x_0$</td>
<td>50.885</td>
</tr>
<tr>
<td>$y_0$</td>
<td>49.900</td>
</tr>
<tr>
<td>$R$</td>
<td>0.0019613</td>
</tr>
<tr>
<td>$p_1$</td>
<td>34.10%</td>
</tr>
<tr>
<td>$p_2$</td>
<td>7.90%</td>
</tr>
<tr>
<td>$P_3$</td>
<td>58.01%</td>
</tr>
<tr>
<td>$\Delta \Phi_{12}$</td>
<td>0.3166 π</td>
</tr>
<tr>
<td>$\Delta \Phi_{13}$</td>
<td>0.5439 π</td>
</tr>
<tr>
<td>$\Delta \Phi_{23}$</td>
<td>0.1892 π</td>
</tr>
</tbody>
</table>

Table 3.3 Revised Parameters in Modal Decomposition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (π)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \Phi_{12}$</td>
<td>0.3293</td>
</tr>
<tr>
<td>$\Delta \Phi_{13}$</td>
<td>0.5312</td>
</tr>
<tr>
<td>$\Delta \Phi_{23}$</td>
<td>0.2019</td>
</tr>
</tbody>
</table>
The core radius is measured as $4.1 \pm 0.1\,\mu m$ according to the microscope image of cleaved SMF-28e fiber as shown in Fig. 3.13, which agrees with the specification sheet provided by the vendor. Using the extracted value of $V=2.7916$, ...
the NA=0.1114–0.117 from Equation 2.12. The NA of SMF-28e fiber at 1053 nm is not available. From the refractive index difference, which is 0.36%, and the refractive index of fiber cladding at 1053 nm, which is 1.45, the NA is estimated to be around 0.12, which is close to the experimental result. Using the extracted value $|M|a_{core} = 782.70 \ \mu\text{m}$, $|M| = 186$–196, which agrees with the Section 3.3 result that $|M| = 183$–187.

![Fig. 3.13 Microscope image of cleaved SMF-28e fiber.](image)

### 3.5 Conclusions

In this chapter, new methods for precise modal decomposition are developed for both field distribution with phase information and intensity distribution without phase information, and are extended to single-mode fiber characterization.
The procedure can be summarized as follows,

1. Calculate different modes sets from varied parameters.

2. Calculate the residues for these modes sets with extracted field distribution of the beam or with the measured intensity through transformation of coordinates.

3. Find the minimum of the residues and the corresponding parameters, and the modal coefficients, the modal power weights, and the modal phase differences if applicable.

Experiments are carried out for single-mode characterization and modal decomposition in few-mode fibers. The experimental results of the parameters agree with the nominal values or measure values.
References


Chapter Four

Helical-Core Fibers

Helical-core fiber is a novel LMA fiber. By integrating coiling directly into the fabrication of the fibers, HOMs in helical-core fibers are suppressed without the penalties introduced by coiling. In this chapter, an improved bending-loss model is developed, and the mode-area scaling properties of helical-core fibers are explored numerically and verified experimentally.

In Section 4.1, a bend-loss model is developed that is appropriate for the small bending radii expected for helical-core fibers. The HOM suppression is compared against the fundamental mode loss for increasing core diameters to explore the mode-area scaling limits with single-mode operation in Section 4.2. The properties of a helical-core fiber are measured in Section 4.3.

4.1 Semi-Analytical Model

The fundamental quantities required for understanding the helical-core fibers are the losses associated with the desired fundamental mode and HOMs. Of all the HOMs, the LP$_{11}$ mode has the lowest susceptibility to bending loss. Thus, to the first order, the beam quality of the laser is determined by the loss of the LP$_{11}$ mode, while
the efficiency of the fiber laser or amplifier is determined by the loss of the fundamental mode.

The curvature radius of the helix is a simple geometrical relationship given by [1]

\[ R_s = \frac{Q}{\sin^2 \theta} = Q + \frac{P^2}{4\pi^2 Q} \]  

where \( Q \) is the offset of the helix measured from the center of the fiber axis (as shown in Fig. 1) and \( \theta \) is the trajectory angle of the helix defined by \( \tan(\theta) = \frac{2\pi Q}{P} \), where \( P \) is the pitch of the helix. Nominally, this relationship can be utilized in conjunction with a bending-loss model for straight-core fibers to predict the behavior of helical-core fibers.

Many bend-loss models are available for a step-index fiber, but only a few of these can be applied to compare the modal bending losses to understand the behavior of helical-core fibers. The most widely used of them is of Marcuse in which the optical field outside of the fiber is expressed in terms of a superposition of cylindrical outgoing waves [2]. The other models include the traveling-wave antenna model, three-dimensional integration model, and diffraction theory model, etc., with different levels of computational intensity [3-5]. Under certain approximations, these models provide results that agree with those of Marcuse’s model.

