

# Diffracted Efficiencies for Optical Wavelength Gratings with Arbitrary Groove Shapes are Predicted and Compared to Measurements

J. HASSETT, R. BONI, and D. H. FROULA

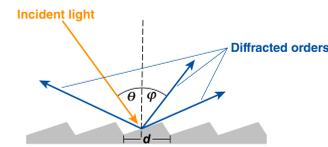
University of Rochester, Laboratory for Laser Energetics

## Summary/Conclusions

- A grating-analysis method based on coordinate transformation is implemented in *MATLAB*
- The results of the analysis method agree with the conducted measurements of the grating efficiency
- Electron microscope images suggest that scattering effects in mechanically ruled blazed gratings are the cause of discrepancies between measurements and theory
- Holographic gratings produce less scattered light and efficiency measurements for these gratings agree very closely with theoretical predictions

## Theory

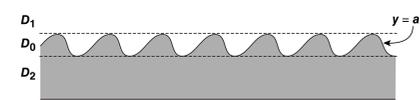
Solutions to grating-efficiency calculations exist in numerical form; there are no closed-form solutions



$$\frac{m\lambda}{d} = \sin(\theta) + \sin(\phi)$$

- The grating equation predicts angles of diffracted orders based on groove spacing
- It fails to predict the efficiencies of propagating light
- Diffraction efficiencies depend on groove shape, material properties of the grating, polarization, incident angle, and wavelength
- Maxwell's equations are solved at the surface boundary of the grooves
- The solution is found by carrying out many numerical calculations via a computer

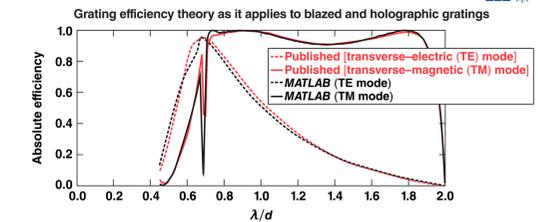
Numerical methods of computation have been developed to solve Maxwell's equations at the grating surface boundary



$$\text{in } D_1 \text{ and } D_2: \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2 \mu \epsilon \right) H_z = 0 \quad H_z = \sum_m A_m^{(p)} e^{i\beta_m x + i\beta_m^{(p)} y}$$

- Helmholtz's equation is written for transverse-magnetic polarization, where  $\epsilon$  is the relative permittivity of propagating material
- In  $D_1$  and  $D_2$ , constant permittivity is assumed and solutions can be written in Rayleigh Expansions
- In  $D_0$ ,  $\epsilon$  varies in two-dimensional space
- This creates a more-challenging problem with several solution methods

The C method grating analysis has been implemented in *MATLAB* and agrees well with published grating-efficiency curves



- First order
- 26.75° blaze angle
- Littrow mount
- Perfectly conductive grating material

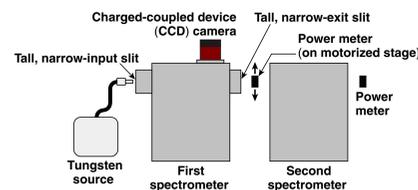
## Abstract

Optical diffraction gratings are commonly used to disperse broadband radiation. Construction of these devices involves the generation of periodic structures, which reciprocate the optical energy incident on the device into predictable locations in space based on the wavelength, groove spacing, and incident angle of radiation. A simple closed-form equation accurately describes this phenomenon. The issue of predicting the energy distribution across this continuous spectrum of diffracted light is much more complex. Only numerical analysis methods that satisfy the boundary conditions along the grating surface offer solutions to this problem. These methods are highly involved and require detailed information regarding the groove shape of the gratings. As a result, many experiments involving diffraction gratings are designed using limited efficiency information. This information is usually provided in the form of an efficiency plot for the ideal Littrow scenario, which is rarely achieved in practical setups.

In response to this issue, a computationally efficient method for predicting efficiencies for the simplest case of a surface-relief grating is presented and implemented. The method is capable of determining diffraction grating efficiencies for planar periodic gratings and is extended to handle arbitrary groove shapes and grating materials. Theoretical results compare well with conducted measurements.

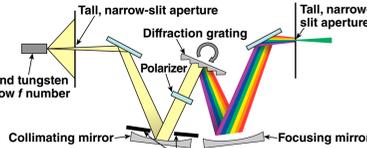
## Measurement setup

An experiment is conducted to measure the efficiency of blazed and holographic planar diffraction gratings across multiple diffracted orders



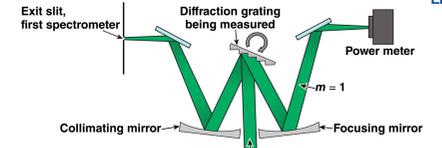
- The first spectrometer outputs the narrow-frequency band; the second spectrometer measures the grating efficiency
- Power loss is measured across the second spectrometer relative to loss when the diffraction grating is replaced with an aluminum mirror
- Mirror reflectivity is measured by Lambda 900, allowing for absolute grating-efficiency measurement

Monochromatic (~2-nm-bandwidth), polarized  $f/20$  light exiting first spectrometer is an ideal source for grating-efficiency measurements



- A broadband light source illuminates an ~200- $\mu\text{m}$ -wide, 8-mm-tall slit at the first spectrometer image plane
- An absorbing mask on the collimating mirror illuminates 68- $\times$  68-mm grating with 15- $\times$  15-mm square beam, avoiding overfilling and clipping of the polarizer and spectrometer mirrors
- The collimated beam passes through a calcite polarizing prism
- Broadband light is separated in angle space by a diffraction grating on a rotatable stage
- An exit-slit aperture at the focus limits output to a narrow frequency band

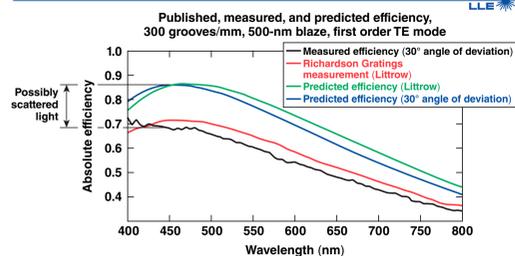
Polarized, monochromatic (~2-nm-bandwidth) light is passed through a second spectrometer; power readings are taken at the entrance and exit



- A motorized stage makes it possible for the first power meter to be placed into and out of the beam path
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- Monochromatic light is diffracted by the grating under test; order is selected by a rotatable turret
- No exit slit is used because the 10-mm-diam power meter captures the entire focused beam
- A mask, however, is placed in front of the exit power meter in order to prevent scattered and stray light from reaching the detector

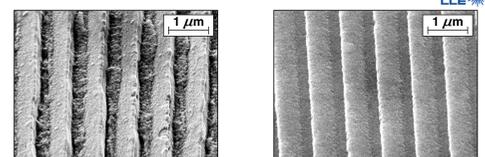
## Results

Theoretical calculations accurately predict the measured efficiencies for blaze gratings with an offset likely caused by scattered light



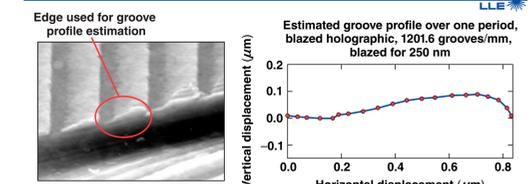
- The diffraction grating is aluminum-coated glass from Richardson Gratings
- Blaze angle of 4.3°, 300 grooves per millimeter, transverse-electric polarization

Mechanically ruled and replicated blazed gratings generate scattered light that is likely the cause of discrepancies between theory and measurements



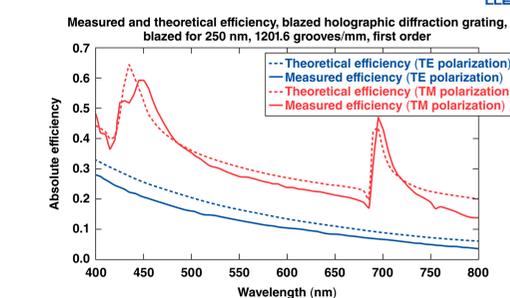
- Grooves are ruled into a master grating with diamond tip
- Imperfections in mechanical process result in small divots and scratches, along with imperfect groove spacing
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Electron microscope images of diffraction-grating defects allow for more-accurate modeling of groove profiles