By matching the boundary at the core–cladding interface, the amount of power carried by the radiating cylindrical waves is calculated. For simplicity, several
approximations are made in this model. First, the directionality of the radiation fields are approximated as delta functions to simplify the required integration. Physically, this translates to constraining the radiative energy to exist only within the plane of the bend. Additionally, the Bessel functions describing the modes within the fiber core are replaced with the Debye approximate

$$H^{(2)}_{\mu}(\xi) \approx i e^{i\mu(\theta - \tanh \theta)} \left( \frac{1}{2\pi \mu \tanh \theta} \right)^{1/2}$$

for simplicity of calculation.

While Marcuse’s model allowed for direct computation of bending loss, the accuracy of this model is insufficient for many applications. The use of the Debye approximation is not required since it leads to significant error and Bessel functions can be readily calculated. Additionally, constraining the radiation to the plane of the bend is simply incorrect in describing the bending loss for small-bend radii. In this case, the total internal reflection (TIR) condition required for waveguiding is violated in a large angular spread of incident angles at the interface of the fiber core.

Several models have been developed to improve the accuracy of Marcuse’s model. Kaufman et al. refined Marcuse’s model by eliminating the Debye approximation and deriving an analytical expression for one of the integrals that was previously approximated by delta-function radiation directionality [6].

Field deformation induced by bending has been modeled in different ways and numerical simulations show that mode deformation can have a significant impact on both peak intensity and bending loss [7-9]. However, since no significant field deformation has been observed experimentally in either coiled fibers or helical-core
fibers [10, 11], this effect is not included in this work. The loss mechanism is restricted to radiative loss in this work. Mode scattering or mode mixing is not considered.

In the model derived here, Marcuse’s method is utilized without Debye and delta-function approximations. In this way, the model is valid in the region of small-bend radii, as expected to be required for helical-core fibers. The geometry for calculating the radiating fields is shown in Fig. 4.1; the field outside the fiber core is matched to cylindrical outgoing waves at the core–cladding interface.

![Fig. 4.1 A fiber is bent into a torus with a radius of curvature $R_B$. The dotted surface indicates the cylindrical matching surface that is tangential to the torus at radius $r=R_B+a$.](image)

For weak guidance, $(n_2 - n_1)/n_1 << 1$, the $E_z$ and $E_\Phi$ components of the LP$_{\nu m}$-mode electric field for $r' > a$ are described by the equations
\[ E_z = AK_{\nu}(\gamma_{\nu}, r') \cos(\nu \phi') e^{i\beta_{\nu} R_B \phi}, \]

\[ E_{\phi} = -\frac{i \gamma_{\nu}}{2n_2 k_0} A \left\{ K_{\nu+1}(\gamma_{\nu}, r') \sin[(\nu+1) \phi'] - K_{\nu-1}(\gamma_{\nu}, r') \sin[(\nu-1) \phi'] \right\} e^{i\beta_{\nu} R_B \phi}, \] (4.2)

where \( \nu \) and \( m \) are the azimuthal and radial mode numbers, \( \gamma_{\nu} = (\beta_{\nu}^2 - n_2^2 k_0^2)^{1/2} \) is the attenuation coefficient of the modes in the cladding, \( K_{\nu} \) is the modified Bessel function of the second kind, \( \beta_{\nu} \) is the propagation coefficient, \( R_B \) is the bend radius, \( n_2 \) is the refractive index of the cladding extending to infinity, and \( k_0 = \omega/c = 2\pi/\lambda \) is the wave number in free space.

The field amplitude \( A \) is related to the power \( P \) carried by the mode as

\[ A = \frac{2 \gamma_{\nu}}{V} \frac{J_{\nu}(\kappa_{\nu} a)}{K_{\nu}(\gamma_{\nu} a)} \left\{ \frac{Z_0 P k_0}{e_{\nu} \pi \beta_{\nu} J_{\nu-1}(\kappa_{\nu} a) J_{\nu+1}(\kappa_{\nu} a)} \right\}^{1/2}, \] (4.3)

where \( \kappa_{\nu} = \left( n_1^2 k_0^2 - \beta_{\nu}^2 \right)^{1/2} \) is the transverse wave vector, \( e_{\nu} = 2 \) for \( \nu = 0 \) or 1 for \( \nu \geq 1 \) accounts for the mode degeneracy, \( Z_0 = \sqrt{\mu_0/\varepsilon_0} \) is the plane-wave impedance in vacuum, \( V = k_0^2 a^2 \left( n_1^2 - n_2^2 \right) \) is the normalized frequency, \( J_{\nu} \) is the Bessel function of the first kind, and \( n_1 \) is the refractive index of the core.