- Ideally, the groove shape would be determined through a profile image that views the grooves from the edge of the grating
- This would require grinding and polishing of the substrate edge, which would likely cause permanent damage to the grating
- The groove shape is instead estimated from SEM images of grating defects (e.g., a scratch intentionally placed near the edge of the grating)
- The edge of the groove is traced in SEM with a series of points; corrections are performed for viewing angle and estimated slice angle

Groove profile imaging leads to improved efficiency predictions for holographic gratings with reduced scattering properties

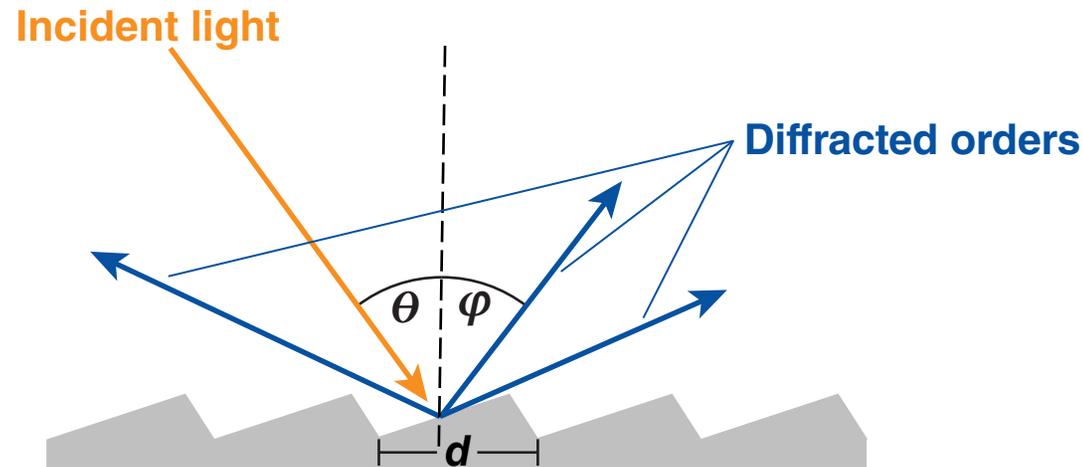


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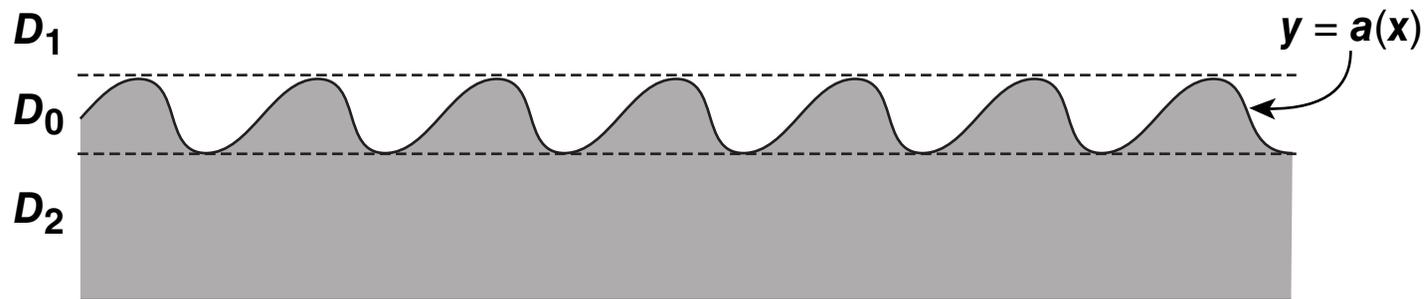
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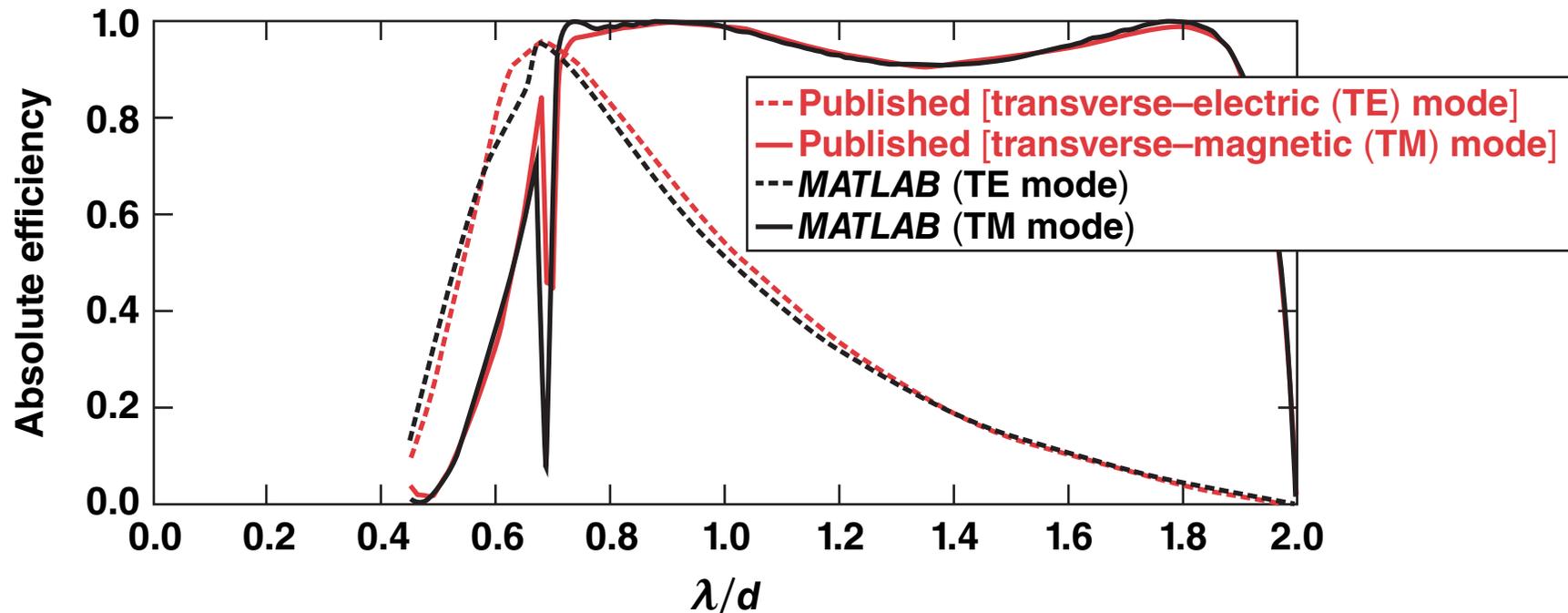
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Grating efficiency theory as it applies to blazed and holographic gratings



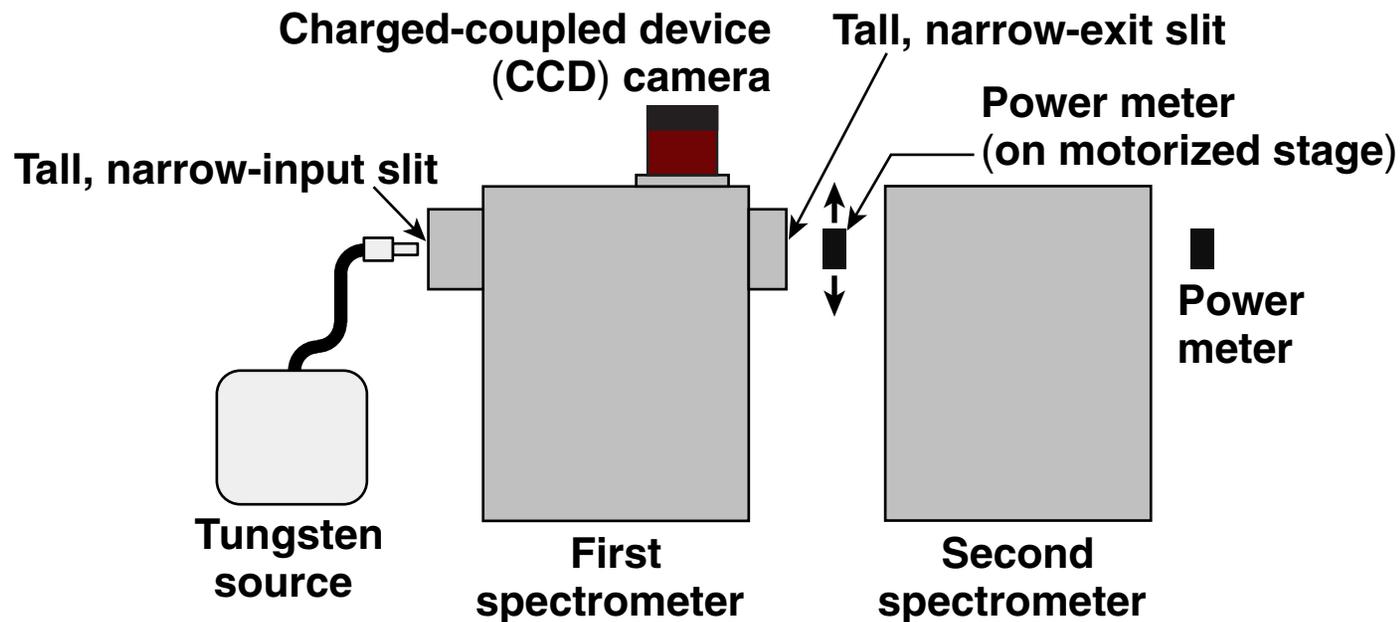
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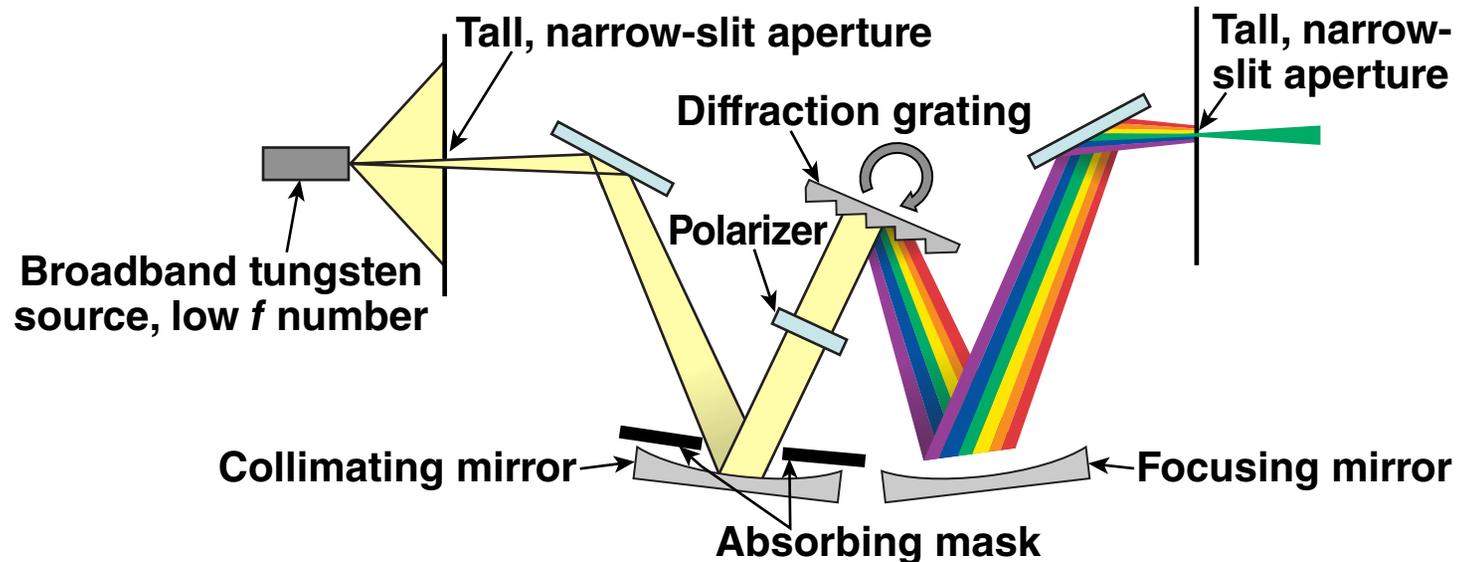
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