The electric field outside of the core can be expressed in terms of a superposition of cylindrical outgoing waves [2]:
\[ E_z = \int_{-n_0}^{n_0} \left[ c_1(\beta) e_z \beta_1 + c_2(\beta) e_z \beta_2 \right] d\beta, \]

\[ E_\phi = \int_{-n_0}^{n_0} \left[ c_1(\beta) e_\phi \beta_1 + c_2(\beta) e_\phi \beta_2 \right] d\beta, \]  

(4.4)

where \( c_1 \) and \( c_2 \) are the expansion coefficients. The \( z \) components of the cylindrical electric and magnetic fields can be expressed by

\[ e_{z}\beta_j = e_{zj} e^{i\beta z} = B_j H^{(2)}_{\mu} (\rho r) e^{i\mu \phi} e^{i\beta z}, \]

\[ h_{z}\beta_j = h_{zj} e^{i\beta z} = B_j F_j H^{(2)}_{\mu} (\rho r) e^{i\mu \phi} e^{i\beta z}, \]  

(4.5)

where \( B_j \) is the amplitude coefficient related to the normalization of the modes,

\[ \rho = \left( n_z^2 k_0^2 - \beta^2 \right)^{1/2}, \]

and \( F_j = (n/Z_0) \Gamma_j \) is the amplitude coefficients related to the mutually orthogonality of the modes such that

\[ \Gamma_1 = \frac{H^{(2)}_{\mu} \left[ \rho (R_B + a) \right]}{H^{(2)}_{\mu} \left[ \rho (R_B + a) \right]} \text{ and } \]

\[ \Gamma_2 = \frac{H^{(2)}_{\mu} \left[ \rho (R_B + a) \right]}{H^{(2)}_{\mu} \left[ \rho (R_B + a) \right]}. \]  

(4.6)
Here, $H^{(2)}_\mu$ is the Hankel function of the second kind, the prime indicates differentiation with respect to the argument, and $\mu = \beta \nu R_B$, assuming the fields of the curved guide near the core as the field of the straight fiber.

The other field components can be obtained from the $z$ components by differentiation;

$$
\varepsilon_{\phi j} = \varepsilon_{\phi j} e^{i\beta z} = \left[ - \frac{\mu \beta}{\rho^2 r} H^{(1)}_\mu (\rho r) - \frac{i \mu_0 \omega}{\rho} F_j H^{(1)}_\mu (\rho r) \right] B_j e^{i\mu \phi} e^{i\beta z},
$$

$$
h_{\phi j} = h_{\phi j} e^{i\beta z} = \left[ - \frac{\mu \beta}{\rho^2 r} F_j H^{(1)}_\mu (\rho r) - \frac{i \varepsilon_0 n^2 \omega}{\rho} H^{(1)}_\mu (\rho r) \right] B_j e^{i\mu \phi} e^{i\beta z},
$$

$$
\varepsilon_{r j} = \varepsilon_{r j} e^{i\beta z} = \left[ \frac{i \beta}{\rho} H^{(1)}_\mu (\rho r) - \frac{\mu_0 \omega}{\rho^2 r} F_j H^{(1)}_\mu (\rho r) \right] B_j e^{i\mu \phi} e^{i\beta z},
$$

$$
h_{r j} = h_{r j} e^{i\beta z} = \left[ \frac{i \beta}{\rho} F_j H^{(1)}_\mu (\rho r) - \frac{\varepsilon_0 n^2 \omega}{\rho^2 r} H^{(1)}_\mu (\rho r) \right] B_j e^{i\mu \phi} e^{i\beta z},
$$

where $r = R_B + a$ at the matching surface. The normalization of the modes is given by
\[
\left[ \int_{-\infty}^{\infty} dz \left[ \varepsilon_{\beta j} h_{\beta j}^* - \varepsilon_{\bar{\beta} j} \bar{h}_{\beta j}^* \right] \right] \\
\left[ \frac{\mu \beta}{\rho^2 r} H_{\mu}^{(1)} (\rho r) - \frac{i \mu_0 \omega}{\rho} F_j H_{\mu}^{(1)} (\rho r) \right] \\
\times B_j e^{i \mu \phi} e^{i \beta z} B_j^* F_j H_{\mu}^{(1)} (\rho r') e^{-i \mu \phi} e^{-i \beta z} \\
- B_j H_{\mu}^{(1)} (\rho r) e^{i \mu \phi} e^{i \beta z} \left[ - \frac{\mu \beta'}{\rho^{2} r} F_j H_{\mu}^{(1)*} (\rho' r) \right] \\
+ \frac{-i \varepsilon_0 \mu_0^2 \omega}{\rho'} H_{\mu}^{(1)*} (\rho' r) B_j^* e^{-i \mu \phi} e^{-i \beta z} \\
= (2\pi) \delta (\beta - \beta') N_{\beta \beta' j j'} \\
= (2\pi) \delta (\beta - \beta') B_j B_j^* \\
\left[ \frac{\beta'}{\rho^2} - \frac{\beta}{\rho^2} \right] H_{\mu}^{(1)} \left[ \rho (R_B + a) \right] H_{\mu}^{(1)*} \left[ \rho' (R_B + a) \right] \\
+ i \varepsilon_0 \mu_0^2 \omega \left[ H_{\mu}^{(1)} \left[ \rho (R_B + a) \right] H_{\mu}^{(1)*} \left[ \rho' (R_B + a) \right] \right] \\
- \frac{\Gamma_j \Gamma_j^* H_{\mu}^{(1)*} \left[ \rho (R_B + a) \right] H_{\mu}^{(1)} \left[ \rho' (R_B + a) \right]}{\rho} \\
\right]
\]