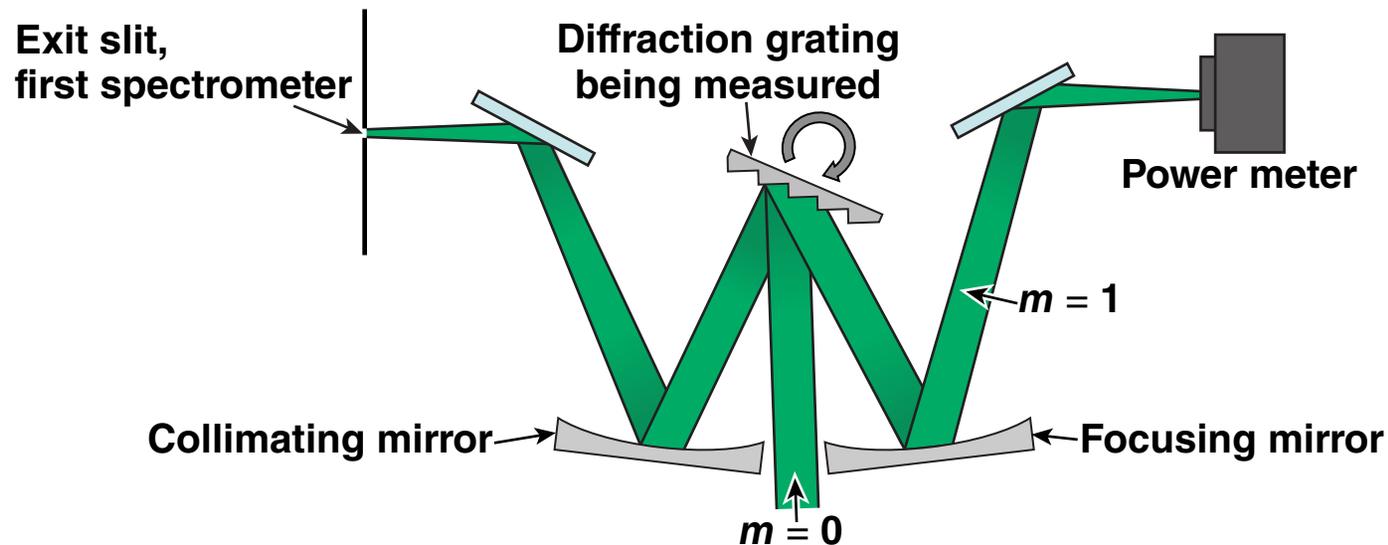
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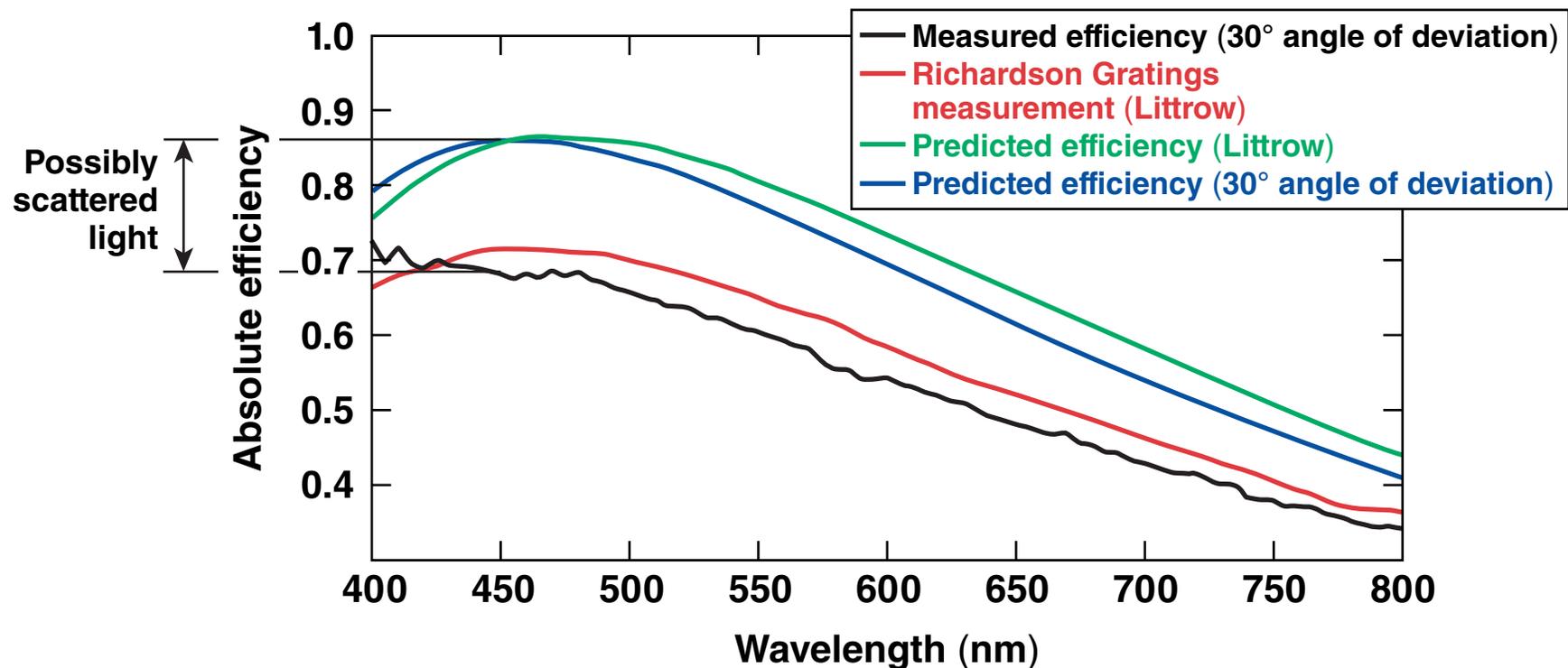
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