(4.8)

The mutual orthogonality of the modes is given by
The expansion coefficients are then determined by

\[
\begin{align*}
\int_{-\infty}^{\infty} \left[ E_{\phi} \phi_{z, j}^{*} - E_{\phi} \phi_{\beta, j}^{*} \right] dz \\
= \frac{2}{\pi} \sum_{j=1}^{n_{k0}} (2\pi) \delta_{j, \beta'} \frac{2i\varepsilon_0 n^2 \omega}{\rho} H_{\mu}^{(1)} (\rho (R_B + a)) H_{\mu}^{(1)} (\rho (R_B + a)) (1 - \Gamma_{j}^{*}) c_{j} (\beta').
\end{align*}
\]

(4.9)

Now the superposition of the cylindrical waves is required to match the guided modes at the matching surface with

\[
\begin{align*}
c_{j} (\beta) &= \frac{\int_{-\infty}^{\infty} \left[ E_{\phi} \phi_{z, j}^{*} - E_{\phi} \phi_{\beta, j}^{*} \right] dz}{N_{\beta, j}}, \\
N_{\beta, j} &= (2\pi) B_{j} \frac{2i\varepsilon_0 n^2 \omega}{\rho} H_{\mu}^{(1)} (\rho (R_B + a)) H_{\mu}^{(1)} (\rho (R_B + a)) N_{\beta, j}, \\
N_{\beta, 1} &= (1 - \Gamma_{1}^{*}), \\
N_{\beta, 2} &= (1 - \Gamma_{2}^{*}).
\end{align*}
\]

(4.10)
\[ r' = \left( a^2 + z^2 \right)^{1/2}, \]

(4.11)

\[ \phi' = \arctan \left( \frac{z}{a} \right). \]

The expansion coefficients \( c_1 \) and \( c_2 \) can be determined analytically with the help of the orthogonal relations [6]

\[
C_j = \frac{1}{N_{\beta j}} \int_{-\infty}^{\infty} \left( E_{\phi} h^*_j - E_z h^*_{\phi,j} \right) dz
\]

\[
= \frac{1}{N_{\beta j}} \left[ \left( i\gamma_{\nu} \frac{K_{\nu+1}(\gamma_{\nu} r') \sin[(\nu + 1)\phi']} {2nk_0} - K_{\nu-1}(\gamma_{\nu} r') \sin[(\nu - 1)\phi'] \right) \times e^{i\beta R_{\nu}^*} B_j^* e^{-i\mu \phi} e^{-i\beta z} 
- A K_{\nu} (\gamma_{\nu} r') \cos(\nu\phi') e^{i\beta R_{\nu}^*} 
\times \left[ -\frac{\mu \beta}{\rho^2 r} F_j^* H_{\mu}^{(1)*} + \frac{-i\varepsilon_0 n_2 \omega}{\rho} H_{\mu}^{(1)*} \right] B_j^* e^{-i\mu \phi} e^{-i\beta z} \right] dz
\]

\[
= \frac{i\varepsilon_0 n_2 \omega A B_j^*}{\rho^2 N_{\beta j}} \left[ \left[ \rho H_{\mu}^{(1)*} - \frac{i\mu \beta}{\varepsilon_0 n^2 \omega} F_j^* H_{\mu}^{(1)*} \right] \times \int_{-\infty}^{\infty} K_{\nu} \left[ \gamma_{\nu} \left( a^2 + z^2 \right)^{1/2} \right] \cos[\arctan(z/a)] e^{-i\beta z} dz 
\right]
\]

(4.12)
The integrals of these new sinusoidal terms can be solved as

\[
\begin{align*}
\int_{-\infty}^{\infty} & \left[ K_{\nu+1} \left[ \gamma_{\nu} \left( a^2 + z^2 \right)^{1/2} \right] \sin \left[ (\nu + 1) \arctan \left( z / a \right) \right] \right] e^{-i\beta z} dz \\
= & \frac{\pi}{2i} \gamma_{\nu}^{-\nu} e^{-a(\gamma_{\nu}^2 + \beta^2)^{1/2}} \left( \gamma_{\nu}^2 + \beta^2 \right)^{-1/2} \left\{ \begin{array}{c}
\nu - 1 \left[ \left( \gamma_{\nu}^2 + \beta^2 \right)^{1/2} + \beta \right]^{\nu + 1} - \left[ \left( \gamma_{\nu}^2 + \beta^2 \right)^{1/2} - \beta \right]^{\nu + 1} \\
- \nu \left[ \left( \gamma_{\nu}^2 + \beta^2 \right)^{1/2} + \beta \right]^{\nu - 1} - \left[ \left( \gamma_{\nu}^2 + \beta^2 \right)^{1/2} - \beta \right]^{\nu - 1}
\end{array} \right\},
\end{align*}
\] (4.13)