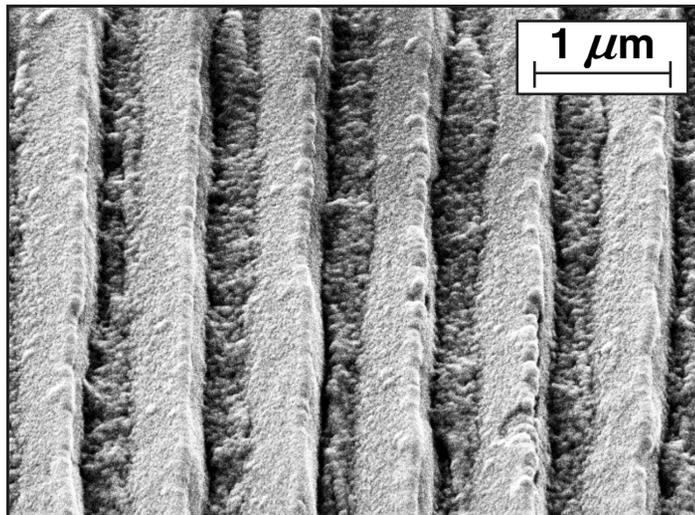
# Theoretical calculations accurately predict the measured efficiencies for blaze gratings with an offset likely caused by scattered light