This expression includes the integral of sinusoidal terms previously neglected by Marcuse and Kaufman. It is the negligence of these terms that leads to the unsuitability of this model for small-bend radii. Similarly, the integrals of cosine terms can be solved as [6]

\[
\begin{align*}
\int_{-\infty}^{\infty} & \left[ K_{\nu} \left[ \gamma_{\nu} \left( a^2 + z^2 \right)^{1/2} \right] \cos \left[ \nu \arctan \left( z / a \right) \right] \right] e^{-i\beta z} dz \\
= & \frac{\pi}{2} \gamma_{\nu}^{-\nu} e^{-a(\gamma_{\nu}^2 + \beta^2)^{1/2}} \left( \gamma_{\nu}^2 + \beta^2 \right)^{-1/2} \left\{ \left[ \left( \gamma_{\nu}^2 + \beta^2 \right)^{1/2} + \beta \right]^{\nu} + \left[ \left( \gamma_{\nu}^2 + \beta^2 \right)^{1/2} - \beta \right]^{\nu} \right\} \\
\end{align*}
\] (4.14)

Finally, the expansion coefficients can be expressed as
The bending loss is obtained by calculating the amount of power outflow per unit length divided by the total power carried in the fiber; $2\alpha = \Delta P/PL$, where $L$ is the length of the fiber. Utilizing Eqs. 4.2–4.15 and taking the limit as $r$ goes to infinity yields

$$2\alpha = \frac{\Delta P}{PL}$$

$$\text{Re} \int_{-\infty}^{\infty} \left[ \hat{r} \cdot (\vec{E} \times \vec{H}^*) \pi r \right]_{r \to \infty} \, dz$$

$$= \frac{2\pi e_r R \left[ n\gamma_r a J_v(\kappa_r a) / \frac{\Sigma_{j=1}^{\infty} \Gamma_j \gamma_{j-1}}{N_{\beta_j}} \right]^2}{4\rho \beta_j H_{\mu j}^{(1)} N_{\beta_j}} \int_{-1}^{1} \text{Re} \left\{ \frac{I_{\nu\beta j} I_{\nu\beta j'} (1 + \Gamma_j \Gamma_{j'})}{(\rho a)^2 \left[ \rho (R + a) \right]^2 N_{\beta_j} N_{\beta_{j'}}} \right\} \, dx.$$
where $x$ is defined as $x = \beta/n_2k_0$.

By using some numerical computation, this model yields an exact solution instead of relying on the delta-function approximation for plane radiation and the Debye approximation. In this way, the Marcuse’s model is extended to include the loss of light radiating outside the plane of the bend, as is appropriate for small-bend radii.

### 4.2 Theoretical Results and Discussions

Figure 4.2 shows the comparison of the Marcuse and Kaufman models and our improved model by plotting the bending loss as a function of bend radius for the LP$_{01}$ and LP$_{11}$ modes.

![Fig. 4.2 The bending loss as a function of bend radius for the LP$_{01}$ and LP$_{11}$ modes for different models. The wavelength, the core diameter, and the NA are 1053 nm, 60 μm, and 0.1, respectively.](image-url)
Based on Fig. 4.2, it is clear that Marcuse’s model significantly overestimates the bending loss. This is consistent with experimental results, which showed significantly lower loss than that predicted from Marcuse’s model [11,12]. Kaufman’s model, on the other hand, only somewhat underestimates the bending loss. Because of the relationship between the helical-core fiber parameters and the bend radii given by Equation 4.1, however, the behavior of the helical-core fiber is sensitive to the precise value of these parameters. As such, the most accurate model is required to design helical-core LMA fibers.

It should be noted that all three of these models consider only one orientation of higher-order modes. Marcuse showed that the orthogonal orientations have, in fact, different bend loss at all bend radii [5]. The same model shows, however, that the low-loss LP\textsubscript{11} mode has lower loss than the LP\textsubscript{01} mode. This is a theoretical anomaly and has not been observed under any experimental coiling conditions that, in contrast, show that the tightest coils produce the LP\textsubscript{01} mode. One possible explanation for this theoretical behavior is that the modeling method is inconsistent and cannot properly account for the polarity where the null is on the bend axis. Additionally, the strong coupling between the two degenerate modes of orthogonal polarities may be significant contributors. In the discussion that follows, only the high-loss polarity is considered in order to be consistent with experimental results.

The improved bend-loss model is first applied to a helical-core fiber with a core diameter of 60 \textmu m and a NA of 0.1. Fig. 4.3 shows bending loss as a function of the helix pitch $P$ for the LP\textsubscript{01} (solid) and LP\textsubscript{11} (dashed) modes for different values of
the core offset $Q$. It is clear from the plot that low loss (<1 dB/m) can be obtained for the fundamental mode while retaining high loss for the other modes. Figure 4.3 also shows that the model behavior of the fiber is heavily reliant on the exact value of the pitch. This dependence translates directly to fabrication tolerance, which can be alleviated by using a large-clad fiber. The larger $Q$ becomes, the larger the range of pitch that is available for low-loss, single-mode operation. This heavy reliance on pitch also places a requirement on the accuracy of any model utilized to design helical-core fibers.

Fig. 4.3 The bending loss as a function of the helix pitch for the LP$_{01}$ (solid) and LP$_{11}$ (dashed) modes for different core-offset values. The wavelength, the core diameter, and the NA are 1053nm, 60µm, and 0.1 respectively.
Fig. 4.4 shows the results for bending loss as a function of the helix pitch $P$ for the LP$_{01}$ and LP$_{11}$ modes for 40 $\mu$m core offset $Q$. From Fig. 4.4, 7.2-mm pitch can offer both low loss for the fundamental mode and high mode discrimination.

Fig. 4.4 The bending loss as a function of the helix pitch for LP$_{01}$ (solid) and LP$_{11}$ (dashed) modes. The wavelength, the core diameter, the core offset and the NA are 1053 nm, 40 $\mu$m, 100 $\mu$m and 0.1 respectively.

The modal loss difference is crucial to the performance of the helical-core fibers. Therefore, it is also instructive to understand the interplay of the loss of the LP$_{01}$ mode with the modal discrimination, defined here as the loss difference between the LP$_{11}$ and LP$_{01}$ modes. Fig 4.5 shows the modal discrimination as a function of fundamental mode loss for different values of core diameter. As the core diameter increases, the modal discrimination decreases. The modal discrimination is more than eight times the loss of the fundamental mode for cores as large as 40 $\mu$m. For cores as
large as 100 $\mu$m, the modal discrimination is higher than the available gain in ytterbium-doped fibers. This plot demonstrates the viability of helical-core LMA fiber lasers and amplifiers for single-mode operation.

It is also evident from Figs. 4.3 and 4.5 that for large core diameters, good mode discrimination cannot be obtained for arbitrarily low fundamental mode loss. However, by allowing a tolerable loss for the fundamental mode, for example 1 dB/m, core diameters can be scaled up to 100 $\mu$m.

Setting the fundamental mode loss to 1 dB/m, the modal discrimination is calculated as a function of NA for different core diameter values. Fig. 4.6 shows that as the NA increases, the modal discrimination decreases. Even so, in the range of NA
from 0.05 to 0.20, even up to a 100-\(\mu\)m core, the modal discrimination is higher than the available gain in ytterbium-doped fibers. Therefore, helical fiber core designs can escape from the NA ranges that are difficult to fabricate (~0.05 to 0.08) to the NA ranges that are more practical (>0.08).

![Fig. 4.6 The modal discrimination as a function of the NA for different core-diameter values. The wavelength and LP\(_{01}\) bending loss are 1053 nm and 1 dB/m, respectively.](image)

Commercially available ytterbium-doped fibers have gains in the range of 1 to 10 dB/m, depending on the dopant density, the pumping level, and the saturation effects. The maximal gain is about 5 dB/m for a typical commercially available fiber. Fixing the modal discrimination at 5 dB/m to ensure the suppression of the LP\(_{11}\) mode, the dependence of the helix pitch on the core diameter and the NA is shown in Fig. 4.7. From this graph, it is clear that a helical-core fiber can be designed to yield good mode discrimination (as shown in Figs. 4.3 and 4.5), regardless of the core
diameter or the NA, by simply adjusting the pitch of the helix. The same is not true for straight-core fibers that are coiled. The vertical axis on the right of the plot is the bend radius required for a straight-core fiber to obtain the same performance as the helical-core fiber. For a fiber with a cladding diameter of 250 µm, a coiling radius less than 2 cm will likely lead to long-term fiber degradation. For high-power fiber amplifiers, cladding diameters are becoming increasingly larger to accommodate more pump light and therefore require even larger bend radii for long-term viability.

Fig. 4.7 clearly shows the advantages of helical-core fibers to coiled conventional fiber amplifier designs.

![Fig. 4.7](image)

Fig. 4.7 The helix pitch (left) and the bend radius (right) as functions of the core diameter for different NA. The wavelength, the core offset, and the modal discrimination are 1053nm, 100 µm, and 5dB/m respectively.
For broadband amplification or lasing operation of helical-core fibers, it is necessary to understand the spectral behavior. Fig. 4.8 shows the dependence of the bending loss on the wavelength. Fixing the bend radius to yield the fundamental mode loss values of 0.5, 1.0, and 1.5 dB/m at 1053 nm, the fundamental mode loss and the modal discrimination are plotted versus the wavelength at a range of 1 µm to 1.1 µm, covering the range of optical gain in Yb-doped fiber. As the wavelength decreases, the LP_{01} mode loss decreases, but the modal discrimination is also reduced. Therefore, for amplification or lasing at wavelengths other than designed, the beam quality may suffer accordingly.