Published, measured, and predicted efficiency,  
300 grooves/mm, 500-nm blaze, first order TE mode

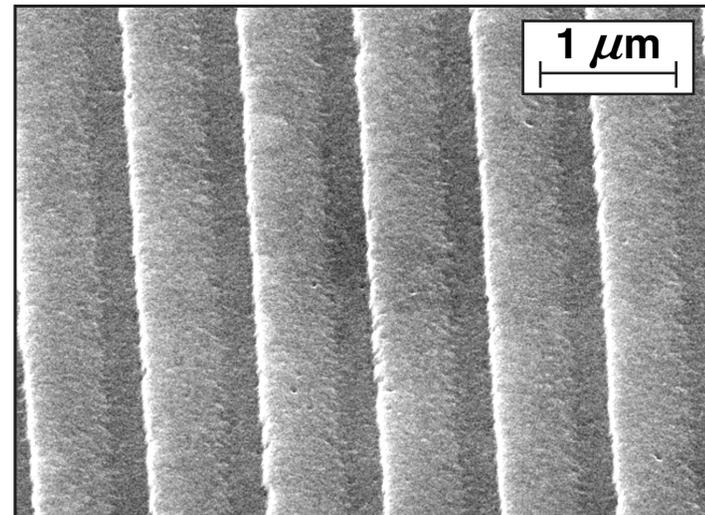


- The diffraction grating is aluminum-coated glass from Richardson Gratings
- Blaze angle of  $4.3^\circ$ , 300 grooves per millimeter, transverse–electric polarization