![Graphs showing the fundamental mode loss and modal discrimination](image)

Fig. 4.8 (a) The fundamental mode loss and (b) the modal discrimination as functions of wavelength for different fundamental mode losses at 1053 nm. The core diameter and the NA are 40 µm and 0.1, respectively.

These results demonstrate the possibility of helical-core fibers exceeding the performance of coiled multimode fibers as fiber lasers while simultaneously
mitigating the detrimental effects of coiling and low NA. More importantly, the calculations presented here represent a drastic change from commercially available large-mode-area fiber designs. These active fibers have $V$ numbers in the range of 3 to 6, core diameters of 20 to 30 $\mu$m, and low NA of 0.05 to 0.06. A helical-core structure removes the requirement for low NA and allows environmentally robust fibers to be fabricated. The helical fibers presented here have $V$ numbers as large as 60 with a considerably larger diameter than commercially available fibers; this indicates a significant increase in nonlinear thresholds and a NA twice as large as the commercial product. Furthermore, while the calculations presented here are for simple step-index fibers, the helical-core fiber geometry can also be utilized on other LMA fiber designs. It should also be noted that since the helical-core geometry is only included in the model through Equation 4.1, the scaling results described in Figs. 4.5-4.8 are also valid for coiled step-index fibers.

### 4.3 Measurement and Results

A passive helical-core fiber with 7.2-mm helical pitch was manufactured at Nufern, Inc. This fiber can offer both low loss for the fundamental mode and high mode discrimination as shown in Fig. 4.4. The fiber specifications, beam quality and the propagation loss of this fiber were measured.
4.3.1 Measurements of Fiber Specifications

Although the parameters of the fiber have been provided by Nufern, as listed in Table 4.1, some parameters were measured and calculated, as listed in Table 4.2. EOP and BOP are defined as the end of pull and beginning of pull, respectively, indicating at what point of the fiber draw the parameters were measured. Note that the parameters measured at a random point in the fiber do not agree precisely with the EOP and BOP values supplied by the vendor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(μm)</th>
<th>Uncertainty(μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOP</td>
<td>110.00</td>
<td></td>
</tr>
<tr>
<td>BOP</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td>Core offset</td>
<td>100.6</td>
<td>2.5</td>
</tr>
<tr>
<td>Core diameter</td>
<td>40.3</td>
<td>1</td>
</tr>
<tr>
<td>Cladding diameter</td>
<td>400</td>
<td>10</td>
</tr>
<tr>
<td>Core helical Pitch</td>
<td>7.2 mm</td>
<td></td>
</tr>
<tr>
<td>$n_{clad}$</td>
<td>1.4574@633nm</td>
<td></td>
</tr>
<tr>
<td>NA</td>
<td>0.1053@633nm</td>
<td></td>
</tr>
</tbody>
</table>
The cladding diameter was measured after the coating was stripped. The core diameter and the core offset were measured according to the microscope image of cleaved fiber, as shown in Fig. 4.9, so these two parameters are correlated. The refractive index of the cladding and the NA were calculated at the measurement wavelength.

Figure 4.10 shows a CCD image of the cleaved end of the fiber with light launched into the core. By allowing the core light to saturate the CCD, light leaking out of the fiber core due to bend loss reveals the helical trajectory inside this fiber very clearly.

Fig. 4.9 Microscope image of cleaved helical-core fiber.

Fig.4.10 CCD Image of helical trajectory inside a helical-core fiber.
4.3.2 Beam Quality Measurement

The experiment arrangement of the beam-quality measurement is shown schematically in Fig. 4.11. A single-frequency narrow-linewidth DFB fiber laser is used as optical source to provide 1053-nm light and delivered with a single-mode fiber. Because the core axis is not parallel to the fiber axis in helical-core fibers, the single-mode fiber is butt-coupled to the perpendicularly cleaved helical-core fiber at an angle with the aid of an XYZ-tip-tilt stage.

The output beam from the helical-core fiber was collimated by lenses and adjusted with mirrors. The beam widths were measured by a CCD camera placed on a track using a commercial beam measurement package (Spiricon). $M^2$ is calculated to be $1.14 \pm 0.05$, which indicates a good beam quality at the output of the helical-core fiber.

![Experimental configuration of beam-quality factor measurement.](image-url)
4.3.3 Loss Measurement

The fundamental-mode loss in the helical-core fiber was measured by the cut-back method: the fiber was cut from the output end backward, and the output power was measured every time it was cut. Over 1-meter of the fiber was left uncut to ensure filtering of HOMs such that only the fundamental mode loss is measured.