# Mechanically ruled and replicated blazed gratings generate scattered light that is likely the cause of discrepancies between theory and measurements



Scanning electron microscope (SEM) image of mechanically ruled, blazed diffraction grating, 1200 grooves/mm, 36.8° blaze angle

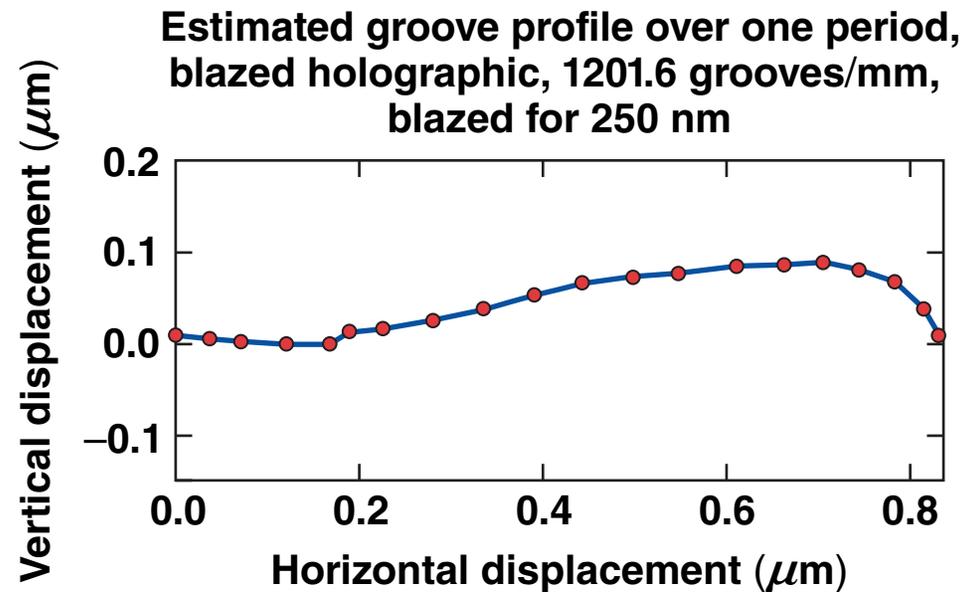
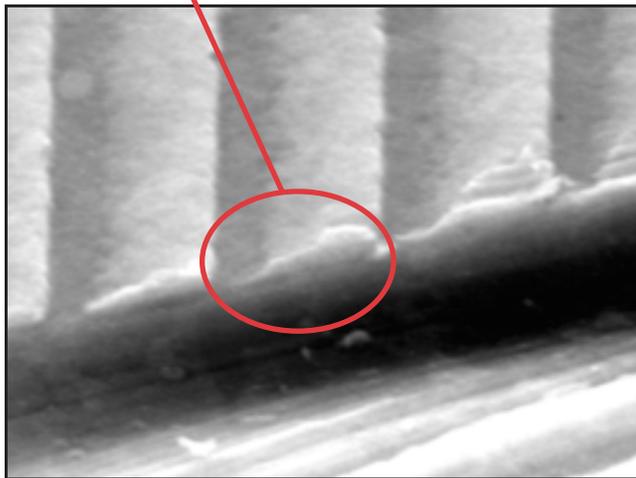


SEM image of blazed holographic diffraction grating, 1201.6 grooves/mm, blazed for 250 nm

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Edge used for groove profile estimation



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# Groove profile imaging leads to improved efficiency predictions for holographic gratings with reduced scattering properties

Measured and theoretical efficiency, blazed holographic diffraction grating, blazed for 250 nm, 1201.6 grooves/mm, first order