The experimental arrangement of the fundamental-mode loss measurement is schematically shown in Fig. 4.12. A single-frequency narrow-linewidth distributed feedback (DFB) fiber laser is used as optical source to provide 1053-nm light and delivered with a single-mode fiber. The single-mode fiber is butt-coupled to the perpendicularly cleaved helical-core fiber at an angle with the aid of an XYZ-tip-tilt stage.

![Fig. 4.12 Experiment arrangement of the fundamental-mode loss measurement](image)

The fundamental-mode loss is measured to be 0.048±0.005 dB/m. For comparison, this value is plotted with the theoretical results from both models as functions of the core diameter and the core offset within the range of their uncertainties, as shown in Fig. 4.13.
Fig. 4.13 Comparison of experimental results of the fundamental-mode loss with the theoretical results from different models.

Fig. 4.13 shows that the experimental result falls far below Marcuse’s model but well within the range of theoretical result from the improved model. The figure also reveals that small changes in parameters will induce large changes in fundamental mode loss, as large as an order of magnitude. This heavy reliance on parameters places a strict requirement on the accuracy of predictive models, fabrication control, and experimental measurement of the fiber parameters. Although the improved model matches the experimental result much better than Marcuse’s model, more accurate measurements are still needed for these parameters.
4.4 Conclusions

In conclusion, an improved semi-analytic bend-loss model is derived that allows for the propagation of radiated fields outside the plane of the fiber bend. This allows for the modeling of small-bend radii for which the waveguide condition for TIR is violated in a large angular spread of incident angles at the interface of the fiber core. This improved model is applied to large-mode-area helical-core fibers (which require small-bend radii) for use as high-power fiber lasers and amplifiers, which enable bending loss to be utilized for mode selection without the deleterious effects of coiling straight-core fibers.

A comparison of the fundamental-mode propagation loss between theory and experiments shows that this improved model is well matched using parameters of the fabricated helical-core fiber, but that more accurate measurements are needed for all parameters.
References


Chapter Five

Conclusions

5.1 Thesis Summary

In this thesis, the transverse modes in large-mode-area (LMA) fibers are explored. The transverse spatial-hole burning (TSHB) effect in LMA fibers is investigated in an amplified spontaneous emission (ASE) source based on an ytterbium-doped (YD) double-clad (DC) LMA fiber. The beam-quality factor and output power are measured as functions of input pump power. The beam quality is optimized when the gain becomes saturated. A model using spatially resolved gain and transverse-mode decomposition of the optical field showed that TSHB was responsible for the observed behavior. A simplified model without TSHB failed to predict the observed behavior of beam quality. A comparison of both models shows TSHB is critical for properly modeling beam quality in LMA fiber amplifiers.

To obtain accurate results of modal decomposition in multimode and LMA fibers, new precise modal decomposition methods, for field distribution with phase information and intensity distribution without phase information respectively, are developed. These methods are also extended to single-mode fibers for characterization purposes. In these new methods, different mode sets are calculated using varied sets of fiber parameters, and the modal power weights of each set are calculated using the experimentally extracted field or intensity distributions of the
beam. Through finding the minimal residue of the mode sets, the corresponding modal power weighting, phase differences, and even experimental fiber parameters are extracted. The single-mode fiber characterization and modal decomposition in few-mode fibers experiments are carried out based on these methods and the experimental results of the parameters agree well with the nominal or measured values.

For small-bend radii, the waveguide condition for total internal reflection is violated in a large angular spread of incident angles at the interface of the fiber core. To account for this, an improved semi-analytic bend-loss model is derived that allows for the propagation of radiated fields outside the plane of the fiber bend. This new model is applied to LMA helical-core fibers (which require small-bend radii) for use as high-power fiber lasers and amplifiers, which enable bending loss to be utilized for mode selection without the deleterious effects of coiling straight-core fibers. A comparison of the fundamental-mode propagation loss between theory and experiments shows that this improved model is well matched using parameters of the fabricated helical-core fiber, but that more accurate measurements are needed for all parameters.

5.1 Future Work

In Chapter 2, the ASE is simplified as a single spectral mode since the bandwidth of ASE is relatively narrow, as shown in Fig. 2.4. However, the bandwidths of most commercial ASE sources are much broader than that of the
source studied in Chapter 2. Since the number of modes supported in a multimode fiber depends on the operating wavelength, it would be meaningful to incorporate both multiple spectral modes and multiple spatial modes in the numerical simulations in order to account for the broad spectral range in commercial ASE LMA-fiber sources.

The experiments of modal decomposition on multimode fibers can be carried out using the method proposed in Chapter 3. For this purpose, genetic algorithms need to be developed for this modal decomposition method to reduce computation times for a large number of modes. Also, more advanced imaging systems should be developed to reduce the possible aberrations induced from the optical elements.

The modal decomposition method can be carried out on helical-core fibers to determine the fiber parameters accurately and obtain the output modal power weights. By controlling the input power weights (launching into specific modes of the helical-core fiber), the higher-order modal loss can be obtained and therefore the accuracy of the improved bend-loss model can be verified for higher-order modes.